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On the Possibility of Volcanic Hot Springs of Meteoric and Magmatic Origin and Their Probable Life Span*

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Abstract

Some theoretical discussions under rough assumptions were offered on the life span of the volcanic hot springs of magmatic water origin and of meteoric water origin, and also on the possible discharge rate of heat, volume out-put and temperature of these hot springs in comparison with the observed values.

In Chapter II, cooling of a sheet-like magma intruded horizontally into the shallow part of Earth's crust was theoretically discussed. Secular change of temperature of the magma and that of temperature gradient at the ground surface were calculated.

The conclusion was applied to the problem of the cooling of a magma of circular cylindrical form in Chapter III. The order of magnitude of probable life span for the above-postulated two kinds of volcanic hot springs was estimated in Chapter IV. In chapters V and VI, discharged heat, volume out-put and temperature of these hot springs were discussed. It is concluded that two kinds of volcanic hot springs are possible from the points of view of heat energy, volume out-put and temperature, and that the order of magnitude of life span of a volcanic hot spring group may be at most two million years.

§ I. Introduction.

It is generally acknowledged by geologists and geochemists as well as geophysicists that many hot springs might have an intimate relation with underground igneous rock masses in a molten state which were intruded in the Earth's crust in relatively recent geological epochs, though the age of the intrusions is not very clearly determined.

There are about one thousand hot spring localities with about ten thousand hot spring orifices in Japan. It could be said that almost all of them are located in the neighborhood of Quaternary volcanoes. To ascertain the facts more clearly, the writer examined the relation between the number of hot spring localities and the number of Quaternary volcanoes contained in respective grid areas which were formed by dividing the whole area of Japan

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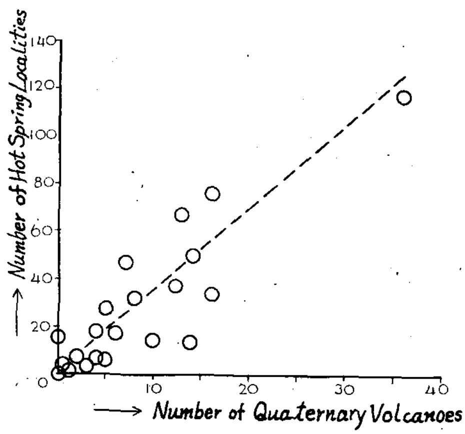

Fig. 1. Relation between number of hot spring localities and number of Quaternary volcanoes in Japan.

at every 2° of latitude and of longitude. Fig. 1 indicates this relation, taking the number of Quaternary volcanoes as abscissa and the corresponding number of hot spring localities as ordinate. From the figure, one can recognize the positive correlation, which indicates that many hot springs in Japan are "volcanic hot springs"; a part or all of the water and heat of these hot springs may originate from the magma chambers of the volcanoes.

In spite of this statistical fact, some parts of these springs in Japan have been supposed by several geologists to be related to the Lower and Middle Miocene volcanism of nearly 20 or 30 million years ago. There are respectively 32 and 50 hot springs in Manchuria and Korea. Many of these have also been supposed to be related to the intruded masses of Upper Jurassic granite porphyry of about 150 million years ago in Manchuria or those of Middle or Upper Cretaceous granite porphyry of about 100 million years ago in Korea. Are these igneous masses intruded as long ago as 20–150 million years a possible heat supply for present hot springs?

If it be assumed that hot springs originated from magma which is supposed to lie at several kilometers depth below a volcanic mass, two kinds of hot spring may be inferred to exist as noticed by D. E. White[1]: in one, all or at least a part of the water and heat is derived directly as ascending steam from the magma chamber, and in the other, all the water is meteoric in origin and all or almost all of the heat may be conducted from the magma chamber. The writer, in this paper, will distinguish the latter as "volcanic hot spring of meteoric (water) origin" from the former which may be called "volcanic hot spring of magmatic (water) origin". How long is it possible for these hot springs to exist and how much heat energy do they require?

The writer is not acquainted with any quantitative discussions of the above-mentioned questions. The problems may be too difficult to deal with exactly, but it may be possible to determine the rough magnitude of the values, if an appropriately simple model of a magma chamber and its mechanics are given.

The writer first discusses theoretically the cooling of sheet-like magma of a certain thickness which is intruded horizontally at shallow depth in the Earth's crust. Next, applying this results to the volcanic magma chamber, he estimates roughly, as the first approach for this problem, the order of magnitude of life spans for volcanic hot springs of magmatic origin and those of meteoric origin. And he also discusses the possibility of the existence of these hot springs, by comparing correspondingly the calculated values of heat energy transported from molten magma or solidified magma by an ascending steam or heat conduction, the calculated water discharge and the calculated water temperature with those of the observed heat energy, the actually observed water discharge and the observed water temperature of several hot spring localities.

§ II. Cooling of a sheet-like magma intruded horizontally into the shallow part of Earth's crust

1) Preliminary consideration on turbulence and convection in the magma.

J.C. Jagger[2] once discussed the cooling of a sheet-like magma of high temperature intruded into an infinite solid body of constant temperature, where no convection occurred in the magma. At first, considering the case in which the intruded semi-infinite mass of magma contacts by a plane boundary the semi-infinite rock mass of initially constant temperature, he got an exact mathematical solution. This solution was applied to the cooling of a sheet-like magma of a certain thickness, but in this case no exact solution could be obtained and some special cases were treated numerically. The cooling of a sheet on which a finite thickness of rock cover existed was discussed numerically in a special case where the cover had half the thickness of the sheet-like magma. He also discussed the cooling of a sheet-like magma of a certain thickenss which intrudes into an infinite rock mass of constant temperature, where perfect convection existed in the magma. Comparing this extreme case of perfect convection with the above-mentioned similar case of no convection, Jagger recognized that the calculated time interval of molten magma with perfect vertical convection was a half value of that with no vertical convection. He inclined to the view that the case of no convection may be probable in reality.

But, it is not at all clear at present whether or not vertical convection may exist in the magma chamber. The writer undertook an approximate approach to this problem as follows :-

When magma is intruded in the shallow part of the Earth's crust, the cooling rate of that magma from its upper surface should be larger than from its lower surface. There is, then, a possibility of turbulence in the upper and middle layers of magma in its molten stage of high temperature and low viscosity.

Now, imagine a small crystal formed in the neighborhood of the upper surface due to the cooling of magma, and assume the form of the crystal to be spherical with the radius r. The falling velocity of the crystal in the magma due to gravity v may be given by Stokes' law :

$$v=\frac{2r^2 g(\rho-\rho')}{9\mu} \tag{1}$$

where g is the gravity acceleration, ρ and ρ' the densities of the crystal and molten magma, and μ the absolute viscosity of magma.

On the other hand, if the thin layer of solidified magma dI in thickness is macroscopically formed at the upper surface in the infinitesimal time dt, the solidification velocity dI/dt is given by

$$\frac{\mathrm{d}I}{\mathrm{d}t}=\frac{\kappa}{L\rho}\left|\frac{\partial\theta}{\partial z}\right|_{z=I}-\frac{\nu}{L\rho}\left|\frac{\partial u}{\partial z}\right|_{z=I} \tag{2}$$

where ρ and L are respectively the density and the latent heat of solidified magma, κ the heat conductivity of the upper layer of Earth's curst, $|\partial\theta/\partial z|_{z=I}$ and $|\partial u/\partial z|_{z=I}$ are respectively the temperature gradients in the rock cover and in magma at the boundary between the magma and the rock cover, and ν the eddy diffusivity in magma. The values of $|\partial\theta/\partial z|_{z=I}$ and $|\partial u/\partial z|_{z=I}$ are always positive in this problem, then

$$\frac{\mathrm{d}I}{\mathrm{d}t}\leq\frac{\kappa}{L\rho}\left|\frac{\partial\theta}{\partial z}\right|_{z=I}. \tag{2$'$}$$

Now, assume $r=0.1$ cm, $g=980$ cm/sec^2, $\rho=2.7$, $\rho'=2.6$, $\kappa=5.4\times10^{-3}$, $L=90$ cal/gr, $\mu=10^3$ poise (this is the value of viscosity of the lava of Ohshima volcano, Japan, at temperature 1200°C measured by K.Kani[3]), and $|\partial\theta/\partial z|_{z=I}$ $\simeq u/I$ (u the temperature of magma at the upper boundary, I the thickness of the upper country rock, and this relation may be nearly valid in the stationary state), $u=1200$°C, $I=5\times10^5$ cm. Substitute these values into (1) and (2)$'$, then one gets

$$v\simeq2\times10^{-4}\ \text{cm/sec}\simeq70\ \text{cm/year},$$

and $\quad (\mathrm{d}I/\mathrm{d}t)\leq(\kappa/L\rho)|\partial\theta/\partial z|_{z=I}\simeq5.5\times10^{-8}$ cm/sec$\simeq0.02$ m/year.

As the value of v is very large in comparison with the value of dI/dt at about 1200°C, all the crystals formed in the neighborhood of the upper boundary move downward to the layer of higher temperature and are remelted. In other words, heat flows from the deeper layer toward the upper surface due to turbulence caused by the falling of crystals. If $v \leq (dI/dt)$, all the crystals formed may be caught up by the boundary layer, that is, this situation corresponds to the case of no turbulence.

Now, put $v = dI/dt$ and calculate approximately the value of the critical viscosity μ_c which gives the critical condition of no turbulence in magma from (1) and (2) neglecting the effect of the second term of right hand side of (2), and obtain

$$\mu_c \gtrsim 4\times 10^6 \text{ poise.}$$

If one adopts $r=1$ cm, one gets $\mu_c \gtrsim 4\times 10^8$ poise.

According to the result of an experiment by S. Sakuma[4], the viscosity of the fused basaltic lava of Ohshima volcano at 1 atomospheric pressure takes the value of 10^{15} poise at 900°C, 10^2 poise at temperature higher than 1200°C, and it changes abruptly at about 1100°C.

From this result one can estimate the critical temperature u_c which corresponds to $\mu_c \gtrsim 4\times 10^6 \sim 4\times 10^8$ poise as $u_c \lesssim 1100$°C.

But, if it is taken into consideration that the original magma which contains volatile substances may be less viscous than the solidified lava, the values of μ_c and u_c for the original magma may take smaller values than the above-mentioned ones.

Natural glasses in a volcanic region which are supposed to keep the volatile substances of the original magma may be the most suitable samples for this discussion. For this reason, adopting the value of viscosity of the natural glasses of the volcanic magma in Japan measured by S. Sakuma and T. Murase[5], the writer estimated u_c, in the same way, as 1100°$\sim$970°C for $\mu_c = 4\times 10^6 \sim 4\times 10^8$ poise.

According to the reliable results of observations on temperature of lava flows carried out just after eruptions of many volcanoes in the past, temperature lies in the range from about 1200°C for basaltic lavas of Hawaii, Parictin and Ohshima volcanoes to about 950°C for silica-rich lava of Showa-shinzan dome in Use volcano, Japan. If it be permitted to assume that the original basaltic magma in the magma chamber which has the initial temperature of 1200°C changes to the silica-rich magma of 900°C in the course of magmatic differentiation accompanied by the decrease of temperature, one may conclude that the state of the magma may be fairly turbulent in the earlier stage and less turbulent or non-turbulent in the later stage.

Then, the actual problem is no longer the extreme case of no turbulence and no convection nor that of perfect turbulence and perfect convection, but it is an intermediate case between these two extreme cases. But, theoretical calculation of such a complicated model must be more difficult to solve than that of the extreme case of no turbulence. Assuming the existence of a large effect of volatile substances on the viscosity of magma, that is, on the temperature range of turbulence, the writer here selects a simple model of perfectly turbulent magma. The approximate mathematical solution can be worked out for the cooling of sheet-like magma of this type which is intruded horizontally in the shallow part of the Earth's crust.

2) The assumed model and assumptions pertaining to it.

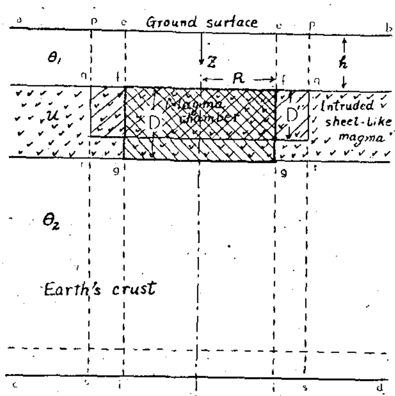

Fig. 2. A schematic map of the assumed model.

The assumed model is illustrated in Fig. 2 where ab and cd are respectively the ground surface and the lower surface of the Earth's crust, the initial thickness of which is l. Let z-axis be taken in the vertical direction downwards from the surface of the ground.

Assumptions pertaining to this model are as follows:

(i) The ground surface and the lower surface of the Earth's crust are respectively kept in constant temperatures T_1 and T_2, where $(T_2-T_1)/l=\alpha$, α being the normal temperature gradient of the Earth's crust at the ground surface. The temperature distribution in the Earth's crust is assumed as $\theta=f(z)$ at the moment just before the intrusion of magma.

(ii) Molten magma of temperature u_0 and thickness D is intruded at time origin $t=0$ in the shallow part of the Earth's crust extending infinitely in horizontal direction. The intrusion is assumed to take place instantaneously, that is, in a time which is very short compared to the time of solidification.

(iii) The intruded magma is assumed to be so well stirred by turbulence and convection that it is kept at constant temperature u at time t of the molten stage, but the temperature is gradually diminished from u_0 to u_1 due to heat dissipation from the upper and the lower boundaries to the Earth's crust.

