A Method of Computer Simulation through Modified Signal Flow Graphs and Operator Concepts and its Application to Synthesis of Heating-Equipment Capacities

Shigeru Matsuura[1]

Faculty of Engineering
Hokkaido University
Sapporo, Japan

In order to facilitate simulation of a physical system, direct simulation on an analog computer through a signal flow graph obtained directly from a schematic diagram in the physical system is used in this paper. Physical meanings of the method are confirmed and modified in view of algorithm as follows: (1) By creating a summing point which defines a signal, a wrong signal flow graph resulting from two definitions of a signal is avoided and an inversion law and interconnection of subgraphs are clarified. (2) A scaling method in s-domain is studied only by the use of a translation of a modified signal flow graph; It is made possible to obtain the completed program on an analog computer in which mutual relations between a signal and a scaled signal (a machine variable) are elucidated. (3) An operator concept is developed into the digital domain and a physical meaning in a closed loop is confirmed, so that simulation on digital computers by same methods as in the above-mentioned simulation on analog computers is made possible.
As an application, a synthesis of heating-equipment capacities is performed together with the confirmation of troublesome points in the actual operation, where not only a building but also automatically controlled heating equipments are simulated.

Key Words: Algorithm, closed loop, definition point, digital operation, dynamic balance, initial value, integral operator, modified signal flow graph, operator concepts, warming up load, scaling in s-domain, space series.

1. Introduction

An environmental design related to buildings would be an optimization of the ways of combination of components (said to be a structure of system) and their values in a buildings system containing equipments, which adjusts its entire balance under certain specified conditions. Considering the use of a computer from the point of view of design, therefore, it is desirable to use it synthetically rather than analytically, that is, it is necessary to be able to talk with the computer. As one of the useful means for it, there exists such simulation as correspondence of components in a system one-to-one, because a building system becomes large-scaled, complicated, high-priced and made-to-order, so that experiments of actual systems are impossible. For the sake of its usefulness, the technique of simulation has been widely used in fields of electronics, automatic-control, chemical process and so on.

The types of computers used in simulation are analog type, digital type and hybrid type (which is the combination of previous 2 types). Considering them from the standpoint of simulation, there, it shows the problems as follows: (1) There are differences of the models (expressions) as languages and ways of thinking depend on the kind of computers. (2) Advanced knowledges and techniques are required for simulation. (3) As the analog computer has the limited usages except for a differential analyzer or a simulator, it is necessary to consider simulation on the digital computer. However, the large-scaled and high-speed machine is required in its case.

As a countermeasure for the above mentioned, it is necessary to consider the next points: (1) Investigating programming-rules through models in use of the same concept which is independent of the kind of computers. (2) Using symbolism with sufficient informations in expression of system and its description. (3) A symbolism with algorithm which leads automatically from description to program. (4) In synthesis, it is necessary that a program one-to-one corresponds to a system in parts and the program is newly made by interconnection and division according to changing of the structure in system by interconnection and division, and also values on the program can be easily changed. (5) Finding out physical meanings and investigating calculation rules which calculation accuracy is not less than it was before even if simple procedures are used for the purpose of using a small machine such as a mini computer or a desk calculator.

This paper deals with a new method of computer simulation with algorithm which automatically gets to a simulation program through a model from an object system.

2. Method of Computer Simulation

It is well known that phenomena of system should be expressed in use of elements and a pair of across variable and through variable with time. For instance, heat conduction phenomena can be expressed as simultaneous differential equations using thermal resistance and thermal capacity as elements, and using temperature and heat flow as across variable and through variable with time. In these equations, relations among components such as wall and boiler which actually construct the system, namely, system structure is not clear.

There are graphical symbolisms as a way of expressing this structure, namely a physical network model, a block diagram, a signal flow graph, an analog computer diagram and so on. These are diagrammed to emphasize different aspects respectively. It is performed to symbolize many informations as to relations of actual components and each element. However, informations for the casual relation in each variable are not diagrammed. The signal flow graph and the block diagram are expressed in regard to the casual relation, but the relation to the actual object becomes weak rather than the physical network model. In the above expressions, direct informations of time are lost. The analog computer diagram has a nature of emphasizing element itself, its own function in itself and connection with other elements.

