Distributive justice and social conflict in an AK model

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Abstract

We introduce distributive justice into a standard macroeconomic model with agent heterogeneity in terms of capital endowments. The labour share is the primitive in the distributive justice game. We provide a clear definition of distributive social conflict. This conflict is resolved by two positive and three normative methods. The two positive ones are 'average voter' in the probalistic voting model and Nash bargaining, encapsulating electoral politics and socio-political bargaining; two normative (justice) criteria are the Utilitarian and the Rawlsian. We also propose a novel criterion of 'justice as minimal social friction'. The labour share in turn affects growth in the 'AK' production economy. Thus, in socio-economic equilibrium, growth and the labour share are jointly determined. Greater impatience, status comparisons, wealth inequality or a decline in productivity exacerbate social conflict. Status comparisons and wealth inequality raise the labour share under all positive and normative criteria. At least if the capital-rich individuals' overall socio-political strength is higher than that of workers, both positive methods imply a smaller labour share and more inequality than do all our three criteria of distributive justice.

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1. Introduction

This paper introduces distributive justice into a standard macroeconomic model of growth and distribution. Distributive justice has occupied philosophers for a long time (see for instance Barry, 1991; Lamont and Favor, 2017; Olsaretti, 2018). In economics, it has been examined mainly in the field of social choice (e.g. Fleurbeay, 2019; Roemer, 1998). Macroeconomics has an important role to play, too, as any criterion of justice will, if implemented, provide a different set of incentives and will therefore alter economic behaviour. There will likely be effects on (e.g.) labour supply and growth. Arguments in the public discourse often emphasise the growth effects of justice proposals. In turn, the macroeconomic outcomes will provide feedback effects on what set of rules is considered just. Thus, a macroeconomic setup with a production economy at its core can both contribute to and benefit from analyses of distributive justice. To our knowledge, such a framework is lacking; this paper is an attempt to bridge this gap. In the process, we make a number of novelties and contributions, explained below. The synthesis between macroeconomy and distributive justice should be relevant in addressing topics of increasing concern such as inequality, which is generally rising among advanced economies (e.g. Piketty, 2014; Atkinson, 2015).

Integrated into our analysis are two concepts: social conflict and the labour share. Inequality gives rise to distributive social conflict (e.g., Acemoglu, 2009, Ch. 22). This conflict is multi-faceted but ever-present (Bernard, 1983; Dahrendorf, 2007), whereby various actors or groups attempt to capture a larger share of the output either directly (through wage negotiations or price rises given the nominal wages) or indirectly by manipulating the political system in order to benefit from taxes, favourable transfers, laws, regulations, and other redistributive policies. Our analysis uses social conflict as the starting point from which the distributive justice game unfolds. A clear definition of this concept and integration into a macroeconomic framework is one novelty of this paper. Arguably, heightened social conflict may be behind the recent rise of anti-systemic vote (see Sandel, 2019).

Furthermore, we are concerned with the ‘functional’ distribution of income, i.e. its division into labour or capital income. The capital-labour split is often thought to be one of the ‘constants’ of macroeconomics and growth theory at about one-to-two; recently, however, there have been indications that the capital share is rising at the expense of the labour share (Piketty, 2014; Karabarbounis and Neiman, 2014), a development for which a consensus explanation has yet to emerge. Factor shares are determined by a whole range of institutional features and policies related to the product market (monopoly power, regulation - Gollin, 2002; Blanchard and Giavazzi, 2003), labour market 'institutions' (trade unions, employment protection, minimum wage laws - Nickel, 1997; Nickel, Nunziata and Ochel, 2005), taxation (for redistribution or welfare programmes - Alvaredo et al., 2011). The empirical significance of such 'institutions' is still being debated in a large literature, but the safest is to adopt a middle position – they do have some relevance. Such institutions have both efficiency and distributive implications. The efficiency implications are well known (and summarised by the fact that such institutions are often called 'imperfections', implying that they are detrimental to efficiency). But such institutions, as well as taxation, have distributional implications, too; in particular, they affect the labour share. Furthermore, all these policies and regulations are contestable, they are among the subjects of social conflict mentioned above; they are determined both by direct action (e.g. politics, strikes) and also indirectly by expert opinion, the media, and ideology. Ultimately, they are a matter of societal choice, so that, by determining those institutions, in whatever way, social partners also determine factor shares. Thus, the labour share (and its flip side, the capital share) can serve as a summary indicator of such institutions – a further novelty.[1] This allows us to introduce distributive justice in an aggregative framework, as we need not consider in detail all the wealth of individual institutions. The bottomline and the starting point of the present paper is that the labour share is endogenous and the choice variable in the distributive justice game. Hence, we also contribute to the debates on the labour share.