The materials solidified from the magma due to decrease in the temperature of the magma are assumed to be uniformly suspended in the magma and the thickness of magma D is always kept constant through the molten stage. The solidification of magma is finished at $t=t_1$ and $u=u_1$.

(iv) The mass of the solidified materials γ per unit mass of the magma except volatile substances due to temperature decrease from u_0 to u, is assumed to be given by

$$\gamma=\frac{u_0-u}{u_0-u_1}. \tag{3}$$

This assumption means that the mass of the materials contained per unit mass of the magma $d\gamma$, which has its melting point in the temperature range from u to $u+du$, is always constant and independent of temperature through the molten stage of magma, that is,

$$\frac{d\gamma}{du}=-\frac{1}{u_0-u_1}. \tag{3$'$}$$

(v) Also the latent heat of melting of the magma ε is assumed to be a constant value independent of temperature. This value of ε is equal to the heat required to melt a unit mass of the solidified magma which contains many different minerals.

(vi) A unit mass of the molten part of magma is assumed to contain always a constant amount of water β_0 which corresponds to the saturated water solubility in the magma. Then, the mass of water vapor β which is contained in a unit mass of the magma at temperature u is given from equation (3) in the following.

$$\beta=\beta_0(1-\gamma)=\beta_0\frac{u-u_1}{u_0-u_1}. \tag{4}$$

This equation means that the mass of the free water vapor discharged from a unit mass of the magma due to the unit temperature decrease is always constant and independent of temperature. The effective water vapor contained in a magma vanishes at temperature u_1.

The free water vapor discharged from the magma is assumed to escape instantaneously through fissures in the Earth's crust to the atmosphere, giving no effective heat to the country rocks.

(vii) The thermal properties of the Earth's crust and the solidified magma are taken to be the same and to be independent of temperature.

viii) In some volcanic magma chamber, the thickness D may be decreased in the course of cooling due to several discharges of lava to the ground surface accompanying volcanic eruptions. The ratio of the total amount of discharge to the total mass of the magma might be roughly, at maximum, 20~10%, as estimated from the values of vertical depression of caldera and the thickness of the magma chamber. This effect is neglected in this paper.

Let the upper part of the Earth's crust bounded by the ground surface and the upper surface of the magma be called "Layer I", the layer of the molten magma "Layer II", and the lower part of the Earth's crust below the lower surface of the magma be denoted by "Layer III". The other variables and constants used in this calculation are according to the following notation:

θ_1 : the temperature of a point at depth z from the ground surface at a time t in Layer I,

θ_2 : the temperature of a point at depth z from the ground surface at a time t in Layer III,

z : the depth of a point measured from the ground surface, but z in Layer III is measured neglecting the thickness D of Layer II in the molten stage of magma,

h : the thickness of Layer I,

ρ, c, κ and k^2 : respectively the density, the specific heat, the coefficient of heat conductivity and the thermal diffusivity ($k^2=\kappa/\rho c$) of the Earth's crust and the solidified magma,

ρ', c', and ε : respectively the mean density, the effective specific heat and the mean latent heat of the molten magma,

c_1 and c_2 : respectively the mean specific heat of liquid part of the molten magma and that of water vapor in the magma.

3) Theoretical calculation in the molten stage of magma.

Taking a vertical column which has a unit cross section and a length D in the molten magma, the temperature decrease of the magma in this column due to heat dissipation through the upper and the lower surfaces to Layer I and Layer III per unit time is considered in the following equation.

$$-\{D\rho'(1-\beta_0)c\gamma+D\rho'(1-\beta_0)(1-\gamma)c_1+D\rho'\beta c_2\}\frac{\mathrm{d}u}{\mathrm{d}t}$$
$$+D\rho'(1-\beta_0)\varepsilon\frac{\mathrm{d}\gamma}{\mathrm{d}t}=-\kappa\left\{\left(\frac{\partial\theta_2}{\partial z}\right)_{z=h}-\left(\frac{\partial\theta_1}{\partial z}\right)_{z=h}\right\}. \quad (5)$$

The right hand side of the equation is the heat dissipated from the lower and the upper surface of the column by heat conduction. The terms in the bracket of the left hand side of the equation are respectively the heat loss of the solid particles suspended in the magma, that of the liquid part of the magma and that of the water vapor. The second term of the left hand side is the heat discharged by solidification of the mass $d\gamma/dt$ of the liquid magma which has the melting point u.

Substituting equations (3) and (4) into the equation (5), one gets

$$D\rho'(1-\beta_0)\left\{c_1+\frac{\varepsilon}{u_0-u_1}-(c_1-c)\frac{u_0-u}{u_0-u_1}\right.$$
$$\left.+\frac{c_2\beta_0}{1-\beta_0}\frac{u-u_1}{u_0-u_1}\right\}\frac{du}{dt}=\kappa\left\{\left(\frac{\partial\theta_2}{\partial z}\right)_{z=h}-\left(\frac{\partial\theta_1}{\partial z}\right)_{z=h}\right\}. \qquad (5)'$$

Of the four terms in the coefficient of du/dt, which will be denoted by $D\rho' c'$ in the following, the third term $(1-\beta_0)(c-{}_1c)(u_0-u)/(u_0-u_1)$ and the fourth term $c_2\beta_0(u-u_1)/(u_0-u_1)$ prove to be so small compared to the first or the second term that $(u_0-u)/(u_0-u_1)$ or $(u-u_1)/(u_0-u_1)$ may be replaced by its mean value 1/2. If one takes the values of the constants which will be discussed below in Section 5), and substitute the values

$$(1-\beta_0)c_1=(1-0.02)\times 0.25=0.245$$
$$\frac{(1-\beta_0)\varepsilon}{u_0-u_1}=\frac{(1-0.02)\times 90}{1200-900}=0.294$$
$$-(1-\beta_0)(c_1-c)\frac{u_0-u}{u_0-u_1}=-(1-0.02)(0.25-0.20)\times\frac{1}{2}=-0.024$$
$$c_2\beta_0\frac{u-u_1}{u_0-u_1}=0.30\times 0.02\times\frac{1}{2}=0.003\,,$$

it results that

$$c'\cong(1-\beta_0)\left\{c_1+\frac{\varepsilon}{u_0-u_1}-\frac{1}{2}(c_1-c)+\frac{1}{2}\frac{c_2\beta_0}{1-\beta_0}\right\}=0.518\,.$$

Since the temperature range that comes into question is from u_0 to u_1, the maximum percentage error of c' to be induced by the replacement of c' by the mean value of c', is

$$\frac{\Delta c'}{c'}=\frac{(1-\beta_0)\left(c_1-c+\dfrac{c_2\beta_0}{1-\beta_0}\right)}{2c'}\simeq 0.052 \text{ or } 5.2\%\,,$$

which justifies the above-mentioned approximation.

Then, the fundamental equation of the cooling of the magma in its molten stage is given by

$$\frac{du}{dt}=\frac{\kappa}{D\rho' c'}\left\{\left(\frac{\partial\theta_2}{\partial z}\right)_{z=h}-\left(\frac{\partial\theta_1}{\partial z}\right)_{z=h}\right\}. \qquad (6)$$

And, the fundamental equations of heat conduction in the Layer I and III are indicated by

$$\frac{\partial \theta_1}{\partial t} = k^2 \frac{\partial^2 \theta_1}{\partial z^2} \tag{7}$$

and

$$\frac{\partial \theta_2}{\partial t} = k^2 \frac{\partial^2 \theta_2}{\partial z^2} \tag{8}$$

where $k^2 = \kappa/\rho c$.

The boundary condition of this model is shown by

$$\left.\begin{array}{ll} z=0\,; & \theta_1 = T_1 \\ z=l\,; & \theta_2 = T_2 \\ z=h\,; & \theta_1=\theta_2=u\,. \end{array}\right\} \tag{9}$$

The initial condition of the problem is denoted by

$$t=0\,;\ \begin{cases} \theta_1 = \mathrm{f}_1(z) \\ \theta_2 = \mathrm{f}_2(z) \\ u = u_0\,. \end{cases} \tag{10}$$

The solutions[6] of equations (6), (7) and (8) which satisfy the conditions (9) and (10) are given by

$$u = T_3 + 2\sum_{i=1}^{\infty} \frac{\left\{\frac{\rho' c'}{\rho c} D(u_0 - T_3) + \frac{1}{\sin \mu_i h}\int_0^h \mathrm{F}_1(z)\sin\mu_i z\,\mathrm{d}z + \frac{1}{\sin\mu_i(l-h)}\int_0^{l-h}\mathrm{F}_2(l-z)\sin\mu_i z\,\mathrm{d}z\right\}\mathrm{e}^{-k^2\mu_i^2 t}}{\frac{\rho' c'}{\rho c}D + \frac{h}{\sin^2\mu_i h} + \frac{l-h}{\sin^2\mu_i(l-h)}}, \tag{11}$$

$$\theta_1 = T_1 + a z + 2\sum_{i=1}^{\infty} \frac{\left\{\frac{\rho' c'}{\rho c} D(u_0 - T_3) + \frac{1}{\sin \mu_i h}\int_0^h \mathrm{F}_1(z)\sin\mu_i z\,\mathrm{d}z + \frac{1}{\sin\mu_i(l-h)}\int_0^{l-h}\mathrm{F}_2(l-z)\sin\mu_i z\,\mathrm{d}z\right\}\sin\mu_i z\cdot\mathrm{e}^{-k^2\mu_i^2 t}}{\left\{\frac{\rho' c'}{\rho c}D + \frac{h}{\sin^2\mu_i h} + \frac{l-h}{\sin^2\mu_i(l-h)}\right\}\sin\mu_i h}, \tag{12}$$

$$\theta_2 = T_1 + a z + 2\sum_{i=1}^{\infty} \frac{\left\{\frac{\rho' c'}{\rho c} D(u_0 - T_3) + \frac{1}{\sin \mu_i h}\int_0^h \mathrm{F}_1(z)\sin\mu_i z\,\mathrm{d}z + \frac{1}{\sin\mu_i(l-h)}\int_0^{l-h}\mathrm{F}_2(l-z)\sin\mu_i z\,\mathrm{d}z\right\}\sin\mu_i(l-z)\cdot\mathrm{e}^{-k^2\mu_i^2 t}}{\left\{\frac{\rho' c'}{\rho c}D + \frac{h}{\sin^2\mu_i h} + \frac{l-h}{\sin^2\mu_i(l-h)}\right\}\sin\mu_i(l-h)}, \tag{13}$$

where μ_i are the infinite numbers of roots of the following trancendental equation

$$\cot \mu_i (l-h)+\cot \mu_i h = \frac{\rho' c'}{\rho c} D \mu_i , \tag{14}$$

and

$$\left.\begin{aligned} T_3 &= T_1+a h , \\ F_1 (z) &= f_1 (z)-(T_1+a z) \\ F_2 (z) &= f_2 (z)-(T_1+a z) . \end{aligned}\right\} \tag{15}$$

In the special case in which the temperature increased by the normal temperature gradient a toward the vertically downward direction in the Earth's crust before the time of the magma intrusion, that is f(z), the value is indicated by

$$f_1 (z) = f_2 (z) = T_1+a z . \tag{16}$$

The solutions (11), (12) and (13) are written as

$$u = T_3+2 \frac{\rho' c'}{\rho c} D (u_0-T_3) \sum_{i=1}^{\infty} \frac{e^{-k^2\mu_i^2 t}}{\frac{\rho' c'}{\rho c} D + \frac{h}{\sin^2 \mu_i h} + \frac{l-h}{\sin^2 \mu_i (l-h)}} , \tag{11$'$}$$

$$\theta_1 = T_1+a z+2 \frac{\rho' c'}{\rho c} D (u_0-T_3) \sum_{i=1}^{\infty} \frac{\sin \mu_i z \cdot e^{-k^2\mu_i^2 t}}{\left\{\frac{\rho' c'}{\rho c} D+ \frac{h}{\sin^2 \mu_i h} + \frac{l-h}{\sin^2 \mu_i (l-h)}\right\} \sin \mu_i h} , \tag{12$'$}$$

$$\theta_2 = T_1+a z+2 \frac{\rho' c'}{\rho c} D (u_0-T_3) \sum_{i=1}^{\infty} \frac{\sin \mu_i (l-z) \cdot e^{-k^2\mu_i^2 t}}{\left\{\frac{\rho' c'}{\rho c} D+ \frac{h}{\sin^2 \mu_i h} + \frac{l-h}{\sin^2 \mu_i (l-h)}\right\} \sin \mu_i (l-h)} . \tag{13$'$}$$

Then, the time interval of the molten stage of the magma t_1 can be required, substituting the value of u_1 into u of equation (11) or (11)$'$. Putting the required value of t_1 into equations (12) or (12)$'$ and (13) or 13)$'$, one obtains the vertical temperature distributions in Layers I and III.

The temperature gradient at the surface of the ground in this special case is given by

$$\left(\frac{\partial \theta_1}{\partial z}\right)_{z=0} = a+2 \frac{\rho' c'}{\rho c} D (u_0-T_3) \sum_{i=1}^{\infty} \frac{\mu_i e^{-k^2\mu_i^2 t_1}}{\left\{\frac{\rho' c'}{\rho c} D+ \frac{h}{\sin^2 \mu_i h} + \frac{l-h}{\sin^2 \mu_i (l-h)}\right\} \sin \mu_i h} . \tag{17}$$

4) Theoretical calculation on the solidified stage of magma.

