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RESEARCH ARTICLE

Characteristics of lateral-shearing nulling interferometry by use of double Fresnel rhombs

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Nulling interferometer for directly imaging exoplanets with a segmented-mirror telescope is investigated. Lateral shearing interferometry is applied to a segmented-mirror telescope such as the Thirty Meter Telescope. Use of a pair of double Fresnel rhombs in Mach-Zehnder interferometer achieves achromatic nulling and lateral shearing simultaneously. In this paper, computer simulations of the lateral interferometry with the Fresnel rhombs in near infrared region are carried out to analyze effects of segmentation errors, tip-tilt errors, amplitude and optical-path difference errors on its nulling performance.

Keywords: Exoplanet; nulling interferometry; shearing interferometer; Fresnel rhomb; TMT; E-ELT

1. Introduction

There have been several reports on direct imaging of exoplanets [1–4]. It is expected that much more exoplanets will be directly imaged with large telescopes such as the Thirty Meter Telescope (TMT) and the European Extremely Large Telescope (E-ELT) [5, 6]. Telescopes larger than 8 m will be constructed with segmented mirrors. Direct imaging methods to cope with segmented mirrors are required for efficient observations. Various methods have been developed for direct imaging of exoplanets [7–10]. However, many of them are useful for a circular aperture without shade of a secondary mirror and spiders.

Shao et al. have studied a lateral shearing interferometer to null starlight [11]. Lateral shearing interferometry is an attractive method for a segmented mirror telescope, because a lateral shear with units of one segment can accommodate gaps between the segments. The light from a star-planet system is divided into two and combined with π phase difference for the starlight. The interferometer should work with broadband light to detect faint planetary light. An ordinary method to realize π phase shift for broadband light is to use several dispersive materials. However, use of dispersive materials usually limits the bandwidth of useful light. Murakami & Baba proposed the achromatic lateral-shearing nulling interferometer using Savart plates and polarizers to realize lateral shearing and the achromatic π phase shift at the expense of the loss of incoming light [12].

Fresnel rhombs will be promising optical components for providing π phase shift for nulling interferometers [13–15]. A pair of double Fresnel rhombs has been proposed for the lateral-shearing nulling interferometer to realize simultaneously lateral shearing and broadband π phase shift [16].

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In this paper, we carried out computer simulations of the lateral-shearing nulling interferometer with a pair of double Fresnel rhombs taking several instrumental errors into account. The optical setup considered here is based on the Mach-Zehnder interferometer. A pair of double Fresnel rhombs is inserted into each arm of the Mach-Zehnder interferometer. The π phase difference is produced with total internal reflections inside Fresnel rhombs. One pair of Fresnel rhombs is set to be perpendicular to the other one. Thus, the π phase shift is attained for both p- and s-polarizations. A lateral shear is accomplished by using Fresnel rhombs with different sizes. The amount of the shear depends on the difference of the Fresnel rhombs' sizes. Here it should be noted that the path length inside one Fresnel rhomb is constant and independent on the incident beam position as described below.

We investigate the performance of the interferometer through computer simulations. We assume the TMT as a model of segmented mirror telescopes. The wavelength region considered here is K-band ($\lambda = 2.0 - 2.4 \mu\text{m}$). Random misplacements of segmented mirrors and tip-tilt errors degrade the performance of the interferometer. Therefore, in this paper, these effects are investigated. In addition, we also mention results of computer simulations for investigating effects of amplitude errors and optical-path difference errors on the nulling performance.

