Jour. Fac. Sci., Hokkaido Univ., Ser. VII (Geophysics), Vol. 11, No. 6, 845-880, 2002.

Spherical Terrain Corrections for Gravity Anomaly Using a Digital Elevation Model Gridded with Nodes at Every 50 m

Akihiko Yamamoto

Institute of Seismology and Volcanology, Hokkaido University, Sapporo, Japan

(Received December 15, 2001)

Abstract

A comprehensive description is given of a method to compute spherical terrain corrections for gravity data using a digital elevation model (DEM) spaced at every 50 m, provided by the Geographical Survey Institute of Japan. The present method is based on a standard right rectangular prism model in the distant area from a gravity station. On the other hand, surface topography is strictly approximated by truncated cones and/or cylindrically-truncated rectangular prisms for the first nine square compartments of DEM as for the nearby terrain. Analytical solutions are used to calculate the gravity effects due to a right rectangular prism, a truncated cone, and a cylindrically-truncated rectangular prism. We discuss the spherical effect of the earth and consider the earth's sphericity strictly in computations of terrain corrections on the basis of the idea that gravity correction is separated into two steps; (1) directly apply the spherical Bouguer correction within a finite distance from a gravity station, and (2) apply the spherical terrain correction for departures of the actual earth's surface from an idealized spherical surface. In this way, a complete program ("TC50") based on the present procedures is written in standard C language. The overall accuracy to compute spherical terrain corrections, using a DEM gridded with nodes at every 50 m, is evaluated by comparing several numerical results for practical field data. Further, spherical terrain corrections are computed as a function of a radius (R) within which terrain effects are systematically estimated in order to clarify quantitatively to what spatial extent the terrain corrections should be made. In addition, variations of spherical Bouguer correction subtracted by spherical terrain correction are also focused to evaluate the actual behaviour of the program and spatial extent of the terrain corrections. The numerical results suggest that the relative difference of terrain corrections between at R km and $R+10$ km even at stations more than 3,000 m high does not exceed 0.5 mgal if we take $R>80$ km. Finally, terrain correction values by the proposed technique and other methods are compared for thousands of actual field data obtained in plains and mountainous regions with rugged topography. It is shown that TC50 computes spherical terrain corrections with relative difference within about 2 mgals from those computed by other methods.

1. Introduction

We require terrain corrections to evaluate complete Bouguer anomalies particularly for gravity measurements made in a rugged topography, where the magnitude of terrain corrections can be comparable to Bouguer anomalies. A simple Bouguer anomaly, ignoring the shape of the topography, tends to overcompensate gravity measurements made near topographic features. The terrain corrections adjust for this overcompensation and is an essential step in reducing gravity measurements made in places of moderate to extreme topographic relief. In a conventional calculation for flat slab model, gravity terrain corrections account for the upward pull of surface material which are higher than a gravity station and the lack of downward pull from open space (material deficit) which is lower than the station. In this way, terrain correction is well known to be always positive. At first when a digital computer is not available, terrain correction was by far an awkward and time-consuming task and so-called Hayford-Bowie (1912) or Hammer's (1939) chart was used manually, usually with topographic maps. This involves centering a transparent overlay over each station on a topographic maps. Sandberg (1958) used an inclined plane top in each cylindrical compartment as an approximation to more accurately represent the topography near a gravity station. Recently, Olivier and Simard (1981) presented a variation of the Hammer's chart technique which uses an algorithm for a segment annulus with conical top and horizontal bottom. Barrows and Fett (1991) reported a manual technique based on the algorithm of Olivier and Simard (1981), that uses a sloping wedge. Spielman and Ponce (1984) described the development of terrain models by hand with templates and topographic maps.

Nowadays, however, with an increasing availability of a digital elevation model (DEM), terrain corrections can be computed automatically using a high-speed computer. Topographic corrections computed using a DEM are possibly more accurate than manual techniques because of inherent subjectivity involved in the selection of the average elevation within the template compartments. A vast literature has numerous contributions dealing with topographic corrections. In general, topographic correction has been computed by modeling the physical topographic relief using a collection of prisms (Nagy, 1966a), where elevation data of a DEM are normally defined on a regularly spaced lattice. The analytical solution for a right rectangular prism is given by many authors (Nagy, 1966b; Banerjee and Gupta, 1977; Garcia-Abdeslem, 1992). A complete usage of a DEM in topographic corrections was first done by Kane (1962) in which elevations were defined in a regular grid. Granar (1976) and Ketelaar (1976) independently developed a method in which the terrain around a gravity station is divided into circular zones filled with square prisms with sloping upper surface. Granar (1987) modified his method and developed a hybrid method that combines the Hammer's chart method to evaluate the near-station effect up to 100 m with a partitioning of the terrain into rectangular prisms elsewhere. Alternatively, terrain modeling using Gaussian quadrature integration has been reported by Mateskon and von Frese (1985). Herrera-Barrientos and Fernandez (1991) used Gaussian surface-fitting to obtain the average elevation within the Hammer compartments for terrain corrections. More recently, Garcia-Abdeslem and Martin-Atienza (2001) developed a method based on a forward solution to compute the gravity effect due to a rectangular prism of uniform mass density that is flat at its base but has a non-flat top. Li and Sideris (1994) investigated the effect of different topographic representations that are suitable for efficient processing of high volumes of DEM data on terrain corrections, and developed a set of new formulae corresponding to the mass prism topographic model, which can be evaluated efficiently with the 2-D FFT method. Parker (1995, 1996) developed a modern technique in the Fourier domain for calculating the gravitational attraction of a layer with an irregular top surface for application in terrain corrections for land and marine gravity stations.

In Japan, most researchers have used a DEM gridded at 2,000 m (e.g. Hagiwara, 1967) or 500 m (e.g. Kono and Kubo, 1983) developed by the Geographical Survey Institute of Japan (GSI). Hagiwara (1967) was first to calculate nation-wide terrain corrections in Japan using a DEM gridded at every 2,000 m, by approximating near-station and distant topography with a stack of pentahedral material and line-mass, respectively. In 1975, a DEM gridded at every 250 m (so-called KS-110) was provided by GSI. Since then, KS-110 has been frequently used for topographic corrections in Japan. Usage of KS-110 in gravimetric reductions has been the accepted standard of terrain corrections in Japan. Tamada (1979) expressed much concern about the effect of the earth's curvature for terrain corrections as a step of gravity reductions. Nozaki (1981) developed a program for spherical terrain corrections using KS-110, in which the surface topography was modeled by a pentahedron in the near-station zone (up to 250 m), while rectangular prisms were used in the distant zones. Yamamoto et al. (1982) also developed a method, using KS-110, by modeling the surface relief in the near-station zone (up to 200 m) using a combination of truncated cones and cylindrically-truncated rectangular prisms to more accurately represent the topography near a gravity station, with a partitioning of the terrain into rectangular prisms elsewhere (hereafter referred to as "*Method Y*"). Another procedure to compute the effect of surface topography, basically similar to Hagiwara (1967), was reported by Kono and Kubo (1983), where a meshed mean-height-data technique was employed incorporating a DEM gridded at every 500 m. Similarly, Katsura et al. (1987) made a program for terrain corrections, using KS-110-1 (revised version of KS-110), where near-station topography was approximated by pentahedra as employed in Hagiwara (1967) and Kono and Kubo (1983). On the other hand, Komazawa (1988) modeled the surface relief using a set of flat-top annular prisms for terrain corrections. In his model terrain regions were subdivided into four parts, according to whose distance from a station KS-110 was reformatted by partitioning the terrain into annular prisms. Recently, GSI (2001) developed and published the digital elevation model spaced at every 50 meters on a grid (hereafter referred to as DEM50) for more accurate representation of the physical land-relief in Japan.

The aim of this paper is to present a method for computing spherical terrain corrections using DEM50 with a combination of rectangular prisms, truncated cones and cylindrically-truncated rectangular prisms. In the present method the sphericity of the earth is strictly taken into account, according to which the vertical component of gravity attractions due to the topographic relief is calculated separately in actual computation. Also the results using KS-110 and DEM50 are compared with some field examples.

2. Digital Elevation Model (DEM)

Precise topographic corrections require an accurate representation of the surface relief from which the gravity effects are calculated. The accuracy of terrain corrections strongly depends on how well a DEM represents the real topography, particularly in the vicinity of a gravity station with a rugged topography, as well as how compatible are a DEM and the elevations at the gravity stations. In Japan, GSI developed "the Numerical Land Information: Land form (mean, maximum, minimum, relief energy)" called KS-114-1 in early 1970s, in which the surface topography was gridded at every 500 m and geographical coordinates of a regular grid are given at every 20×22.5 seconds of arc ($\sim 500 \times 500\ m^2$) along NS (meridian line) and EW (parallel line) directions, respectively. Successively, GSI provided a DEM called KS-110 as "the National Land Numerical Information" in 1975, in which the surface topography of land area in Japan was gridded with nodes at every 250 m. KS-110 represents the topography by a regular grid of orthometric elevations in geographical coordinates in the Tokyo Datum geodetic system. Geographical coordinates of a regular grid of KS-110 are given at every 7.5×11.25 seconds of arc ($\sim 250 \times 250$ m^2) along NS and EW directions, respectively. The original version of KS-110 is known to have several bugs and GSI published a revised version called KS-110-1 in 1983.