Observing these expressions from a view of simulation programming on the computer, with respect to the analog diagram, simulation programming seems to have been accomplished at one sight. However, as the analog computer diagram is usually introduced in terms of an expression of simulatanеous differential equations, the system structure is lost and the correspondence of system one-to-one in parts can not be found. Simulation in use of this procedure requires to supplement informations through thoughts of the structure. Therefore, it is very useful to obtain a simulation diagram by making the best use of the characteristics of each graphical symbolism. That is; at first, the physical network model is introduced by a schematic graph [1][2] which indicates actual system; and then, it is transformed into the signal flow graph [2] or the block diagram; and finally, the simulation diagram is obtained. However, it is not always said to accomplish algorithm of final processes. Symbolism applicable to both computers is, therefore, developed under considerations of terms of physical concepts as follows:

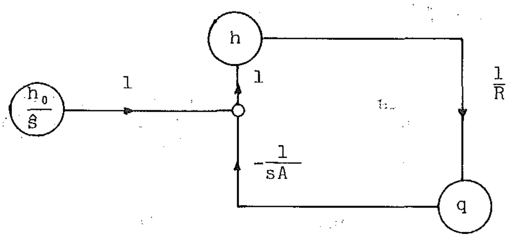
2.1. Concepts of Signal and Operator

Observing the relations between variables and elements from a new standpoint, variables are signals which transmit in a system and the signal is modified by the element, so that it becomes the next signal in succession. Elements should be thought as operators. Under such thoughts, what is diagrammed in s-domain is a signal flow graph. However, it should be noted that descriptions corresponding to a system have many equivalent signal flow graphs, but physical meaning of the graph is clear only when the graph is described in the form of 1/s concerning time, namely, in the form of an integral operator, because physical phenomena may be said in general to depend on the past and the conservation of energy principle.

---

[2] Figures in brackets indicate the literature references at the end of this paper.

As a simple example to clarify the above mentioned, it is considered that water is discharged from water tank (across sectional area A) using a pipe (resistance R). This is applied also to the case in which heat is discharged only by ventilation out of the room. When the water level is h, its initial value is h_0 and outgoing water flow is q, the modified signal flow graph expression of this system is given as eq (1).

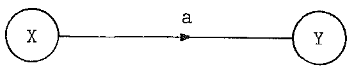


(1)

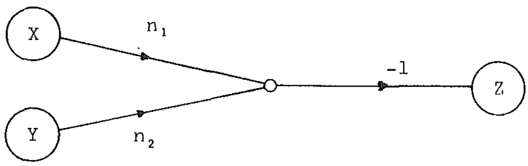
where, the signal flow graph is modified as follows: The signals are enclosed by a large circle to be distinguished from transmittance (operator) and the new summing points shown by small circles are made, they are also definition points in signals. Observing the definition as to h, it is equivalent to the next equation.

$$h = -\frac{q}{sA} + \frac{h_0}{s} \tag{2}$$

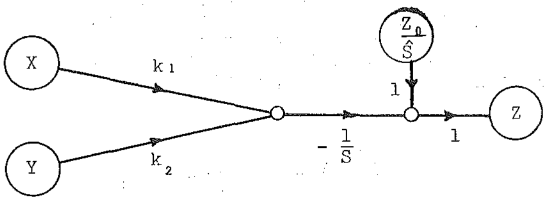
By the modifications, the next merits may occur. By means of separating and symbolyzing definition points in such a way as each a signal has only one definition point (by elimination of signals on the way, it is not prevented that the signal has sequentially two more definition points), misses in two definitions of a signal, when the flow graph is drew, can be prevented. Physical meaning of interconnections in system is in a concordance with across variables and a continuity of through variables. An interconnection in sub-graphs (which correspond to sub-system) attending an interconnection of sub-system requires that in any interconnection the across variables are connected by a branch with transmittance 1 and the through variable is defined by another through variables, connected by each branch so that continuity conditions may be held. In this case, if a signal has two definition points through the interconnection, here, a branch of either definition point is inversed (in which 1/S is left as it is) as a definition point for a different signal, and two definitions resulting newly from it is inversed in succession until reaches a signal having no a definition point. Furthermore, by creating this summing point, the analog computer diagram can be easily expressed by modified signal flow graphs. Namely, a potentiometer, a summing amplifier and a summing integrator are expressed as eqs (3), (4) and (5). By using them, an analog simulation diagram corresponding to a system one-to-one can be made only by equivalent transformations of the graph.



