Loss of self-control in intertemporal choice may be attributable to logarithmic time-perception.
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**Summary**

Impulsivity and loss of self-control in drug-dependent patients have been associated with the manner in which they discount delayed rewards. Although drugs of abuse have been shown to modify perceived time duration, little is known regarding the relationship between impulsive decision-making in intertemporal choice and estimation of time-duration.

In classical economic theory, it has been hypothesized that people discount future reward value exponentially. In exponential discounting, a temporal discounting rate is constant over time, which has been referred to as dynamic consistency. However, accumulating empirical evidence in biology, psychopharmacology, behavioral neuroscience, and neuroeconomics does not support the hypothesis. Rather, dynamically inconsistent manners of discounting delayed rewards, e.g., hyperbolic discounting, have been repeatedly observed in humans and non-human animals. In spite of recent advances in neuroimaging and neuropsychopharmacological study, the reason why humans and animals discount delayed rewards hyperbolically is unknown. In this study, we hypothesized that empirically-observed dynamical inconsistency in intertemporal choice may result from errors in the perception of time duration. It is proposed that perception of temporal duration following Weber's law might explain the dynamical inconsistency. Possible future study directions for elucidating neural mechanisms underlying inconsistent intertemporal choice are discussed.
**Background**

Discounting of delayed rewards refers to the observation that the value of a delayed reward is discounted (reduced in value or considered to be worth less) compared to the value of an immediate reward [1,2]. Studies in psychopharmacology, psychiatry, behavioral neuroscience, and economics have been focused on how subjects discount delayed rewards. Notably, it has repeatedly been demonstrated that substance abusers more steeply discount delayed rewards than non-drug dependent subjects [1].

According to classical economic theory, including a rational addiction theory, it has been assumed that individuals discount delayed reward in a rational manner [2]. This type of discounting (exponential discounting) follows the exponential equation:

\[ V(D) = A \exp(-kD) \]  

(Equation 1)

where \( V \) is the subjective value of a reward, \( A \) is the (objective) amount of the reward, and \( D \) is the length of delay until the delivery of reward. The free parameter \( k \) is an index of the degree of discounting, i.e., larger \( k \) values correspond to steeper delay discounting. However, subsequent empirical studies have revealed that the following (general) hyperbolic equation fits the behavioral data better than Equation 1 [1,2]:

\[ V(D) = \frac{A}{1 + jD^s} \]  

(Equation 2)

where \( j \) and \( s \) are free parameters. Note that when \( s=1 \), the function is referred to as a simple hyperbolic equation. Discounting delayed rewards following this hyperbolic equation is called “hyperbolic discounting”. Larger \( j \) and \( s \) values again correspond to steeper delay discounting. A remarkable distinction between the exponential and hyperbolic discounting exists in the time-course of a discounting rate, defined as \( (dV/dD)/V \) [2]. Specifically, in hyperbolic discounting, the discounting rate is a decreasing function of delay, resulting in “preference reversal”, which is an example of dynamically inconsistent behavior and loss of self-control (commonly observed in drug addicts) [1]; while in exponential discounting, the discounting rate is independent of delay and keeps constant, which is called dynamical consistency [1,2].

Recently, in the emerging field of neuroeconomics [3], how neural substrates mediate discounting of delayed rewards attracts much attention. For instance, dopaminergic reward-processing brain regions, orbitofrontal and limbic regions play pivotal roles in delay discounting [4,5]. We have also examined a neuroendocrine correlate of delay discounting [6]. However, in spite of extensive neuropsychopharmacological and neuroimaging studies, it is still unknown why individuals discount delayed rewards hyperbolically, rather than exponentially [4]. In this paper, we propose that the empirically-observed inconsistency in discounting may result from errors in the estimation of time-duration.
**Hypothesis**

The psychophysicists Weber and Fechner proposed that the external stimulus (e.g., loudness) is scaled into a logarithmic internal representation of sensation (Weber’s law), rather than a linear internal representation [7]. Some recent studies further suggest that the mental timer also seems to be logarithmic, rather than linear, following Weber’s law [8,9], although it is still controversial whether time estimation is processed in distributed neural networks or in central time-keeping neural circuitry [9]. Therefore, it is reasonable to suppose that discounting of delayed rewards with logarithmic time-perception differs from that with linear time-perception. Let \( \tau \) be logarithmically perceived (subjective) time-duration which can be represented as:

\[
\tau(D) = \alpha \ln (1+\beta D)
\]  
(Equation 3)

where \( \alpha \) and \( \beta \) denote constants independent of \( D \) and \( \tau \). Note that \( \tau(0)=0 \) and \( \tau \) is no less than 0. Suppose that individuals try to discount delayed rewards exponentially, with this type of logarithmic time-perception. In this case, \( D \) in Equation 1 is replaced with \( \tau \). Then, exponential discounting with Weber-type time-perception follows the exponential function with \( \tau \):

\[
V = A \exp(-k\tau) \quad \text{(Equation 1')} \]

If this equation is expressed in terms of \( D \),

\[
V(D) = A \exp\{-k\alpha \ln (1+\beta D)\} \quad \text{(from Equation 3)}
= A \exp\{-\ln(1+\beta D)^{k\alpha}\}
= \frac{A}{(1+\beta D)^{k\alpha}}.
\]  
(Equation 4)

Here, if we denote \( j=\beta \) and \( s=k\alpha \), Equation 4 (exponential discounting with logarithmic time-perception) is the same as Equation 2 (the general hyperbolic function). As can be seen from the derivation of this equation, even if subjects with logarithmic time-perception try to discount delayed rewards exponentially (i.e., in a dynamically consistent manner), rather than hyperbolically, their actual discounting of delayed rewards may follow the hyperbolic function, possibly due to an error in time-perception which follows Weber’s law.

**Several neuropsychopharmacological findings supporting our hypothesis**

Neuropsychopharmacological studies have revealed that both acute and chronic administration of dopaminergic drugs (e.g., alcohol, heroin, and nicotine) dramatically affect individual’s degree of discounting delayed rewards [1]. For instance, parameters of hyperbolic discounting (e.g., \( \beta \) in Equation 4) have been shown to be increased in
drug addicts, which is supposed to associate with their impulsive decision-making in intertemporal choice and loss of self-control [1,10-13]. Interestingly, it has been reported that dopaminergic drugs also markedly affect subject’s time-perception [14-16]. Together, it is possible that dopaminergic drugs modulate neural processing underlying time-perception and increase non-linearity of time-perception, resulting in exaggerated inconsistency in discounting and loss of self-control.

Conclusions
Relations between non-linearity of time-perception and subject’s parameters in discounting equations should be empirically investigated in future studies, in order to test our hypothesis. Studies employing substance abusers and administrations of dopaminergic drugs would be especially important.

References