Fairly recently, GSI (2001) has newly published DEM50, gridded with nodes at every 50 m, to more accurately represent the real topography in which geographical coordinates of a regular grid are given at every 1.5×2.25 seconds of arc ($\sim 50 \times 50$ m^2) along NS and EW directions, respectively. DEM50 is the most precise nation-wide DEM ever known in Japan, except for the GISMAP terrain, having a grid spacing of 10 m, by Hokkaido Chizu Co., Ltd. which is constructed simply by gridding from contour lines directly read on 1/25,000 scale topographic maps provided by GSI. Fig. 1 demonstrates an example on how well a digital topography represents the real topography around Daisetsu mountainous area in Hokkaido, Japan, using KS-110-1 (Fig. 1a) and DEM50 (Fig. 1b), respectively. Colored image of the topographic relief is shown at the bottom and its 3-D representation is also illustrated at the top. Azimuth and elevation of the view point of 3-D image are 140° clockwise from north and 35°, respectively. Location of Mt. Asahidake is marked in the figures. Grid size of these figures is reduced to 0.01 minutes (=0.6 seconds) of arc ($\sim$16 meters) both along NS and EW directions by interpolating a regular grid of each DEM. In all figures of Fig. 1 the digital topography is illuminated by the light from the due north direction. Note that the sharpness of visual image is significantly enhanced by the illumination. We can see DEM50 well represents the real topographic relief in Fig. 1b, while it is obvious that the topography by KS-110-1 is poorly resolved and no improvements to represent the real topography can be achieved by interpolating a regular grid to reduce its size in KS-110-1 spaced at 250 m as shown in Fig. 1a. This is quite natural because elevation data of KS-110-1 do not contain any information at wavelengths shorter than twice the grid spacing ($\sim$250 meters). It is clear that DEM50 is undoubtdely suitable to cases where the topographic relief is very rugged. However, it is worth noting that the earlier versions of DEM50 are known to have several bugs in elevation that may cause wrong estimates of terrain corrections in some cases. We can confirm this by showing one example. Fig. 2 illustrates an example of digital topography with spurious elevation data around the Akaishi Mountains, Central Japan. Tiny green dots lying on a regular grid show a DEM evenly spaced at 50 m. In Fig. 2 the digital topography is also illuminated by the light from the due north direction. An uneven topography, having spurious elevation, can be

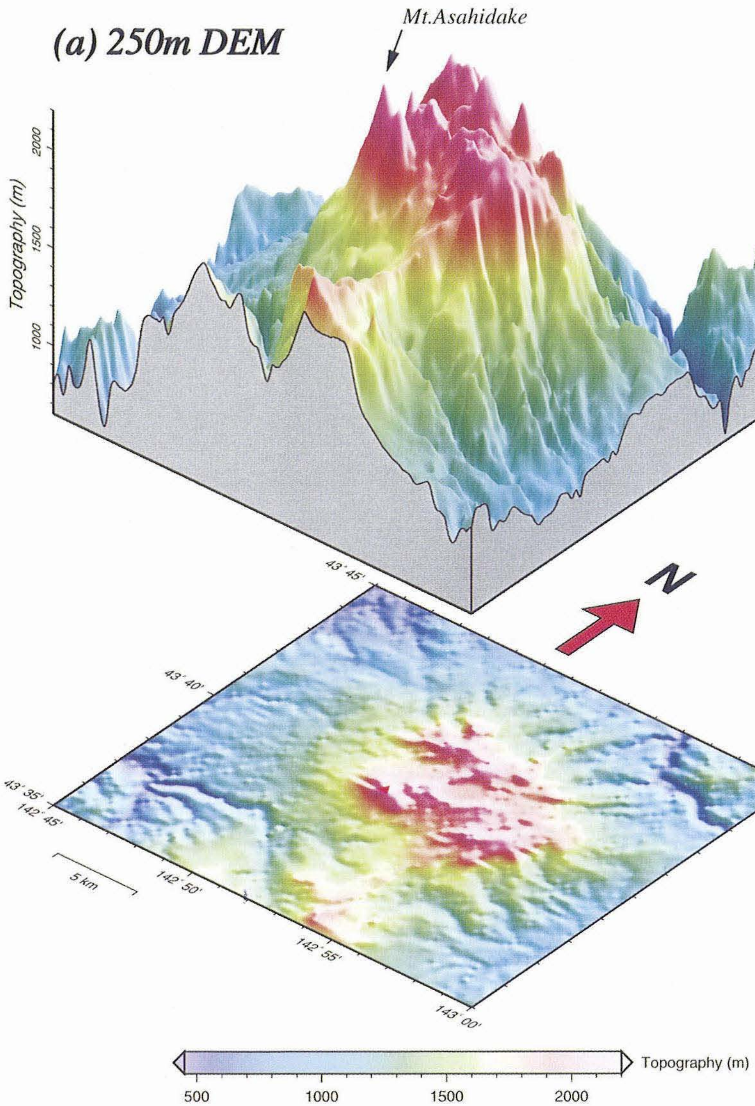


Fig. 1a. An example of digital topography around Daisetsu mountainous area in Hokkaido, Japan, using KS-110-1 (mesh size $\sim 250\times 250\ \mathrm{m}^2$), where colored image of the topographic relief (bottom) and its 3-D representation (top) are illustrated. Azimuth and elevation of the view point of 3-D image are 140° clockwise from north and 35°, respectively. Grid size is reduced to 0.01 minutes of arc (~16 meters) both along north and east directions by interpolating a regular grid of KS-110-1. Location of Mt. Asahidake is marked by the red triangle (bottom) and the black arrow (top). The digital topography is illuminated by the light from the due north direction.

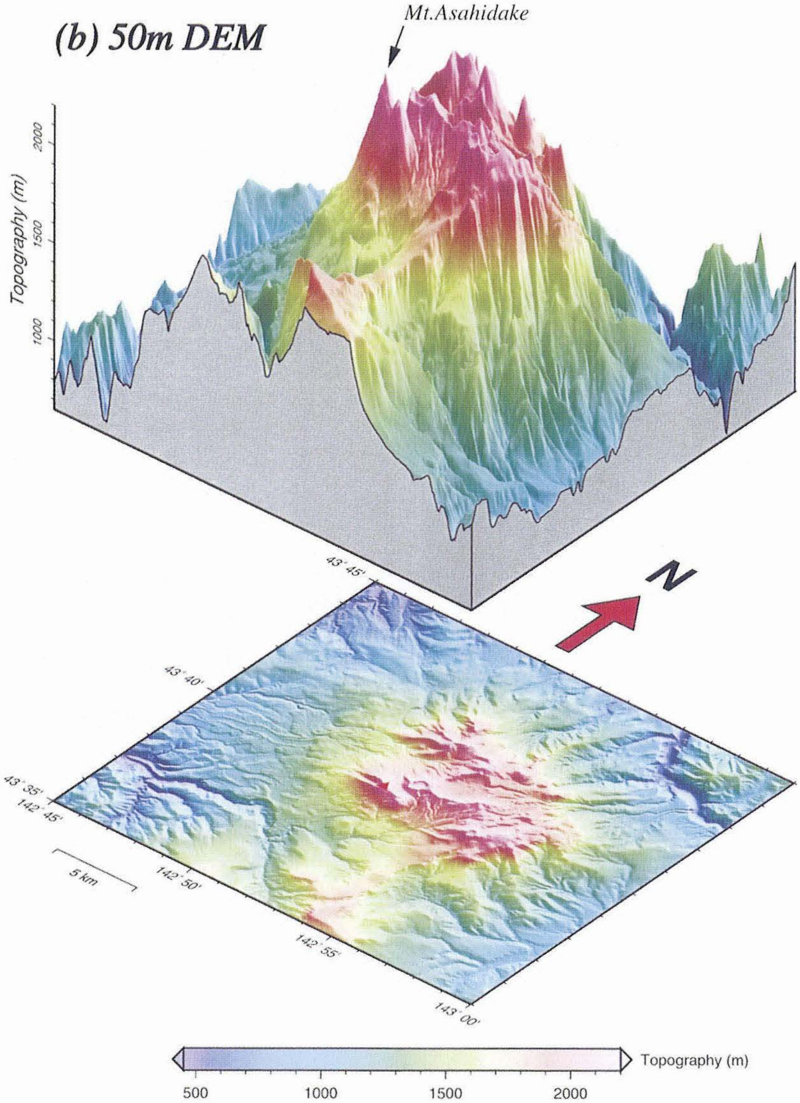


Fig. 1b. Same as Fig. 1a, but using DEM50 (mesh size $\sim 50 \times 50$ m^2) instead of KS-110.

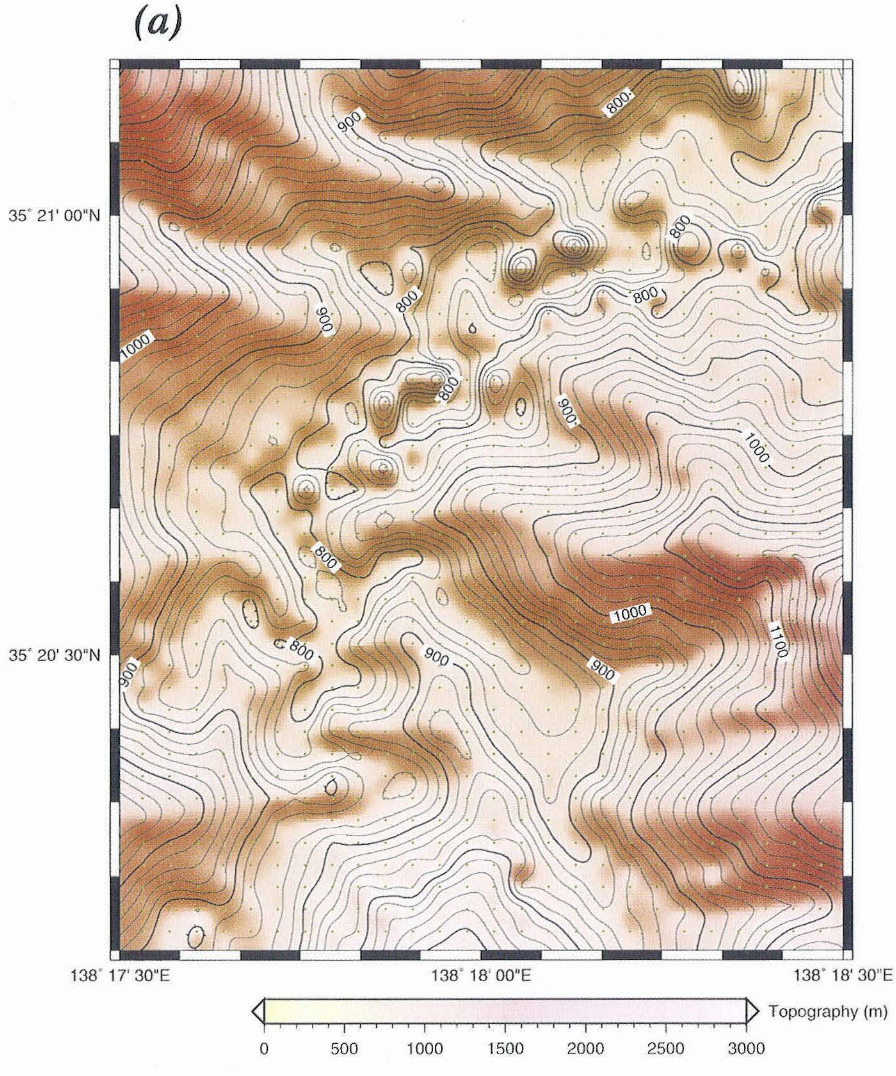


Fig. 2a. An example of digital topography for the earlier version of DEM50 having spurious elevations around the Akaishi Mountains, Central Japan. Contour interval is 20 meters. Tiny green dots plotted on a regular grid show DEM data spaced at every 50 m. These bugs will lead to a serious error in estimates of terrain corrections in some cases.