After the magma is completely solidified, the thermal properties of Layers, I, II and III are all equal to each other under assumption vii) in Section 2) above. If z-axis is taken in vertically downward direction from the ground surface, if the time origin $t'=0$ is taken at the time when the magma is just solidified, and if the temperature at a depth of z from the ground surface is denoted by θ, the fundamental equation is given by

$$\frac{\partial \theta}{\partial t'} = k^2 \frac{\partial^2 \theta}{\partial z^2}, \tag{18}$$

where
$$k^2 = \frac{\kappa}{\rho c}.$$

The boundary conditions in this case are indicated by

$$\left.\begin{array}{ll} z=0; & \theta = T_1 \\ z = D+l = L; & \theta = T_2. \end{array}\right\} \tag{19}$$

Now, put the vertical temperature distributions in Layers I, II and III at the time of the complete solidification of magma $t=t_1$ which were obtained in Section 3) as $\varphi_1(z)$, u_1 and $\varphi_2(z)$ respectively. Then the initial conditions for the solidified stage of the magma are indicated by

$$t'=0; \begin{cases} z = 0 \sim h; & \theta = \varphi_1(z) \\ z = h \sim (h+D); & \theta = u_1 \\ z = (h+D) \sim L; & \theta = \varphi_2(z). \end{cases} \tag{20}$$

According to H.S. Carslaw[7)], the solution of equation (18) which satisfies the boundary conditions (19) and the initial conditions (20) is given by

$$\theta = T_1 + \frac{T_2 - T_1}{L} z + \frac{2}{L} \sum_{s=1}^{\infty} \sin\left(\frac{s\pi}{L} z\right) \cdot e^{-k^2(s\pi/L)^2 t'} \left\{ \frac{T_2 \cos s\pi - T_1}{\left(\frac{s\pi}{L}\right)} + \int_0^h \varphi_1(z) \sin\left(\frac{s\pi}{L} z\right) dz + u_1 \int_h^{h+D} \sin\left(\frac{s\pi}{L} z\right) dz + \int_{h+D}^{L} \varphi_2(z) \sin\left(\frac{s\pi}{L} z\right) dz \right\} \tag{21}$$

where s are the positive integers.

The temperature gradient at the surface of the ground is written by

$$\left(\frac{d\theta}{dz}\right)_{z=0} = \frac{T_2 - T_1}{L} + \frac{2}{L} \sum_{s=1}^{\infty} \left(\frac{s\pi}{L}\right) e^{-k^2(s\pi/L)^2 t'} \left\{ \frac{T_2 \cos s\pi - T_1}{\left(\frac{s\pi}{L}\right)} + \int_0^h \varphi_1(z) \sin\left(\frac{s\pi}{L} z\right) dz + u_1 \int_h^{h+D} \sin\left(\frac{s\pi}{L} z\right) dz + \int_{h+D}^{L} \varphi_2(z) \sin\left(\frac{s\pi}{L} z\right) dz \right\} \tag{22}$$

In a special case, if $\varphi_1(z)$ and $\varphi_2(z)$ are shown by the following equations that were substituted for $t=t_1$ in equations (12)′ and (13)′,

$$\left.\begin{aligned}
\varphi_1(z) &= \sum_{i=1}^{\infty} G_i \frac{\sin \mu_i z}{\sin \mu_i h} + T_1 + a z \\
\varphi_2(z) &= \sum_{i=1}^{\infty} G_i \frac{\sin \mu_i (L-z)}{\sin \mu_i (l-h)} + T_1 + a(z-D) \\
\text{where} \quad G_i &= \frac{2\frac{\rho' c'}{\rho c} D (u_0 - T_3)\, e^{-k^2 \mu_i^2 t_1}}{\frac{\rho' c'}{\rho c} D + \frac{h}{\sin^2 \mu_i h} + \frac{l-h}{\sin^2 \mu_i (l-h)}},
\end{aligned}\right\} \quad (23)$$

solutions (21) and (22) are written as follows:

$$\theta = T_1 + \frac{T_2 - T_1}{L} z + \frac{2}{L} \sum_{s=1}^{\infty} B_{si} \sin \lambda_s z \cdot e^{-k^2 \lambda_s^2 t'} \quad (21)$$

and

$$\left(\frac{\partial \theta}{\partial z}\right)_{z=0} = \frac{T_2 - T_1}{L} + \frac{2}{L} \sum_{s=1}^{\infty} B_{si} \lambda_s \cdot e^{-k^2 \lambda_s^2 t'} \quad (22)$$

where

$$\lambda_s \equiv \frac{s\pi}{L}$$

and

$$B_{si} = \frac{a}{\lambda_s^2} \{\sin \lambda_s h - \sin \lambda_s (h+D)\} + \{\cos \lambda_s h - \cos \lambda_s (h+D)\} \left(\frac{u_1 - T_3}{\lambda_s} + \sum_{i=1}^{\infty} \frac{\lambda_s G_i}{\mu_i^2 - \lambda_s^2}\right) - \sum_{i=1}^{\infty} \frac{\mu_i G_i}{\mu_i^2 - \lambda_s^2} \{\cot \mu_i h \sin \lambda_s h + \cot \mu_i (l-h) \cdot \sin \lambda_s (h+D)\} \quad (24)$$

5) Evaluation of the theory.

The adopted numerical values of the constants are indicated in Table I.

Table I.

Constant	Unit	Numerical value	Constant	Unit	Numerical value
T_1	°C	0	ρ'	gr/cm	2.61
T_2	°C	1200	c_1	cal/deg.gr	0.25
l	km	40	c_2	cal/deg.gr	0.30
α	deg/cm	30×10^{-5}	β_0		0.02
u_0	°C	1200	ε	cal/gr	90
u_1	°C	900	h	km	2.5∼20
κ	cal/deg.cm.sec	5.4×10^{-3}	D	km	2.5∼10
ρ	gr/cm³	2.70			
c	cal/deg.gr	0.20			

Some explanations are given as follows:

(i) The initial thickness of the Earth's crust l and the temperature at the lower surface of the Earth's crust are respectively assumed as 40 km and 1200°C, but these assumptions are not very important compared with the value of the normal temperature gradient $\alpha=30°C/km$. As illustrated in Fig. 5, the effect of the lower surface of the Earth's crust upon the cooling of magma is very small within the time interval of 10^6 years.

(ii) The range of temperature of the molten magma $u_0 \sim u_1$ was assumed as 1200°C$\sim$900°C, in view of the temperature of lava observed by many authorities just after the eruptions of active volcanoes as already mentioned in II-1).

Table II. Change of temperature of

D (km)	h (km)	t_1 ($10^4\times$ years)	Time in molten stage t ($\times 10^4$ years) 0	2	4	5	6	10	15	20	30	40	50	60	70	80
2.5	2.5	3.15	1200	955												
	5.0	4.15	1200	983	904											
	10.0	6.3	1200	1021	952		905									
	20.0	18.5	1200			1006		952	918	894						
5.0	2.5	12.0	1200			1004		925								
	5.0	17.0	1200	1103		1034		965	916							
	8.0	19.0	1200			1033		970	926							
	10.0	22.0	1200			1047		985		912						
	15.0	42.7	1200			1152		1085		1001	947	908				
	20.0	75.0	1200			1093		1053		1005	974	952	933	918	905	
10.0	2.5	34.5	1200			1109		1062		987	925					
	5.0	55.0	1200			1118		1075		1018	983	944	910			
	10.0	96.0	1200					1115		1064		1000		957		923

Table III. Temperature gradient at ground

D (km)	h (km)	T_1 ($\times 10^4$ years)	Time in the molten stage t ($\times 10^4$ years) 0	2	4	5	10	15	20	30	40	50	60	80	Time t_1 / 0
2.5	2.5	3.15	30	144	×										256
	5.0	4.15	30		39	×									40
5.0	2.5	12.0	30			(334)	(364)	×							(360)
	5.0	17.0	30	49		61	119	155	×						163
	8.0	19.0	30			32	35	46	×						57
	10.0	22.0	30				32		37	×					39
10	2.5	34.5	30			388	431			378	×				367
	5.0	55.0	30			37	110		184	199		190	×		187
	10.0	96.0	30						34		63		81	89	90

N.B. { * relatively low temperature
— maximum value of $(d\theta/dz)_{z=0}$

(iii) The mean latent heat of solidification of the magma is taken as 90 cal/gr. Once, J. Joly adopted the same value for the latent heat of basalt in his famous radioactive decay theory of mountain formation. J. C. Jagger took the value as 80–100 cal/gr in his evaluation of the coloing of sheet-like magma. Applying the values of the respective latent heats for minerals, the writer tried to calculate the value for an andesite of Hakone volcano, Japan, on which H. Kuno[8] estimated the mode of the rock, and got 91 cal/gr.
(iv) According to R. W. Goranson's experiment[9], the saturated water content of a granite indicates the values of 5.7–4.8 % due to the change of

magma (°C) with time.

Time in solidified stage t' ($\times 10^4$ years)														z_0 (km)
0	5	10	15	20	30	40	50	60	80	100	120	180	200	
900	700	562	470	403	316		227			153				3.75
900	737	633	569	524	460		379			283				6.25
900														
900														
900	869	792		656		485		387	325	284				5.0
900	882	837		757		638		555	494	448	412			7.5
900	876	839		772		682		622	578	542	513			10.5
900		884		793		718		669	633	605	581			12.5
900														
900														
900														
900		895		868		799		734	678	631		542	480	10.0
900		897		885		852		820	791	765		709	664	15.0

surface $(d\theta/dz)_{z=0}$ (in 10^{-5} °C/cm) with time.

in the solidified stage t' ($\times 10^4$ years)													Duration of hot spring activity refer to t' ($\times 10^4$ years)	T_2 ($\times 10^4$ years)	T_3 ($\times 10^4$ years)
5	10	15	20	30	40	50	60	80	100	120	150	200			
285	221	174	141	103		68			43				−2.5∼65	68	68
89	116	120	115	100		77			51				2.5∼74.5	72	76
308	247		182		120		90	73	62				−10∼105	115	117
175	170		150		118		96	81	71	64			−12∼133	145	150
70	78		86		86		80	72	66	61			1.5∼121.5	120*	140
	51		59		67		67	64	61	58			2.1∼106	85*	107∼128
	245		186		134		107	86	75		56	45	−33∼140	173	175
	172		151		122		104	91	82		66	56	−47∼179	226	234
	91		90		86		81	76	71		62	55	−57∼163	220*	259

temperature from 900°~1200°C at constant pressure of 980 bars. Basalt is supposed by petrologists to be the parent magma, but the water content of basaltic magma is, as far as the writer is aware, unknown at present. Then, the writer adopted the mean value of the above-mentioned water contents 5.3%, that is, a value of β_0 is assumed as 2.0% by weight in this calculation.
(v) Specific heat of water vapor contained in molten magma c_2 is also unknown. The value 0.30 which is extrapolated from the values at 100°~400°C in normal pressure is adopted. But, the fourth term of the fundamental equation (5)' which contains c_2 is so small compared with other terms that the effect may be negligible, even if the value of c_2 contains moderate errors.
(vi) The values of the depth h and the thickness D of the intruded sheet-like magma necessary to evaluate this problem are assumed in the extents of 2.5~10 km as indicated below in §III.

Tabulated in Tables II and III is the result of evaluation use being made of these constants for the case of the intruded sheet-like magma in the shallow part of the Earth's crust, where the initial condition is given by equation (16), that is, the normal temperature gradient exists just before the intrusion of magma.

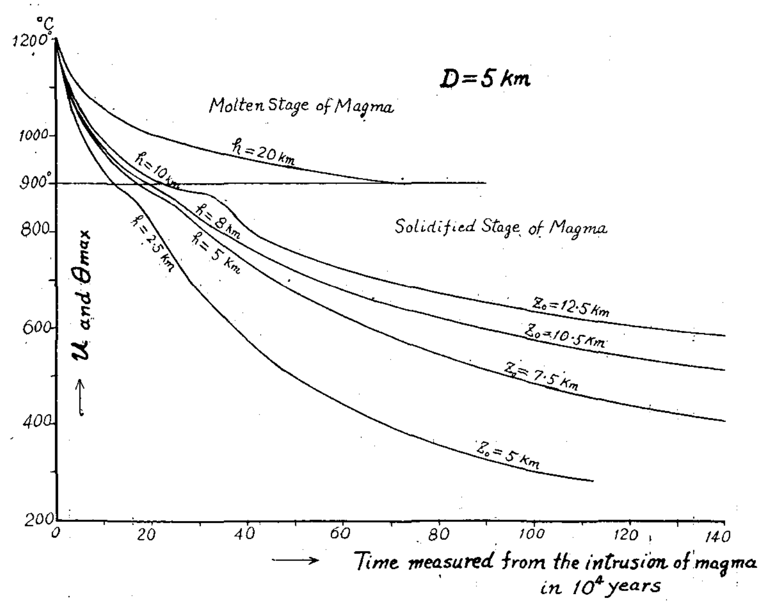

Fig. 3. An example of cooling of magma with time for $D=5$ km and $h=2.5\sim20$ km. z_0 in the figure indicates mean depth of magma from the ground surface, where temperature shows nearly θ_{max}.

Fig. 3 is an example of the cooling of magma with time for $D=5$ km and $h=2.5\sim20$ km. The temperature decrease with time is nearly exponential. The time interval t_1 from the magma intrusion to the complete solidification of the magma at 900°C can be obtained from the figure. Fig. 4 shows the change of the molten interval t_1 of the sheet-like magma with different values of D and h.

