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The Viscous Behavior of Tarumai dome lava and Oshima lava

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Abstract

Viscous behavior as a function of temperature was determined near the softening range by an ordinary static method for Tarumai (or Tarumae) dome lava and Oshima lava. In these experiments viscosity exhibited a slow, approximately linear decrease with increasing temperature until a certain temperature range characteristic of each rock was reached; from then viscosity decreased very rapidly, and the lavas showed the same behavior as Showa-Shinzan new lava, that is, the more rapidly the specimens were brought to high temperature, the lower their apparent viscosities became. Also the writer compared the results obtained by the present reheating experiment with those concerning remelting experiment. The comparison showed the former viscosity to be higher than the latter. In explanation of these results, it should be noticed that the mechanical effect of the crystal in rocks may influence the viscous behavior of a crystal-liquid mixture, besides the influence of the change of chemical composition of rock melts.

The writer discusses whether there is any significance to be attached to the present experiments in order to gain a better understanding of some matters related to the restoration of rocks to the viscosity possessed at or previous to the time of extrusion.

1. Introduction

A cursory comparison of viscosity obtained by reheating experiment¹⁾ and that by remelting experiment²⁾ for Oshima lava seems to establish a link between them within the narrow limits of 50 degrees above 1100°C. The possibility of such a link, however, is not supported by data obtained in the field;³⁾ the experimental data are higher than the field data, ca. 10⁷ poises at about 1000°C. What does the difference signify? Kani²⁾ ascribed the sudden increase of viscosity below 1250°C to crystallization of plagioclase and magnetite. Volarovic et al.⁴⁾ mentioned that rocks having a low viscosity begin to crystallize during measurement, then their viscosity gradually increases at a stable temperature. Also Preston⁵⁾ studied a similar effect of crystallization on the viscosities of two glasses below the liquidus temperature.

In the present paper the writer reports the results of measurement of

viscosity of Tarumai (or Tarumae) dome lava of which silica content is found between that of Showa-Shinzan new lava and Oshima (or Oo-sima) lava. In addition the writer measured the viscosity at higher temperature than that of Sakuma's measurement¹⁾. Remelting experiment for Tarumai dome lava has been carried out already by Kani & Hosokawa⁶⁾, hence some considerations are presented on the above-stated difference and on some matters touching upon the viscosity of lava at or previous to the time of extrusion by the comparison of the values obtained at the same temperature.

In the 1900's one saw the formation of two volcanic domes in Japan under one's own eyes, viz., Tarumai dome⁷⁾ and Showa-Shinzan dome⁸⁾. Tanakadate⁹⁾ distinguished volcanic domes recently formed in Japan by the modes of their formation, one of them may be called the "Tarumai type" and the other the "Ousu type" to which latter Showa-Shinzan dome may belong. The difference of modes of their formation suggests that the viscosity of respective lavas plays an important role. In this sense it is very interesting to compare the results for Tarumai dome lava with the writer's previously reported results for Showa-Shinzan new lava¹⁰⁾.

2. Method and Specimens

The method of viscosity measurement applicable to the present experiment is the same as that described in the previous paper. Temperature is raised at a constant and definite rate.

Thus viscosity is derived from the following equation:

Specimen	Tarumai dome lava	Oshima 1950 lava	
SiO	57.40	52.02	
$\mathrm{Al_2O_3}$	16.84	15.83	
Fe_2O_3 .	3.68	2.28	
${ m FeO}$	5.96	10.80	
. MgO	3.28	4.47	
CaO	6.60	9.48	
Na_2O	2.88	1.58	
$\mathrm{K}_2\mathrm{\ddot{O}}$	1.21	0.29	
H ₂ O+	0.00	0.99	
$H_2^{\bullet}O^{-}$	0.36	0.24	
TiO_2	0.65	1.52	
P_2O_5	tr.	n.d.	
MnŎ	1.08	0.09	
S	0.02		
Total	99.96	99.59	
Analyst	K. KANI	J. OSSAKA	

Table I. Chemical Composition of Specimens (wt. %).

$$\eta = (1/3) \left\{ (wl^3g/4 \ bd^3) + (5 \ wl^4 \ g/32 \ bd^3) \right\} (dt/dX)$$

where all notations are the same as listed in the previous paper.

