<table>
<thead>
<tr>
<th>項目</th>
<th>内容</th>
</tr>
</thead>
<tbody>
<tr>
<td>タイトル</td>
<td>じゃばら折りの一般化とその複雑さの研究</td>
</tr>
<tr>
<td>作者</td>
<td>上原 隆平</td>
</tr>
<tr>
<td>引用</td>
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<td>ファイル情報</td>
<td>10_all.pdf</td>
</tr>
</tbody>
</table>

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じゃばら折りの一般化とその複雑さの研究

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概要

日本での折紙の研究は、数学的な観点からの研究が先行してきた。しかし Lang, Domain, O’Rourke といった研究者の活躍により、最近 computational origami という言葉が市民権をもつようある。しかし理論計算機科学としての枠組みは、まだまだ未整備である。折紙を計算機科学の対象としたときの単純なモデルとは何だろうか。ここではごく単純な折紙モデルとして、次のものを考える。入力として与えられるのは、長さ n+1 の紙と、長さ n の M、V 上の文字列である。ここで M は山折り、V は谷折りを意味している。つまり長さ n+1 の紙の上に、等間隔に山/谷が指定されている。この文字列にしたがって紙を折る問題を考える。まず時間計算量に対して、折りの回数が自然に対応すると考えることができる。この観点から、じゃばら折りとその一般化に対して、講演者は、ほぼ最適な折りアルゴリズムを設計した。では領域計算量に対応する概念は何だろうか。これに対してはみんなが合意する概念は存在しない。講演者は、折り目に揃まる紙の枚数を基準として提案し、これに関する最適化問題を研究している。部分的な解は得られているものの、まだ残された研究課題の方が多いのが実状である。
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or, ask Google with "uehara origami"

Kawasaki Rose

http://www.jaist.ac.jp/~uehara/etc/origami/

1. At 4th Japan Meeting on Origami in Science, Mathematics, and Education, 2008/6/22,
   - Toshikazu Kawasaki, a mathematician and designer of Kawasaki rose, said that
     “For a mathematician, it is OK if solution exists.”
   - A computer scientist, or I, cannot agree;
     “For a mathematician, it is OK if solution exists.”

   Is there any good problem just about computational cost??

2. From the viewpoint of Computer Science
   - Two resources of Origami model?
     1. time...the number of foldings (operations)
     2. space...minimization of "stretch" that is
       the number of papers between two hinged papers

   Complexity of folding ( )

   From the viewpoint of Computer Science
   - Two resources of a computation model;
     1. time: the number of steps of operations
     2. space: the number of memory cells required to compute

   Pleat folding = 1D Origami
   - Alternating foldings of Mountain and Valley
   - Basic tool of Origami
   - Many applications
     ➢ Extension to General Patterns and consider its complexity...

   Quite Important!!

http://www.jaist.ac.jp/~uehara/etc/origami/
New open problem

- Least **stretch** folding problem
  - **Input**: Paper of length \( n+1 \) and \( s \in \{ M, V \}^x \)
  - **Output**: folded paper according to \( s \)
  - **Goal**: Find a good folded state with few **stretch**
    - At each crease, the number of papers between the papers hinged at the crease is **stretch**.
    - Two minimization problems; max/total(average)
      - minimize maximum
      - minimize total (=average)

It seems simple, so easy??

Not!

7/33

New open problem

- Simple non-trivial example
  - **Input**: MM/MM/MM/MM/MM
  - **Goal**: Find a good folded state with few **stretch**
    - The unique solution having min. max. value 3
      - total=13
    - The unique solution having min. total value 11
      - total=13

8/33

New open problem

- Least **stretch** folding problem
  - **Input**: Paper of length \( n+1 \) and \( s \in \{ M, V \}^x \)
  - **Output**: folded paper according to \( s \)
  - **Goal**: Find a good folded state with few **stretch**
    - Two minimization problems; max/total(average)
      - A few facts;
        - a pattern has a unique folded state iff it is pleats
        - solutions of \{min max\} and \{min total\} are different depending on a crease pattern.
        - there is a pattern having exponential combinations

MVMVMVMVVMVVMVVMVVMV

9/33

Least stretch folding problem

- Open problem:
  - Tractable/Intractable?
    - \#P-hard?
      - Poly-time solvable by Dynamic Programming?
        - up to now, I have no answer to this question ;-
    - Then, exhaust search technique, that produces all possible folding ways, works?

exponential pattern

Not!

10/33

Least stretch folding problem

1. Average number of folding ways for a random pattern = \( f(n)/2^n \), where

   \[ f(n) = \# \text{ of folding ways (or folded states) of a paper of length } n+1 \text{ (summation for all possible patterns)} \]

   I give some bounds of \( f(n) \):
   - "The On-Line Encyclopedia of Integer Sequences" tells us up to \( n=28 \) (by enumeration);
     \( f(n) \sim O(3.3^n) \).
   - I have upper/lower bounds;
     - upper bound: \( f(n)=O(4^n) \)
     - lower bound: \( f(n)=\Omega(3.07^n) \)

11/33

Least stretch folding problem

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     - upper bound: \( f(n)=O(4^n) \)
     - lower bound: \( f(n)=\Omega(3.07^n) \)

12/33

69
Least stretch folding problem

Upper bound \( f(n) = \Omega(4^n) \) comes from the Catalan number.

\[ \text{Proof} \] If the paper of length \( n+1 \) is folded, the crease points should be nested.

\[ \text{Proof} \] We consider “folding of the last \( k+1 \) unit papers”; the crease points should be nested.