(3)



(4)



(5)

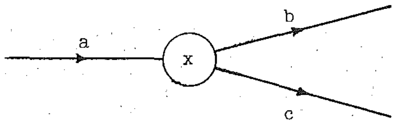
However, if a scaling change is not done in the analog computer programming, it is not that the program is perfect. A perfect analog simulation in only a signal flow graph is not always done. It means that there remains problems of algorithm. Next, a scaling in s-domain is considered.

2.3. Scaling in S-domain

A scaling has two kinds, first is to transform variables in a system into machine variables which are non-dimension and smaller than 1. Transforming them in s-domain by considering a magnitude scale factor α with dimensions, it becomes as follows:

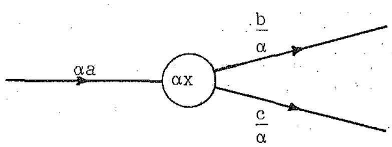
$$X = \alpha x \tag{6}$$

For the purpose of representing this relation on a graph it is necessary to add a new rule, that is, by multiplying a signal by α, the procedure in such a way that a transmittance of incoming branch is multiplied by α and that of outgoing branch is multiplied by $1/\alpha$ is required so that the graph is equivalent to the original graph.



(7)

is equal to



(8)

Next, it is thought to transform concerning time in use of time scale factor β for simulation within time or frequency adapted to the machine.

At first, in t-domain

$$\tau = \beta t \tag{9}$$

transforming (4) into s-domain

$$\frac{1}{S^2} = \beta \frac{1}{s^2} \tag{10}$$

where t, s correspond to real time and τ , S to machine time. And according to Mixnsky's expression [3]

$$\tau = \frac{1}{S^2} \tag{11}$$

$$t = \frac{1}{s^2} \tag{12}$$

Observing both sides of these equations from the point of view of a dimension, they do not coincide because 1/s is said to have a dimension of t.

The next points are considered to clarify this problem. Observing a signal as the result of operators acting on a unit impulse, in a usual description of function in s-domain, operators alone are expressed and the unit impulse is not expressed. Therefore, this unit impulse having value 1 and the dimension 1/t (because the unit impulse is thought as a limit of a pulse in the width Δt and the height $1/\Delta t$) should be affixed to the right hand of eqs (11) and (12) and both equations coincide also in the dimension. Hereafter, to distinguish clearly a signal from a group of operator, the signal is written in the form of affixing 1 having a dimension of 1/t. Considering physical meaning of time scale change in eq (9), the phenomena is extended by β in time. As an area of a unit impulse must be always 1, the unit impulse $\mathbf{I}$ in machine time is given as:

$$\mathbf{I} = \frac{1}{\beta} \tag{13}$$

A unit step function which results in one integral operator acting on the unit impulse is expressed as, $1/s$, $\mathbf{I}/S$, respectively. As this unit step function is an infinite step without concerning time scale change, they must be equal. And show them as $1/\hat{s}$.