In the previous paper the present writer who assented to Sakuma's discussion described that the lag of deformation due to elastic after-effect is negligible if the temperature are higher than 800 °C. But the approximate value of the retardation time (τ') used in Sakuma's discussion is based on his previous ex-

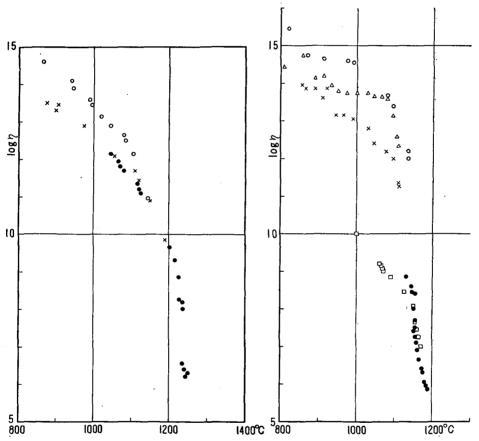


Fig. 1. Viscosity-temperature curve at various heating rates for Tarumai dome lava.

Open circles: Asymptotic value at the given temperature.

Crosses: 600°C/h. Solid circles: 1000°C/h.

Fig. 2. Viscosity-temperature curve for Oshima 1950 lava.

Open circles: asymptotic value

100°C/h. 600°C/h.}(after Sakuma) Triangles : Crosses:

Solid circles: 1000°C/h.

Squares: results on specimen having

40% porosity.

periment. Therefore it is inappropriate to use this retardation time contained both the lag of deformation due to elastic after-effect and the change of other physical property due to the thermal history of specimen. Considering these effects the writer uses in the present paper such notation as η^{600} which denotes the apparent viscosity obtained under the heating rate of 600 °C/hour.

Chemical and petrographical data were published by Ishikawa¹¹⁾ for Tarumai dome lava and by Tsuya and Morimoto¹²⁾ for Oshima lava. The chemical composition of the specimens is given in Table I.

The percentage of error in the results secured due to all factors contained in the above-stated equation was less than \pm 5% in poises below about 1150°C and \pm 10% above about 1150°C owing to the non-uniformity of thermal expansion of specimens. These errors admit of discussion which is presented below in connection with the logarithm scale in this paper. The obtained viscosity coefficient of commercial glass fiber compared with that obtained by an elongation method differed by less than \pm 10% in viscosity range 10^{13} -10^6 poises. This comparison perhaps gives a significant measure of the accuracy of the present method.

3. Results

Results obtained at the various heating rates in the temperature range of about $800\text{--}1400^{\circ}\text{C}$ are graphically represented in Figs. 1 and 2 where temperature (T) is plotted along the abscissa, while the logarithm of viscosity ($\log \eta$) along the ordinate. They were almost the same as those in the case of Showa-Shinzan new lava. Oshima lava below about 1100°C data are from Sakuma¹).

In each figure the viscosity of each lava exhibits a slow, approximately linear decrease in a low-temperature range followed by a more rapid non-linear decrease at high temperatures; then at still higher temperatures the rate of decrease is much greater, for example, for Oshima lava within the narrow range of 50°C at temperatures above 1100°C.

When the specimens below a certain temperature were dealt with under various rates of increase of temperature, viz., $1000^{\circ}\text{C}/hour$ and $600^{\circ}\text{C}/hour$, viscosity was observed to depend not only on temperature (T), but also on heating rate (dT/dt); the more rapidly the specimens were brought to high temperature, the more fluidal they became, viz., $\eta^{1000} < \eta^{600}$. For Tarumai dome lava, Fig. 3 shows the relation between viscosity and various heating rates. This figure shows that η^{600} is almost the lowest limit of viscosity of specimens, that is, $\eta^{600} \approx \eta^{\infty}$. The solid lines are the curves of viscosity obtained

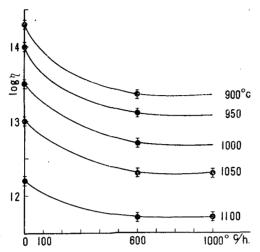


Fig. 3. Viscosity-heating rates curve for Tarumai dome lava.

by the empirical formula

$$\log \eta = A + B \exp(-C\dot{T})$$

where A, B and C are the factors determined by temperature. Now let it be taken in the above formula that η^0 is the viscosity at $\dot{T}{=}0$. η^0 above a certain temperature accords with η^{∞} . Up to the present stage of studies this formula is only a mathematical expression. It is desirable to make further investigations in order to learn whether there is any physical meaning to be attached to this formula.

