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On the Thermal Conductivity of Soil

By

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By the ÅNGSTROM's method, a new apparatus for measuring the thermal diffusivity of soil was designed. In order to give a boundary condition, the apparatus was constructed to change the applied voltage of a heater. The Slidac and a special cam was used for this purpose. The specific heat of soil was measured hy the method of mixture. Thermal diffusivity of a volcano-geneous soil was measured with respect to the water content. Thermal conductivity was calculated from the diffusivity and the specific heat thus obtained.

1. Introduction

Among the various methods for measuring the thermal conductivity of soil, we adopted ÅNGSTROM's method, because it can be conveniently applied to the sample of soil in the state of natural packing.

The solution of the differential equation of the heat conduction in a linear form

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} \tag{1}$$

is known to be

$$u = u_0 e^{-x\sqrt{\frac{\pi}{kT}}} \sin\left(\frac{2\pi}{T} t - x\sqrt{\frac{\pi}{kT}}\right) \tag{2}$$

when the boundary condition is given as below,

$$u_{x=0} = u_0 \sin \frac{2\pi}{T} t \tag{3}$$

T is the period of the temperature wave, and k is the thermal diffusivity of the sample of soil.

The temperature waves are measured at the two points, x_1, x_2, distant from the surface, and the amplitudes A_1, A_2 and the phases

ε_1, ε_2 are obtained. Then the thermal diffusivity k is given by the equations

$$k = \frac{\pi (x_1 - x_2)^2}{T (ln\, A_1 / A_2)^2} = \frac{\pi (x_1 - x_2)^2}{T (\varepsilon_1 - \varepsilon_2)^2} \qquad (4)$$

The thermal conductivity K of the soil is given as

$$K = c\rho k \qquad (5)$$

where c is the specific heat and ρ is the density of the sample.

For giving the boundary condition as shown by (3), an electric resistance has usually been used, which gives a sinusoidal form of heat generation in the heater. King used a cam[1], and Tadokoro designed a plectrum resistance[2] for this purpose. But, in either of them, it must be noticed that the combined value $R = r + r'$ is effective in controlling the heating current I, where r is the series resistance inserted and r' is the resistance of heating coil. Therefore, the change of r' for various samples will give rise to the change of r: that is the change of the design of the cam or of the plectrum resistance. This is the most unconvenient piont of these methods. Furthermore, it is laborious and troublesome to build up the resistance r.

Mr. T. Ishida, therefore, proposed a new method[3]. The heat generated in the heating coil is expressed by

$$H = V^2 / R$$

The Slidac, a handy autotransformer for laboratory use, is used for varying the applied voltage V of the alternating current continuously, so that H can be changed in sine form with respect to time.

2. The Apparatus*

The Cam. It was verified that a linear relation holds with suffcient accuracy between the out-put voltage and the rotation angle ω of the knob of a slidac. Then the problem is solved if we rotate the knob in such a way that the square of ω is sinusoidal. Winding a string round the knob as shown in Fig. 1. we can change the rotation of the knob into a linear

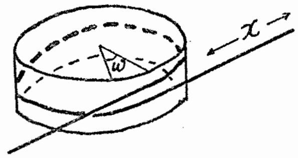

Fig. 1. Knob and string

* Most of the apparatus was originally designed by Mr. T. Ishida

motion, and vice versa. In order to make the square of the displacement of the string sinusoidal, a special cam was disigned. For this purpose, we express the displacement x of the string by

$$x = a \sqrt{1 + \sin\theta} \ .$$

The form of the cam is given in polar coordinate as follows;

$$r = b + x = b + a \sqrt{1 + \sin\theta}$$

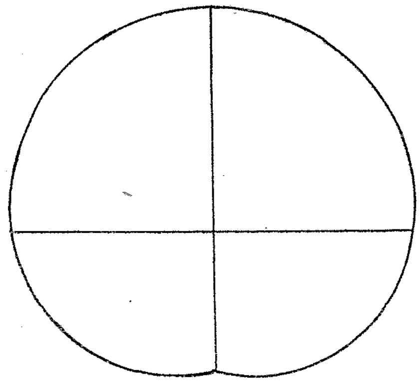

Fig. 2. The form of the cam.

