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MIXED POISSON DETERIORATION MODEL: APPLYING FOR PEELING/FALLING OF CONCRETE

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ABSTRACT

In this study, the authors develop a model of the generating process on peeling/falling (P/F) of protective covering concrete as a counting process. The model is able to consider a time-independent factor (random event) and a time-dependent factor (deterioration event) of P/F process. In detail, the random event is presented by the homogeneous Poisson process, the deterioration event is presented by the non-homogeneous Poisson process where arrival rates are changed with time. Then, a mixed Poisson deterioration process is formulated by both the homogeneous and the non-homogeneous Poisson process. Lastly, the appropriateness of the proposed model is discussed empirically, by applying it to actual inspection data.

By estimating the mixed Poisson deterioration model, it is possible to predict the generating process on P/L of protective covering concrete. The proposed model is able to represent three things at the same time. Firstly, the number of P/F concrete increases if an inspection interval increases. Secondly, the model formulates the deterioration event that the number of P/F concrete increases with accelerating speed if an inspection interval increases, which is unable to be presented by homogeneous Poisson model. Lastly, the model stands for the random event that is generated even if the service life is small, which is unable to be presented by non-homogeneous Poisson model. Moreover, the model makes possible to classify observed data stochastically by the generating factor which is the random event or the deterioration event.

Keywords: bridge management, Mixed Poisson deterioration model, peeling/falling of concrete.

1. INTRODUCTION

In recent years, with the aging, infrastructures which require countermeasures such as repair or reinforcement has been increasing rapidly. Especially, it is necessary for damage which can have third party damage to take measures as soon as possible even though it has a small impact on the structural safety of the infrastructure. Structural administrator is committed to ensuring the safety of

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infrastructure in order to prevent third party damage with carrying out a visual inspection at fixed intervals. However, in the maintenance of infrastructure, inspection costs are not so small that it is essential to examine an efficient inspection and repair method on the assumption that safety is ensured.

In order to prevent third party damage caused by infrastructure aging, it is important to early detection of damage to the structure. The more frequent inspections of infrastructure, the risk of leaving damage for a long time will be reduced. On the other hand, frequent inspections increase maintenance costs because inspection costs are increasing. Moreover, as an aging infrastructure, there is possibility of increasing in incidence of damage. Therefore, even though set the same inspection intervals, it is also conceivable that the number of damage is increasing together with the aging infrastructure. In order to corresponding to third parson damage reasonably, it will be necessary to evaluate the risk of damage according to the service years.

In this study, the authors develop a model of the generating process on peeling/falling (P/F) of protective covering concrete as a counting process. This mixed Poisson deterioration model composed by Poisson model and non-homogenous Poisson model (McNeil et al. 2005) is able to consider both a time-independent factor (random event) and a time-dependent factor (deterioration event). In addition, P/F generating probability is estimated by using the mixed Poisson deterioration model and empirical inspection data.

2. BASIC IDEAS OF THIS STUDY

2.1. Statistical deterioration prediction based on visual inspection data

In the field of statistical deterioration prediction method, various models based on hazard model have been documented. For instance, Markov hazard model (Tsuda et al. 2005) calculates control limit (life expectancy) with condition states of the visual inspection data corresponding to respective deterioration event. On the other hand, Poisson model (Kaito et al. 2005) is possible to forecast the number of damage event with Poisson distribution which expresses the number of deterioration events and damage events. In these models, structure and environmental conditions of the facility-specific are adopted as the explanatory variables, on the basis of large amount of information related to a deterioration process. Statistical deterioration model is possible to represent an average deterioration path. However, in the case of using a single probabilistic model, it can only predict an average deterioration path. Therefore, there is a limit that useful management information, for micro-level decisions which target individual facilities, cannot be provided. Thus, the deterioration prediction model has been proposed, which is associated with heterogeneity of individual facilities. For example, there are mixed Markov hazard model (Obama et al. 2008) and Random proportional Poisson model (Kaito et al. 2009), which express individual heterogeneity by using gamma distribution. As a result, evaluation of deterioration rate can be carried out for each bridge unit.