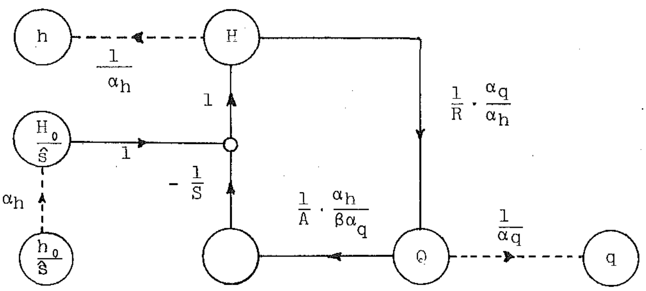
$$\frac{1}{s}1 = \frac{1}{S}\mathbf{I} \equiv \frac{1}{\hat{s}} \tag{14}$$

accordingly

$$\frac{1}{s} = \frac{1}{\beta S} \tag{15}$$

From the above investigation, if time scale change is done directly in s-domain, it will be done by affixing 1 to input signal and also by adapting eqs (14) and (15).

For the purpose of showing a simple example of analog simulation procedures, the graph of the tank model shown already in eq (1) is transformed equivalently in such a way as constructed by analog computer elements given in eqs (3), (4) and (5). And when magnitude and time scale change are done with regard to the above mentioned, the analog simulation program can be accomplished automatically and successively as follows:



(16)

where, the values of H and Q are non-dimension and machine variables smaller than 1 and $1/R \cdot q_q / \alpha h$, $1/A \cdot \alpha h / \beta \alpha_q$ are non-dimension and values smaller than 1 and indicate potentiometer values. The newly added branches shown as dotted line indicate relations between machine variables and variables of the original system (these do not become the object of the simulation). Next, consider the case of digital operation.

2.4. Digital Operation

In the case of an operation on a digital computer, various numerical methods have been developed in regard to information which can be obtained when it is sampled. That is, information related to the structure is weak, so that they don't always adapt to system simulation. For the purpose, the "time series" method [4] and the "thermal response factor" method [5] were published. But it is difficult to adapt the methods to the next cases; namely (1) when systems are interconnected, (2) when the system has non-linearity, (3) when the system has the initial value which represents the past effect, (4) when the problem having non-periodic intermitting heating including off days is solved.

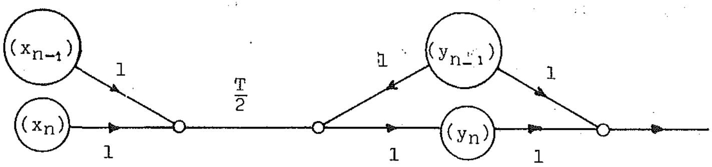
A digital operation of an integral operator is considered, observing that time is represented only by the integral operator 1/S in the simulation diagram corresponding to a system one-to-one. When values of a signal at t=0, T, 2T, ..., etc. (T is time interval) are x_0, x_1, x_2, x_3, ..., etc., the signal x is approximated by straight line segments at each interval. It is expressed as eq (17), which is named "space series".

$$[\ (x_0),\ (x_1),\ (x_2),\ (x_3), \ldots\ldots\ ] \qquad (17)$$

The signal resulting from the integral in eq (17) is expressed by space series as follows:

$$\frac{T}{2}\left[(0),\ (x_1 + x_0),\ (x_2 + x_1 + x_1 + x_0),\ (x_3 + x_2 + x_2 + x_1 + x_1 + x_0), \ldots\right] \qquad (18)$$

As mentioned above, y resulting from the digital operation of 1/s from (n - 1)T to nT is expressed as follows:



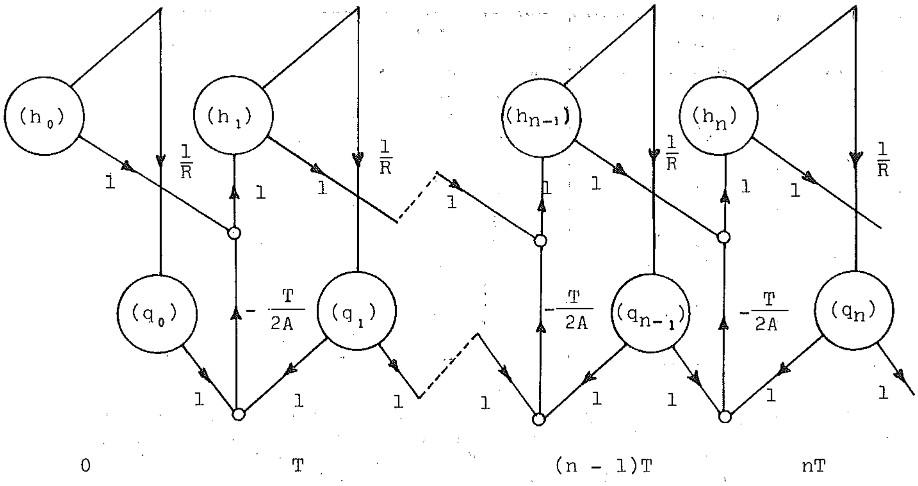
(19)