The form of the cam thus obtained is as shown in Fig. 2., and its actual dimension is tabulated in TABLE I.

TABLE I. *The actual radii of the cam with respect to θ*

$b = 4.90$ cm , $a = 2.10$ cm

$\theta°$		r cm	$\theta°$		r cm
0	180	7.00	180	360	7.00
10	170	7.18	190	350	6.81
20	160	7.33	200	340	6.58
30	150	7.47	210	330	6.39
40	140	7.59	220	320	6.15
50	130	7.69	230	310	5.90
60	120	7.77	240	300	5.67
70	110	7.83	250	290	5.41
80	100	7.86	260	280	5.16
90		7.87	270		4.90

The essential parts of the apparatus are shown in Fig. 3 and the photograph is reproduced in PLATE I. The cam C is driven by a motor, reducing its speed with two warm gears. The period is made to be about 35 minutes. The string S must be free from any elongation or shrinkage. The Koto-string (Koto is the Japanese harp) is used for this purpose. It is wound round the knob K of the slidac and fixed at a point on the knob. Passing through two pulleys P and Q, its one end is fastened to the end of a movable rod R. A

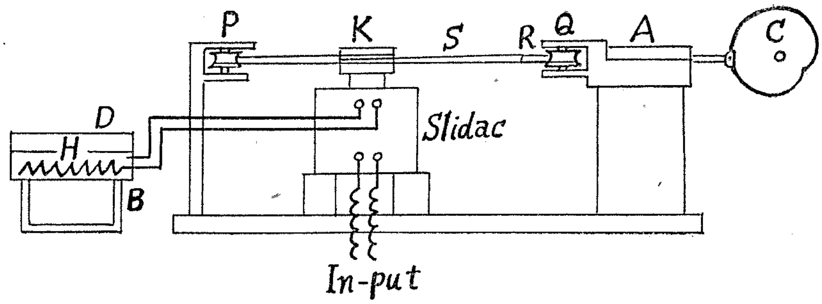

Fig. 3. Apparatus for measuring the thermal diffusivity.
C; cam. A; spring box. P, Q; pulleys.
R; rod. S; string. K; knob. H; heater.
D; heater container. B; specimen box.

spring is inserted in the cylinder A, which pushes this rod against the cam. The out-put voltage is varied between 0 and 22 volts for a complete rotation of the cam.

The Heater and the Specimen. The heater H consists of a coil of nichrom wire. It is buried in the magnesia powder filled in a circular cylinder D. The soil specimen is $10 \times 10\ \mathrm{cm}^2$ in dimension and 5 cm in height, packed in a wooden box B.

3. Details of the Measurements

The Temperature Measurements. The temperature waves were measured at two points, about one centimeter apart, in the center of the specimen. Two copper-constantan thermocouples and a short period galvanometer were used. The temperature could be measured with an accuracy of 1/100°C. The temperature measurements were carried out every minutes, and the values were plotted against time.

The Calorimeter. In order to calculate the thermal conductivity, it is necessary to know the specific heat of the soil. This measurements was carried out with dry soils by an usual calorimeter. The specific heat of wet soils were calculated from these data and that of the water. The steam-bath for heating the soil sample was improved. Fig. 4. shows the design of the calorimeter. When the sample M was sufficiently heated, the cover T was opened, and the sample is made to fall into the calorimeter L. This process could be operated in about 15 seconds, when a sample of about 50 grams was used.

4. Results of the Experiments

One example of the measurements of the temperature waves is shown in Fig. 5. The ordinate is the reading of the galvanometer scale, and the abscissa is the time. Each temperature-time curve shows a perfect sinusoidal form. The period T and the phase difference $\varepsilon_1 - \varepsilon_2$ were measured from wave trains.