2.2. Visual inspection scheme of bridges

Visual inspections for the bridge are carried out every determined interval. In one of the inspection items, there is P/F event of protective covering concrete. As a result of visual inspections, information about individual P/F events is written on an inspection register. Then a rank of condition states is determined. On the other hand, repairing action for P/F events is planned based on a bridge span. Therefore, when operation and maintenance for P/F events are performed, it is important to model generating process about the total number of P/F events. Figure 1 shows generating process about P/F events and visual inspection scheme in a bridge span. Character t represents an elapsed time from commencing services. In the following, the elapsed time t is called "time" simply.

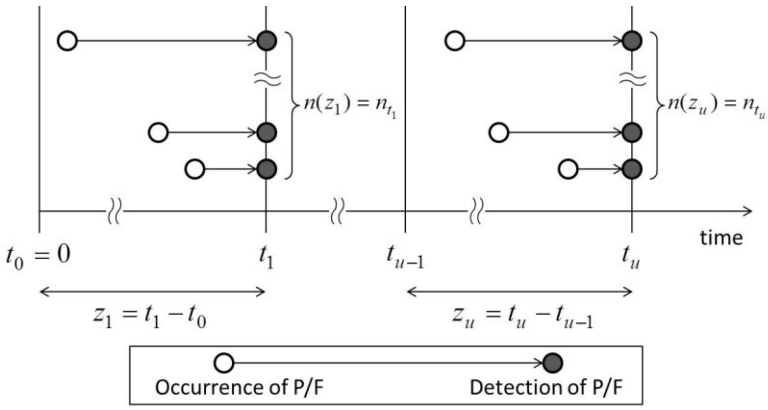

Figure 1: Generation process of P/F

3. MIXED POISSON DETERIORATION MODEL

3.1. Time-independent Poisson model

Consider the time axis starting at the initial point $t=0$. The point on the time axis is "time". Focus on the i th$(i=1,\cdots,I)$ step. Figure 1 shows that inspections are carrying out at the time $t\ =0,t_1,\cdots T_U$ before the current time. Here, inspection interval between the 0 th inspection and the 1st inspection is defined as $z_0(=t_1-t_0)$. In the same way, inspection interval between the u th inspection and the $u+1$th inspection is expressed as $z_u(=t_{u+1}-t_u)$. Focus on the case that the P/F process in the ith span conforms to Poisson process where arrival rate λ_i does not depend on time. In the period $[t_u,t_{u+1}]$ of inspection interval z_u, the probability of the occurrence of n-P/F in the ith span is expressed by the following time-independent Poisson distribution being subject to only inspection interval z_u:

$$\mathrm{P}_1(n_i(z_u)=n)=\frac{(\lambda_i z_u)^n}{n!}\exp(-\lambda_i z_u) \tag{1}$$

Here, λ_i is represented as following formula (2) by using the characteristic parameter $x_i = (x_{1,i}, \cdots, x_{M,i})$ of the i th step such as structure condition or environmental condition and by using unknown parameters vector $\beta_1 = (\beta^1, \cdots, \beta^M)$, and $x_{m,i}(m = 1, \cdots, M)$ is an observed value of m th characteristic variable:

$$\lambda_i = \lambda(x_i, \beta_1) = \exp(x_i \beta_1') \tag{2}$$

$'$ is showing a transpose operation.

3.2. Time-dependent Poisson model

Let us focus on the case that an arrival rate of P/F events is changing by progress of deterioration. Let $\mu_i(t)$ be the arrival rate of P/F events at an arbitrary time t. Moreover, it is assumed that the arrival rate $\mu_i(t)$ is integrable in a period $[0,t]$. Then, a mean arrival rate $\Lambda_i(t)$ in the period $[0,t]$ is expressed as follows:

$$\Lambda_i(t) = \int_0^t \mu_i(\tau) d\tau \tag{3}$$

$\Lambda_i(t)$ is called a mean function because it intends the number of a mean occurrence in the period $[0,t]$. In the time independent Poisson distribution expressed by formula (1), the mean function is represented by $\lambda_i z$. Focus on the period $[t_u, t_{u+1}]$ of the inspection interval $z_u (z_u = t_{u+1} - t_u)$. Then, describe the number of P/F events in the i th span as $n_i(t_u + z_u)$, which is observed newly at time $n_i(t_{u+1})$, which is also expressed $n_i(t_{u+1})$, by using a previous inspection time t_u and by using an inspection interval $z_u = t_{u+1} - t_u$. It is assumed that P/F points found at a previous inspection time are repaired immediately at the time. Then, the probability, that the number of n-P/F is newly observed in the i th span at the time $t_{u+1} = t_u - z_u$, is as follows:

$$\begin{aligned} \mathrm{P}(n_i(t_u + z_u) - n_i(t_u) = n) &= \frac{\{\Lambda_i(t_u + z_u) - \Lambda_i(t_u)\}^n}{n!} \exp[-\{\Lambda_i(t_u + z_u) - \Lambda_i(t_u)\}] \\ &= \frac{\left\{\int_{t_u}^{t_u + z_u} \mu_i(\tau) d\tau\right\}^n}{n!} \exp\left\{-\int_{t_u}^{t_u + z_u} \mu_i(\tau) d\tau\right\} \end{aligned} \tag{4}$$

Formula (4) is extended expression of the Poisson model. Here, time dependent arrival rate is defined as follows:

$$\mu_i(t_u) = \mu(x_i, \beta_2, t_u) = \exp(x_i \beta_2') \cdot \alpha t_u^{\alpha - 1} \tag{5}$$

α is an unknown parameter which expresses time-dependence, and it is equivalent to the acceleration parameter in the Weibull distribution.

3.3. Mixed Poisson model

The actual P/F process is, caused by length of service periods and construction methods, distinguished between random events and deterioration events. However, information given by visual inspection data is such as structural conditions, environmental conditions, inspection times and the number of P/F events in an individual span. In this study, using these inspection datasets, the authors formulates mixed Poisson deterioration model which is possible to express both random events and deterioration events.

As a result of visual inspections, it is assumed that information of the total K inspection samples is obtained. Let $\bar{e}^k$ be information of sample $k(k=1,\cdots,K)$:

$$\bar{e}^k = \left(\bar{n}^k, \bar{z}^k, \bar{t}^k, \bar{x}_{i(k)}\right) \tag{6}$$

where $\bar{n}^k$ is the number of P/F, $\bar{z}^k$ is an inspection interval, $\bar{t}^k$ is years in service, of sample k. $\bar{x}_{i(k)}$ indicates characteristic vector of the span in which a sample k belongs. A population Ξ in which the observed data $\bar{e}$ is extracted is composed by two exclusive subpopulations, Ξ_1 and Ξ_2, with respect to random events and deterioration events. Thought visual inspections for spans, information vector $\bar{e}^k$ about a sample $k(k=1,\cdots,K)$ is earned, and the total $\bar{n}^k$-P/F events are observed. However, it is impossible to classify the P/F events as belonging to random or deterioration events. Then, the pattern, that $\bar{n}^k$-sample is extracted from subpopulation, is classified as one of following $\bar{n}^k+1$ patterns:

1) $\bar{n}^k$-sample is extracted from population Ξ_1. $(h=1)$
2) $(\bar{n}^k-1)$-sample is extracted from population Ξ_1.
   1-sample is extracted from population Ξ_2. $(h=2)$

⋮ ⋮

$\bar{n}^k+1$) $\bar{n}^k$-sample is extracted from population Ξ_2. $(h=\bar{n}^k+1)$

