学　位　論　文　内　容　の　要　旨
Abstract of Doctoral Dissertation

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CR submanifolds in holomorphic statistical manifolds
（正則統計多様体のＣＲ部分多様体論）

CR submanifolds in Kaehler geometry are introduced by A. Bejancu in 1978 as a generalization of both complex submanifolds and totally real submanifolds. Works of K. Yano and M. Kon (1983, 1984) and A. Bejancu (1986) gather the significant results on this topic. M. Djoric and M. Okumura have been intensively investigated CR submanifolds in the complex projective space, with focus on the case when CR submanifolds are of maximal CR dimension. They generalized many important results on real hypersufaces (Y. Tashiro and S. Tachibana in 1963, R. Takagi in 1975, among others), which are typical examples of CR submanifolds of maximal CR dimension.
Soon after CR submanifolds in Kaehler manifolds are introduced, their research is extended to other ambient spaces. There is a large amount of literature when the ambient manifold is a nearly Kaehler or locally conformal Kaehler manifold. We initiate investigation of CR submanifolds in holomorphic statistical manifolds, which are new objects originating from information geometry. Statistical manifolds may be considered as manifolds consisting of certain probability density functions. They are geometrically formulated as Riemannian manifolds with a certain affine connection. Their complex version, i.e. holomorphic statistical manifolds are defined by T. Kurose in 2004.

After obtaining the fundamental equations for CR submanifolds in holomorphic statistical manifolds, we investigate CR submanifolds with umbilical shape operators, semi-parallel totally real submanifolds and real hypersurfaces. We show that the results obtained by O. Kassabov (1986), M. Djoric and M. Okumura (2009) and Y. Tashiro and S. Tachibana (1963) hold in the theory of CR submanifolds in holomorphic statistical manifolds, by finding the corresponding conditions to the ones in the original theory. Our main results are the following.

When M is a CR submanifold of maximal CR dimension in a holomorphic statistical manifold, we naturally get a special normal vector field, which is called the distinguished normal vector field of M. Accordingly, in this setting we have two shape operators A, A*, which are the shape operator in the distinguished normal vector field direction with respect to the affine connection of the ambient space, and the one with respect to the dual connection. Let I denote the identity operator of the tangent space of M. We then obtain

Theorem. *Let M be an n-dimensional CR submanifold of maximal CR dimension in an (n + p)-dimensional holomorphic statistical manifold of constant holomorphic sectional curvature c, and let p < n. If the shape operators of the distinguished normal vector field of M are given as* $A = \alpha I$, $A^* = \beta I$ *for*

functions α and β, then $c = 0$.

Real hypersurfaces are typical examples of CR submanifolds of maximal CR dimension. When M is a real hypersurface, the distinguished normal vector field is a unit vector field to a hypersurface.

On the other hand, we obtain the following for a semi-parallel totally real submanifold of constant curvature. Let (∇, g) be the statistical structure on such a submanifold induced from the ambient space, and f the f-structure of the normal bundle induced from the complex structure J of the ambient space. Let D and D* be the normal connections with respect to the affine connection of the ambient space and the dual connection, respectively.

Theorem. *Let (M', ∇', g', J) be a holomorphic statistical manifold and M a totally real submanifold of M'. Suppose:*

(1) $D_X (fV) = fD^*_X V$ for any tangent vector field X and normal vector field V of M.

(2) (∇, g) is of constant curvature $c \neq 0$.

If M is semi-parallel for ∇', then M is totally geodesic for ∇'.

Finally, we construct holomorphic statistical structures on a domain of R^4, dependent on eight functions, using a g-natural metric. Moreover, holomorphic statistical manifolds of constant holomorphic sectional curvature are also constructed.