2. Optical configuration for shearing and nulling

A Fresnel rhomb provides phase retardation between two orthogonal linear polarizations achromatically by utilizing total internal reflections (TIRs). For realizing a half-wave plate (phase retardation of π), two rhombs are generally used for causing four TIRs. In this case, the phase retardation can be written as

$$\psi = 8 \arctan \left(\frac{\cos \theta \sqrt{\sin^2 \theta - (n_0/n)^2}}{\sin^2 \theta} \right), \quad (1)$$

where θ is an acute angle of the rhombs, n and n_0 are refractive indices of the material and environment [17]. The acute angle θ is set to adjust the retardation to $\psi = \pi$ (i.e., a half-wave plate, HWP) for a specific wavelength. We note here that the retardation slightly depends on wavelength because of a wavelength-dependent refractive index n . When the Fresnel-rhomb HWPs are placed into each arm of an interferometer and their axes are set to be orthogonal each other, achromatic nulling interference can be acquired for unpolarized incoming light as suggested by Mawet et al. [15].

Figure 1 shows a scheme based on the Mach-Zehnder interferometer. A pair of Fresnel rhombs with different sizes is inserted into each arm of the Mach-Zehnder interferometer. One set of the double Fresnel rhombs is oriented perpendicular to the other one. In a polarization interferometric method a polarizer and an analyzer are needed to achieve nulling interference, and use of these optical elements leads to waste of light.

A lateral shearing is realized by using different-size Fresnel rhombs in contact as shown in Fig. 1. The path length ℓ inside one Fresnel rhomb is given through geometrical calculation by

$$\ell = L \sin \theta + W \sin 2\theta, \quad (2)$$

where L , W , and θ are the length, the width, and the acute angle of the rhomb, respectively (see an inset of Fig. 1). Therefore, the path length is independent of the beam incident position. When the widths of two Fresnel rhombs are W and $W + \delta$, the lateral shear is given by $s = 2\delta \sin^2 \theta$ (see also Fig. 1). The amount of the lateral shear is governed by the size difference δ and the acute angle θ . The precise adjustment of the two Fresnel rhombs is not needed, because the relative

Figure 1. Shearing null interferometer by use of a pair of double Fresnel rhombs. Details of the double Fresnel rhomb and a Lyot stop are also shown.

lateral displacement between two Fresnel rhombs does not affect the position of the emerging beam. In this way we can perform both achromatic π phase shift and lateral shearing.

Planetary light can survive, because light from a planet impinges inclined to the interferometer and does not interfere destructively at all the observational wavelength. An intensity of a planetary light is modulated due to the interferometric effect as a function of an angular separation from its parent star. The interferometric signal will be maximum when the star-planet angular separation is $\alpha = \lambda/(2s)$, where λ is a wavelength and s is an amount of the lateral shear. The interferometric signal will decrease when the angular separation is smaller than α . It will be important to set the amount of the lateral shear s appropriately for each target. It should be noted that spatial resolution to image exoplanets also depend on the diameter of the Lyot stop that should be less than the diameter of the reimaged pupil minus the shear amount (see Fig. 1).

Exoplanets will be observed with segmented large telescopes mainly in near infrared. We here investigate the performance of the above mentioned interferometer in K-band ranging from $\lambda = 2.0 \mu\text{m}$ to $2.4 \mu\text{m}$. The key component in the interferometer is a Fresnel rhomb as shown in Fig. 1. The crucial aspect is the characteristic of achromatic phase shift in the specified wavelength region. We found ZnSe is an appropriate material among NaCl, KBr, CsI, LiF, CaF₂, BaF₂, KRS-5, KRS-6, and CdTe. In the following, our computer simulations are conducted by using refractive index of ZnSe as a material of Fresnel rhombs. With regard to achromatic property a King's type is better than an ordinary Fresnel rhomb [18–21]. However, deviation from the optimized incident angle is very critical in the case of the King's type. Here, we adopt ordinary Fresnel rhombs for numerical evaluations.

3. Simulation results

We conduct computer simulations to evaluate the performance of the nulling interferometer presented in the preceding section. The TMT is supposed for a model of a segmented-mirror telescope (an effective diameter of the telescope is $D = 30 \text{ m}$). The TMT will consist of 492

Figure 2. Pupil of segmented mirror telescope and a direction of lateral shear.

Figure 3. Extinction characteristics of the interferometer for a point-like star.

hexagonal elemental mirrors, of which the diagonal length is $d = 1.44$ m. The pupil function of a telescope with hexagonal segmented mirrors can be expressed analytically [22].