The conclusions that arise necessarily from the above-described results are that experiments under constant temperature, after specimen had been rapidly heated to the desired temperature, showed that viscosity increases with time and gradually approaches an asymptotic value as shown in Fig. 4. The results were obtained at a slightly different temperature, but the value of Showa-Shinzan new lava seems to approach an asymptotic value faster than that of Oshima lava does whilist that of Tarumai dome lava is found between them; moreover the asymptotic value is in accord with η^0 .

Attention must be given to the fact that the viscosity at temperature higher than some certain value is not the viscosity of the substantial part of specimens because the volatile materials are vesicated. Then the specimens assume a foamy state containing many bubbles which considerably decrease the viscosity of the specimens. The density variation can be measured after

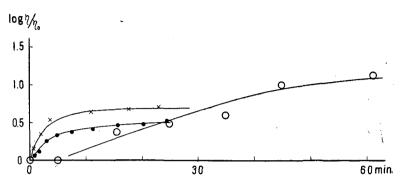


Fig. 4. Viscosity (log η (t=t) -log $\eta_0(t=0)$) versus time curves.

Open circles: Oshima lava at about 1000°C.
Solid circles: Tarumai dome lava at about 1000°C.
Crosses: Showa-Shinzan new lava at about 1010°C.

quenching other specimens which were heated under the same heating rate as the principal specimen used in measuring viscosity (Fig. 5). Recently the variation in viscous behavior as a function of density (or porosity) was carried out by the sonic method with respect to ice and snow in Greenland ice cap. It was found that $\log \eta$ decreases almost linearly with decrease in density (or with increase in porosity) until it reaches 0.55 (or about 40% in porosity) and that the rate increases for the lower densities 13).

In the case of volcanic rocks it was difficult to obtain specimens in which porosity is distributed uniformly. Fortunately for Oshima lava the writer could obtain foamy specimens which had vesicated volatile material at the time of extrusion until their porosity was about 40%. Results plotted in Fig. 2 show that 40% porosity lowers viscosity from 10¹³ to 10¹⁰ poises at 1000°C at which the specimens remain unchanged in their density as shown in Fig. 5. The relation between viscosity and density (or porosity) must be influenced by the mode of vacancy and possibly by size distribution of bubbles.

Knudsen¹⁴) developed an empirical equation form, approximating the apparent dependence of the strength of brittle polycrystalline specimens on the combined effect of porosity and grain size, from concepts and data in the earlier published literature. In his theoretical considerations of the relation of intergranular contact area and specimen porosity, he investigated (A) the relation between porosity and contact area between adjacent spheres within a cubic, orthorhombic, or rhombohedral packing arrangement of coalescing spheres (Fig. 6) and (B) the relation between relative strength and porosity of

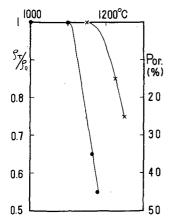


Fig. 5. Density variation by heat-treatment.

Crosses: Tarumai dome lava. Solid circles: Oshima lava.

coalescing spheres based on the idea that if the contact between grains is weaker than the grains, then the specimen strength would be dependent on the area of contact between grains for all ranges of

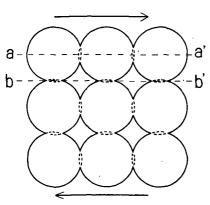


Fig. 7. Imaginary cubic packing arrangement of coalescing spheres.

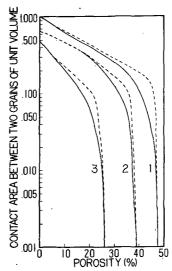


Fig. 6. Relation between porosity and contact area between two adjacent spheres within a cubic (1), orthorhombic (2), or rhombohedral (3) packing arrangement of coalescing spheres.