As Bowles (2008) emphasises, social conflict breeds compromise. 'Distributive justice' is the analysis of how conflict over competing claims on resources is resolved. A simple definition of 'justice' is elusive. Macroeconomics routinely examines questions of social optimality by comparing the equilibrium outcome with the 'command optimum' determined by a social planner. While this conceptual device is useful in representative-agent economies, it is much less clear what criterion the social planner will use in economies with heterogeneous agents. Maximisation of aggregate welfare, the utilitarian criterion, is one, but not the only, possibility. Alternatively, the planner may opt for some version of egalitarianism (famously espoused by Rawls, 1971), or some other criterion. We consider various methods ('criteria'), by which distributive social conflict is resolved. Specifically, we examine two positive and three normative criteria. The positive criteria summarise the outcome (labour share) delivered by key socio-political institutions (more below). The normative criteria

[1] Appendix A articulates how the labour share summarises the product and labour market institutions.

capture how things 'ought to be' according to some criterion of justice, and therefore may be said to be closer to the idea of distributive justice. Our focus and main contribution is to study how distributive social conflict is resolved by the various normative criteria, and how this interacts with the macroeconomy (labour share, growth). Furthermore, by considering the positive criteria, we analyse whether the normative outcomes are likely to be fulfilled in abstract, yet realistic, setups.[2]

The social conflict approach to the labour share echoes some earlier literature, e.g. the socio-economic approach to inflation (Goldthorpe, 1978, 1984; Hirsch, 1978). In macroeconomics, issues involving social conflict have received less attention; notable exceptions include Rowthorn (1977), Benhabib and Rustichini (1996), Rodrik and van Ypersele (2001), Eggert, Itaya and Mino (2011) and Tsoukis and Tournemaine (2011). Acemoglu, Johnson and Robinson (2002) summarises an important programme of research which analyses how the distributional conflict has historically been resolved by institutions, some of which may be regarded as the historical embodiment of the positive and normative methods considered here. Closer to our analysis, a couple of models of the functional distribution of income (Alesina and Rodrik, 1994; Bertola, 1993) have pointed out a basic conflict of interest between workers and owners of capital and analyse its implications for tax policy. In a similar manner, Tornell and Lane (1999) analyse the conflict over fiscal policy and access to a common resource that gives rise to a commons-type problem ('voracity'). Despite such contributions, a synthetic model of justice and the macroeconomy is so far lacking.

Analytically, our model resembles the workhorse model of Garcia-Penalosa and Turnovsky (2006) in key respects (AK production, capital endowment heterogeneity). As already emphasised, the labour share is endogenous, the choice variable in the distributive justice game, not exogenous or determined in Walrasian equilibrium by marginal productivity considerations alone.[3] Agent heterogeneity is captured by two groups of otherwise identical capital-rich and capital-poor agents ('capitalists' and 'workers'). As is well known, the AK model features no transitional dynamics. The absence of dynamics lends this model tractability as the initial heterogeneity related to capital (the only asset) is perpetuated for ever. In every other respect, all agents are identical: they all provide inelastically one unit of labour and have a common rate of time preference. Agents

[2] In a recent contribution, Growiec, McAdam and Muck (2018) consider the social (sub)optimality of the labour share delivered by a decentralised, representative-agent economy. It is suboptimal as this economy features spillovers and externalities among sectors of intermediate goods producers in an endogenous growth setup. The social optimum internalises these externalities. In our setup, the focus is different. There are no externalities, while we introduce inequality in the form of heterogeneity in capital endowments.