For spectral distribution of starlight we suppose a black body radiation at 5800 K, similar to the Sun, and our concerned wavelength region is K-band ($\lambda = 2.0 - 2.4 \mu\text{m}$ with a central wavelength of $\lambda_0 = 2.2 \mu\text{m}$). The direction of lateral shear is shown on a model of TMT as in Fig. 2. The amount of the shear is based on the width of one hexagonal element, and is set to $s = d + d/2 = 2.16$ m in the computer simulations. Figure 3 shows the extinction characteristics for a point-like star [16]. The broken line shows the intensity distribution of the star image without nulling, and the solid line with nulling. A 10^{-7} extinction is attained on the optical axis, and a 10^{-10} extinction is possible at $5 \lambda_0/D$ (corresponding to 76 mil arc seconds in case of TMT). The incident angle of the total reflection in the Fresnel rhomb is 65.10° , which corresponds to θ in Eq.(1). Therefore, the acute angle of the Fresnel rhomb should be this value.

Figure 4. One example of light leakage from random misplacement of segmental mirrors.

Figure 5. Extinction profiles for a point-like star with an x-shearing when there are random misplacements between segmental mirrors up to ± 0.1 mm, ± 0.5 mm, and ± 1.0 mm.

It seems difficult to locate the elementary hexagonal mirrors at their strict regular positions. Light leakage caused by irregular gaps between the segmental hexagonal mirrors deteriorates the nulling performance. To simulate misplacement of segmental mirrors we generate random errors for their central positions over 492 segments up to ± 0.1 mm, ± 0.5 mm, and ± 1.0 mm. One example of light leakage with lateral shearing is shown in Fig. 4 (largely exaggerated for display purpose). We can observe the light leakage along the boundaries of segmental mirrors. Figure 5 shows the intensity distributions of the nulling performance. The extinction profile without misplacement (Fig. 3) is also drawn in Fig. 5 for reference. The deterioration by misplacement of the segmental mirrors is quite severe. For example, the extinction becomes worse than 10^{-6} at $5 \lambda_0/D$ in case of the ± 1.0 mm misplacements.

To solve this problem, light leakage must be blocked with a mask placed on the Lyot stop

Figure 6. Transmission map and directions of tip-tilt error.

Figure 7. Extinction deterioration with tip-tilt errors.

plane (see Fig. 1). The mask should have thin opaque lines along the boundaries of the segmental mirrors. Assuming that the telescope pupil is reimaged on the Lyot stop plane with a diameter of 50 mm, and that the mask is designed to block the boundaries with a width corresponding to 5% of the segmental mirror size, the opaque lines would be very thin with a width of approximately 120 μm . Such a mask would be difficult, but seems to be feasible to be manufactured.

Next, we investigate effects of tip-tilt errors. The extinction for a point-like star on the optical axis is quite deep. On the other hand the extinction becomes deteriorated for off-axis light. Therefore, the tip-tilt error analysis is important to characterize the performance of the interferometer. Tip-tilt errors cause deviation from the optimal total reflection angle in Fresnel rhombs. The phase errors caused by the deviation of the reflection angle can be derived from Eq.(1). Fig-

Figure 6 shows a transmission map for nulling interferometer with lateral (x-direction) shearing of $s = 2.16$ m. Here, the transmittance is defined as an interferometric intensity normalized by that under a constructive condition. Directions of tip-tilt errors assumed in the computer simulations are shown on this figure. In the calculated map, we can see that a transmittance of a planet located at $5 \lambda_0/D$ is very high (> 0.8). We here consider a very small tip-tilt error of 0.0001 arc seconds that corresponds to $0.007 \lambda_0/D$. The extinction deterioration depends on the direction of tip-tilt error. Figure 7 shows the extinctions under the tip-tilt errors in four directions. When the direction of tip-tilt error coincides with that of the lateral shearing, the extinction deterioration is worst. It can be seen that the stellar extinction becomes worse to around 10^{-8} at $5 \lambda_0/D$ in case of shear directions of 0° and 30° . The tip-tilt error in the direction perpendicular to the lateral shearing is insensitive to the extinction deterioration.