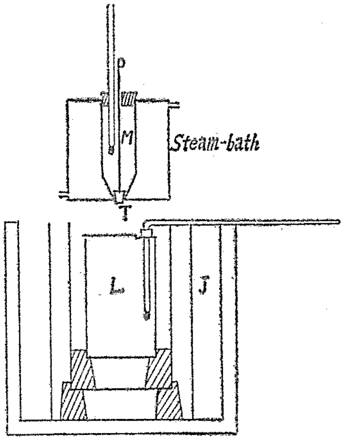

Fig. 4. Calorimeter apparatus.
M; sample. T; cover.
L; calorimeter.
J; water jacket.

We calculated the thermal diffusivity from the phase difference in the equation (4). Considering the deviations of the amplitudes of the waves caused by some unavoidable fluctuation in the in-put voltage, the calculation of diffusivity from phase difference seems to be better than that from the amplitudes ratio.

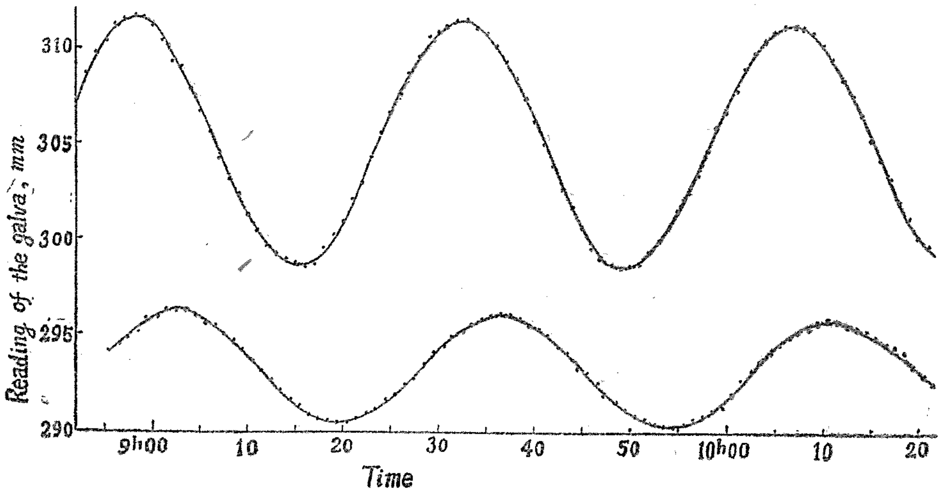

Fig. 5. Temperature-time curves of the experiment No. 15.

Several samples were tested, changing the water content and the rate of packing. One example of the volcanogeneous soil taken at Memuro in Hokkaido is tabulated in TABLE II.

TABLE II. *Results of the measurements of the thermal diffusivity of the volcanogeneous soil.*

Loose packing

Exp. No.	$k \times 10^3$ C.G.S.	$x_1 - x_2$, cm	$\varepsilon_1 - \varepsilon_2$	r	M, g./c.c.
2	1.84	0.80	0.725	9.75	0.39
3	2.24	1.67	1.36	0.41	0.51
5	1.52	0.86	0.870	0.34	0.51
6	1.13	0.65	0.749	0.00	0.60
7	2.13	0.88	0.740	0.67	0.45
8	1.84	1.09	0,990	0.25	0.54
18	2.03	0.84	0.692	0.86	0.45

Close packing

Exp. No.	$k \times 10^3$ C.G.S.	$x_1 - x_2$, cm	$\varepsilon_1 - \varepsilon_2$	r	M, g./c.c.
10	1.83	0.72	0.656	0.21	0.71
11	2.48	1.07	0.837	0.53	0.66
12	2.06	1.21	1.037	0.84	0.69
14	1.31	0.93	1 000	0.00	0.90
15	1.95	0.91	0.803	0.93	0.73
16	2.41	0.85	0.674	0.52	0.64
17	1.87	0.76	0.685	0.40	0.64

r is the moisture ratio, defined as the ratio of the water mass to the dried soil mass.
M is the dried soil mass contained in unit volume of the wet soil sample.