Firstly, in the visual inspection of interval $\bar{z}^k$ carried out at time $\bar{t}^k$, the probability, that $\bar{n}^k$-P/F-sample is observed, is expressed as follows:

$$P\left(n\left(\bar{z}^k, \bar{t}^k = \bar{n}^k \mid \theta\right)\right) = \sum_{h=1}^{\bar{n}^k+1} P_1\left(\bar{n}^k - h + 1 \mid \theta_1\right) P_2\left(h-1 \mid \theta_2, \bar{t}^k\right) \tag{7}$$

where h indicates the extracted pattern. $\theta=(\theta_1,\theta_2)$ are unknown parameter vectors, where $\theta_1=\beta_1$, $\theta_2=(\beta_2,\alpha)$. Additionally, the log likelihood function of the mixed Poisson deterioration model can be expressed by the following formula:

$$\ln\{L(\theta \mid \bar{e})\} = \sum_{k=1}^{K} \ln \sum_{h=1}^{\bar{n}^k+1} P_1\left(\bar{n}^k - h + 1 \mid \theta\right) P_2\left(h-1 \mid \theta\right) \tag{8}$$

By using the log likelihood function (8), it is possible to obtain the maximum likelihood estimate of the parameters θ of the mixed Poisson deterioration model.

4. EMPIRICAL ANALYSIS

4.1. Outline of application cases

The model proposed in this study is applied to the data of visual inspection carried out in certain span of an expressway. The number of target bridges is 484, which was built between 1974 and 2005, respectively. Therefore, the oldest bridge in them has been elapsed for 38 years from operation starting. These bridges are divided into 2,649 spans and the total samples are 3,933.

4.2. Estimation results

Using mixed Poisson deterioration model formulated in Chapter III, it is able to express the generation process of P/F concrete. At this time, seven factors, which was considered to influence generating process of P/F, were adopted as characteristic candidates. This candidates included 1) span length, 2) bridge area, 3) slab thickness, 4) traffic volume, 5) large traffic volume, 6) minimum angle of skew, 7) girder space. As a result of estimation, the model, which minimizes AIC in the model satisfying t-value and sign condition of unknown parameters, was selected as an optimum model. Table 1 shows these results. One characteristic variable of 1) span length was adopted.

Table 1: Estimation results

	constant term	span length	acceleration parameter
β_1 estimator t-value	-1.914 (-50.364)	0.865 (4.3967)	-
β_2 estimator t-value	-6.433 (-70.850)	1.799 (16.394)	2.590 (100.016)
AIC	32,384		

4.3. Mixed Poisson deterioration distribution

As the formula (8), the generating process of P/F concrete is expressed by mixing of the two distributions both time-independent Poisson distribution and time-dependent Poisson distribution. Figure 3 and 4 show results of calculating probability distributions related to a generation of P/F concrete by using Tab.1. Here, Figure 2 shows the distribution of the number of P/F concrete in the case of setting service time $t=0$ and changing inspection interval z , Figure 3 shows the distribution in the case of setting inspection interval $z=5$ and changing service time t. In explanatory notes of Fig.2, 3, the expected values $E[n|z,t]$ of the number of P/F concrete are also shown together. It is noted that the proposed model is able to represent three things at the same time. Firstly, the number of P/F concrete increase if an inspection interval increases. Secondly, the model formulates the deterioration event that the number of P/F concrete increases with accelerating speed if an inspection interval increases, which is unable to be presented by homogeneous Poisson model. Lastly, the model stands for the random event that P/F concrete is generated even if the service life is small, which is unable to be presented by non-homogeneous Poisson model.

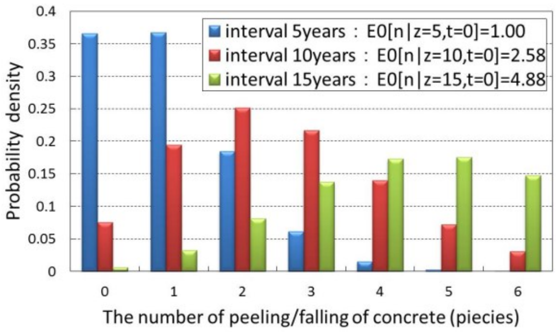

Figure 2: Mixed Poisson distribution (t=0)