[3] Appendix A on microfoundations shows how the labour share as the 'primitive' fully determines all the essential variables such as the output-capital ratio and the interest rate.

consume their identical wage, equal to the labour share (as agents are of unit mass), plus a constant fraction of their capital, as is standard in an AK model. The higher labour share however depresses the marginal profitability of capital and thereby growth, consistently with the Euler equation of consumption growth. So, as the labour income is equal while capital income is not, there are conflicting preferences as to the 'functional' division of national income. Because capital-poor (-rich) agents rely relatively more (less) on labour income, they prefer a higher (lower) labour share. This is the source of conflict in this model. In fact, social conflict is defined as the difference between the partisan preferences of the two groups over the labour share. The Pareto-efficient 'utility possibility frontier' (UPF) shows the trade-off in intertemporal welfare between the two groups as the labour share varies; the various criteria pick up different points on the UPF.

We study how conflict is resolved and the labour share is determined by two socio-political arrangements ('positive criteria') and by three notions of distributive justice ('normative criteria'). The positive criteria are based on the 'average-voter' model, capturing electoral politics, and on the Nash bargaining solution, capturing broad socio-political bargaining between classes. In the former, the electoral outcomes reflect the partisan preferences of the two classes weighted by their sizes. In the latter, the two classes of 'capitalists' and 'workers' bargain directly but the class sizes do not map directly onto electoral outcomes; rather, other intermediate mechanisms like culture, media, the organisational strength of 'corporatist' associations such as labour or employer unions a well as party politics, also play a role. We abstract from details and only postulate an overall socio-political bargaining strength (different from numerical strength) that affects outcomes. In addition to these positive criteria, we also analyse normative criteria of distributive justice such as the utilitarian criterion and Rawls's (1971) maximin criterion that entails maximising the welfare of the weakest.[4] Furthermore, we propose a novel normative criterion of 'justice as minimum social friction', 'friction' defined as the difference in welfare between the two groups normalised by aggregate welfare gains over a certain benchmark. According to this criterion, the social planner should consider both welfare differences and the aggregate welfare.[5] We rank these criteria in terms of labour shares; a pecking order arises that is one of our key results. To pre-amble, almost by definition, the Rawlsian criterion is the most egalitarian (giving the highest labour share), followed by our minimal social friction criterion, followed by the utilitarian outcome and finally by the electoral

[4] We apply the maximin criterion to intertemporal welfare rather than current income (*contra* Rawls, 1971).

[5] Roughly, the three criteria of Rawls, utilitarian and minimum 'friction' correspond to notions of justice as, respectively, egalitarianism (albeit partial), maximisation of the 'public interest', and a 'workable compromise'. These criteria may be thought to be three possible guiding principles for the social planner.

(‘average-voter’) outcome. The Nash solution will be lower than the utilitarian at least if the overall power of capitalists (combining socio-political as well as numerical strength) is equal or higher than that of workers. Under this condition, we show that all positive criteria lead to a lower labour share than all the normative criteria we consider.

Further themes touched upon by our analysis include social interactions (Manski, 2000), featured as status comparisons in individual utility (e.g., Tsoukis, 2007; Tournemaine and Tsoukis, 2008, 2009). This allows us to study how social interactions affect conflict and the factor shares. The strengthening of such status-related consumption externalities leads to higher conflict and increases the labour share under any distributive criterion, as a way of compensation. Furthermore, following Benabou (2000), we may refer to the institutional arrangements (positive criteria) that resolve conflict as the ‘social contract’. As this is part of institutional design and constitutions, we contribute to these strands of literature, too (Aghion, Alesina and Trebbi, 2004; Persson and Tabellini, 2004; Przeworski and Wallerstein, 1982; Tricchi and Vindigni, 2010).

We show that a productivity slow-down, a rise in impatience, a rise in the intensity of status comparisons and greater inequality in capital endowments all lead to heightened social conflict; all are candidates for understanding the conflict that has potentially led to the recent political developments mentioned above. In a longer version of this paper (Tsoukis and Itaya, 2018), we provide a simple numerical illustration for a handful of diverse economies; this shows that, with a set of data on wealth distribution and reasonable assumptions, the model can emulate the labour share found in the data quite well, proving the usefulness of this framework. The dissonance is related to the income Gini coefficient, which seems below what is found in the data. This suggests the need to provide a fuller model of inequality as an extension of the current model.