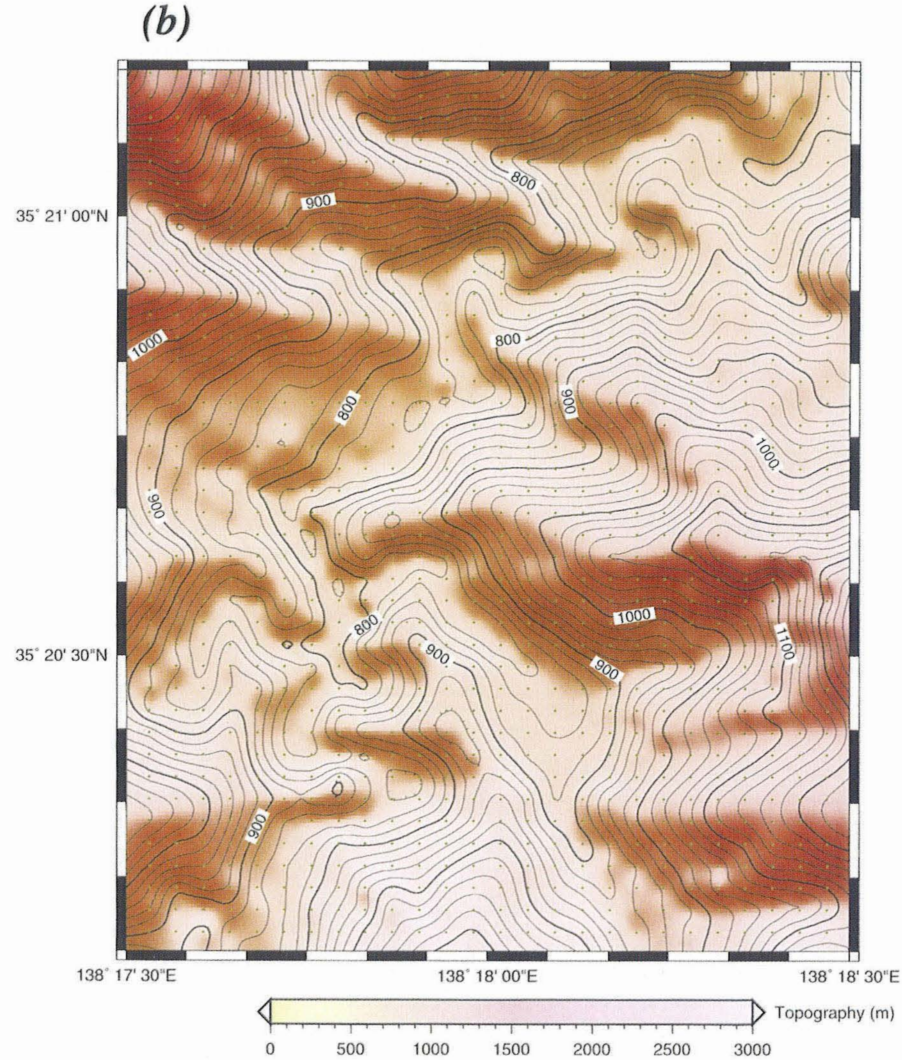


Fig. 2b. Same as Fig. 2a, but for the revised version of DEM50 (Geographical Survey Institute, 2001). Note that the bugs shown in Fig. 2a are successfully fixed.

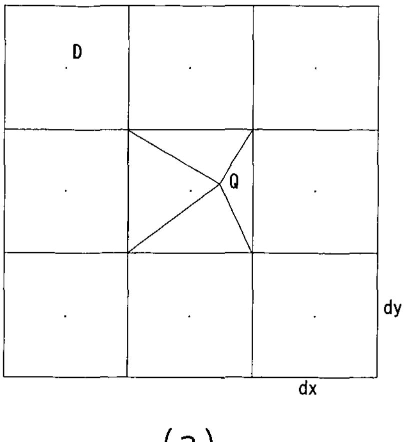
clearly seen along the valley centered in Fig. 2a. These bugs will lead to a serious error in estimates of terrain corrections in some cases. These (and other) bugs are successfully fixed in the latest version (GSI, 2001) as shown in Fig. 2b. We should use the latest version of DEM50 or patches, provided by the Japan Map Center, in which the above (and other) bugs are already fixed. The information on bugs of DEM50 is always available at "http://www.jmc.or.jp/."

3. Theory

This section focuses on the detailed description of the present method for terrain corrections together with some analytic expressions to calculate the gravitational attractions. The earth's sphericity is addressed squarely in a later section.

3.1 Method

Calculations of terrain correction normally include subdividing the range into an outer region within which the contribution of each spot elevation is modeled as a rectangular prism, and an inner region within which both the spot elevations and the probable slopes must be considered in a collection of blocks (right circular cones, cylinders, pentahedra and rectangular prisms etc.). This parameterization allows us to calculate the gravitational attraction caused by each block separately. The sum of the contributions from each of the individual blocks produces total terrain corrections. Modern researchers have used an algorithm for a segmented annulus with conical or flat top and horizontal bottom for inner zone calculations by dividing the area around a gravity station in small compartments bounded by concentric rings and their radii drawn at suitable angular intervals. The mean elevation in each compartment is determined from a DEM or topographic map (e.g. Olivier and Simard, 1981 ; Komazawa, 1988 ; Barrows and Fett, 1991). Some researchers reported the inner zone calculations by fitting polynomial expressions (Gettings, 1982 ; Cogbill, 1990). Plouff (1976) approximated topographic mass as a stack of laminas and derived the gravitational attraction of a layer of finite thickness, with vertical sides and with top and bottom surfaces approximated by polygons. These polygonal layers can be stacked on top of another in order to approximate three dimensional bodies of arbitrary shape to calculate terrain corrections. Tsoulis (2001) presented a method of terrain corrections, using a DEM around the Bavarian Alps, in which the neighbourhoood around the gravity station was modeled using an analytical method, while for the rest far-zones a modified FFT



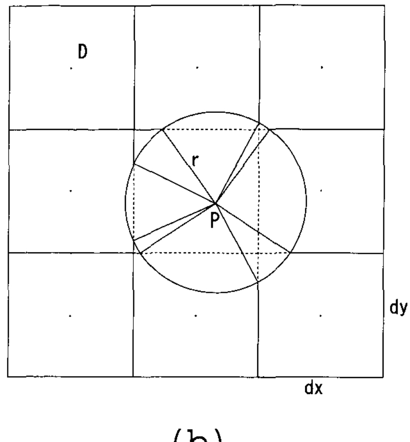
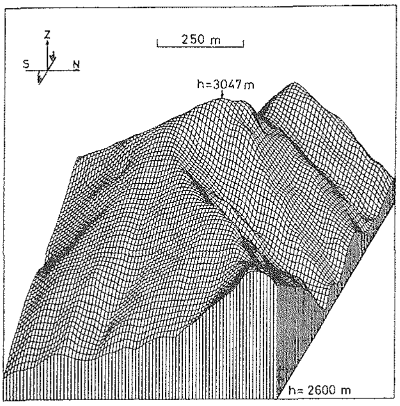


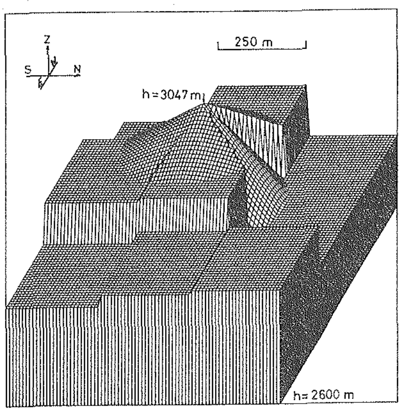
Fig. 3. Schematic diagrams in which surface topography is approximated by two models. (a) Two to four pentahedra are used in a near-station zone within a compartment bounded by a square grid including a gravity station (Q). Each height of the bases of two to four pentahedra is given by the corresponding DEM height (D). (b) A series of (four to eight) cone segments are used in a circular region within a radius of "r" from a gravity station P. The cone segments have a common apex at the station and their basal arc of a cone segment has a common radius of "r." The base of each cone segment has a height defined by the corresponding DEM height (D). In the two models, tiny square dots centering in each compartment show the location of a DEM data. dx and dy represent grid spacings ($\sim 50 \times 50$ m^2) along NS and EW directions, respectively.

series was applied. Generally, there is no compelling reason for a circular division of the terrain except some convenience in manual calculations. For computer-aided procedures a rectangular grid division is preferably used. However, the analytic expression used for calculations for a circular division becomes rather simple (only involving a mathematical function to compute a square root) in modeling terrains by a segmented annulus with flat top and horizontal bottom (Komazawa, 1988).

Hagiwara (1967) and Nozaki (1981) approximated terrains by using two to four pentahedra in a near-station zone within a compartment bounded by a square grid including a gravity station (Q) as shown in Fig. 3a. Tiny square dots centering in each compartment show the location of a DEM data. dx and dy in Fig. 3 represent grid spacings ($\sim 50 \times 50$ m^2) along NS and EW directions, respectively. In their models each height of the bases of two to four pentahedra is given by the corresponding DEM height (D). In the remaining eight compartments they modeled the nearby topography as a right rectangular prism to



(a)



(b)

Fig. 4. Schematic illustration for the correction of topography over the first nine meshes (mesh size $\sim 50 \times 50$ m^2) around a gravity station. (a) Actual topography around Mt.Shiomi, Central Japan, (b) Approximated topography based on the scheme shown in Fig.3b. (After Yamamoto et al., 1982)

estimate terrain effects. Then, they changed the computational procedures according to the distance from a measuring point. Hereafter we call this method "*Method P*."

In the present study we use a combination of truncated circular cone segments, cylindrically-truncated prisms and regular prisms in the near-station zone to estimate terrain effects, incorporating DEM50, on the basis of the technique ("*Method Y*") presented by Yamamoto et al. (1982). Computational procedures are changed according to the distance from a gravity station:

(1) *Zone A.* The topography is approximated by a series of (four to eight) cone segments in a circular region within a radius of "r" meters from a gravity station (P) as shown in Fig. 3b. These cone segments have a common apex at the station and their basal arc of a cone segment has a common radius of "r." The base of each cone segment has a height defined by the corresponding DEM height. This is schematically illustrated in Fig. 4. Generally, "r" takes a value of 30 ~40 meters in the present study. If we take "r" approaching zero, the topography is asymptotically approximated by a rectangular prism. To compute terrain effects for a station measured inside a tunnel, we model the nearby topography in the central compartment including a station by a right rectangular prism (cone segments are not used).

(2) *Zone B.* For the regions outside the Zone A within the nine compartments (see Fig. 3) including the gravity station, the topography is modeled by a rectangular prism or a prism segment truncated by a vertical cylinder of a radius "r" centered on the observation point. We call this prism segment as "cylindrically-truncated prism." Note that all the remaining eight compartments except the central compartment including the station P are modeled by a right rectangular prism if "r" takes a value less than about 25 meters (half the size of DEM grid) when a measuring point lies around the center of the central compartment.

(3) *Zone C.* Within the distance of "Y" km from the station but outside Zone B, the topography in each compartment is modeled by a rectangular right prism. Normally, we take a value of 10~20 km as "Y." Up to Zone C from the gravity station all calculations are made using analytic solutions.