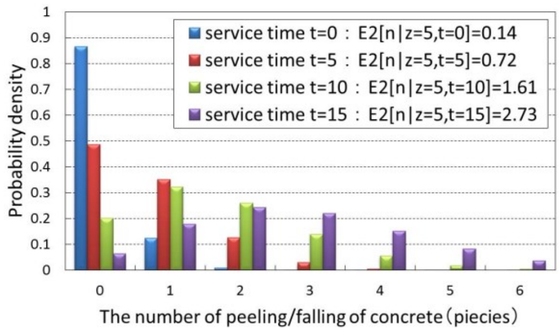

Figure 3: Mixed Poisson distribution (z=5)

4.4. Estimate the mixture ratio

Mixed Poisson deterioration model is able to manage a risk focused on the number of P/F concrete. However, a generating process is difference between random events and deterioration events. Therefore, to manage a risk reasonably, it is need to classify the samples by the generating factor which is the random event or the deterioration event, and it is need to enforce inspection/repair policy in response to the result of the identification. When $\bar{n}$ -P/F samples are observed, a mixture ratio $\pi_h(\bar{n})(h=1,\cdots,\bar{n}+1)$ of an extracted pattern of P/F events is formulated by using Bayes' theorem:

$$\pi_h(\bar{n}) = \frac{P_1(\bar{n}^k - h + 1 \mid \theta)P_2(h-1 \mid \theta)}{\sum_{h=1}^{\bar{n}+1} P_1(\bar{n}^k - h + 1 \mid \theta)P_2(h-1 \mid \theta)} \tag{9}$$

Mixture ratio is the distribution of the extracted pattern h. Figure 4 shows a relation between distributions of mixture ratio and service time t in the case that the 5-sample is observed and inspection interval z is 5 years. Focused on the mixture ratio just after the service start, it can be seen that $\pi_2(5) = 0.33$ and $\pi_3(5) = 0.33$. As a result, it can be found that the 5-sample is extracted by pattern 2 and 3 at 66% probability. By using a mixture ratio, when the number of P/F is observed, it is possible to distinguish between the random events and the deterioration events. From the above, using mixture ratio will be able to carry out risk management reasonably.

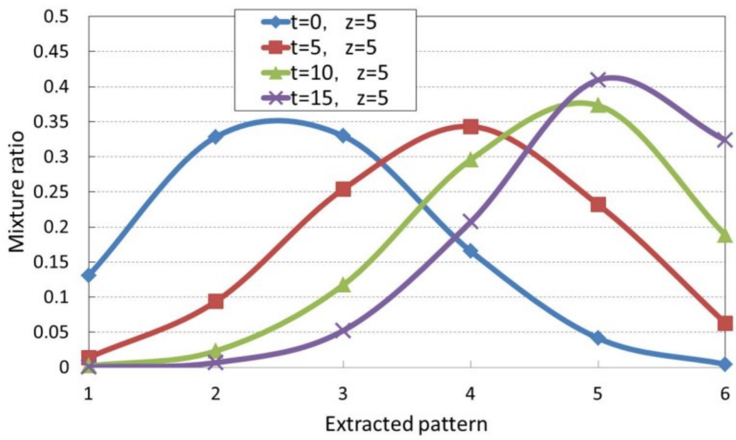

Figure 4: Relation between mixture ratio distribution and service time

5. CONCLUSIONS

In this study, the authors have proposed a method for expressing the number of P/F concrete and also distinguishing between random events and deterioration events of P/F concrete by using mixed Poisson deterioration model. This model has been able to find the knowledge regarding time-dependence of P/F generating process which cannot be concluded on visual inspection data, and also it has been able to propose operation and maintenance plans with respect to inspection interval or repair work corresponding to each slab span.

The methodology proposed in this study is highly practical, but there still remain the following problems to be solved. The first problem is that the analysis target in this study was limited to specific bridges. In order to study a variety of infrastructure facilities, it is essential to collect visual inspection data and accumulate the cases of application of the proposed methodology. The second problem is that it is necessary to develop a methodology of risk management. Specifically, formulate an evaluating index, and calculate expected cost for inspection work based on a generating probability of P/F concrete, it will be able to find an optimum inspection policy which minimize an expected cost under a given confidence level of risk.

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