In order to mitigate the problem of the tip-tilt error, it would be useful to utilize multi-beam interference, that is, to make y-shear in tandem with x-shear [11, 16]. We carried out computer simulations of the four-beam interference with x- and y-shear taking the tip-tilt errors in the same four directions into account. As a result, for the all directions of the tip-tilt errors, $< 10^{-10}$ extinctions are attained on the optical axis, while $< 10^{-13}$ extinctions are possible at $5 \lambda_0/D$. Thus the nulling performance can be greatly improved by the four-beam interference with x- and y-shear. The transmittance of the four-beam nulling interferometer will be very low (about 0.3) at $5 \lambda_0/D$, and is improved to 0.8 at $7.5 \lambda_0/D$. These numerical estimations suggest that an angular resolution of the four-beam nulling interferometer will be degraded as compared to that of the two-beam interferometer. In these numerical estimations, we used amounts of shears in x- and y-directions $s_x = 2.16$ m and $s_y = \sqrt{3}d = 2.49$ m (based on the width of two hexagonal elements). It should be noted that an exoplanet transmission depends on the angular distance and direction from the host star, and the amount of shears.

We also conducted computer simulations taking into account amplitude errors and optical-path difference (OPD) errors. For realizing deep nulling performance, the amplitudes of two beams must be identical, and the OPD between two arms of the interferometer must be zero. Our computer simulations show that the nulling performance would be degraded severely by these errors. For example, the relative intensity would become about 10^{-8} and 4×10^{-9} at $5 \lambda_0/D$, assuming the 0.5% amplitude error and a 1 nm OPD error, respectively. Our simulations suggest that extremely small amplitude and OPD errors would be required for achieving excellent nulling performance such as 10^{-10} at $5 \lambda_0/D$. We note that these requirements would be relaxed by introducing additional high-contrast imaging techniques, such as a pupil apodization, behind the nulling interferometer [23].

4. Conclusion

Achromatic nulling for lateral shearing interferometer is achieved by use of a pair of double Fresnel rhombs. Our computer simulations, assuming the ideal condition, showed that Earth-like planets can be detectable by the TMT equipped with the above mentioned interferometer. It becomes crucial to characterize exoplanets through spectroscopy. Low dispersion spectroscopy would then be incorporated to the interferometer [24–26].

We also carried out the computer simulations taking into account several instrumental errors such as the tip-tilt errors, the misplacements of segmented mirrors, the amplitude and the OPD errors. These results would be useful for constructing the nulling interferometer and evaluating its performance. The computer simulations suggested that extremely small instrumental errors would be required for achieving excellent nulling performance.

The double lateral shearings, namely x- and y-shears, would be an attractive solution for reducing the effect of tip-tilt errors. This can be realized by adding another Mach-Zehnder interferometer for y-shearing to the optical setup shown in Fig. 1. The light leakage due to

misplacements of segmented mirrors can be blocked by an opaque mask along the boundaries of the segmented mirrors. Finally we note that the amplitude and OPD errors must be reduced as small as possible. We expect that their effects on nulling performance would be relaxed, for example, by introducing additional pupil apodization on the Lyot stop plane, which will be interesting future works.