(4) *Zone D.* Within the distance of "W" km from the station but outside Zone C, the topography in each compartment is approximated by a line mass because (1) gravitational attraction decreases rapidly as the distance from a measuring point increases, (2) computation of the attraction due to a right rectangular prism directly using the analytic formula is impractical (it would require much computational CPU time), and (3) the gravity attractions due to a right rectangular prism agree with those due to a line-mass within negligibly small error in sufficient distant area. "W" is normally determined by a trade-off between accuracy we require and computational effort. A value of 50

$\sim$100 km of "W" has been used in general. The estimate of "W" is another problem which will be discussed in a later section.

Hereafter we call the present method, described above, "*Method C*." The analytic solutions used for calculations of gravity attractions in the above regions are summarized below.

3.2 Analytic Expressions

(a) Right Rectangular Prism

The vertical component of the gravitational attraction at the origin due to a right rectangular prism bounded by planes $x=x_1$, $x=x_2$ $(x_2>x_1)$; $y=y_1$, $y=y_2$ $(y_2>y_1)$; and $z=0$, $z=h$ $(h>0)$; is given by

$$g_p=G\rho\{F(x_2,\ y_2,\ h)-F(x_1,\ y_2,\ h)-F(x_2,\ y_1,\ h)+F(x_1,\ y_1,\ h)\}, \tag{1}$$

where G is the universal constant of gravitation, and ρ is the density of the prism (Nagy, 1966b; Banerjee and Gupta, 1977).

The function F is defined by

$$\begin{aligned} F(x,\ y,\ h)=&x\ \ln\left(\frac{y+\sqrt{x^2+y^2}}{y+\sqrt{x^2+y^2+h^2}}\right)\\ &+y\ \ln\left(\frac{x+\sqrt{x^2+y^2}}{x+\sqrt{x^2+y^2+h^2}}\right)\\ &+h\ \arctan\left(\frac{xy}{h\sqrt{x^2+y^2+h^2}}\right). \end{aligned} \tag{2}$$

(b) Line-Mass Approximation for a Right Rectangular Prism

The analytic expression of line mass approximation for the above right rectangular prism is defined by

$$g_\ell=G\rho s_1 s_2\left(\frac{1}{u}-\frac{1}{\sqrt{u^2+h^2}}\right), \tag{3}$$

where

$$\begin{aligned} s_1&=x_2-x_1\\ s_2&=y_2-y_1\\ u&=\sqrt{\left(\frac{x_1+x_2}{2}\right)^2+\left(\frac{y_1+y_2}{2}\right)^2}. \end{aligned}$$

(c) Right Circular Cone

The vertical component of the gravitational attraction at the apex of a circular cone of constant mass density is given by

$$g_{c_1}=2\pi G\rho h\left(1-\frac{h}{\sqrt{a^2+h^2}}\right), \tag{4}$$

where h is the height of the cone and a its radius (e.g. Talwani, 1973; Tsuboi, 1983).

(d) Circular Cone Segment

The vertical component of the gravitational attraction due to a truncated cone segment is given by

$$g_{c_2}=g_{c_1}\cdot\frac{\theta}{2\pi}=\theta G\rho h\left(1-\frac{h}{\sqrt{a^2+h^2}}\right), \tag{5}$$

where θ is the truncated angle.

(e) Cylindrically-Truncated Prism Segment

The vertical component of the gravitational attraction due to a prism segment bounded by planes $x=x_1$, $x=x_2$ $(x_2>x_1)$; $y=y_1$, $y=y_2$ $(y_2>y_1)$; and $z=0$, $z=h$ $(h>0)$; as well as vertically truncated by a cylinder with a radius of "R" $(R>0)$ is given by

$$\begin{aligned}g_q=G\rho\{&H(x_2, y_2, h, R)-H(x_1, y_2, h, R)\\ &-H(x_2, y_1, h, R)+H(x_1, y_1, h, R)\}.\end{aligned} \tag{6}$$

The function H is defined by

$$\begin{aligned}H(x, y, h, R)=&x \ln\left(\frac{R+\sqrt{R^2-x^2}}{\sqrt{R^2+h^2}+\sqrt{R^2-x^2}}\right)\\ &+y \ln\left(\frac{R+\sqrt{R^2-y^2}}{\sqrt{R^2+h^2}+\sqrt{R^2-y^2}}\right)\\ &+(\sqrt{R^2+h^2}-R)\arctan\left(\frac{\sqrt{(R^2-x^2)(R^2-y^2)}-xy}{x\sqrt{R^2-y^2}+y\sqrt{R^2-x^2}}\right)\\ &+h\arctan\left(\frac{h\sqrt{R^2+h^2}(x\sqrt{R^2-y^2}+y\sqrt{R^2-x^2})}{h^2\sqrt{(R^2-x^2)(R^2-y^2)}-xy(R^2+h^2)}\right),\end{aligned} \tag{7}$$

where R is a radius of a vertical cylinder centerd on the measuring point (e.g. Hagiwara, 1978). If we put $R^2=x^2+y^2$ into the equation (7), then it becomes equivalent to (2).

Since elevation values in KS-110 are not defined in the inland water area such as lakes, we need to incorporate the actual elevation manually for these regions before calculation. DEM50, however, involves elevation data also in the inland water area as an actual height of its surface, and we have only to use DEM50 just as it is without any modification in the present approach.

4. Sphericity of the Earth

In a traditional computation, the infinite (or finite) flat-slab model has been adopted for terrain correction since the effect of sphericity of the earth on gravity attraction has been thought to be negligibly small. Actually, Bouguer reduction is not extended to global portions due to the reasons; (1) in cases where relative distant effects are large, errors from nearby terrain are far more significant than those resulting from distant topography, (2) continental elevations over vast regions are generally low (averaging about 680 m), and (3) for many land surveys, much of the distant departure from the normal earth is actually sea water, producing a partial canceling of the effects. Bullard (1936) first pointed out the importance of the sphericity of Bouguer correction, which is known as "Bullard B correction." The purpose of the Bullard B correction as a step in calculating Bouguer anomaly is to convert the geometry for the Bouguer correction from an infinite slab to a spherical cap whose thickness is the elevation of the gravity station. Swick (1942) proposed the three-step correction for gravity reductions; (1) apply the simple infinite-slab Bouguer formula, (2) add a curvature (Bullard B) correction, and (3) apply a terrain (Bullard C) correction for departures of the actual earth's surface from an idealized spherical surface. The first two procedures are functions of elevation and terrain density only, while the third is a function of surrounding topography. Others, such as Takin and Talwani (1966) and Karl (1971), addressed the importance of the earth's curvature for gravity reductions. Vanicek et al. (2001) argued that the Bouguer flat-slab model does not allow for a physically acceptable consideration scheme for the topography. However, curvature corrections are not discussed in the fundamental textbooks such as Jacobs et al. (1959), Nettleton (1976), Telford et al. (1976), Parasnis (1986), Dobrin and Savit (1988), Blakely (1996) etc. More recently, LaFehr (1991b) presented an exact analytic formula for the Bullard B correction. This exact formula yields the startling possibility of as much as a 200 μgal difference from those implied by the Swick tables. Also LaFehr (1991a) emphasized the importance of the gravity reduction standards, incorporating the earth's sphericity, that are needed to improve anomaly quality for interpretation and to facilitate the joining together of different datasets. On the basis of the second Helmert consideration of external topographical masses, Novak et al. (2001) showed that the planar approximation of surface terrains is not adequate and its use can be responsible for the long-wavelength errors in the geoid computations.

In this paper we separate gravity correction into two steps for gravity

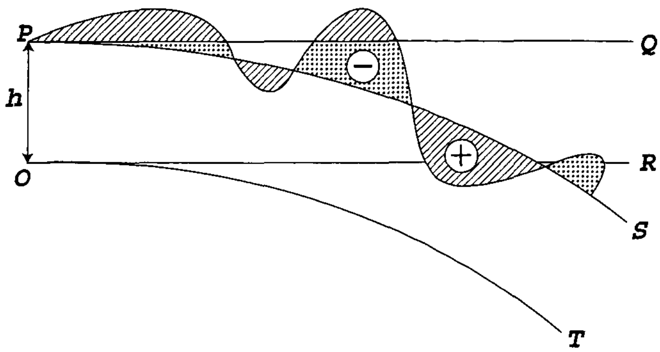


Fig. 5. The geometry of spherical cap in relation to flat Bouguer slab for the earth's sphericity. All dimensions are greatly exaggerated to clearly show the nature of the correction. Hachured and dotted areas have positive and negative signs for terrain corrections, respectively. h : thickness of a spherical cap, P : the location of the station, O : the origin point which passes zero-elevation geoid, PS : the earth's surface which passes the station P, OT : the earth's geoid which passes sea level T, PQ : the horizontal base plane which passes the station P, OR : the horizontal base plane which passes sea level T,

reductions; (1) directly apply the spherical Bouguer correction within a finite distance from a gravity station, and (2) apply the spherical terrain correction for departures of the actual earth's surface from an idealized spherical surface, essentially on the basis of the technique by Yamamoto et al. (1982). In the first step, we compute spherical Bouguer correction using the analytic expression given by Hagiwara (1975). Yamamoto (1984) effectively demonstrated the discrepancy between the conventional flat-slab correction and the spherical Bouguer correction. The finite distance within which the spherical Bouguer and terrain corrections should be made is defined as the arc length, equivalently the solid earth angle subtend by the spherical cap. The earth's sphericity is strictly considered to eliminate the actual terrain effects in the second step. The geometry of spherical cap in relation to flat Bouguer slab is schematically illustrated in Fig. 5. Here all dimensions are greatly exaggerated to clearly show the nature of the correction. P and O are the location of the station and the origin point which passes zero-elevation geoid, respectively.

As shown in Fig. 5, spherical Bouguer correction within a finite area removes all the slab effects of a spherical cap with thickness h, in which the upper surface is defined by PS (the earth's surface which passes the station P). The lower bottom here is defined by the curved plane OT (the earth's geoid which passes sea level T). In our approach, terrain effects are classified in principle as several parts separated by a horizontal line and the curved earth passing through a gravity station; the correction of topography above the horizontal base level (upper surface of flat Bouguer slab) defined by the plane PQ, which passes the station P, in Fig. 5 has positive sign (upward pull), while topography above the station elevation P defined by a curved surface PS, but lower than the upper surface PQ of flat Bouguer slab, should be considered to have negative correction (downward pull). Fig. 5 schematically illustrates this approach in which terrain corrections with positive and negative signs are indicated by a stippled and hachured area, respectively. The terrain correction for topography lower than the elevation of gravity station P is always positive to cancel out the excess Bouguer correction for the lack of downward pull from open space. In actual computation each DEM data is separated to several parts (one to four pieces) to estimate their terrain effects independently according to the above scheme.