Therefore, it is proper to replace all 1/s of the graph with the above equation and calculate it step by step at each interval. However, as exceptions, when the signal value is always zero in the previous interval and it rises to the value (x_0), an operation of $0 + (x_0) = (0)$ must be used, and in a unit impulse, the calculation is such that instantaneously (1) will be preserved.

Next, consider adaptation of the digital operation in eq (1). When eq (1) is solved theoretically with Mason's rule [6], eq (20) is given as:

$$\frac{h}{h_0} = \frac{ST_c}{1 + ST_c} = e^{-\frac{t}{T_c}} \qquad (20)$$

where, T_c = AR is time constant. Equation (1) is expressed in the form of the digital operator using sampling interval T as follows:



(21)

Solving it in accordance with the graph rule at time T

$$\frac{(h_1)}{(h_0)} = \frac{2 - \frac{T}{T_c}}{2 + \frac{T}{T_c}} \tag{22}$$

at time 2T = $2\mu T_c$, where $T/T_c = \mu$

$$\frac{(h_2)}{(h_1)} = \frac{2 - \mu}{2 + \mu} \tag{23}$$

or

$$\frac{(h_2)}{(h_0)} = \left(\frac{2 - \mu}{2 + \mu}\right)^2 \tag{24}$$

and at time nT = $n\mu T_c$

$$\frac{(h_n)}{(h_0)} = \left(\frac{2 - \mu}{2 + \mu}\right)^n \tag{25}$$

Table 1 indicates the calculation result with μ = 1/4 and it agrees to theoretical values very well.

Table 1. Comparison between calculation values in use of $\mu = T/T_c$ = 0.25 and theoretical values.

Time	Time(by T_c)	Calculation Value	Theoretical Value
0	0	1.0000	1.0000
T	$0.25T_c$	0.7777	0.7788
2T	$0.5\ T_c$	0.6049	0.6065
3T	$0.75T_c$	0.4705	0.4723
4T	$1.0\ T_c$	0.3659	0.3678
5T	$1.25T_c$	0.2846	0.2865
6T	$1.5\ T_c$	0.2213	0.2331
7T	$1.75T_c$	0.1721	0.1737
8T	$2.0\ T_c$	0.1339	0.1353
⋮	⋮	⋮	⋮
12T	$3.0\ T_c$	0.0490	0.0497
16T	4.0	0.0179	0.0183

When μ = 2 (namely the interval of 2 T_c), eqs (22) and (23) become zero (these exact values are 0.135335 and 0.018316). And when μ> 2, they oscillate in -, +, -, +, ..., etc., having values smaller than 1. Therefore, it is seen that μ is an index for modelizing a distributed system into a lumped system. That is, time constants in each part should be divided to coincide as much as possible. In order to clarify correspondence to the system, if divisions are done in such a way that time constants in each part have considerable differences , the interval in calculation of each part should be modified in such a way that μ becomes equal in parts. For the purpose of rough calculations, when the calculations are tried with large intervals, it is proper to neglect heat capacities in the part of μ> 2.

3. The Application to Synthesis of Heating Equipment Capacitance

In use of the methodology mentioned above, it is reported to simulate hot water heating in a building on an analog computer. A one-story house (100 m^2) having concrete walls of thickness of 15 cm affixed with glasswool 5 cm is heated by hot-water radiator and the system is represented in figure 1 using physical network model. As the used computer is small, the building is one-room model with one boiler (with hot-water-supply tank inside) having one radiator, and a burner is controlled ON-OFF by room temperature and water temperature in the boiler.