Fig. 5 indicates two examples of the vertical temperature distribution at $t=0$ (time of magma intrusion) and $t=t_1$ (time of complete solidification) for respectively ($D=5$ km, $h=5$ km) and ($D=10$ km, $h=5$ km). The effect of heat due to intrusion of magma is not attained at the lower part of the Earth's crust in the extent of time here considered, as shown from the figure. It means that the initial temperature gradient only is necessary and that the thickness of Layer III and the temperature at the lower surface of the Earth's crust are not important for the cooling of molten magma.

Fig. 6 shows the change of the temperature gradient with time at the ground surface in the molten stage and in the solidified stage of magma for $D=5.0$ km and $h=2.5\sim10$ km.

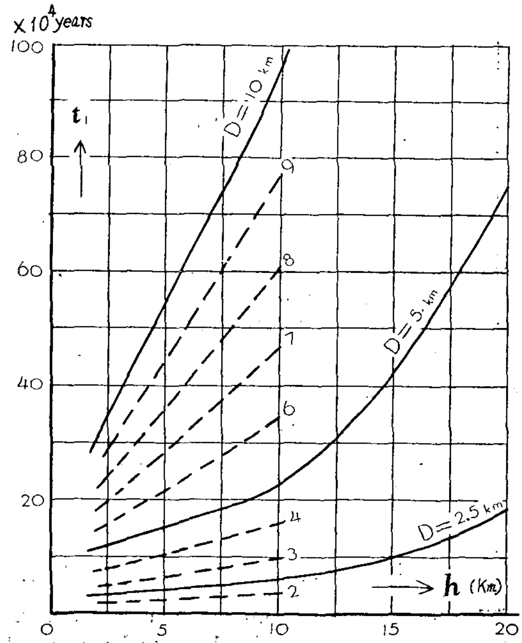

Fig. 4. Change of the molten time interval t_1 of the sheet-like magma for different values of D and h.

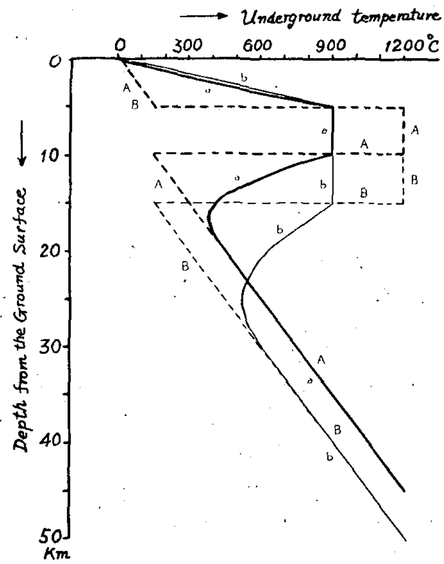

Fig. 5. Examples of the vertical temperature distribution at $t=0$ (time of magma intrusion) and $t=t_1$ (time of complete solidification) for two cases of $D=5$ km, $h=5$ km and $D=10$ km, $h=5$ km.

A : $D=5$ km, $h=5$ km and $t=0$,
a : $D=5$ km, $h=5$ km and $t=15\times10^4$ years,
B : $D=10$ km, $h=5$ km and $t=0$,
b : $D=10$ km, $h=5$ km and $t=55\times10^4$ years.

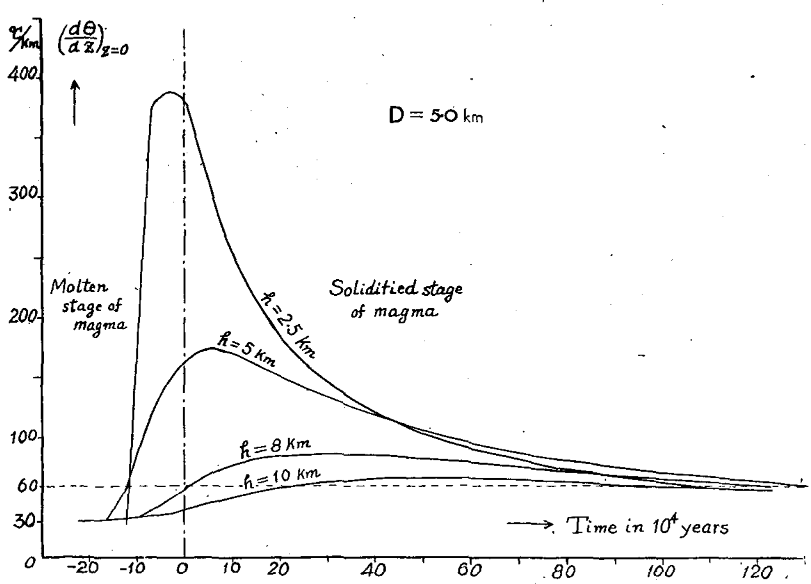

Fig. 6. Change of the vertical temperature gradient with time at the ground surface in the molten stage and in the solidified stage of magma for $D=5.0$ km and $h=2.5\sim10$ km.

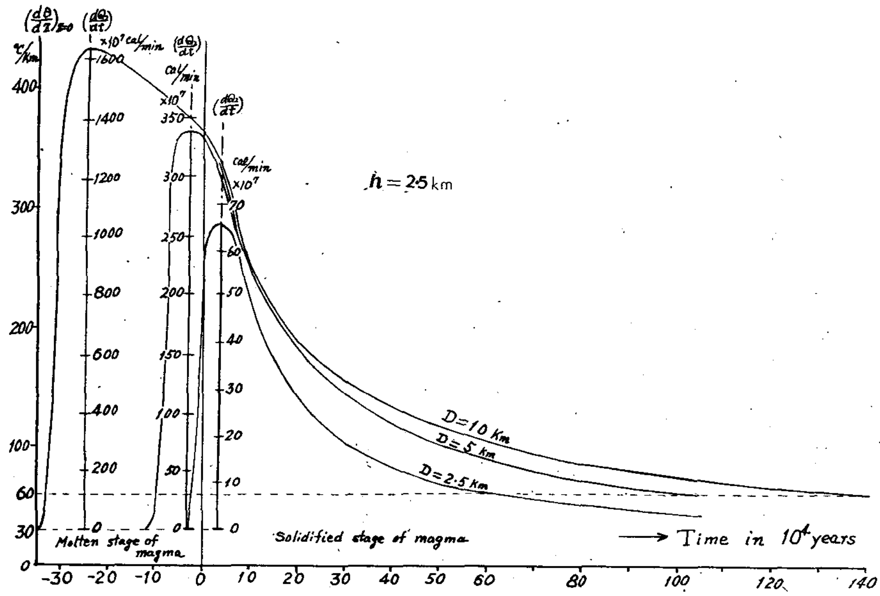

Fig. 7. Change of the vertical temperature gradient with time at the ground surface in the molten stage and in the solidified stage of magma for h=2.5 km and D=2.5~10 km.

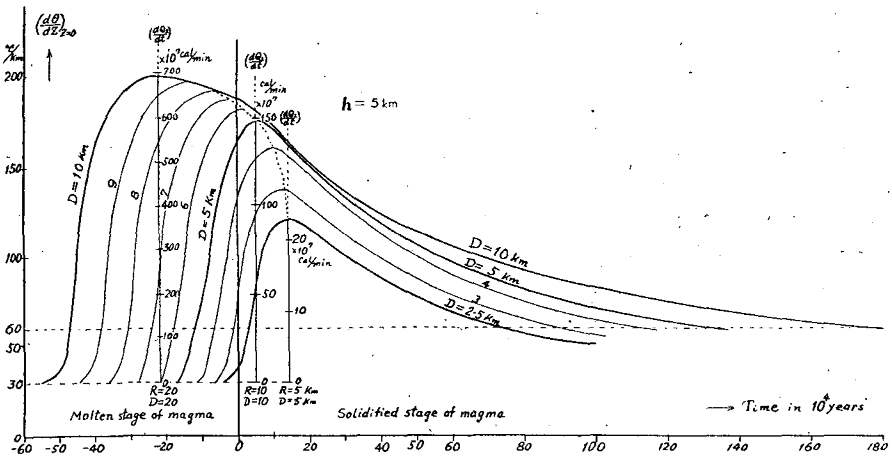

Fig. 8. Change of the vertical temperature gradient with time at the ground surface in the molten stage and in the solidified stage of magma for h=5 km and D=2.5~10 km.

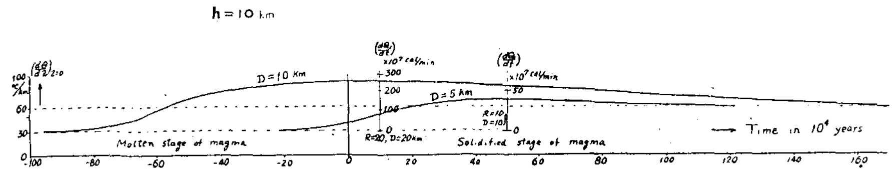

Fig. 9. Change of the vertical temperature gradient with time at the ground surface in the molten stage and in the solidified stage of magma for $h=10$ km and $D=5\sim10$ km.

Fig. 7–9 indicate respectively the change of the temperature gradient with time at the ground surface in the molten stage and in the solidified stage of magma for $h=2.5$ km, $h=5$ km and $h=10$ km and for $D=2.5\sim10$ km.

§III. An application of the above theory to the magma chamber of circular cylindrical form of thickness D and radius R.

1) The real shape of the volcanic magma chamber is not clearly known at present, but it is more reasonable to assume a circular cylindrical form of radius R and thickness D than an infinite sheet-like form of thickness D. If it is assumed that such a vertical cylindrical magma chamber (ffgg) lies at a depth h from the ground surface, and that an adiabatic wall exists along its cylindrical surface from the ground surface to the lower surface of the Earth's crust (efgi in Fig. 2), the above theory on the cooling of the sheet-like magma can be directly applied to such a case. This model is for convencence called "model I" in the following. It means that the heat dissipation from the side wall (fg) of the magma chamber to the country rock and the lateral heat conduction from the part of the Earth's crust just above and just below the magma chamber (ef) and (gi) to the surrounding parts are neglected in considering this model. Then, the result of the above-described calculations on the time interval of the molten stage of magma t_1 and on the temperature gradient in the area πR^2 of the ground surface $(\mathrm{d}\theta/\mathrm{d}z)_{z=0}$ just above the magma chamber at any time of the molten stage and of the solidified stage of magma will give larger values than the real corresponding values in the cylindrical magma chamber which has no adiabatic side wall.

Since it may be difficult to solve the problem considering exactly the effect of heat dissipation from the side wall, it is necessary at the present stage to be content with an estimation of order of magnitude of t_1 and $(\mathrm{d}\theta/\mathrm{d}z)_{z=0}$.

Comparing the heat dissipations from the side wall (fg) of the magma chamber to the country rock with those from the part of the Earth's crust just above and just below the magma chamber (ef) and (gi), the former effect might be larger than the latter, if the thickness of the magma chamber and its depth from ground surface is in nearly the same order of magnitude and the temperature of side wall of magma chamber (fg) is moderately higher than that of (ef) and (gi) as shown in Fig. 5.

Therefore, only the heat dissipation from the side wall of the magma chamber (fg) is taken into consideration for the estimation of the order of magnitude of t_1 and $(\partial\theta/\partial z)_{z=0}$.

For this purpose, a model is introduced here, in which areas of the upper and the lower horizontal surfaces of the magma chamber are increased from the original areas by respectively $2\pi R\cdot D/2$, that is, a half area of the cylindrical side wall of the magma chamber, and in the model the adiabatic side wall (pqrs) is taken to exist just at the outside of the increased area as indicated in Fig. 2. This model is conveniently called "model II" in the following.

The fundamental equation of the cooling of the molten magma in the case of model II is given by

$$\pi R^2 D \rho' c' \frac{\mathrm{d}u}{\mathrm{d}t} = \kappa\left(\pi R^2 + 2\pi R\cdot\frac{D}{2}\right)\left\{\left(\frac{\partial\theta_2}{\partial z}\right)_{z=h} - \left(\frac{\partial\theta_1}{\partial z}\right)_{z=h}\right\}. \tag{25}$$

Then, if one puts

$$D' = \frac{D}{1+\dfrac{D}{R}}, \tag{26}$$

equation (25) takes the same form as the fundamental equation of the cooling of sheet-like magma (6) except that the value of the thickness of Layer II is D' instead of D. Therefore, the result of calculation of the sheet-like magma may be applied to this case, substituting D' instead of D.

The relation among D, R and D' is shown in Table IV from equation (26), when they take respectively the values D=5, 10, 15 and 20 km, R=5, 10 and 20 km.

Changing the form of (26), one gets

Table IV. Effectve thickness of magma chamber D' (km), when R and D take respectively 5, 10, 15 and 20 (km).

D \ R	5 (km)	10 (km)	(20 (km))
5 (km)	2.5	3.33	
10 (km)	3.33	5.0	
15 (km)	3.75	6.0	
(20 (km))			(10.0)

$$\pi R^2 D = \left(\pi R^2 + 2\pi R \cdot \frac{D}{2}\right) D'. \tag{27}$$

The physical meaning of this equation is that the total mass, that is, the total heat energy of the magma of model II at $t=0$ or $t'=0$ must be taken as the same as that of model I. This may be a reasonable assumption. The consequent effect that the total thickness of the Earth's crust is changed from $l+D$ to $l+D'$ is negligibly small as mentioned above in II-5).