5. Method Assessment

According to the procedures fully described in the previous sections, a computer program, called "TC50", to estimate spherical terrain corrections is developed. In this section we discuss an accuracy of spherical terrain corrections by the proposed method and show numerical results for actual field data in which terrain effects are computed for stations measured at typical mountainous areas in Japan. We also investigate the validity of the program and verify its performance in this section. The characteristic features of TC50 in computational view will be summarized briefly in the later section.

5.1 Errors Arising from the Topographic Model

We investigate how the different DEMs unfavorably affect terrain corrections by using the same program. Fig. 6 is a good example which shows a comparison of terrain corrections by "*Method Y*" (Yamamoto et al., 1982) for more than 3,000 gravity data in Hokkaido, Japan, using KS-110 (original version) and KS-110-1 (revised version). Each data is colored according to its elevation. The terrain correction results by using these two DEMs agree well with each other except for several outliers. Note that most of these outliers lie in the lower side of $y=x$ line. This fact indicates that the estimates of terrain corrections at the stations of these outliers by using KS-110 are rather larger than those by KS-110-1. The largest difference in this case is at most about 4 mgals. Consequently, this fact is interpreted as these outliers come from the integration error due to spurious elevations on summing contributions from a

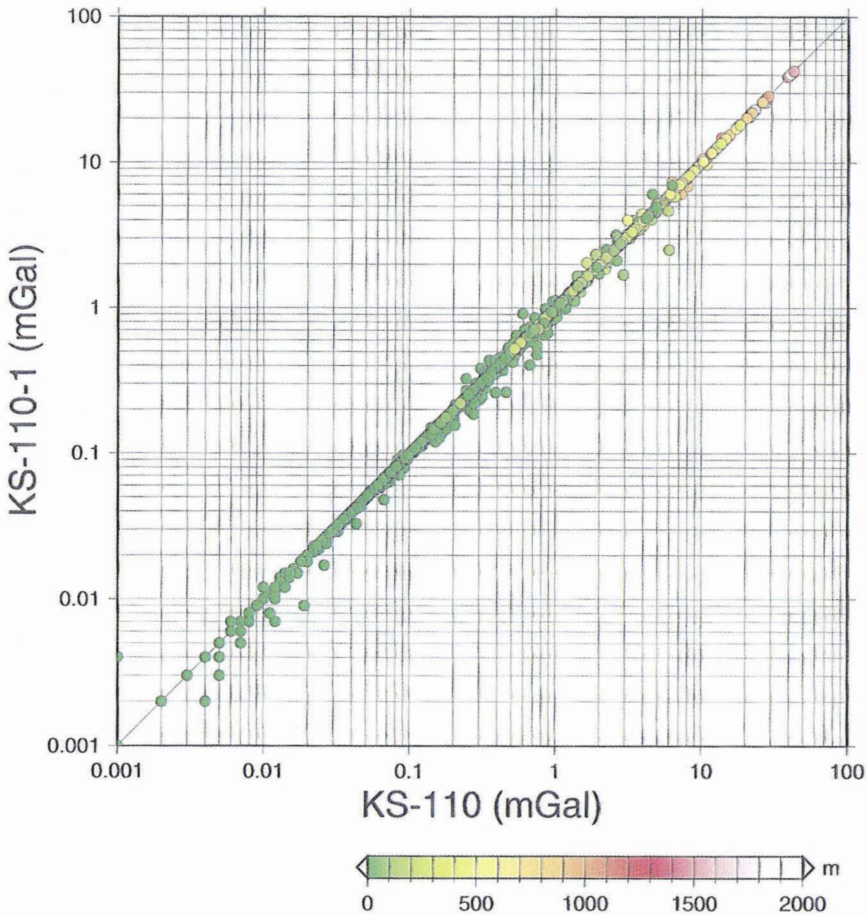


Fig. 6. Comparison of terrain corrections by Yamamoto et al. (1982) for more than 3,000 gravity data in Hokkaido, Japan, using KS-110 (original version) and KS-110-1 (revised version). Each station is colored according to its elevation value. Several outliers come from the discrepancy of elevation values between KS-110 and KS-110-1.

column of material, defined by DEM, under each topographic point. As far as the author investigates, the maximum correction error in Japan due to spurious DEM data can be found at the Central Ranges whose value is about 15 mgals. It should be pointed out that we need to use bug-free DEM in estimating terrain corrections properly.

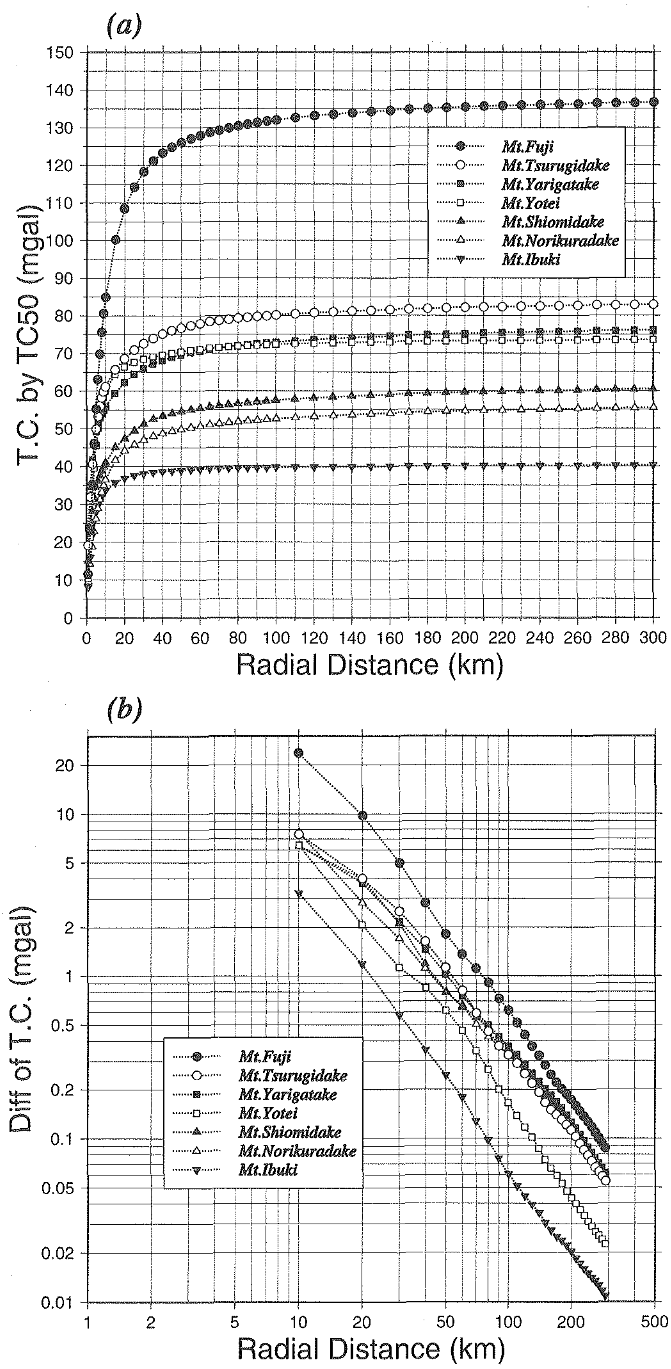
5.2 *Numerical Analysis*

The overall accuracy of the procedure can be estimated in several ways. The most straightforward is to compare the program output with the exact attraction calculated for a simple topographic model. First, computational accuracy of TC50 is checked to see how the results by TC50 agree with the theoretical values of the gravitational attraction due to a flat and cylindrical Bouguer slab with thickness of 1 km and radius of R km within which terrain corrections are computed. The correction density is assumed to be 2.67 g/cm^3. The near-station terrains are approximated by right rectangular prisms (not by cone segments). The computation was performed with radius R successively ranging from 1 to 100 km. The calculated results, replacing elevation difference between a gravity station and each DEM to 1 km, show an extremely good agreement with the theoretical values within 0.00006 mgals (0.06 μgals) for all cases in which the radius R from the station for terrain corrections is ranging from 10 to 100 km. For $R>50$ km, the calculated and theoretical results agree quite well with each other within 0.0028 μgals. On the other hand, the larger discrepancies arise in a few cases in which the radius R is rather smaller. In case of $R=1$ km, the largest error (0.02747 mgals) occurs. For the cases of $1<R<10$ km, the error is less than 0.01 mgals. It is concluded that when we take $R>50$ km (commonly employed value) the accuracy of direct integration using TC50 is estimated to be less than 0.0028 μgals. Also this check serves to show the program has been properly coded. It is suggested that the error which arises from the approximation by modeling near-station topography using truncated cones and/or rectangular prisms is quite small in above cases. The most simple way to improve reliability of the results of the calculation is to employ a DEM gridded with higher dense nodes, since the accuracy of direct integration is fundamentally limited by the finite sampling of topography by a DEM.

By the way, the estimate of radius R to which actual terrain corrections should be extended, including the most rugged topography, is a very fundamental and important problem. This follows a trade-off between accuracy we require in Bouguer anomaly values and computational time. In principle the choice of radius is arbitrary, but it is advantageous to select the value under some criterion. The geodetic principle dictates that terrain correction does not converge unless we extend the correction region to a whole earth. Yamamoto et al. (1982) pointed out that Bouguer anomaly values biased only constantly for the whole area of survey should be used. According to this idea, spherical terrain corrections by TC50 are computed in a real situation for a series of radii up to 300 km within which terrain effects are systematically evaluated. Fig. 7 illustrates the results for seven representative summits mostly in Central Ranges, Japan, where "*Method C*" is employed in the near-station zone. All summits except Mt. Ibuki (1,377 m) and Mt. Yotei (1,898 m) have elevation values more than or nearly equal to 3,000 m and their terrain correction values are generally anticipated to be rather large. Therefore, it would be appropriate to investigate the absolute values of terrain corrections at these summits and their variation as a function of radius R.

As illustrated in Fig. 7a, we can see the largest terrain correction value is obtained for Mt. Fuji (35°21′26.46″N, 138°43′49.98″E, 3,775.6 m), where the value exceeds 135 mgals and converges to about 136.66 mgals for the radius of 300 km. A similar diagram is given by Nozaki (1981) who used KS-110 by modeling the nearby topography as several segments of pentahedron, while as a right rectangular prism in a distant zone. His result for Mt. Fuji shows the convergence value of terrain correction is 135 mgal for the rectangle region of 300×400 km^2. As shown in Fig. 7a, the correction values for each mountain monotonously increase after a sharp rise within a radial distance of about 30~40 km from a station, and the rate of an increase of terrain corrections for each summit is almost constant. The rate of an increase for Mt. Ibuki is almost zero over a radial distance larger than about 80 km, actually, the difference of correction values at $R=80$ km and $R=300$ km is less than 0.5 mgals. Similarly, Mt. Yotei shows an almost constant correction value over a radius larger than about 100 km. It is intriguing that for the other higher mountains terrain correction values increase with almost the same rate.