3.1. Warming up load

As the results of simulation, figure 2 indicates an intermittent operation in which an operation is sixteen hours and a stoppage is eight hours. In this case, an average outside air temperature is -10°C and a calculation load in steady state is 10000 kcal hr^{-1}.

Observing the results, at night the room temperature in stoppage of operation falls from 20°C to 6°C, therefore, it seems as if fuel is saved in general. But judging from figure 2, it is said that the sum of outgoing heat flow falls only a little. The reason is that heat stored in the wall is discharged at night and the heat is compensated during warming up time. It requires about three hours until it reaches 20°C even when a burner of 20000 kcal hr^{-1} (two times of calculation load) is used. In this example, the intermittent operation has not saved even 10 %, compared to the continuous operation and it is clear that the burner output from 2 to 4 times larger than the steady state load, would be required, corresponding to the interval of warming up time. Therefore, considering initial cost, the continuous operation is profitable rather than the intermittent operation.

3.2. The Need of Dynamic Balance of System

Figure 3 indicates ON-OFF of the burner and the boiler water temperature, there, the ratio k of the radiator capacity in steady state to the burner capacity is changed to 1.0, 1.1, 1.2.

As the result, in spite of having no troubles in steady state, it is seen that in transient state of warming up time, the boiler water temperature reaches a limit and the ON-OFF operation begins before the room temperature reaches 20°C. This ON-OFF operation means that the burner output becomes smaller. Therefore, it is necessary to consider not only static balance of system in steady state but also balance in transient state.

3.3. The Drop in Hot Water Supply Temperature and Additional Load in Hot Water Supply

Figure 4 indicates the drop of the hot water supply temperature and an influence on the room temperature when the hot water is supplied in thirty minutes at the rate of 10 ℓ every minute. At that time, there are two cases such as the intermittent operation with the burner output in 20000 kcal hr^{-1} and the continuous operation in 10000 kcal hr^{-1}

From the results of these simulations, it is seen that when limit design of equipment capacities and so on is done, each simulation should be done case by case because the characteristics are different because of the differences of the systems and therefore limit design should be determined after confirmation and investigation of problems.

4. Other Considerations

By means of the concept of the operator (the concept of the very system element itself which is the operator) and the modified signal flow graph (where it indicates that signals are modified by the operators), algorithm was reported where simulation will take place from the environmental system related to building to its simulation automatically and continuously without regard to the type of computer. It will be thought that the description method is also convenient for common expressions of phenomena in fields of environmental engineering such as electricity, electronics, dynamics, fluid dynamics, process and so on.

As the example of synthesis only the methodology using the small analog computer was indicated. If a large-scaled analog computer is used, it is possible to indicate each room. As digital computers occupy the major parts in general, simulation in use of the digital computer should be indicated. Languages oriented conversations with computers are in the stage of development in our laboratory. It will be discussed on another occasion.

5. References

[1] Samuel J. Mason and Henry J. Zimmermann, Electronic circuits, signals, and systems, John Wiley & Sons, Inc. (1960).

[2] Louis P. A. Robichaud, Maurice Boisvert, and Jean Robert, Signal flow graphs and applications, Prentice-Hall, Inc. (1962).

[3] Jan Mikusinski, Rachunek operatorow, Panstwowo Wydawnictwo Naukowe, Warszawa (1957).

[4] A. Tustin, A method of analyzing the behaviour of linear systems in terms of time series, Inst. Elec. Engineers, Vol. 94, Part II-A, No. 1, p. 130-142, (1947).

[5] D. G. Stephenson and G. P. Mitlas, Cooling load calculations by thermal response factor, ASHRAE Transaction, Vol. 73, Part II, p. 72 (1965).

[6] Richard S. Sanford, Physical networks, Prentice-Hall, Inc. (1965).