2) Next, the probable values of the dimensions and depth of the actual volcanic magma chamber are discussed.

i) According to A. Rittmann[10], H. Kuno[11] and T. Rikitake[12], the depth of the volcanic magma chamber h was respectively estimated as 6.5 km in Vesuvius, Italy, from several to 10 km in Hakone volcano and 3.5 km in Ohshima volcano, Japan. Then, it may be taken as h=2.5–10 km in this paper.

ii) On the vertical dimension of the volcanic magma chamber, a few estimations have been made in the past. T. Rikitake once noticed that a sphere of 4 km diameter which had no magnetization lies below Ohshima volcano. G. S. Gorshkov[13] indicated the existence of molten magma chamber of about 25 ~35 km in extension and thickness below the Kliuchevsky volcano, Kamchatka in his seismological investigation. For the present discussion $D=5\sim10$ km ($D=20$ km as the maximum value) will be adopted for the vertical dimension of magma chamber in this paper.

iii) The shape of volcanic calderas is almost circular or elliptic.

This fact may show that the horizontal shape and dimensions of a magma chamber are probably the same as those of its caldera. According to M. Minato and T. Ishikawa[14], the volcanic calderas in northern Japan excepting those of some small dimension have the mean radius of 6~11 km as shown in Table V. Then, the mean radius $R=5\sim10$ km ($R=20$ km as the maximum value) is adopted for the magma chamber in this paper.

Table V. Volcanic Calderas in Northern Japan.

Name of Caldera	*Diameters (km)	*Area (km²)	Mean radius R (km)
Kuccharo (active)**	26.0 × 20.0	430.0	11.7
Akan (active)	24.0 × 13.0	244.0	8.8
Shikotsu (active)	15.0 × 13.0	140.0	6.7
Toya (active)	17.0 × 12.0	133.0	6.5
Towada (extinct)	12.5 × 11.8	117.2	6.1

* According to T. Ishikawa and M. Minato

** Activity of the near-by volcanoes.

§IV. The probable life span of volcanic hot springs.

It was already stated in §I that the volcanic hot springs containing hot springs of effusive rock origin are divided into two kinds, viz., the volcanic hot spring of magmatic water origin and that of meteoric water origin.

1) Probable life span of volcanic hot springs of magmatic water origin.

The duration of life of volcanic hot springs of magmatic origin T_1 is the same as the time interval of molten magma t_1 (Fig. 4), since the free water vapor is assumed to be discharged only from the molten magma.

When the probable depth $h=5\sim10$ km and the probable dimension of the magma chamber $R=5\sim10$ km, $D=5\sim10$ km are adopted, the order of magnitude of the life span of the volcanic hot springs of magmatic origin T_1 is calculated as shown in Fig. 10. As illustrated from the figure, the life span T_1 may extend from several tens of thousands to several hundreds of thousand years.

If it is permitted to assume $R=20$ km, $D=20$ km for the possible largest dimension of the magma chamber, the life span might be attainable to the maximum value from several hundreds of thousands to one million years.

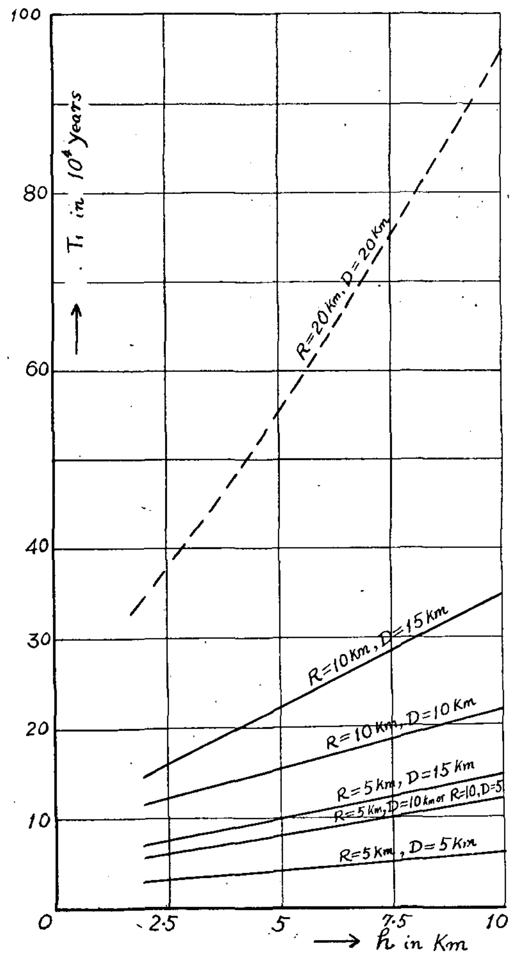

Fig. 10. Order of magnitude of the life span for volcanic hot springs of magmatic water origin.

(2) Probable life span of volcanic hot springs of meteoric origin.

The most important factor necessary to heat the meteoric water which flowed into underground rock is the stationary heat flow from the deep, that is, a larger positive vertical thermal gradient than the normal thermal gradient at the shallow part of the Earth's crust. Since the theoretical or empirical solution of the exact relation between temperature of hot springs and the thermal gradient, and the greatest depth of descending meteoric water are unknown, it is necessary at present to content oneself with taking a criterion to distinguish the hot springs from the normal underground water as more than twice the value of the normal thermal gradient. But, this assumption may be not so absurd, since it is frequently adopted in the practice of hot spring exploration.

Applying this criterion to Figs. 6~9 and using the relation shown in Table IV, the order of magnitude of the life span of volcanic hot springs of meteoric origin T_2 can be calculated. Fig. 11 indicates the life span T_2, when $R=5\sim 10$ km, $D=5$, 10, 15 km, and $h=2.5\sim 10$ km. As illustrated in the figure, the life span T_2 may be within the range from a few tens of thousand to one million and half years. If the largest dimension of magma chamber be supposed as $R=20$ km and $D=20$ km, the life span T_2 might take the value of

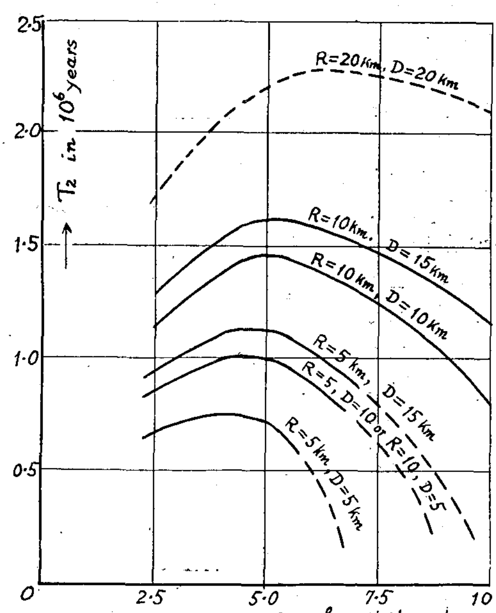

Fig. 11. Order of magnitude of the life span for volcanic hot springs of meteoric water origin.

about two million years.

It may be worthy of note that they are only tepid hot springs in the case of $h=10$ km as shown in Fig. 9.

(3) Probable life span of volcanic hot springs.

If the life interval of volcanic hot springs T_3 is considered without distinguishing two sorts mentioned above, that life span can be given, as shown in Fig. 12, for the values of $R=5$, 10 km, $D=5$, 10, 15 km and $h=2.5\sim10$ km. The life span of volcanic hot springs T_3 may be of the order of magnitude from several hundreds of thousand to 2 million years. If the largest dimension of magma chamber be assumed as $R=20$ km, $D=20$ km, the life span T_3 might take the value of $2\times10^6\sim2.6\times10^6$ years.

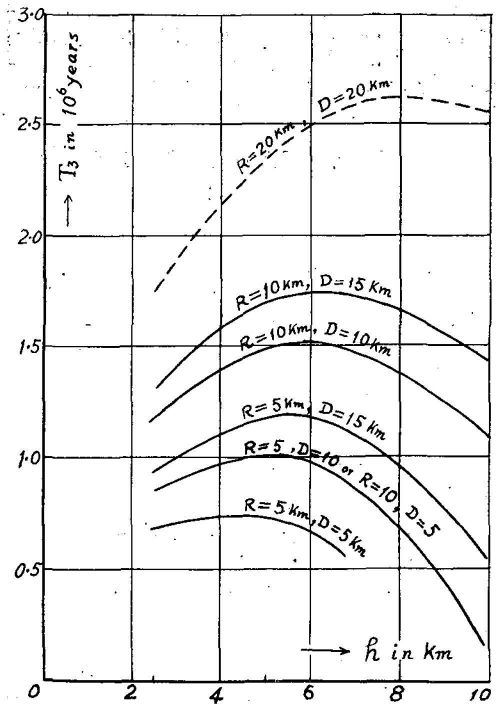

Fig. 12. Order of magnitude of the life span for volcanic hot springs

§V. Heat discharged from the volcanic hot spring areas.

(1) Heat discharged as water vapor from a magma chamber.

It was assumed above in §II-(iv) that the mass of solidified materials crystallized per unit decrease of temperature is always constant and independent of temperature in molten stage of magma; it was further assumed above in §II-(vi) that the molten part of magma contains a constant amount of water β_0 per unit mass, and the free water vapor discharged from the magma escapes through fissures in Layer I to the atmosphere, giving no effective heat to the country rock.

Under these assumptions, the discharged mass of water vapor $\mathrm{d}q/\mathrm{d}t$ from total mass of the magma at time t is given by the following equation from equation (4):

$$\frac{\mathrm{d}q}{\mathrm{d}t}=-\frac{\pi R^2 D\rho'\beta_0}{u_0-u_1}\frac{\mathrm{d}u'}{\mathrm{d}t}. \tag{28}$$

The discharged heat per unit time $\mathrm{d}Q_1/\mathrm{d}t$ at any time t is given by

$$\frac{\mathrm{d}Q_1}{\mathrm{d}t}=\frac{\mathrm{d}q}{\mathrm{d}t}\left[c_v\left\{\left(u'-mh\right)-100\right\}+L'+100\,c\right] \tag{29}$$

where $m=(\sigma g/R')(\gamma'-1/\gamma')$, c_v and c_p respectively are the specific heats of water vapor at constant volume and at constant pressure, $\gamma'=c_p/c_v$, g the gravity acceleration, σ the molecular weight of H_2O, R' the gas constant, L' the latent heat of evaporation of water at 100°C, c the specific heat of liquid water and the term mh is the temperature decrease due to adiabatic cooling. The standard of heat is taken at 0°C.

In equation (29), the effect of heat loss from the side walls of the fissures and the effect of hydrodynamical energy loss are not considered. It is not necessary to consider the former effect in this case, since it is desired to calculate the total sum of heat discharged from the magma chamber. The latter effect is negligibly small.

From equations (28) and (29), one gets

$$\frac{\mathrm{d}Q_1}{\mathrm{d}t}=-\frac{\pi R^2 D\rho'\beta_0}{u_0-u_1}\frac{\mathrm{d}u'}{\mathrm{d}t}\left[c_v\left\{\left(u'-mh\right)-100\right\}+L'+100\,c\right], \tag{30}$$

where u' is given from equation (11)′ substituting D' instead of D for model I, then $\mathrm{d}u'/\mathrm{d}t$ is given by

$$\frac{\mathrm{d}u'}{\mathrm{d}t}=-2\frac{\rho'c'}{\rho c}D'(u_0-T_3)\sum_{i=1}^{\infty}\frac{k^2\mu_i^2\,\mathrm{e}^{-k^2\mu_i^2t}}{\frac{\rho'c'}{\rho c}D'+\frac{h}{\sin^2\mu_i h}+\frac{l-h}{\sin^2\mu_i(l-h)}}. \tag{31}$$

Now, let the following be adopted: $\sigma=18$, $R'=8.32\times10^7$ erg/°K, $L'=539$ cal/gr, $c\simeq1.0$ cal/deg.gr, $c_v=0.29$ cal/deg.gr, $c_p=0.39$ cal/deg.gr and $\beta_0=0.02$.

The values of c_v and c_p are the extrapolated values at 700°C in 1 atmospheric pressure.

Substituting the values in equation (30) change of dQ_1/dt with time is calculated for three cases of ($R=10$ km, $D=10$ km), ($R=5$ km, $D=5$ km) and ($R=20$ km, $D=20$ km) when $h=2.5\sim 10$ km.

The former two cases are indicated by full lines and the last case is shown by broken line in Fig. 13. As illustrated from the figure, the value of dQ_1/dt is very large in the earlier part of the molten stage of magma especially during several scores of thousand years after the magma intrusion and is gradually diminished with time in that stage.

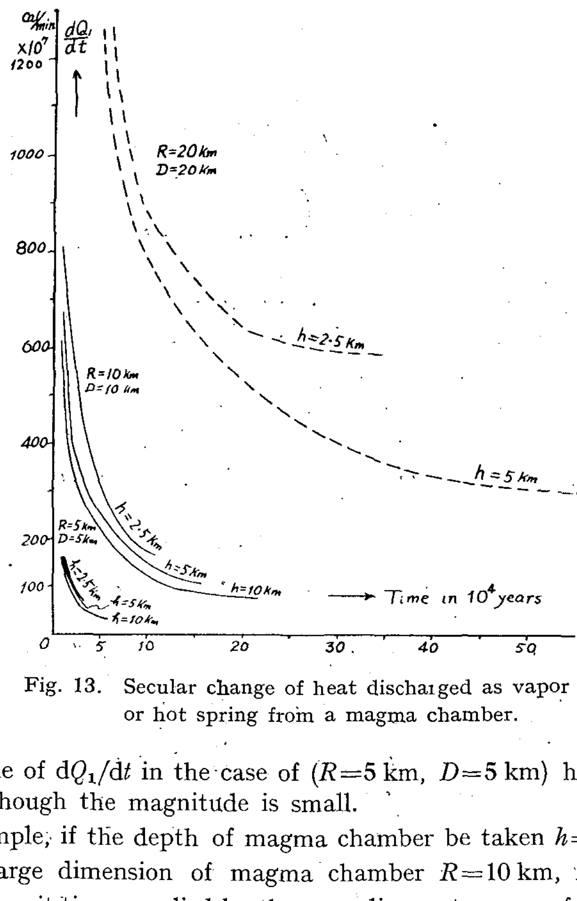

Fig. 13. Secular change of heat discharged as vapor or hot spring from a magma chamber.