Solid lines: Calculated values.

Dotted lines: Hypothetical curves assum-

ing lens shaped bond between grains.

[cited from Knudsen]

porosity. Using similar conceptions the writer considers the relation between viscosity and porosity.

At first if the structure of actual rock specimens is assumed to be composed of an imaginary cubic packing arrangement of coalescing spheres (Fig. 7), then one can use Fig. 6, just as it is, to represent the relation between porosity and contact area. In Fig. 7 average binding force per unit area on b-b' plane would be weaker than that on a-a' plane, hence b-b'

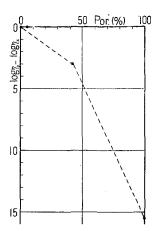


Fig. 8. Relation between porosity and viscosity for Oshima lava.

extent of the contact. The structure of actual rock specimens is much too disordered to permit an analysis of the relations between viscosity, porosity, and contact area. However, if one takes the viscosity of water vapor at 1000°C : ca. $10^{-3}-10^{-4}$ poises, as the viscosity of 100% porosity specimens, the variation in the viscosity of porous rock specimens with changing porosity as first approximation would behave in the manner as shown in Fig. 8.

If the relation for all present specimens which are less 40% in porosity value shows similar linear behavior, that is, $\Delta \log \eta = \log \eta_0(0\% Por.) - \log \eta_x (x\% Por.) = Ax$, where x is porosity (%) and A = 3/40, the present results as to viscosity evaluation for each lava may be cor-

plane is higher than the other in mobility under a given shearing stress. Thus one can conclude that the portion of the curves between maximum viscosity and 10% of maximum are nearly linear based on the assumption that viscosity is proportional to the

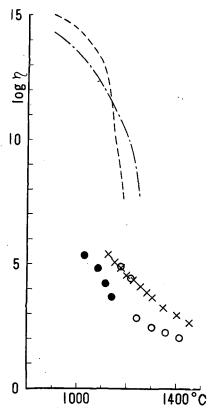


Fig. 9. Corrected data and remelting experiment data.

Dotted line: Oshima lava.
Chained line: Tarumai dome lava.
Open circles: Oshima lava (Kani).
Crosses: Tarumai dome lava
(Kani and Hosokawa).
Solid circles: field data for Oshima
lava (Minakami).

rected using the viscosity-porosity relation in order to investigate the viscosity of crystal-liquid mixture exclusive of gas phase. Fig. 9 shows the corrected results of two specimens for η^0 and also the results of remelting experiment by Kani and Hosokawa⁶. They used Tarumai dome lava and Oshima lava which is the lava of the outer somma of Oshima, but the chemical composition of the latter is almost the same as that of the specimens used in the present experiment.

4. Discussion

In Fig. 9 it is very interesting that viscosity obtained at comparatively higher temperatures is much higher for silica-rich specimens than for silica-poor ones, but the results obtained at temperatures below about 1100°C under the same rates of increase of temperature are of almost the same value though the silica content of each rock is different. Accordingly results obtained by present experiment below about 1100°C do not depend on the chemical composition, but they may suggest that the mechanical interaction between the crystalline parts and glassy parts in the rocks plays an important role as the writer stated regarding the viscosity of Showa-Shinzan new lava.

Comparison of the corrected data of reheating experiment with those of remelting experiment for Tarumai dome lava in Fig. 9, shows evidently that the former shows much higher values than the latter in the range 1100–1200°C. Needless to say, special regard should be paid to the effect of composition on viscosity of silicate melts of which the composition is changed by crystallization during cooling. In addition to this, it should also be noticed that the viscosity of a suspension is represented as a function of the suspended amount by a formula which indicates that the suspended material increases the viscosity of liquid¹⁵).