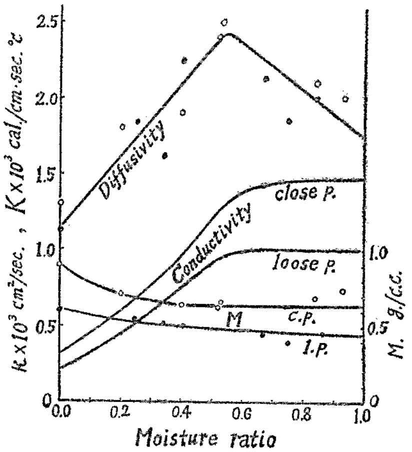

Fig. 6. Thermal diffusivity, conductivity and dried soil mass per unit volume of wet soil with respect to the moisture ratio.

The results is also shown in Fig. 6. The black dots represent the values for loose packing, and the white ones for close packing. In this case, the thermal diffusivity looks to be independent of the degree of packing. These values almost agree with the values obtained in field observation at Memuro[4].

$c\rho$ is the specific heat per unit volume. Neglecting the specific heat of air space, it is calculated as the sum of the specific heat of soil material and that of water contained in unit volume of the soil sample. The former is the product of the specific heat of dry soil s by the mass M of dried soil in unit volume of the sample. The water content of unit volume is obtained by rM. Therefore, the following relation is obtained,

$$c\rho = sM + rM$$

From the calorimeter measurements, we obtained 0.32 cal/g°C as the value of s. Actually M varies with r. We obtained the value M with respect to r from the smooth curve drawn in Fig. 6., which was plotted from TABLE II. These results were tabulated in TABLE III. with those of the volume specific heat.

TABLE III. *Calculated volume specific heat etc.* $s = 0.32$

r	M		rM		$c\rho$	
	loose	close	loose	close	loose	close
0.0	0.61	0.99	0.00	0.00	0.20	0.29
0.1	0.57	0.78	0.057	0.078	0.24	0.33
0.2	0.54	0.71	0.109	0.142	0.28	0.37
0.3	0.52	0.66	0.156	0.098	0.32	0.41
0.4	0.50	0.64	0.20	0.26	0.36	0.46
0.5	0.48	0.64	0.24	0.32	0.39	0.53
0.6	0.46	0.64	0.28	0.33	0.42	0.59
0.7	0.45	0.64	0.32	0.45	0.46	0.66
0.8	0.44	0.64	0.35	0.51	0.49	0.72
0.9	0.44	0.64	0.40	0.58	0.54	0.78
1.0	0.44	0.64	0.44	0.64	0.58	0.85

The value of $c\rho$ is a function of the degree of packing, so the thermal conductivity is a function of water content and the degree of packing. The final results are shown in TABLE IV. and Fig. 6.

TABLE IV. *Calculation of the thermal conductivity K of the volcanogeneous soil.*

r	$k \times 10^3$ C.G.S., represented by the curve in Fig. 6	$K \times 10^3$ C.G.S.	
		loose	close
0.0	1.13	0.22	0.32
0.1	1.39	0.33	0.42

r	$k \times 10^3$ C.G.S., represented by the curve in Fig. 6.	$K \times 10^3$ C.G.S.	
		loose	close
0.2	1.63	0.46	0.60
0.3	1.86	0.60	0.76
0.4	2.11	0.76	0.97
0.5	2.34	0.81	1.23
0.6	2.37	1.00	1.40
0.7	2.20	1.01	1.44
0.8	2.06	1.01	1.47
0.9	1.88	1.01	1.47
1.0	1.75	1.01	1.47

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