The value of dQ_1/dt in the case of ($R=5$ km, $D=5$ km) has also similar inclination, though the magnitude is small.

For example, if the depth of magma chamber be taken $h=5$ km and the moderately large dimension of magma chamber $R=10$ km, $D=10$ km, the total heat per unit time supplied by the ascending water vapor from the magma chamber takes a value of more than 100×10^7 cal/min through the molten stage of magma, a value of more than 200×10^7 cal/min during several tens of thousand years after the time of formation of the magma chamber, and exceeds 400×10^7 cal/min within 20 thousand years after the magma intrusion.

In this paper, the free water vapor discharged from the magma was assumed to escape continuously through fissures in the Earth's crust to the atmosphere, but in actual fact the discharge of water vapor may perhaps occur cyclically, that is, the fissures might be frequently closed by deposits and some other materials, and opened again by steam explosions when the vapor pressure in the magma chamber is increased to a certain high pressure. Then, the discharged rate dQ_1/dt in this case after the opening of fissures may take a larger value than that of the corresponding time shown in Fig. 13.

(2) Heat flow at the ground surface conducted by rock from a magma chamber.

The heat per unit time conducted through Layer I from the magma chamber to the ground surface dQ_2/dt is given by

$$\frac{dQ_2}{dt}=\kappa\pi R^2\left\{\left(\frac{\partial\theta}{\partial z}\right)_{z=0}-\alpha\right\}, \tag{32}$$

where κ represents the heat conductivity of the Earth's crust, α the normal thermal gradient in the Earth's crust, and $(\partial\theta/\partial z)_{z=0}$ the thermal gradient at the ground surface indicated by equation (17) in the molten stage and by equation (22) in the solidified stage of magma. But, the value of D in these equations must be respectively taken as the effective thickness D' indicated by equation (26) instead of the actual thickness D of the magma chamber.

Adopting $\kappa=0.0054$ cal/deg.cm.sec, $\alpha=30\times10^{-5}$ deg/cm, the radius of the magma chamber $R=5$ or 10 km, $D=5\sim15$km, the depth of magma chamber $h=2.5\sim10$ km, one may calculate dQ_2/dt from equation (32) as shown in Fig. 7 for $h=2.5$ km, in Fig. 8 for $h=5$ km and in Fig. 9 for $h=10$ km.

Three scales attached to ordinates at the mode of the curves in the figures are respectively the values of dQ_2/dt for three cases of ($R=20$ km, $D=20$ km), ($R=10$ km, $D=10$ km) and ($R=5$ km, $D=5$ km).

For example, if one considers the normal magma chamber, dimensions of which are in the extent from ($R=10$ km, $D=10$ km) to ($R=5$ km, $D=5$ km), the discharge rate of heat dQ_2/dt is, as seen from the figures, very small in the earlier part of the molten stage of magma and gradually increases to the maximum value at the later part of the molten stage or the earlier part of the solidified stage. The time interval in which dQ_2/dt exceeds 100×10^7 cal/min is about 40×10^4 years in the case ($R=10$ km, $D=10$ km and $h=2.5\sim$ 5 km). After that, heat discharge diminishes gradually, but it is maintained during more than 1 million years after the complete solidification of magma.

In the case (h=10 km, R=5~10 km and D=5~10 km), the amount of dQ_2/dt is so small that hot springs of moderately high temperature which were defined above in §IV do not originate in this case, as seen in Fig. 9.

In a special case in which the dimensions of the magma chamber are the largest value as (R=20 km, D=20 km) and the depth of the magma chamber is h=2.5~5 km, dQ_2/dt shows the maximum value at the earlier part or the middle part of the molten stage of magma and maintains high discharge rate of heat of more than 500×10^7 cal/min during about 60×10^4 years as shown in Figs. 7 and 8.

The discharge rate of heat dQ_2/dt is, in general, smaller than dQ_1/dt, though on the contrary, the life span of the former is very long compared with that of the latter.

A certain fraction of the heat flow from the magma is absorbed by meteoric water which descended into the shallow part of the Earth's crust and is discharged to the ground surface as volcanic hot springs of meteoric origin.

It is generally recognized by geologists that hot springs are one of the post-volcanic phenomena. This may be true in regard to the volcanic hot springs of meteoric origin in which the depth of magma chamber h is less than 8 km and the dimensions of magma chamber are in the extent of (R=10 km,

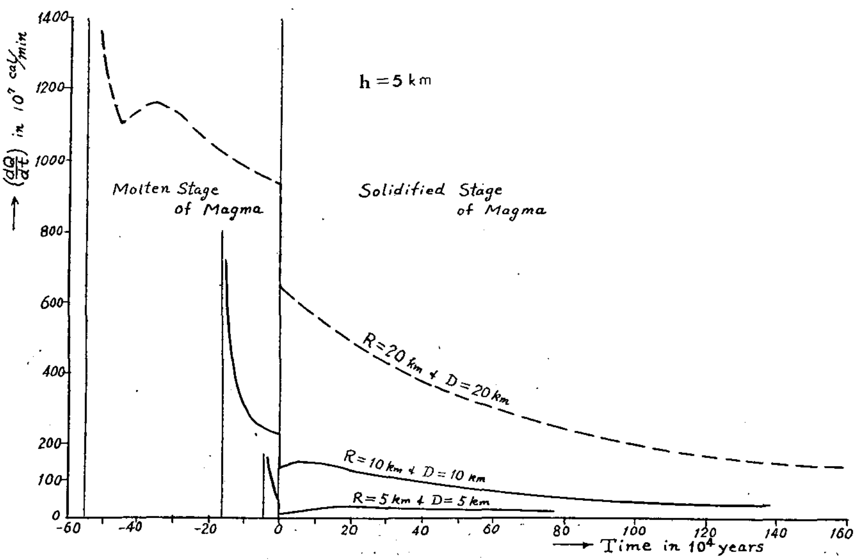

Fig. 14. Secular change of the sum of discharge rate of heat $\left(\frac{dQ_1}{dt}\right)$ and $\left(\frac{dQ_2}{dt}\right)$.

$D=10$ km) to ($R=5$ km, $D=5$ km) as discussed above. Fig. 14 shows the change of the sum of dQ_1/dt and dQ_2/dt with time in the cases of ($R=20$ km, $D=20$ km,) ($R=10$ km, $D=10$ km) and ($R=5$ km, $D=5$ km).

(3) The observed discharge rate of heat from hot spring localities and comparison with the calculated values of two kinds of volcanic hot springs.

The total discharge rate of heat dQ/dt from a hot spring locality is the sum of the discharge rate of heat flowing out as steam or hot water from orifices in that locality dq_1/dt and the heat generated from the ground surface of the locality by heat conduction dq_2/dt. The results of measurements made by many observers in the past are summarized in Table VI.

Table VI. Observed discharge rates of heat from hot spring localities in 10^7cal/min unit.

Locality	$\frac{dq_1}{dt}$	$\frac{dq_2}{dt}$	$\frac{dQ}{dt}$	Maximum temperature in °C	Note
Wairakai Area (New Zealand)[15]	660			270	
Upper Basin, Yellowstone (U.S.A.)[15]	540			180	
Mammoth & Hot River Area (〃)[15]	200			75	
Frying Pan Lake (New Zealand)[15]	140			55	
St. Vincent (West Indies)[16]	110			200 ca	
Solfatara, Dominica (West Indies)[16]	103				
Jigokudoni Noboribetsu (Japan)[17]	49	18	152	> 150	Kuttara Volcanic Area
Oyunuma, Noboribetsu (Japan)[17]	84			112	
Karurusu (〃)[17]	0.8	0.2		56	
Jōzankei (〃)[17]	58	2	60	89	
Qualibau Solfatara, St. Lucia (W.I.)[16]	52				
Norris Basin, Yellowstone (U.S.A)[15]	48			205	
Kawayu (Japan)[17]	37	10	47	> 120	
Steamboat, Nevada (U.S.A.)[15]	42				
Baba, Niseko (Japan)[17]	27	1	28		
Doyako (Japan)[17]	12	1.1	13	50	
Sōunkyo (〃)[17]	8.6	2.8	11	94	
Nukabira (〃)[17]	2.8	4.0	7	70	
Tokachigawa (〃)[17]	5.8			50	
Mont, Serrat (West Indies)[16]	2				
Akankohan (Japan)[17]	1.4	3.4	5	99	

As seen in the table, the discharge rate of heat of a hot spring locality ranges from 1×10^7 cal/min to 700×10^7 cal/min, corresponding to its hot spring activity. For example, Karurusu (Japan) which discharges 1×10^7 cal/min is a small-scale hot spring locality having several hot springs of 56°C at maximum in a small area of about 0.005 km²; the Upper Geyser Basin in Yellowstone National Park in the U.S.A. which shows 540×10^7 cal/min in its discharge rate of heat is a large-scale hot spring locality having perhaps a few hundred orifices of boiling water in the large area of about 10 km².

To compare the observed value of dQ/dt with the calculated values of dQ_1/dt and dQ_2/dt, consider that in a volcanic hot spring of magmatic origin the calculated value dQ_1/dt may be applied at maximum, but in a volcanic hot spring of meteoric origin only a certain fraction of the calculated value of dQ_2/dt may be applied generally. This is true because the circulation area of a system of meteroic water may be confined to a relatively small area compared with the area of the upper surface of the magma chamber.

Comparison of the observed discharge rate of heat with the calculated rate of heat with consideration of the above mentioned fact, indicates fair agreement in volcanic hot springs of magmatic origin and also those of meteoric origin, that is, the observed discharge rates lie in the extent of the calculated discharge rates for both kinds of volcanic hot springs.

The discussion may be first presented for the case of the normal dimensions of magma chamber (R=10 km, D=10 km)～(R=5 km, D=5 km).

The hot spring locality, in which its observed discharged rate of heat is less than 100×10^7 cal/min, may possibly have considerably long time duration for cases of both volcanic hot springs of magmatic and those of meteoric origin, if the depth of the magma chamber is assumed as less than several kilometers. The case in which the observed discharge rate is 100～200×10^7 cal/min, may be possible during a certain interval after the magma intrusion in a volcanic hot spring of magmatic origin; but it may be possible only in the limited interval of the later stage of molten magma and of the earlier stage of solidified stage of magma in a volcanic hot spring of meteoric origin in the case of (R=10 km, D=10 km); it may be impossible in the case of (R=5 km, D=5 km), as shown in Figs. 7 and 8. Especially, the hot spring locality with observed discharge rate of more than 200×10^7 cal/min may be possible only during a short interval in the earlier part of molten stage of magma or at least in the periods just after volcanic eruptions as mentioned above in §V-1).

The possibility of the existence of a hot spring of meteoric origin is doubtful when the depth of magma chamber is larger than 8 km, but on the contrary, if the depth of the chamber is as small as h=2.5 km, hot springs of meteoric origin having large discharge rates of heat are possible, though their life spans would be comparatively short as illustrated in Fig. 7. In the Yellowstone area U.S.A. of total discharge rate of heat may be more than 800×10^7 cal/min as seen in Table VI. It is acknowledged that the hot spring activity had begun already at the Middle Quaternary epoch of 40×10^4～50×10^4 years ago, since the three or four layers of tillite[18] were overlain the deposits of geyserite. To explain these facts without contradiction, an assumption is postulated that the largest class of magma chamber of (nearly R≒20 km,

$D=20$ km) as shown in Fig. 13 and Fig. 14 exists at a depth of $2.5\sim5$ km below the ground surface of this area and the present status is at the later part of molten stage or the earlier part of solidified stage of magma.

§VI. Temperature and volume out-put of voçanic hot springs.

In this chapter, the writer estimates theoretically the rough magnitude of temperature and volume out-put for volcanic hot springs of magmatic origin and those of meteoric origin, and discusses the possibility of the existence of such hot springs.

(1) Temperature and volume out-put of volcanic hot springs of magmatic origin.

As mentioned above in §V-(1), the discharged mass of water vapor q' from total mass of magma per unit time at time t is given from equation (28) as follows :

$$q' = -\frac{\pi R^2 D \rho' \beta_0}{u_0 - u_1} \frac{\mathrm{d} u'}{\mathrm{d} t}, \tag{33}$$

where the rate of temperature decrease of magma $\mathrm{d}u'/\mathrm{d}t$ is given by equation (31) in §V-(1).