Using the same specimens as in measurement of the viscosity of the melts of Tarumai dome lava, Kani and Hosokawa investigated the order of crystallization of minerals in the melts; They exposed the specimen for about 1 or 2.5 hours to a constant temperature and decided the sort of minerals by examining the optical properties of those products under the polarization microscope after quenching. Results obtained are given in Table II; they show that the point of discontinuity in the viscosity-temperature curve accords with the crystallization temperature of magnetite and plagioclase, and suggest that the crystallization of crystal in melts increases the viscosity thereof. In view of the occurrence of such effects the viscosity below the liquidus temperature may be subject to the cooling rate of rocks, that is to say, the faster the cooling

Table II. Order of crystallization of minerals in melts for Tarumai dome lava. (After Kani and Hosokawa)

Temp. in °C	Results	
1300	Glass	
1290	,,	
1280	"	
1270	,,	
1260	Magnetite + Glass	
1250	Magnetite + Plagioclase + Glass	
1240	, ,	
1230	"	
1220	"	
1210	"	
1200	· <i>"</i>	
1187	"	
1175	<i>II</i>	
1163	"	
1150	"	

speed, the less the volume of crystals; then the viscosity becomes lower and may show such viscosity as that of glass. Thus the remelting experiment shows that if the rock melts were held for a only comparatively short times at temperatures lower than the liquidus temperature, then the viscosity-temperature curves obtained are merely continuations. The reheating experiment shows that the viscosity of the resulting crystal-liquid mixture increases with the increase of the crystal contained in rock melts.

Consideration with respect to the change of chemical composition and the mechanical effect due to the crystallization of crystal in melts makes the meaning of the present reheating experiment clear. A schematical diagram of viscosity-temperature of time curve is shown in Fig. 10. When rock melt is cooled in glassy state, temperature change in viscosity of the melt may be characteristic of glassy materials as shown in Fig. 10, (A). The results obtained for obsidian are the same 16). In general, however, rock contains many minerals and glassy part, hence when rock melt is kept at a constant temperature (T_1) below liquidus temperature, the melt may crystallize to reach perfect thermodynamical equilibrium and then the produced crystal mechanically increases the viscosity which becomes constant when the equilibrium is reached. Accordingly viscosity-time curve is indicated by the arrow (a-a') as shown in Fig. 10, (B). The cases of (T_2) and (T_3) are in the same. In natural cooling the temperatures of rock melt goes down gradually, then the relation of viscosity-temperature or the time curve for rock melt may trace the path of (a-b-c) in this figure and it may arrive at a point (d). The specimens treated in present reheating experiment are in the state (d), and

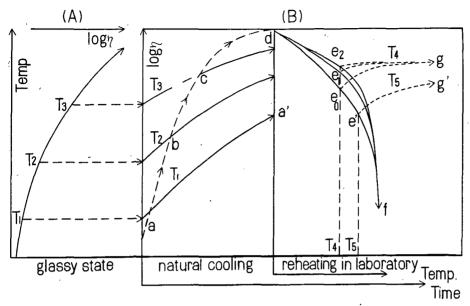


Fig. 10. Schematical viscosity-temperature or time diagram.

the reheating experiment begins at this point (d). Various heating rates show different data; $(d-e_0-f)$, $(d-e_1-f)$ or $(d-e_2-f)$. If temperature of specimens is kept constant at (e_0) , the viscosity of the specimens increases with the passage of time; viscosity-time curve is indicated by the arrow (e_0-g) . The case of (e_1) , (e_2) and (e') are the same.

The physical state of the specimen which shows τ^0 , therefore, approximately corresponds to the thermodynamical equiliblium at the given temperature; then one can conclude that τ^0 is the upper limit of viscosity which the specimens show at the given laboratory temperature, and the lag of deformation does not exist. Of course, because it is unnecessary in the cooling process under natural conditions to consider the crystallization by reheating in laboratory, τ^0 is the upper limit of the value which showed in nature at the time of extrusion.

If the present considerations are confined to the matter of the viscosity of the specimens which are almost split into the phase of crystalline rock + volcanic gasses¹⁷⁾, it is possible to conclude that the results obtained η^0 , by reheating experiment attain at least approximately the magnitude of the viscosity of the specimens at the time of extrusion. This is so in the case of Showa-Shinzan dome lava which is of a high crystalinity. In the case of Tarumai dome lava, however, it is not easy to make these calculation, because

one does not know the cooling rate of the rock and the crystallization rate of rock minerals at the cooling stage. Accordingly one only can conclude that the viscosity of Tarumai dome lava at the time of extrusion was somewhere between τ^0 and the data obtained by remelting experiment. The following rough estimation based on field observations at the time of extrusion, supports belief that the value may be confined within above-stated limits.