In addition, we focus on a variation of the correction differences obtained at the different two radii, because such variation is thought to be a good index to investigate what spatial extent the terrain corrections should be required. Fig. 7b illustrates the variation of the relative differences of terrain corrections between at the radii R km and $R+10$ km, plotted as a function of a radial distance R whose range is successively varied from 10 to 290 km. Note that each curve monotonously (or exponentially) decreases as R increases. The radial distance $R_{0.5}$ at which the terrain correction takes a value 0.5 mgals smaller than that at $R_{0.5}+10$ km is about 120 km for Mt. fuji as illustrated in Fig. 7b. Mt. Yotei and Mt. Ibuki indicate the values of 60 and 35 km for $R_{0.5}$, respectively. All the other mountains show the values of 70~80 km for $R_{0.5}$. These results imply that if we take $R>80$ km the relative difference of terrain corrections between at R km and $R+10$ km even at stations more than 3,000 m


(a)
T.C. by TC50 (mgal)
Radial Distance (km)
Mt.Fuji
Mt.Tsurugidake
Mt.Yarigatake
Mt.Yotei
Mt.Shiomidake
Mt.Norikuradake
Mt.Ibuki
(b)
Diff of T.C. (mgal)
Radial Distance (km)

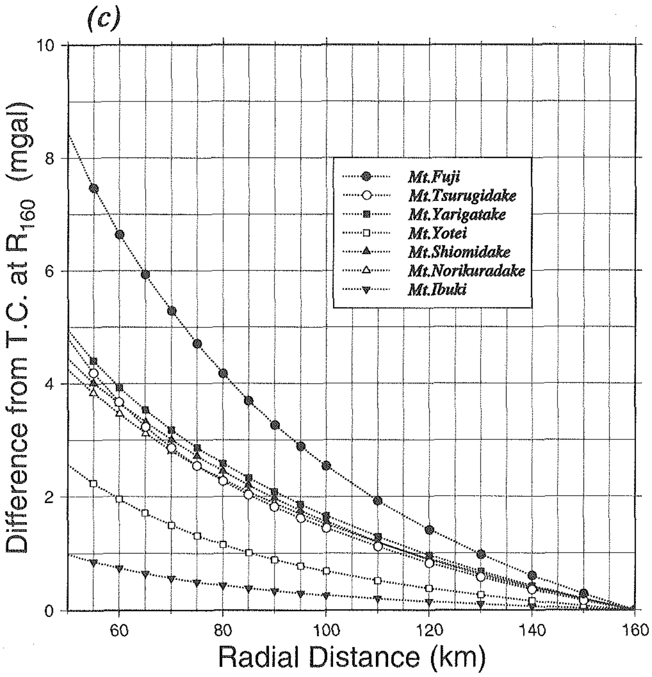
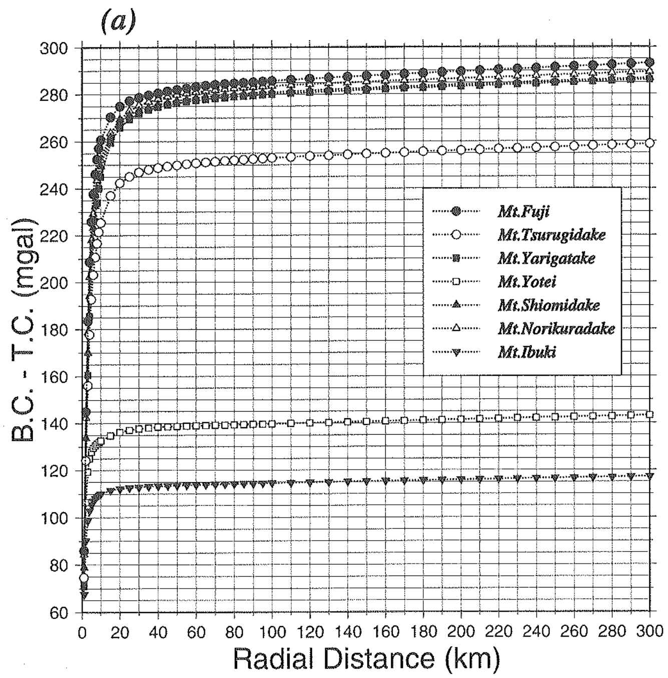
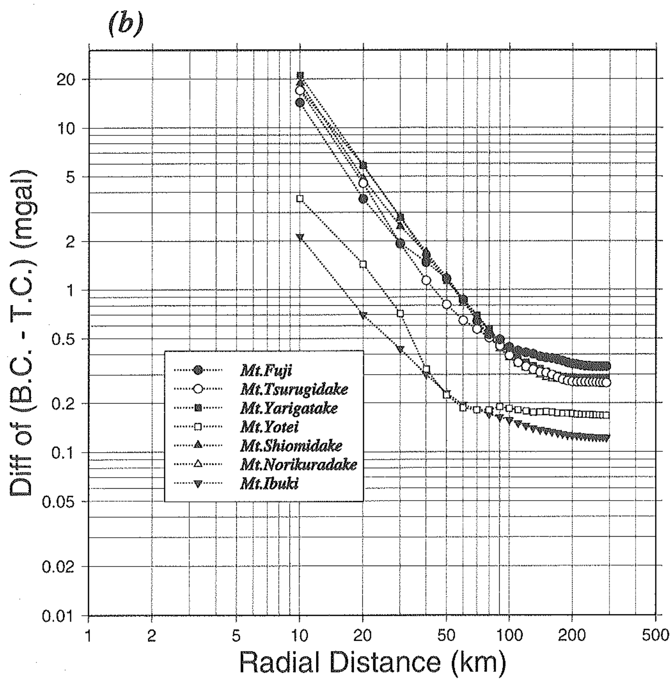


Fig. 7. (a) Spherical terrain corrections by TC50 as a function of the radial distance R within which terrain effects are computed for seven representative summits mostly in Central Ranges, Japan. Near-station topography is approximated by modeling cone segments as shown in Fig. 3b. The correction values for each mountains monotonously increase after a sharp rise within $R=30\sim40$ km from a station. The rate of an increase of terrain corrections for each summit is almost constant. Elevation values (in meter): 3,775.6 (Mt. Fuji), 2,998 (Mt. Tsurugidake), 3,180 (Mt. Yarigatake), 1,898 (Mt. Yotei), 3,047 (Mt. Shiomidake), 3,026 (Mt. Norikuradake), and 1,377.06 (Mt. Ibuki). (b) Variation of the relative differences of terrain corrections between at the radius R km and $R+10$ km is plotted as a function of a radial distance R. Notice that each curve monotonously decreases as R increases. (c) Absolute difference of terrain corrections at R from those at $R=160$ km is plotted as a function of radius R. The radius $R=160$ km is assumed to be the ultimate one within which the gravity effects are constantly biased.


(a)
B.C. - T.C. (mgal)
Radial Distance (km)
Mt.Fuji
Mt.Tsurugidake
Mt.Yarigatake
Mt.Yotei
Mt.Shiomidake
Mt.Norikuradake
Mt.Ibuki


(b)
Diff of (B.C. - T.C.) (mgal)
Radial Distance (km)
Mt.Fuji
Mt.Tsurugidake
Mt.Yarigatake
Mt.Yotei
Mt.Shiomidake
Mt.Norikuradake
Mt.Ibuki

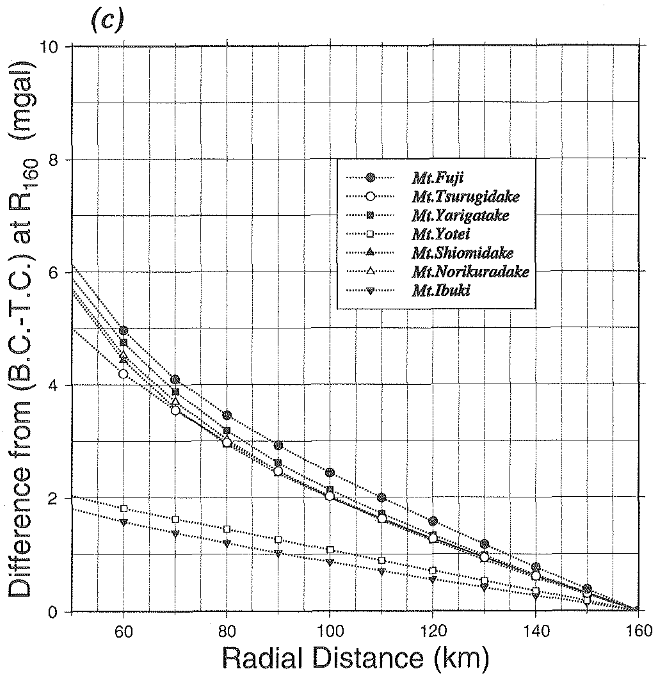


Fig. 8. (a) The correction term ($B_s - T_s$) for the seven stations shown in Fig. 7 is plotted as a function of a radial distance whose range is successively varied from 1 to 300 km, where B_s and T_s are spherical Bouguer correction and spherical terrain correction, respectively. ($B_s - T_s$) values for all the stations approach the one with the radius R=300 km as R increases. (b) Variation of the relative differences of ($B_s - T_s$) at the radial distance R km and R+10 km is plotted as a function of a radial distance R. Note that ($B_s - T_s$) term of all stations clearly converges to an almost constant value over R more than 200 km. (c) Absolute difference of ($B_s - T_s$) values at R from those at R=160 is plotted as a function of radius. The radius R=160 is assumed as the ultimate radius within which the gravity effects are constantly biased.

high does not exceed 0.5 mgal. Further, the absolute difference of terrain corrections at R km from those at $R=160$ km is plotted as a function of radius R in Fig. 7c, where we assume that the radius $R=160$ km is the ultimate one within which the gravity effects are constantly biased. It should be notable here that all stations, except for Mt. Fuji, indicate that the difference between at R km and $R=160$ km becomes less than 1.5 mgal if we roughly take $R>100$ km.

5.3 *Variation of (B_s-T_s) term*

Next, we compare the variation of spherical Bouguer correction (B_s) subtracted by spherical terrain correction (T_s) as a function of a radial distance for the area of terrain correction, since we need to know the behavior of (B_s-T_s) (1) to evaluate Bouguer anomaly properly, and (2) to estimate the reasonable spatial extent of the terrain corrections. B_s is calculated using the formula given by Hagiwara (1978). Fig. 8a illustrates the variation of the correction term (B_s-T_s) for the above seven stations, plotted as a function of the same radial distance as shown in Fig. 7a.

It is also found that correction values rapidly increase within a radial distance of about 10~20 km from a station. After this sharp rise, the correction values for each mountain increase very slowly. Notice that (B_s-T_s) values for all stations approach the one with the radius $R=300$ km as R increases.