A fissure which discharges free water vapor from magma chamber to atmsophere is assumed to be a vertical cylindrical tube of radius r_0. Consider the upward direction of the central axis of the tube as z-axis, take the co-ordinate origin at the lower end of the tube, and put r-axis in radial direction from the central axis of the tube. If the effect of energy loss due to friction of the side wall and the change of kinetic energy of water vapor in the tube are assumed to be negligibly small, the fundamental equation of temperature change of ascending water vapor through this tube in stationary state is roughly given by

$$\frac{\mathrm{d} T}{\mathrm{d} z} = \frac{2\pi r_0 \kappa}{c_v q'} \left(\frac{\partial \theta}{\partial r}\right)_{r=r_0} - m, \tag{34}$$

where $m=(\sigma g/R')(\gamma'-1)/\gamma'$, σ the molecular weight of H_2O, R' the gas constant, c_v, c_p are respectively the specific heat of vapor at constant volume and at constant pressure, $\gamma'=c_p/c_v$, κ the heat conductivity of country rock, g the gravity acceleration. $(\partial\theta/\partial r)_{r=r0}$ the thermal gradient of country rock in the radial direction at the tube boundary, and $\mathrm{d}T/\mathrm{d}z$ the rate of temperature change of water vapor per unit ascent. The first and the second terms of the right hand side of the equation indicate respectively the effect of heat dissipation from the side wall of the tube to country rock and the effect of adiabatic cooling. The change of temperature due to adiabatic cooling is explained as follows:

The water vapor is assumed as an ideal gas, then $pv=R'K$ holds in this case, where p the pressure of water vapor, v the volume of 1 gram-molecule of H_2O and $v=\sigma/\rho$, ρ the density of water vapor and K the absolute temperature. On the other hand, the equation of adiabatic change $p^{1/\gamma'}v=C$ may hold in this case, where C is a constant. From these equations,

$$K=C\,R'\,p^{(\gamma'-1)/\gamma'}$$

is obtained. The change of kinetic energy of water vapor on the way of ascending is small compared with other terms, then $\mathrm{d}p=\rho g\,\mathrm{d}z$ is assumed.

From these equations, $\mathrm{d}T/\mathrm{d}z$ in the case of no heat-loss from the side wall of the tube may be written as

$$\frac{\mathrm{d}K}{\mathrm{d}z}=\frac{\mathrm{d}T}{\mathrm{d}z}=\frac{\sigma}{R'}\frac{\gamma'-1}{\gamma'}g=m\,.$$

Exact solution of equation (34) is difficult to obtain owing to the fact that c_v and c_p are not perfectly independent of temperature T, but for the purpose of estimating the approximate value of temperature at the ground surface, it may be permitted to assume c_v and c_p as constant values and also to adopt the approximate value of $(\partial\theta/\partial r)_{r=r_0}$ as follows:

The equation of heat conduction in the country rock of the tube in stationary state is given by

$$\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}+\frac{\partial^2\theta}{\partial z^2}=0$$

In the rock surrounding the tube, the thermal gradient in the radial direction is large compared with that in the vertical direction. Then, neglecting the third term of the equation in the extent $r_0<r<l$, one gets

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}r^2}+\frac{1}{r}\frac{\mathrm{d}\theta}{\mathrm{d}r}=0 \qquad \text{for } r_0<r<l\,. \tag{35}$$

The writer also assumes that the temperature of the country rock in the extent $r\geqq l$ is always indicated by

$$\theta=u\left(1-\frac{z}{h}\right) \qquad \text{for } r\geqq l\,. \tag{36}$$

Equation (36) means that high temperature of ascending water vapor in the tube exerts no important effect upon the temperature of rock in the region $r>l$.

The solution of equation (35) satisfying the boundary conditions ($r=r_0$; $\theta=T$) and (36) may be written as

$$\theta = T + \frac{T-u\left(1-\frac{z}{h}\right)}{\log\frac{r_0}{l}}\log\frac{r}{r_0}. \tag{37}$$

$$\therefore\ \left(\frac{\partial\theta}{\partial r}\right)_{r=r_0} = \frac{T-u\left(1-\frac{z}{h}\right)}{r_0\log\frac{r_0}{l}}. \tag{38}$$

Substituting (38) into equation (34), one gets

$$\frac{\mathrm{d}T}{\mathrm{d}z} + \frac{2\pi\kappa}{c_v q'\left(-\log\frac{r_0}{l}\right)}T = \frac{2\pi\kappa u}{c_v q'\left(-\log\frac{r_0}{l}\right)}\left(1-\frac{z}{h}\right) - m. \tag{39}$$

The solution of this equation satisfying the boundary condition ($z=0$; $T=u$) is given by

$$T = u\left(1-\frac{z}{h}\right) + \frac{c_v q'\left(-\log\frac{r_0}{l}\right)}{2\pi\kappa h}(u-mh)\left\{1-\exp\left(-\frac{2\pi\kappa}{c_v q'\left(-\log\frac{r_0}{l}\right)}z\right)\right\}. \tag{40}$$

Then, the temperature of water vapor at the ground surface is indicated by

$$T_h = \frac{c_v q'\left(-\log\frac{r_0}{l}\right)}{2\pi\kappa h}(u-mh)\left\{1-\exp\left(-\frac{2\pi\kappa h}{c_v q'\left(-\log\frac{r_0}{l}\right)}\right)\right\}. \tag{41}$$

In any case, discharge rate of vapor q' and temperature of vapor at the ground surface T_h are respectively shown by equations (33) and (41). Discharge rate of vapor q'' in the case of Model II can be calculated by equation (33), substituting the value of $\mathrm{d}u'/\mathrm{d}t$ corresponding to D' which is indicated by equation (26) above in §III. Temperature of vapor at the ground surface T_h' can be calculated by equation (41) subsituting q'' in place of q'.

Now, let the depth of magma chamber be taken h=5 km, the dimension of magma chamber (R=20 km, D=20 km), (R=10 km, D=10 km) and (R=5 km, D=5 km), ρ'=2.60, u_0=1200°C, u_1=900°C, the saturated water content per unit mass of liquid part of magma by weight β_0=0.02, the specific heat of water vapor at constant volume c_v=0.29 cal/deg.gr and at constant pressure c_p = 0.39 cal/deg.gr (at 700°C), σ = 18, R' = 8.32 × 10^7erg/°K, the gravity acceleration $g=980\,\text{cm/sec}^2$, $\frac{r_0}{l}=\frac{1}{5000}$*, and the heat conductivity of the Earth's crust $\kappa=5.4\times10^{-3}$ cal/deg·cm·sec. Changes of q'' and T_h' with time are calculated as shown in Fig. 15. Thick and thin lines in the figure show respectively the changes of T_h' and q''. As illustrated in the figure, volume out-put and temperature of hot springs of magmatic origin are large in the earlier part of molten stage of magma and gradually diminish with time. If the effects of mixing of cold underground water with the ascending water vapor and of the energy loss due to the friction between ascending vapor

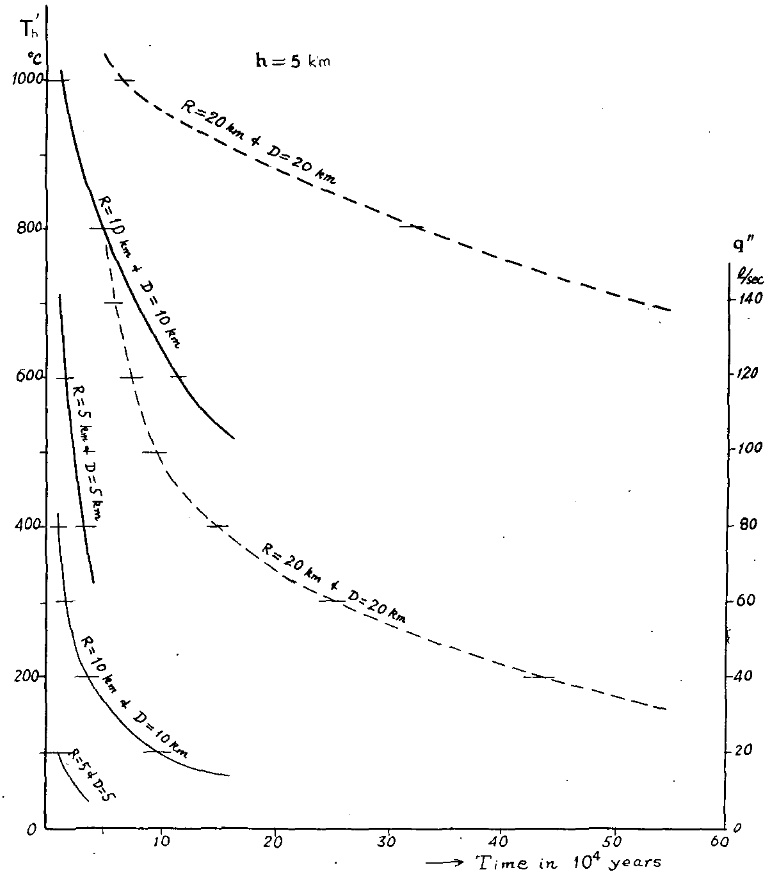

Fig. 15. Secular change of orifice temperature T_h' and of volume out-put q'' of volcanic hot springs of magmatic origin.

* According to G. Okamoto[19], effect of temperature of ascending hot water in a vertical pipe upon country rock is negligible without this distance.

and side wall are negligibly small, temperature of hot springs of magmatic origin at the ground surface is, in general, higher than the critical temperature of water (374°C). Such high temperatures are occasionally observed in fumaroles in the neighborhood of a crater of an active volcano. Temperature of less than the critical temperature of water may be expected for the smaller value of R and the larger values of D and h, especially in the later part of the molten stage of magma. Such temperatures are sometimes observed in some fumaroles and in the underground of several active boiling springs.

Comparison of the calculated volume out-put with the observed total volume out-put of hot springs in a locality in Table VII, enables it to be said that the former is of the same order of magnitude as the latter.

Table VII. Observed maximum temperature and volume out-put from fumarole and hot spring localities.

Locality		Maximum temperature in °C	Discharge rate (l/sec)	Total	Note
Izu-Ooshima Volcano (Japan), fumarole[21]		800~50			
Usu Volcano (Japan)	Kamenoko-iwa, Showa-dome[20]	314	8.5	60	
	fissure, Showa-dome[20]		1.2		
	Dōyako hot spring[17]	50	50		
Tarumae Volcano (Japan), fissure, east side of dome[19]		286	3.9		
Meakan Volcano (Jadan)	Obuki-fumarole[20]	126	190	194	
	Akankohan hot spring[17]	99	4		
Kuccharo Volcano (Japan)	Iwosan-fumarole[22]	> 120	—	> 170	
	Kawayu hot spring[17]		170		
Wairakai Area (New Zealand)[15]		270			
Yellowstone Area[16] (U.S.A.)	Norris Basin	205			
	Upper Basin	180			
Kuttara Volcono (Japan)[17]	Jigokudani	> 150	56~36	157~137	
	Ōyunuma	112	101		
Stemboat (U.S.A)[16]		172			
Baba, Niseko (Japan)[17]		> 120	8		
Sōunkyo Hot spring (〃)[17]		94	22		
Tokachigawa Hot spring (〃)[17]		50	31		

2) Relation between temperature and volume out-put of volcanic hot springs of meteoric origin.

Heat flow at the ground surface conducted by rock cover from a magma chamber has already been discussed in §V–(2). In this case, if underground water exists in the shallow part of the rock cover, its temperature should be higher than that of normal underground water due to the effect of large heat flow in comparison with normal value. The writer tried to estimate the order of magnitude of temperature of underground water considering the following simplest model.

A high temperature plane surface of θ_0°C and a horizontal aquifer of slight thickness are assumed to exist respectively at depths h and d from the ground surface of temperature 0°C, where $0<d<h$. Let it be considered that ground water of temperature T_0 and of flow rate per unit width of an aquifer V supplied from one end of the aquifer and discharged from the other end after passing slowly through. The total length of the aquifer is divided into n sections of equal length ΔL perpendicular to flow direction of the underground water as shown in Fig. 16. Denote by ΔT_n the increase of water temperature due to vertical heat flow from the deep for the n-th part.

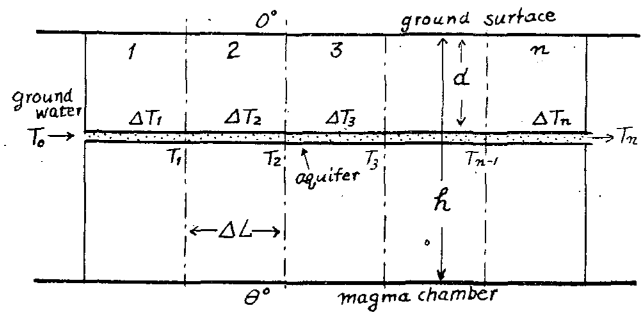

Fig. 16. A schematic map of the model.