The relationship between the coefficient of viscosity and the pressure (ΔP) necessary for driving the lava through a tube of radius (r) and length (l) at a mean velocity (\bar{v}) may be written in the form

$$\Delta P = 8 \, \eta \, \overline{v} \, l/r^2 + m \, \rho \, \overline{v}^2 \,,$$

where $m\approx 1$. After Oinoue⁷⁾, Tarumai dome of which mass is about $2\times 10^7\,\mathrm{m}^3$ was built within 2 days and the radius of crater-bottom where dome lava flowed out was about 30 m previous to the time of extrusion. In above expression if one takes r=100–20 m (the mean radius of the dome is about 200 m), $l=5-50\,\mathrm{km}$, $dP=5\times 10^2-5\times 10^3$ atm. and $\rho=2$, $\eta=10^{12}-10^7$ poises. Though one does not know the temperature of the dome at the time of extrusion, if it was below 1200°C the limits of the viscosity calculated would be consistent with those obtained in the present experiment.

The case of Oshima lava makes the consideration still more difficult. The viscosity of Oshima lava during the cooling process under natural conditions is also illustrated in Fig. 9 for comparison³⁾. There is a considerable difference in viscosity, ca. 107 poises at about 1000°C., in other words difference in temperature of, ca. 150°C, at 10⁵ poises between the field data and the reheating experiment data. The activity of Oshima lava flow may belong to the phase of lava+floating phenocrysts+gasses, hence one must consider the following effects in order to explain the above-noted differences: i) Magma which was under sufficiently high pressures before the eruption undergoes degasation due to the release of the external pressure by the opening of an abyssal fissure; the magma takes a foamy form which alteration decreases mechanically the viscosity of the melts. For example, 40% porosity decreases the viscosity to 1/1000 as shown in Figs. 2 and 6. If one explains the abovementioned difference in viscosity, ca. 107 poises, with due regard to only the mechanical effect of porosity, the difference suggests that the flowing lava at the time of extrusion contained about 60% porosity. If this 60% porosity is composed of water vapor, it follows that the flowing lava at 1000°C contained about 0.6% (wt.) of water vapor at 50 bars, about 6% at 5000 bars and about 10% at 1000 bars at the time of extrusion. ii) The effective role of water is to lower cyrstallization temperatures. Goranson¹⁸⁾ determined phase equilibria

relations for the albite-water and orthoclase-water systems from 800 to 1200°C and pressures to 4000 bars. Using his data if the freezing-point curve of Oshima lava behaves similarly to that of albite, a difference in temperature, ca. 150°C at 10⁵ poises suggests that the flowing lava previous to the time of extrusion was under 500 bars and flowed out as the result of the opening of an abyssal fissure. Albite under 500 bars and 1200°C can contain about 3% (wt.) of water.

However, all liquids show a roughly exponential increase in viscosity with increasing pressure¹⁹⁾, and the viscosity of rock was found to decrease by the dissolution of water²⁰⁾, then all will not come right in the end unless the porosity and water content of lava under high pressures can be observed at or previous to the time of extrusion.

5. Conclusion

In this paper, the writer has suggested that the mechanical effect of crystal on viscosity below the liquidus temperature may play also an important role in order to explain the difference of visocity obtained by the present reheating experiment and remelting experiment. However, on the basis of only the experimental results as they are, one can not enter into further discussion required to make sure all matters pertaining to the restoration of lavas to the viscosity possessed at or previous to the time of extrusion. It is impossible because no experiments on the crystallization rate of minerals under water-vapor pressure have ever been performed.

The present writer believes that some difficult problems therewith connected will be settled somehow or other and will be brought a satisfactory solution, step by step, in future.

Acknowledgement. In concluding, the writer wishes to express his cordial thanks for the facilities and advices accorded to him by the late Prof. S. Sakuma and by Dr. I. Yokoyama.