Similarly in Fig. 7b, we also focus on a variation of the relative differences of (B_s-T_s) term at the different radial distances. Fig. 8b demonstrates the variation of the relative differences of (B_s-T_s) at the radial distance R km and $R+10$ km, plotted as a function of a radial distance R. It should be noted that the relative difference of (B_s-T_s) term of all stations clearly converges to an almost constant value over R more than 200 km. The radial distance $R_{0.5}$ at which (B_s-T_s) term takes a value 0.5 mgals smaller than that at $R_{0.5}+10$ km can be investigated here. Surprisingly, all the mountains except for Mt. Ibuki and Mt. Yotei show the same value of 80~90 km for $R_{0.5}$ as shown in Fig. 8b. Mt. Ibuki and Mt. Yotei give lower values of 30 km and 40 km for $R_{0.5}$, respectively. Thus these results strongly suggest that the gravity reduction, involving (B_s-T_s) term, can be done within 0.5 mgal difference if we take $R>90$ km.

If we choose the (B_s-T_s) value at Bullard B radius (about 160 km) as the ultimate radius R_u within which the gravity effects are constantly biased, the radial distance $R_{1.5}$ at which (B_s-T_s) term takes a value 1.5 mgals smaller than that at R_u is another criterion to investigate the requirement of spatial extent of the terrain corrections. Assuming $R_u=160$ km here, the absolute difference of (B_s-T_s) values at R from those at R_{160} is plotted as a function of radius R in Fig. 8c. All stations show the difference of less than 2.0 mgal if we roughly take $R>110$ km as shown in Fig. 8c, where R is ranging from 50 to 160 km. Mt. Fuji gives a largest value of about 120 km for $R_{1.5}$ while Mt. Ibuki shows a smallest one of 60 km. The values of $R_{1.5}$ for another mountains become about 110 km. These results suggest that if we take $R>110$ km for the correction radius, TC50 gives the terrain corrections with a relative error less than 1.5 mgals even at stations more than 3,000 m except for Mt. Fuji.

6. Discussions

In this section we investigate the performance of TC50 for the actual field data. We quantitatively compare the numerical results by TC50 ("*Method C*") with those by other programs by "*Method Y*" (Yamamoto et al., 1982) and the revised method of Komazawa (1988), both of which use KS-110 (or KS-110-1) systematically. A correction density for all cases is assumed to be 2.67 g/cm^3. For purposes of comparison, we employed terrain corrections in a circular region within a radius of 100 km from a gravity station according to discussions in the previous section. The first two examples show a comparison between the present "*Method C*" and "*Method Y*" (Yamamoto et al., 1982) in which these two methods are applied to a few thousands of gravity data in mountainous and flat-land areas, respectively. On the other hand, TC50 ("*Method C*") and the revised version of Komazawa (1988) are compared in the third example.

Fig. 9 illustrates the comparison of terrain corrections calculated by TC50 and "*Method Y*" (Yamamoto et al., 1982) for the field data published by Shichi and Yamamoto (2001a). TC50 and "*Method Y*" employs a DEM spaced at 50 m and 250 m (KS-110-1), respectively. 5,000 gravity stations are selected here in a way that limits the terrain correction values, calculated by "*Method Y*", larger than about 8 mgals (the largest one is 60.043 mgals at Mt. Shiomidake). Each station is colored according to its elevation. Many stations have their elevation values larger than 1,000 m. These gravity data are distributed in a typical mountainous region which consists of the three largest mountain ranges (Akaishi, Kiso and Hida) in Japan, having the width of $2^\circ\times2^\circ$ ($\sim$200 km$\times$200km). All mountains, except Mt. Fuji, whose altitude exceeds 3,000 m in Japan gather in this area. The terrain corrections by TC50 are computed within a circular region with a radial distance R of 100 km, while "*Method Y*" employs $R=80$ km.

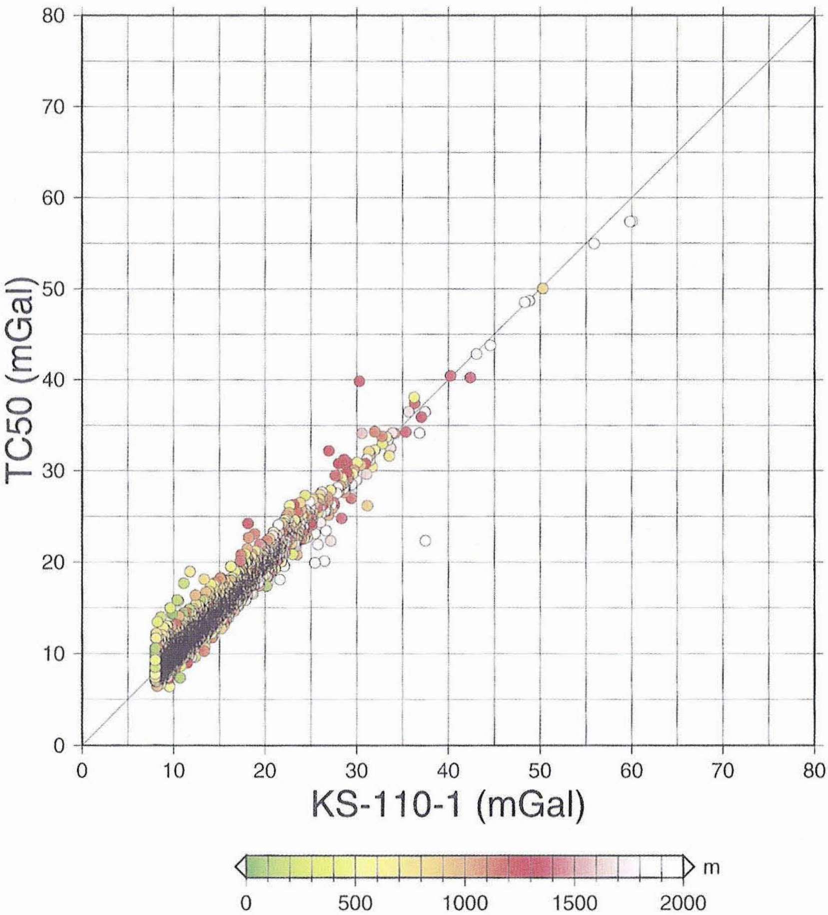


Fig. 9. Comparison of terrain corrections calculated by TC50 and "*Method Y*" (Yamamoto et al., 1982) for the field data in mountainous regions published by Shichi and Yamamoto (2001a). TC50 and "*Method Y*" employ a DEM spaced at 50 m and 250 m (KS-110-1), respectively. Terrain corrections by TC50 are computed within $R=100$ km, while "*Method Y*" computes terrain corrections up to $R=80$ km. 5,000 gravity stations are used whose terrain correction values by "*Method Y*" are larger than about 8 mgals (the largest one is 60.043 mgals at Mt. Shiomi). Each station is colored according to its elevation.

As shown in Fig. 9, almost all the correction values agree well with each other within 2~3 mgals, although the estimates by TC50 tend to become slightly larger than those by "*Method Y*." Although TC50 takes a value of $R=100$ km and its gravity effects might be somewhat larger than those by "*Method Y*" for $R=80$ km, the difference of terrain corrections at $R=80$ km and $R=100$ km,

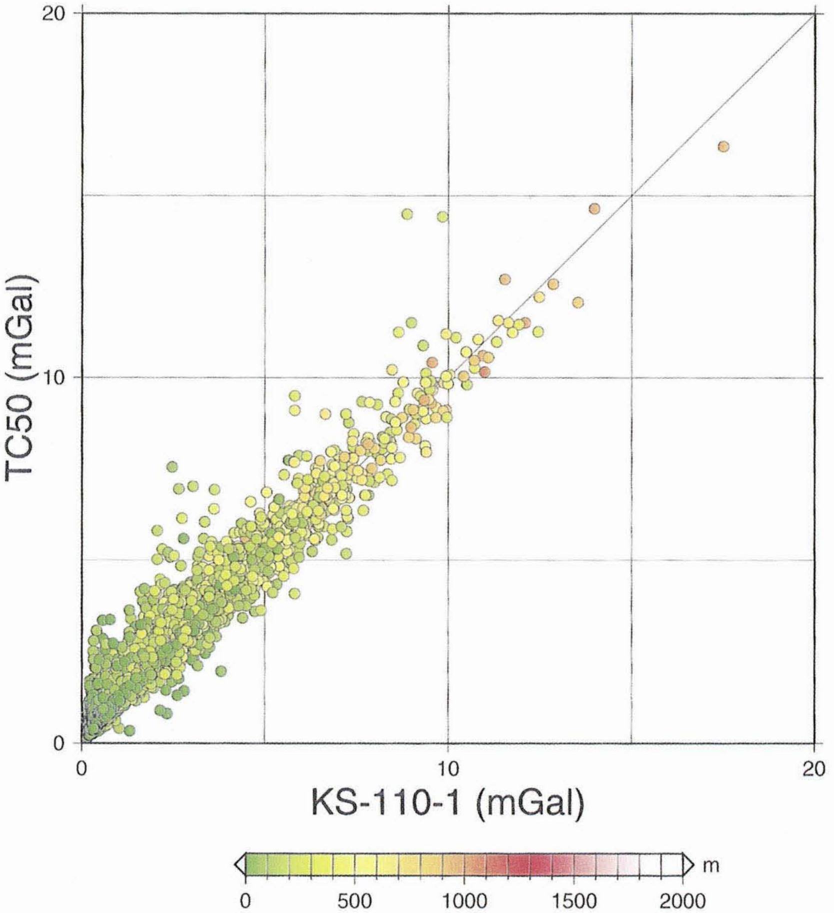


Fig. 10. Comparison of terrain corrections calculated by TC50 and "*Method Y*" (Yamamoto et al., 1982) for the 5,861 gravity data around the Kinki district, southwest Japan (Shichi and Yamamoto, 2001a, 2001b). Most of the stations are distributed in flat-land, having an elevation value smaller than 500 m. The terrain corrections by TC50 are computed within $R=100$ km, while "*Method Y*" computes terrain corrections up to $R=80$ km.

using TC50, is at most 1 mgal except for Mt. Fuji as shown in Fig. 7b. This can be interpreted as somewhat larger differences more than 2~3 mgals may be due to less accuracy of elevation values of DEM (KS-110-1) itself. Several outliers can also be seen with a difference larger than 5 mgals in Fig. 9. These outliers probably come from spurious elevation values of DEM spaced at 250 m. The number of gravity stations in which terrain corrections using TC50 differ from those using "*Method Y*" by larger than 3 mgals amounts to be only 58 stations. Also 256 and 1,254 gravity stations show the terrain correction difference between TC50 and "*Method Y*" by larger than 2 and 1 mgals, respectively. These facts indicate that 94.88% of 5,000 gravity data, located in typical mountainous regions in Japan, is in extremely good agreement with each other in terrain correction values by TC50 and "*Method Y*" within 2 mgals.