Put T_n, ρ and c respectively as the temperature, density and specific heat of discharged water, $(\mathrm{d}\theta/\mathrm{d}z)_{u,n}$ and $(\mathrm{d}\theta/\mathrm{d}z)_{l,n}$ respectively the vertical thermal gradients of country rock at the upper and the lower surfaces of the aquifer, κ the heat conductivity of country rock, and α the stationary thermal gradient in the upper Earth's crust in the case where no aquifer exists. Then, relation between the temperature of discharged water and the thermal gradient of the country rock in stationary state with reference to the n-th section is given by

$$V\rho c\,\Delta T_n=\kappa\,\Delta L\left\{\left(\frac{\mathrm{d}\theta}{\mathrm{d}z}\right)_{l,n}-\left(\frac{\mathrm{d}\theta}{\mathrm{d}z}\right)_{u,n}\right\}. \tag{42}$$

Substituting the approximate values $\left(\frac{\mathrm{d}\theta}{\mathrm{d}z}\right)_{u,n}=\frac{T_{n-1}+\Delta T_n}{d}$ and $\left(\frac{\mathrm{d}\theta}{\mathrm{d}z}\right)_{l,n}=\frac{\theta_0-(T_{n-1}+\Delta T_n)}{h-d}$ instead of $\left(\frac{\mathrm{d}\theta}{\mathrm{d}z}\right)_{u,n}$ and $\left(\frac{\mathrm{d}\theta}{\mathrm{d}z}\right)_{l,n}$ in this equation, one gets

$$\left(\frac{V\rho c}{\kappa\Delta L}+\lambda\right)\Delta T_n=\left(\frac{\theta_0}{h-d}-\lambda T_{n-1}\right),$$

where

$$\lambda\equiv\frac{h}{d(h-d)}.$$

If one puts $\frac{1}{\beta} \equiv \left(\frac{V\rho c}{\kappa \Delta L} + \lambda\right)$, $\gamma \equiv \frac{\theta_0}{h-d} - \lambda T_0$ and substitutes $n=1, 2, 3, \cdots\cdots n$ into the equation (42), he obtains

$$\begin{aligned}
\Delta T_1 &= \gamma \beta \\
\Delta T_2 &= \gamma \beta (1-\lambda \beta) \\
\Delta T_3 &= \gamma \beta (1-\lambda \beta)^2 \\
&\cdots\cdots\cdots\cdots \\
\Delta T_n &= \gamma \beta (1-\lambda \beta)^{n-1} .
\end{aligned}$$

Then the temperature of discharged water T_n is expressed by

$$\begin{aligned}
T_n = T_0 + \sum_{n=1}^{n} \Delta T_n &= T_0 + \frac{\gamma}{\lambda} \left\{1-(1-\lambda \beta)^n\right\} \\
&= T_0 + (a h - T_0) \left[1 - \left\{\frac{1}{1+\dfrac{\kappa h \Delta L}{V \rho c d (h-d)}}\right\}^n\right] . \qquad (43)
\end{aligned}$$

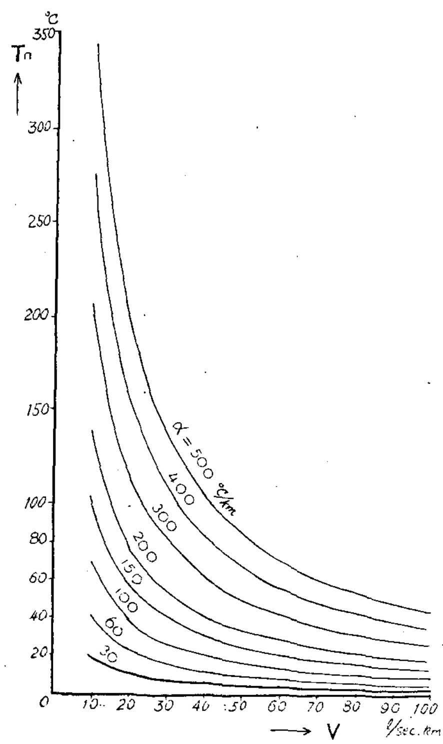

Fig. 17. Relation between temperature of discharged underground water T_n and the volume out-put V.

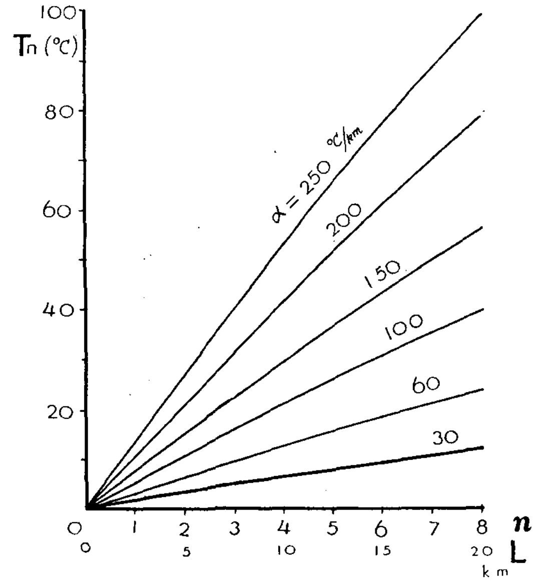

Fig. 18. Change of temperature of the discharged ground water T_n with increase of the length of aquifer $L=n.\Delta L$.

In special case when $1 \gg \chi \equiv \dfrac{\kappa h \Delta L}{V \rho c d (h-d)}$, equation (43) is transformed to

$$T_n = \alpha d \left\{ n\chi - \frac{n(n+1)}{2}\chi^2 \right\} + T_0 \left\{ 1 - n\chi + \frac{n(n+1)}{2}\chi^2 \right\}. \qquad (43)'$$

For example, if numerical values of the constants are adopted as $\rho c \simeq 1$, $\kappa = 5.4 \times 10^{-3}$ cal/deg·cm·sec, $h = 5$ km, $d = 2$ km, $\alpha = 30 \sim 500$°C/km, $\Delta L =$ 2.5km, $V = 10 \sim 100$ litres/sec·km and $n = 4$ (length of the aquifer $L = 10$ km), the temperature of dischanged water is calculated as shown in Fig. 17 when $T_0=0$. Fig. 18 shows the change of T_n with n, when $T_0=0$ and $V=40$ l/sec·km.

The value of χ is given as $0.1 \sim 0.01$ in the above evaluation, then (43)′ holds to the extent that the value of V is more than 20 l/sec.km.

Table VIII. Correction for T_n when $T_0 \neq 0$.

T_0 \ V	10l/sec·km	20	40	60	80	100 l/sec·km
0°C	0.0	0.0	0.0	0.0	0.0	0.0
5	3.3	4.0	4.5	4.6	4.7	4.8
10	6.6	8.1	9.0	9.3	9.5	9.6
15	9.8	12.1	13.4	13.9	14.2	14.3

The correction for T_n in the case where T_0 is not zero is given by Table VIII from (43)'.

Thin curves in Fig. 17 represent change of temperature of discharged water with the discharge rate V for $\alpha = 500\sim60°C/km$, and the thick curve ($\alpha=30$ C°/km) corresponds nearly to the normal place having no special heat source as a magma chamber.

If the discharge rate of water V becomes zero the temperature tends to αd which is the maximum temperature of discharged water, but it diminishes suddenly with increase of discharge rate and tends to zero for a certain value of thermal gradient.

The order of magnitude of flow rate V in an actual aquifer is estimated as follows: In Kuccharo Caldera, Hokkaido, there are Kawayu, Nibushi, Sunayu, Ikenoyu, Ponto and Wakoto hot springs, the total number of orifices of these hot springs is about 70 and total sum of the volume out-put is estimated as 2×10^2 l/sec. If the area of the upper surface of the magma chamber which is supposed to exist at several kilometers below the caldera is assumed to be the same as the area of the caldera (430 km²), and if the existence of these hot springs depends upon the above-mentioned mechanism, the value of V may take, in this case, roughly $10\sim50$ l/sec.km for the length of aquifer $L=$ 10 km and the ratio of area of the aquifer to the area of the magma chamber $1/2\sim1/10$.

In Japan, there are about ten thousand hot spring orifices in the over-all hot spring area of about 6×10^4 km²; total discharge of hot water is estimated as about 3×10^4 litres/sec. Then, if the total area of these hot spring aquifers is assumed as $1/2\sim1/10$ of the total hot spring area and if the length of an aquifer is about 10 km, the value of V becomes $10\sim50$ l/sec.km.

In an actual case of meteoric water, the effect should be considered of heating or cooling caused by descending or ascending of ground water through rock between the ground surface and the aquifer. But, any such effect is neglected in above calculation. To counter balance such neglect, the initial temperature of ground water was adopted as T_0, though the actual numerical value of T_0 is unknown.

It may be more practical to estimate the rough order of magnitude of the increase of temperature, if $T_0=0$ is taken and the sum of the legnth of aquifer and the depth of the aquifer from the ground surface is taken as $(L+d)$ instead of the actual length of the aquifer L in equation (43). This assumption may be reasonable when the horizontal length of aquifer is very large in comparison with the depth of the aquifer.

For this reason, Fig. 17 may give the rough order of magnitude of the increase of temperature when the meteoric water intruded at a depth of 2 km flows 8 km along a horizontal aquifer and is discharged again to the ground surface.

As illustrated in Fig. 17, there is hardly any possibility that a hot spring of meteoric water origin of more than 40°C in temperature is originated due to the normal geothermal gradient of about 30°C/km, even if the initial temperature of the meteoric water be considered as 10°C. But, if the geothermal gradient is larger than 100°C/km and discharge rate of water is less than 20 l/sec.km, the increase of temperature may take a larger value than 40°C as shown in the figure.

When a geothermal gradient is more than 200°C/km, a hot spring of superheated vapor is possible for a small value of discharge rate.

On the contrary, when a geothermal gradient is 60~100°C/km, only a tepid spring is possible except in a case in which its discharge rate is less than 15 l/sec.km.

At present, it is difficult to distinguish positively the hot springs of meteoric origin from those of magmatic origin for the actual cases of respective hot springs in spite of the above-mentioned theoretical possibility. But, some facts are known that suggest statistically the existence of this kind of hot spring. An example is described in the following.

Taking the chemical residue due to evaporation of respective hot spring in abscissa and the corresponding temperature in ordinate, the writer plotted in Fig. 19 by numerals the number of hot springs contained in respective grid areas which were formed by dividing the total area of the map at every 10°C in temperature and at every 0.2 g/l in chemical residue, and drew curves which show respectively the same frequency of hot springs 2.5, 5, 10 and 20 for all of the hot springs listed in the publications of the Hygienic Laboratory of

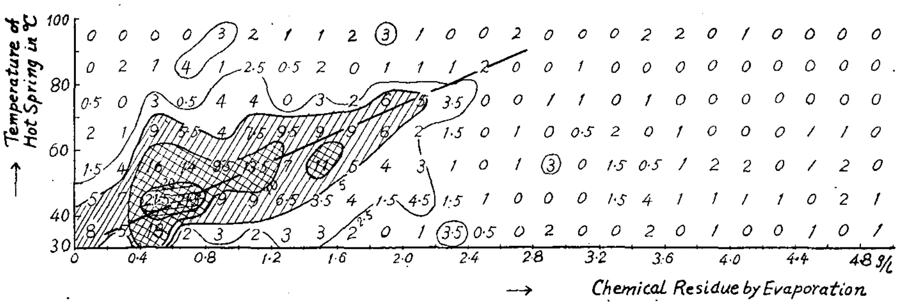

Fig. 19. Relation between temperature of a hot spring and corresponding chemical residue due to evaporation in Japan.

Japan. In Fig. 19, the writer shows conveniently only a part of graph of less than 5 g/l in chemical residue. The equal frequency curves indicate, as shown, the tendency that the total amount of chemical constituents contained in a unit volume of a hot spring increases with the rise of temperature of the hot spring. But, when all the hot springs containing those of larger chemical residue than 5 g/l were included, such definite inclination could not be found.

If it is assumed that the meteoric water intruded into the underground rock at several kilometers below the ground surface might be, in general, a chemically thinner solution than the magmatic water and that larger the length of path through which the meteoric water flow, the higher the temperature and the larger the concentration of chemical components, hot springs indicating a positive correlation between the chemical residue and the corresponding temperature in Fig. 19 might be said to be hot springs of meteoric origin.

VII. Concluding remarks.

Some theoretical discussions were presented, in this paper, under rough assumptions on the life spans of the volcanic hot springs of magmatic water origin and of meteoric water origin, and also on the possible discharge rate of heat, volume out-put and temperature of these hot springs. The chief conclusions are summarized as follows:

1) Volcanic hot springs of meteoric water origin as well as those of magmatic water origin are possible from the points of view of heat energy, volume out-put and temperature, if the depth of the magma chamber h is less than 10 km.

2) The life span T_1 of a volcanic hot spring of magmatic water origin may be in the range from several tens of thousand to several hundreds of thousand years for the probable depth $h=5\sim10$ km, and the probable dimensions of the magma chamber $R=5\sim10$ km, $D=5\sim15$ km.

3) The life span T_2 of a volcanic hot spring of meteoric water origin may be within a few tens of thousand to one and a half million years for $R=5\sim10$ km, $D=5\sim15$ km and $h=2.5\sim10$ km.

4) The duration of life T_3 of a volcanic hot spring which is a combination of the above-noted two sorts of hot springs may be of the order of magnitude from several hundreds of thousand to 2 million years for the same dimensions of magma chamber. Then, it is concluded that the present hot spring activities seen in the Circum-Pacific zone may presumably be caused by the vulcanism which occurred in the later stage of the Cenozoic.

An exact discussion of these problems is difficult at present on account of lack of knowledge of the magma chamber, of the phenomena which take place in it and also on account of the difficulty of mathematical calculation. The writer wishes to request readers to be generous in attitude toward his primitive treatment of the problem in this paper under rough assumptions, since it is the first trial in this field.

In this paper, the discussion was based upon the assumptions that the temperature of total solidification of magma is 900°C and that a large part of water contained in the magma is discharged in its molten stage. But, it is well known that some kinds of magma intruded deep in the Earth's crust keep the original water content and have, in the course of magmatic differentiation, a pegmatite period, in which the magma maintains a liquid state even to a relatively low temperature of about 500°C. It seems highly desirable to investigate in future the possibility of occurrence and the life span of hot springs originated from these kinds of magma.

In conclusion it should be said that almost all of this work was done in the School of Mineral Sciences, Stanford University, U.S.A. during a stay from December, 1958, to November, 1959. The writer wishes, here, to express his hearty thanks to Professor Charles F. Park, Professor Joshua L. Soske, Professor George A. Thompson, Professor Stanley N. Davis, Professor Robert R. Compton and Professor Konrad B. Krauskopf for their kind advice and discussion concerning this work. A part of the expense of this work was defrayed from the Funds for Scientific Research from the Ministry of Education, Japanese Government.

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