The remainder of the paper is organised as follows. In Section 2, we develop the model. In Section 3, we develop the UPF and highlight the nature of conflict. In Sections 4 and 5, we analyse the positive and normative outcomes. We conclude in Section 6. Appendix A elaborates on the microfoundations how to derive the production function which is used thorough the paper, while Appendix B expands on the nature of our proposed criterion of minimal social friction. An Online Appendix shows more details on the derivation of key results.

2. A model of growth and the functional distribution of income

We postulate an economy in continuous time with a unit-mass of infinitely-lived individuals, who are identical in all respects except their endowments of physical capital (which is the only form of asset). All individuals supply inelastically a fixed amount of labour (normalised to one unit), for which they receive an identical wage. If we indicate the constant share of output that labour receives by $0<\gamma<1$, the total wage bill is γy, where y (without any superscript) is aggregate output; with unit-labour, γy also equals each individual's labour earnings.[6] All individuals rent their capital to firms in competitive rental markets. For this, they receive remuneration for their capital in proportion to their relative capital holdings, k^m/k;[7] the total amount is equal to a fraction $1-\gamma$ of output. Our point of departure is that the time-invariant labour share ($0<\gamma<1$) is the choice variable, determined by the application of the various positive and normative criteria we consider.

The production function is of the AK form, with an addition. Apart from the distributional effects of policies and laws, there are also static efficiency implications: As is well known, output is maximised (given the predetermined capital) under competitive markets: For example (Blanchard and Kiyotaki, 1987), monopoly power reduces the level of output. The basic wage-price setting model also suggests that employment declines with the strength of labour market institutions (or distortions) such as trade unions and labour market regulations (e.g., Nickell, 1997); although the strength of these effects empirically remains debatable. In principle, taxation also provides disincentives and may reduce output. We summarise these effects by saying that the output-capital ratio declines whenever the labour share deviates from its competitive level (λ).[8] In a schematic way, we write the production function (see Appendix *A* on how to derive (1)):

$$y = Ak\left(1 - \frac{\varphi(\gamma-\lambda)^2}{2}\right), \qquad 0\leq\varphi<2\,, \tag{1}$$

where *A* is productivity, and φ captures the extent of the static inefficiency induced by any laws, policies or taxation that deviate from the competitive benchmark.

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[6] These stylised assumptions are made for tractability and for sharper focus on the issues that concern us. Appendix A considers detailed microfoundations based on flexible labour, a CES production function, monopoly power in the product market, and labour market institutions. The messages are that: (a) the stylised model here is fully consistent with rigorous microfoundations, (b) the change of labour share can be effected through policies, laws and regulations applying to product and labour market institutions, as well as taxation, all of which are subject to societal choice as mentioned in the Introduction, (c) the labour share is a sufficient statistic summarising such institutions as it fully determines the output-capital ratio and the interest rate. Equation (1) is a way of introducing the key points of the richer framework into the AK framework.

[7] The notational convention is that individual variables are with superscripts, e.g. c^m, whereas means of variables (which equal the aggregate due to unit mass) are without superscripts, e.g. c. Also, time (t) is omitted except when necessary.

[8] To be more precise, λ is the competitive labour share (Λ) adjusted by monopoly power (μ): $\lambda\equiv\Lambda/(1+\mu)$; see Appendix A.

We turn now to the budget constraint of individual $m \in [0, 1]$. The unit mass of individuals implies that aggregate and average values coincide, so the terms are used interchangeably. Total income (y^m) is made up of two elements: Firstly, labour earnings (γy), which are equal to all individuals, as all supply an equal amount of labour; the return to capital, which equals $(1-\gamma)y/k$ and is remitted in proportion to capital ownership by each individual (k^m). In turn, income is either consumed (c^m) or used to purchase further capital:

$$\dot{k}^m = y^m - c^m = \gamma y + (1-\gamma) y \left(\frac{k^m}{k}\right) - c^m \ . \tag{2}$$

As mentioned, there is no transitional dynamics in AK models. The steady state and the balanced-growth path holds at all times; therefore, capital holdings of all individuals grow at the constant, common rate g, i.e. $\dot{k}^m = gk^m$, for all m. The growth rate g will be endogenized below. Therefore:

$$gk^m = \gamma y + (1-\gamma)\left(\frac{y}{k}\right) k^m - c^m \ . \tag{2'}$$

Each agent maximises intertemporal utility:

$$U^m = \int_0^\infty \exp\{-\rho t\}\, u^m(t) dt, \quad \rho>0 \ , \tag{3}$$

where $\rho>0$ is the common instantaneous rate of time preference. Instantaneous utility, $u^m(t)$, is:

$$u^m(t) = \log(c^m(t) - \alpha c(t)), \qquad 0<\alpha<1 \ , \tag{4}$$

where $c(t)$ represents aggregate/average consumption. By $0<\alpha<1$, we capture the status or 'keeping up with the Joneses' externality relative to mean consumption mentioned in the Introduction; a rise in mean consumption decreases, *ceteris paribus*, individual utility.