Thus, we have \( f(n) \geq \left( g(k) \right)^{k+1} = \left( \frac{(k+1)!}{k!} \right)^{k+1} \).

Combination of \( n \) pairs of \( \text{Catalan Number} \ C_{n}^{k} \)

Least stretch folding problem

Lower bound \( f(n) = \Omega(3.07^n) \).

\[ \text{Proof} \] We consider “folding of the last \( k+1 \) unit papers”; the crease points should be nested.

We let

\( f(n) \): the number of folding ways of length \( n+1 \)

\( g(k) \): the number of folding ways of length \( k+1 \) s.t. the leftmost endpoint is not covered

Then, we have \( f(n) \geq g(k)^{k+1} \).

Open problem and Future work

Least stretch folding problem:

- Tractable/Intractable?
- \( \text{NP}-\text{hard} \)?
- Poly-time solvable by Dynamic Programming?
- Fixed parameter tractable for “stretch-free”? (a)
- What is the most complex pattern of M/V that has the most feasible folded states?
- Extension to...
  - non-unit length (operation-restricted linkage)
  - 2D (a kind of map folding)
- What “space complexity” of Origami is?
- area for folding?
- Future work

Open problem and Future work:

- Ext. of Least stretch folding problem:
  - non-unit length
    - First of all, there is a pattern that cannot be folded;
  - revisit unit length paper...
    - For any pattern, can it be folded?
      - “Yes”; by repeating “end-fold”
    - For any “folded state” of length \( l \), can it be folded?
    - If you allow any folding, it is “Yes” by
      - unlocked linkage in 2D ... reverse of bloom
      - universal theorem ... adding many creases
    - Is it “Yes” under simpler/reasonable model????

Universal theorem:

- Universal theorem of unit length folding in a simple folding model;
  - Input: a folded state of a paper strip of length \( n+1 \) to unit length
  - Question: Is it foldable on a suitable set of operations?
  - Simple folding model
  - 1. (from flat state)
  - 2. pick up a point
  - 3. valley fold most inner layers
  - 4. to flat state
Universal theorem of linkage…

- Universal theorem of unit length folding in a simple folding model;
  - Any flat folded state of unit length can be made by a sequence of simple foldings of length at most 2n.

- Related results;
  - Any M/V pattern can be flat folded by repeating "end-folding"
  - If the creases are not unit-length, some folded state cannot be folded by simple foldings:
  - If "any folding" is allowed, "Yes" even in non-unit length
    - comes from "no locked linkage in 2D"!!!

Let’s turn to the folding complexity of pleat folding

- Repeating of mountain and valley foldings
- Basic operation in some origami
- Many applications

Pleat folding

- Pleat folding (in 1D)
  - Repeating “folding in half” is the best way to make many creases

  • Naive algorithm: n time folding is a trivial solution
  • We have to fold at least \( \log n \) times to make \( n \) creases
  • More efficient ways…?
  • General Mountain/Valley pattern?

  • proposed at Open Problem Session on CCCG 2008 by R. Uehara.

Open:
- The number of unfolding can be \( n \)?

Characterization of “non-unfoldable” (non-unit length) origami by simple (un)folding.

- In simple folding model, no locked flat tree in 2D if edges are unit length?
• Complexity of Pleat Folding

[Main Motivation] Do we have to make \( n \) foldings to make a pleat folding with \( n \) creases??

1. The answer is "No"!
   - Any pattern can be made by \( \lceil n/2 \rceil + \lceil \log_2 n \rceil \) foldings
2. Can we make a pleat folding in \( o(n) \) foldings?
   - Yes! It can be folded in \( \Theta(\log^2 n) \) foldings.
3. Lower bound; \( \log n \)
   - \( \Omega(\log \log n) \) lower bound for pleat folding!!

http://www.jaist.ac.jp/~uehara/etc/origami/
Lower bound of Unit FP

[Thm] Almost all patterns but $o(2^n)$ exceptions require $\Omega(n/\log n)$ foldings.

[Proof] A simple counting argument:

1. # patterns with $n$ creases $> 2^n/4 \approx 2^n/3$
2. # patterns after $k$ foldings $< (2 \times n) \times (2 \times n+1) \times (2 \times n+2) \times \ldots \times (2 \times n+1)^k$

We cannot fold most patterns after at most $k$ foldings if

$$\sum_{i=1}^{k} (2n(n+1))^i \leq 2^{n+1} \cdot (2n(n+1))^k < 2^n.$$

Letting $n \geq 2$, $k = O\left(\frac{n}{\log n}\right)$ we have $(2n(n+1))^i = o(2^n)$

Open Problems

- Pleat foldings
  - Make upper bound $O(\log^2 n)$ and lower bound $\Omega(\log n / \log \log n)$
  - "Almost all patterns are difficult", but...
  - No explicit M/V pattern that requires $(cn/\log n)$ foldings
- When "unfolding cost" is counted in...
  - Minimize #foldings + #unfoldings

Any pattern can be folded in $cn/\log n$ foldings

- Split into chunks of size $b$
  1. Each chunk is small and easy to fold
  2. # kinds of different $b$ are not so big

Main alg.

1. pile the chunks of pattern $b$ and mountain fold them
2. fix the reverse chunks
3. fix the boundaries

Minimize #foldings + #unfoldings

Analysis is omitted