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References

- SAKUMA, S.: Effect of Thermal History on Viscosity of Oosima-lavas. Bull. Earthq. Res. Inst., 32, (1954), 215-230.
- 2) KANI, K.: The measurement of the Viscosity of Basalt Glass at High Tempera-

- ture, II. Proc. Imp. Acad. Japan, 10 (1934), 79-82.
- MINAKAMI, T.: On the Temperature and Viscosity of the Fresh Lava Extruded in the 1951 Oo-sima Eruption. Bull. Earthq. Res. Inst., 29 (1951), 485-498.
- 4) Volarovic, M.P., D.M. Tolstoj and L.I. Korcemkin: A Study of the Viscosity of Molten Lavas from Mount Alaghez. Comptes Rendus (Doklady) Acad. Sci. URSS. Vol. I (X), No. 8 (85), (1936), 333-336.
- PRESTON, E.: The Viscosity of the Soda-Silica Glasses at High Temperatures and Its Bearing on their Constitution. Jour. Soc. Glass Tech., 22 (1938), 45-81.
- 6) Kani, K. and K. Hosokawa: On the Viscosities of Silicate Rock-Forming Minerals and Igneous Rocks. Res. Electrotech. Laboratory (in Japanese), No. 391, (1936). 1–105.
- OINOUE, Y.: Activity of Tarumai-dake in 1909. Rep. Imp. Earthq. Investig. Comm. (in Japanese), 64 (1909), 1~23.
- MINAKAMI, T., T. ISHIKAWA and K. YAGI.: The 1944 eruption of Volcano Usu in Hokkaido, Japan; history and mechanism of formation of the new dome "Syowa-Sinzan". Bull. Volcan., Ser. II, 11 (1951), 45-157.
- 9) TANAKADAT H.: Two Types of Volcanic Dome in Japan. Proc. Fourth Pac. Sci. Congr. II B. (1929), 1–10.
- Meurase, T.: Viscous Behavior of Showa-Shinzan New Lava. Jour. Fac. Sci. Hokkaido Univ., Ser. VII, 2 (1958), 93-101.
- ISHIKAWA, T.: The chemical characteristics of the lavas from Volcano Tarumai, Hokkaido, Japan, Jour. Fac. Sci. Hokkaido Univ., Ser. IV, 7 (1952), 107-135.
- 12) Тsuya, H. and R. Morimoto: Petrography of the 1950-lavas of Oshima volcano, Seven Izu Islands, Japan. Bull. Earthq. Res. Inst., 29 (1951), 563-570.
- 13) NAKAYA, U.: Visco-elastic Properties of Snow and Ice in Greenland Ice Cap. Jour. Fac. Sci. Hokkaido Univ., Ser., II, 5 (1958), 119–163.
- 14) Knudsen, F.P.: Dependence of Mechanical Strength of Brittle Polycrystalline Specimens on Porosity and Grain Size. Amer. Ceram. Soc., 42 (1959), 376–387.
- 15) HAPPEL, J.: Viscosity of Suspensions of Uniform Spheres. Jour. Appl. Phys., 28 (1957), 1288–1292.
- Murase, T.: A Note on Visco-elastic Behavior of Obsidian. Jour. Fac. Sci. Hokkaido Univ., Ser. VII, 2 (1958), 103-113.
- 17) RITTMANN, A.: Physico-chemical interpretation of the terms magma, migma, crust and substratum. Bull. Volcan., Ser. II, 19 (1958), 85–102.
- 18) GORANSON, R.W.: Silicate-water Systems: Phase Equilibria in the NaAlSi₃O₈-H₂O and KAlSi₃O₈-H₂O Systems at High Temperatures and Pressures. Amer. Jour. Sci., 5th. Ser., 35 (1938), 71-91.
- BRIDGMAN, P.W.: The Physics of High Pressure, Bell & Sons, London, 1949.
 330–356.
- 20) SABATIER, G.: Influence de la teneur en eau sur la viscosité d'une rétinite, verre ayant la composition chimique d'un granite. Comptes rendus, 242 (1956), 1340-1342.