A similar comparison is given in Fig. 10, in which terrain corrections are also calculated by TC50 and "*Method Y*" for 5,861 gravity data in flat-land area around the Kinki district, southwest Japan (Shichi and Yamamoto, 2001a, 2001b). Most of these stations have an elevation value smaller than 500 m (flat-land). Similarly as shown in Fig. 9, Fig. 10 illustrates that the estimates by TC50 also tend to become slightly larger than those by "*Method Y*." Most of all the correction values show a good agreement within about 2 mgals except for several outliers which probably come from the discrepancy of elevation values of DEM. Gravity stations whose terrain correction values using TC50 differ from those using "*Method Y*" by larger than 3 mgals number only 11. Further, 45 and 310 stations show the terrain correction difference between TC50 and "*Method Y*" by larger than 2 and 1 mgals, respectively. As a result, 99.23% of 5,861 gravity data shows an extremely good agreement in terrain correction values by TC50 and "*Method Y*" within 2 mgals. The above two examples demonstrate the extreme validity of DEM50 and TC50.

Finally, we show another example in which TC50 and the method proposed by Komazawa (1988) are applied to the 1,000 gravity data published by Geological Survey of Japan (GSJ) (2000). Actually, terrain corrections for the gravity data published by GSJ (2000) were carried out by the revised version (hereafter referred to as "*Method K*") of Komazawa (1988) by the use of KS-110-1 (Komazawa, 2001). We selected gravity stations for comparison such that their terrain correction values, calculated by "*Method K*", are larger than 20.67 mgals (the largest one is 131.401 mgals at Mt. Fuji). Each station is colored according to its elevation. Notice that many stations have their elevation values larger than 1,500 m.

The result is illustrated in Fig. 11. We can see the correction values show a good agreement with each other within several mgals, as shown in Fig. 11. The only one exception can be recognized as an outlier having a largest difference of correction values. This gravity station (37°46.32′N, 139°55.60′E, 501.037 m) shows the correction values of 27.4997 mgals by "*Method K*", and TC50 ("*Method C*") evaluates its terrain correction as 18.291 mgals. The difference in this case amounts to be 9.209 mgal. "*Method Y*" gives 16.5 mgals

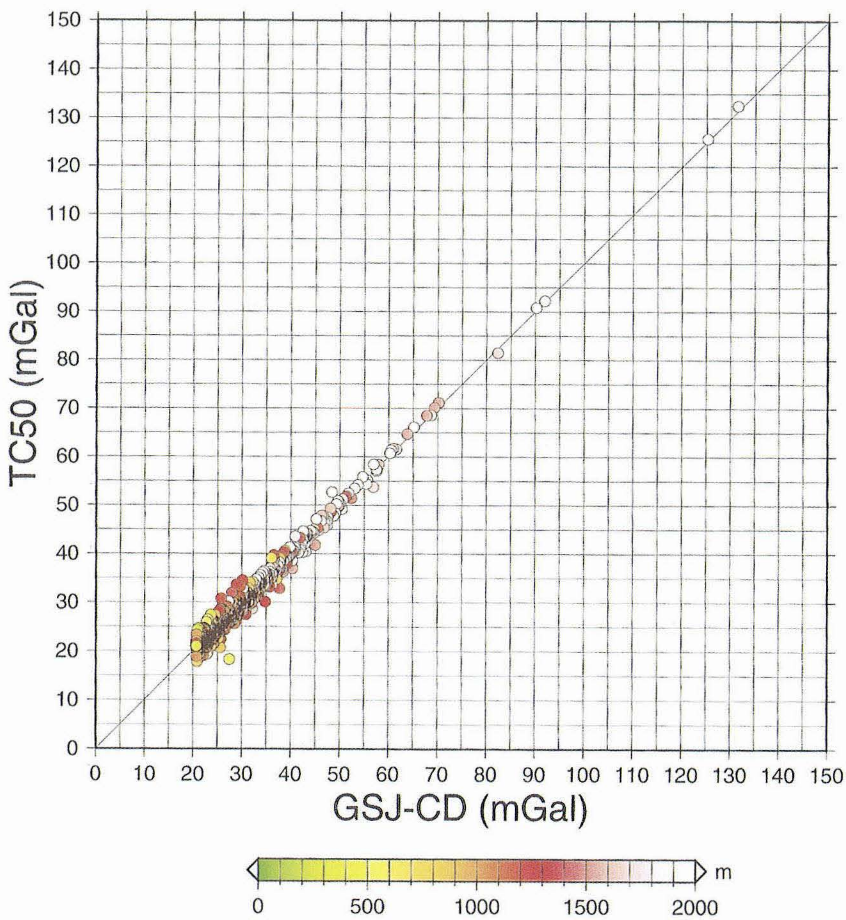


Fig. 11. Comparison of terrain corrections calculated by TC50 and "*Method K*" (Komazawa, 1988) for the 1,000 gravity data published by Geological Survey of Japan (GSJ) (2000). Terrain corrections by TC50 and "*Method K*" are computed within $R=100$ km and $R=60$ km, respectively. Gravity stations whose terrain correction values by "*Method K*" are larger than 20.67 mgals (the largest one is 131.401 mgals at Mt. Fuji) are used. Each station is colored according to its elevation. 90.6% of 1,000 gravity data shows a good agreement in terrain correction values by these two methods within 2 mgals. The only one exception can be recognized as the outlier having a largest difference (9.209 mgals) of correction values.

as the terrain correction value for this station. Komazawa (2001) interpreted the difference between TC50 and "*Method K*" as a less accuracy of KS-110-1. Excluding this station in Fig. 11, 30 gravity stations show the correction difference larger than 3 mgals between the methods of TC50 and "*Method K*."

The numbers of gravity stations that show the terrain correction difference between TC50 and "*Method K*" by larger than 2 and 1 mgals are 94 and 319, respectively. This fact suggests that 90.6% of 1,000 gravity data shows a good agreement in terrain correction values by TC50 and "*Method K*" within 2 mgals.

The estimated values using TC50 do not show any fluctuations in all three cases even for mountainous or flat-land regions. This implies that TC50 gives quite stable estimates of terrain corrections which do not seriously depend on the ruggedness of topography. The above results convincingly suggest that TC50 computes terrain corrections with a relative accuracy within 2 mgals compared to other methods even if applied to the data in mountainous regions where large terrain corrections are anticipated.

7. Computational Remarks

TC50 is a straightforward imlementation of the procedures presented in this paper and written in ANSI-standard C language with about 13,000 steps, incorporating the "*Method C*" as well as the "*Method P*." As written, this means that gravity effects based on both spherical-cap theory and finite flat-slab scheme, optionally with line-mass approximations, can be computed selectively. The program only requires the file that defines the input data involving the coordinates of gravity stations besides a DEM file. TC50 is configured to make it possible to calculate terrain corrections for a gravity data measured inside a tunnel or in the open air. Also a variety of computer algorithms to facilitate the computation for terrain effects in gravity measurements are flexibly implemented in TC50. As a strictly computational viewpoint, TC50 is designed to compute terrain corrections by reading DEM files only once, and their gravity effects are successively added up for each station. Actually, this technique effectively reduces a computational time. TC50 does not require any limitation on the important variables. For example, it is not necessary to statically allocate an array size for the maximum number of stations in compilation, since the required memory is dynamically and automatically allocated whenever it is needed, and disposed when it becomes free. This implies that the only limitation is hardware resources such as capacity of memories and hard disks etc. However, the program is rigorously designed for DEM50 provided by GSI and is difficult to be used as it is under a DEM whose format is essentially different from that of DEM50. As for a computational CPU time, the program requires a CPU time of about 60 seconds per one station under a standard PC (Pentium-III, 1 GHz) to compute spherical terrain corrections out to $R=80$ km. For numerical economy, computational time is, of course, considerably reduced when as many gravity data as possible are applied to TC50 at the same time. All the numerical calculations reported in this paper have been performed under the Linux Operating System (kernel : 2.2.19) where TC50 was compiled using the GNU Compiler Collection (GCC, version : 2.95.3). Source codes of TC50 developed here are available upon request to the author (star@eos.hokudai.ac.jp) via email.

8. Conclusions

We have developed an improved method to compute spherical terrain corrections for a gravity survey using DEM50 equally spaced on a regular grid at 50 m. Unlike other techniques, this method is designed to use a combination of truncated circular cone segments, cylindrically-truncated prisms and regular prisms to approximate a near-station topography, incorporating DEM50, essentially on the basis of the technique proposed by Yamamoto et al. (1982). In addition, the earth's sphericity is strictly considered in computations of terrain corrections, employing the idea that gravity correction is separated into two steps ; (1) directly apply the spherical Bouguer correction within a finite distance from a gravity station, and (2) apply the spherical terrain correction for departures of the actual earth's surface from an idealized spherical surface. A complete program called "TC50" is developed in standard C language to compute spherical terrain corrections based on the present technique. The theoretical values of the gravitational attraction due to a finite and cylindrical flat-slab with thickness of 1 km are quantitatively compared with those by TC50. Computational accuracy of TC50 is achieved to be within 0.0028 μgals for this case in a circular region with radial distance more than 50 km. To clarify quantitatively what spatial extent the terrain corrections should be required, spherical terrain corrections are computed as a function of the radius (R) within which terrain effects are systematically evaluated. Also a variation of spherical Bouguer correction (B_s) subtracted by spherical terrain correction (T_s) is focused as a function of R, since the right interpretation of the behavior of ($B_s - T_s$) term is quite important (1) to evaluate Bouguer anomaly properly, and (2) to estimate the correct spatial extent of the terrain corrections. The results suggest that if we take $R > 80$ km the relative difference of terrain corrections between at R km and $R+10$ km even at stations more than 3,000 m high does not exceed 0.5 mgal. Also it is concluded that if we take a value of $R = 110$ km, the proposed method gives the terrain corrections with a relative accuracy of 2 *mgals even at stations more than 3,000 m except for Mt. Fuji, assuming that the* ultimate radius is 160 km within which the gravity effects are constantly biased. Finally, we compared terrain correction values by the proposed method with those by other methods for thousands of actual field data obtained in plains and mountainous regions with rugged topography. It is conclusively demonstrated that the program computes spherical terrain corrections with relative difference within about 2 mgals from those computed by other methods.

Acknowledgements

I wish to thank Messrs. Isao Kimura and Yoshifumi Hiraoka of the Geographical Survey Institute of Japan (GSI) who kindly offered various information about the Digital Map 50 m Grid (Elevation) provided by GSI. I am grateful to Prof. Ryuichi Shichi of Chubu University for his invaluable suggestions to improve the manuscript. Thanks are also expressed to Dr. Masao Komazawa of the Geological Survey of Japan (GSJ/AIST) for useful information about terrain corrections involved in GSJ (2000). Production of most of the diagrams was facilitated by the freely available GMT (Generic Mapping Tools) software (Wessel and Smith, 1995 ; Available as http://gmt.soest.hawaii.edu/). This work was funded in part by Grant-in-Aid for scientific research from the Ministry of Education, Culture, Sports, Science and Technology of Japan (No. 13640428).

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