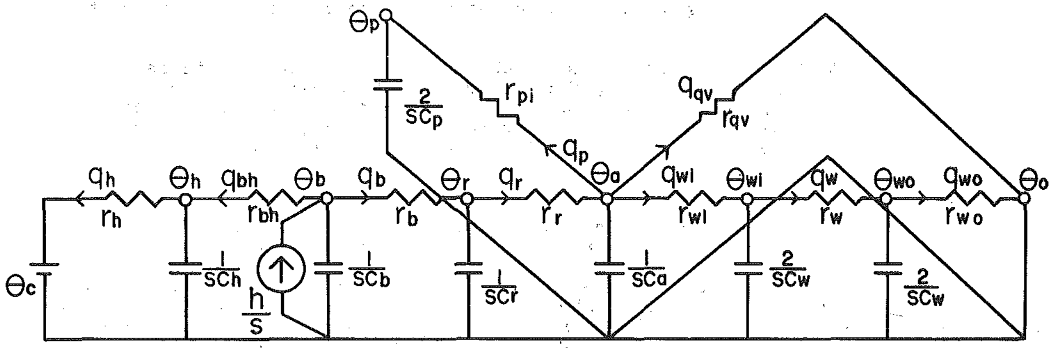


Fig 1 Physical Network Model
θ: temperature, C: heat capacitance, h: burner output,
q: heat flow, r: resistance
(Subscripts)
a: room, b: boiler, c: cold water, g: glass, h: heat water supply,
i: inside, o: outside, p: room wall, r: radiator, v: ventilation,
w: wall

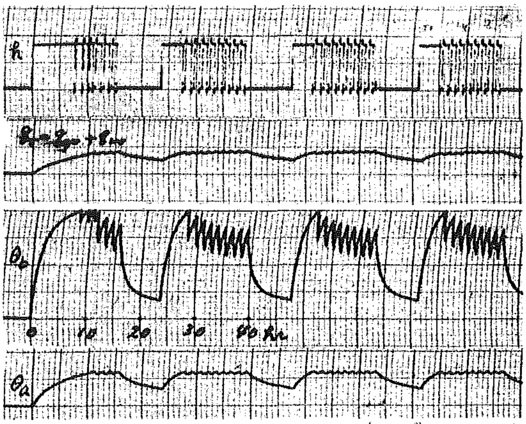


Fig. 2 Response concerning h, Q_{qr}, θ_b and θ_a for Intermittent Operation

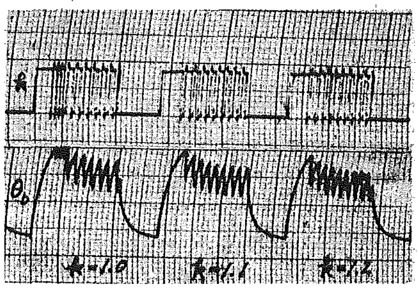


Fig. 3 Response concerning h and θ_b for Intermittent Operation when k = 1.0, 1.1 and 1.2

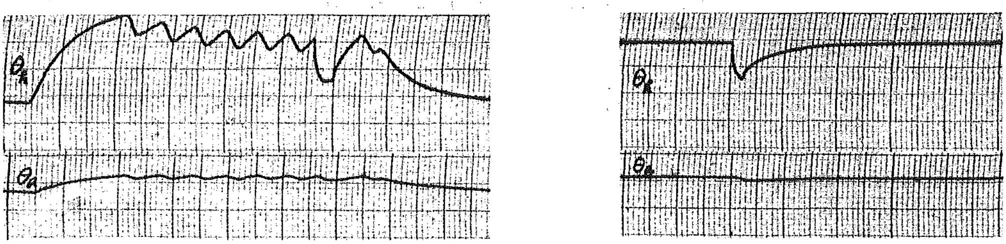


Fig. 4 (a) Response concerning θ_h and θ_a for Intermittent Operation with $h = 20000\ \mathrm{kcal\ hr^{-1}}$
(b) Response concerning θ_h and θ_a for Continuity Operation with $h = 10000\ \mathrm{kcal\ hr^{-1}}$