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References

- [1] Kalas, P.; Graham, J.; Chiang, E.; Fitzgerald, M.; Clampin, M.; Kite, E.; Stapelfeldt, K.; Marois, C.; Krist, J. *Science* **2008**, *322* (Nov 28), 1345–1348.
- [2] Marois, C.; Macintosh, B.; Barman, T.; Zuckerman, B.; Song, I.; Patience, J.; Lafreniere, D.; Doyon, R. *Science* **2008**, *322* (Nov 28), 1348–1352.
- [3] Serabyn, E.; Mawet, D.; Burruss, R. *Nature* **2010**, *464* (April 15), 1018–1020.
- [4] Lagrange, A.-M.; Bonnefoy, M.; Chauvin, G.; Apai, D.; Ehrenreich, D.; Boccaletti, A.; Gratadour, D.; Rouan, D.; Mouillet, D.; Lacour, S.; Kasper, M. *Science*, **2010**, *329* (July 2), 57–59.
- [5] <http://www.tmt.org/>
- [6] <http://www.eso.org/public/teles-instr/e-elt.html>
- [7] Roddier, F.; Roddier, C. *Publ. Astron. Soc. Pacific* **1997**, *109* (July), 815–820.
- [8] Rouan, D.; Riaud, P.; Boccaletti, A.; Clenet, Y.; Labeyrie, A. *Publ. Astron. Soc. Pacific* **2000**, *112* (Nov), 1479–1486.
- [9] Baba, N.; Murakami, N.; Ishigaki, T.; Hashimoto, N. *Opt. Lett.* **2002**, *27* (Aug 15), 1373–1375.
- [10] Foo, G.; Palacios, D.; Swartzlander, Jr., G. *Opt. Lett.* **2005**, *30* (Dec 15), 3308–3310.
- [11] Shao, M.; Wallace, J.; Levine, B.; Liu, D. *Proc. SPIE* **2004**, *5487* (Oct), 1296–1303.
- [12] Murakami, N.; Baba, N. *Opt. Lett.* **2010**, *35* (Sept 15), 3003–3005.
- [13] Baba, N.; Murakami, N.; Ishigaki, T. *Opt. Lett.* **2001**, *26* (Aug 1), 1167–1169.
- [14] Baba, N.; Murakami, N.; Ishigaki, T. *Proc. SPIE* **2003**, *4838* (Feb), 1451–1458.
- [15] Mawet, D.; Hanot, C.; Lenaerts, C.; Riaud, P.; Defrere, D.; Vandormael, D.; Loieq, J.; Fleury, K.; Plesseria, J.-Y.; Surdej, J.; Habraken, S. *Opt. Express* **2007**, *15* (Oct 1), 12850–12865.
- [16] Baba, N.; Kobayashi, K.; Kogoma, Y.; Murakami, N. *Opt. Lett.* **2011**, *36* (Sept 15), 3611–3613.
- [17] Born, M.; Wolf, E. Reflection and refraction of a plane wave In: *Principles of Optics*, 7th ed.; Cambridge University press: 1997, 49–53.
- [18] King, R. J. *Sci. Instrum.* **1966**, *43* (Sept), 617–622.
- [19] Bennet, J. *Appl. Opt.* **1970**, *9* (Sept 1), 2123–2129.
- [20] Clapham, P.; Downs, M.; King, R. *Appl. Opt.* **1969**, *8* (Oct 1), 1965–1974.
- [21] Baba, N.; Furuta, H.; Murakami, N., "Geometric phase modulator for IR nulling stellar interferometer," in *Advanced Infrared Technology and Applications 2007*, Strojnik, M., Ed., 147–152, Leon, Mexico (2008).
- [22] Chanan, G.; Troy, M. *Appl. Opt.* **1999**, *38* (Nov 1), 6642–6647.
- [23] Nishikawa, J.; Kotani, T.; Murakami, N.; Baba, N.; Itoh, Y.; Tamura, M.; *Astron. Astrophys.* **2005**, *435* (May), 379–384.
- [24] Zubko, N.; Baba, N.; Morisaki, S.; Murakami, N. *Opt. Express* **2007**, *15* (Sept 17), 12189–12196.
- [25] Zubko, N.; Baba, N.; Shibuya, H.; Murakami, N. *Opt. Lett.* **2009**, *34* (Aug 15), 2432–2434.
- [26] Baba, N.; Zubko, N.; Shibuya, H.; Murakami, N. *J. Mod. Opt.* **2010**, *57* (Oct), 1803–1807.