Maximisation of (3) with (4) takes place subject to the budget constraint (2'), production (1), the initial levels of k^m for all m and the standard transversality condition:

$$\lim_{t\to\infty} \exp\{-\rho t\} \frac{\partial u^m(t)}{\partial c^m} k^m(t) = 0 \ .$$

The resulting Euler equation is standard, therefore details are omitted:

$$\frac{\dot{c}^m - \alpha\dot{c}}{c^m - \alpha} = r - \rho \ . \tag{5}$$

The only admissible possibility is a balanced-growth path with a constant growth rate, g, common across all individuals (m), and across both consumption and capital.[9] The resulting constant growth rate simplifies the

[9] In other words, the economy jumps immediately to a balanced-growth path characterised by $\frac{\dot{c}^m}{c^m} = \frac{\dot{c}}{c}$ without transitional dynamics. A unique balanced growth rate is required as different steady-state growth rates would imply that the individual with the highest growth rate would asymptotically own the capital of the entire economy; while the absence of transitional dynamics is delivered by the form of the Euler equation (5') where only growth rates appear.

analysis, as it implies both the absence of transitional dynamics and constant relative capital holdings. Thus, the initial heterogeneity in asset endowments is preserved unaltered for ever: The individual capital ownership at any time equals the relative endowment ($\chi^m \equiv k^m/k$) multiplied by aggregate capital (k), which in turn grows at a constant rate (g). So, the status-induced consumption externality has no implication for growth in the absence of flexible labour (e.g., Tsoukis, 2007). These results are all standard in the AK setup. As mentioned, firms readily hire all capital.

Moreover, the factor shares, however determined, are time-invariant: the chosen γ stays constant through time. Thus, considering $g \equiv \frac{\dot{c}^m}{c^m} = \frac{\dot{c}}{c} = \frac{\dot{k}^m}{k^m} = \frac{\dot{k}}{k}$, growth becomes:

$$g = r - \rho \,. \tag{5'}$$

The interest rate is:[10]

$$r = (1-\gamma)y/k. \tag{6}$$

Inserting (6) and (5') into the individual budget constraint (2'), we get:

$$c^m = \gamma y + \rho k^m \,. \tag{7}$$

It is well known that in this economy, the optimal consumption of each individual is given by their labour earnings plus a constant fraction of their capital holdings determined by the discount rate (Bertola, Foellmi and Zweimüller, 2006). To appreciate the results that follow, it is worth noting from (7) that the capital-poor (rich) individuals rely relatively more (less) on the labour share (γy) to support their consumption.

We now describe capital endowments, the only source of agent heterogeneity. To ensure tractability, we assume that individuals belong to two classes of fixed size, $m=i,j$. Without loss of generality, class i (the 'capitalists') are relatively well endowed with capital, whereas class j (the 'workers') are poorly so; thus, the time-invariant relative capital holdings ($\chi^m \equiv k^m/k$) are such that $\chi^i > 1 > \chi^j > 0$. The size of the capitalist class is $0<\theta<1$, while the worker class is of size $1-\theta$; noting the unit size of the total labour force, we have the identity $\theta\chi^i + (1-\theta)\chi^j = 1$. We assume that the worker class is more numerous than the capitalist class, so $1-\theta>0.5$, a realistic assumption in view of the fact that wealth ownership is usually right-skewed. This is crystallised below:

[10] To gain intuition, let us note that, assuming an AK production function in the spirit of Romer (1986) or Barro (1990), the competitive marginal product of capital would be $(1-\lambda)y/k$. Here, λ is replaced by the endogenously determined labour share γ. This is fully supported by rigorous microfoundations, as shown in Appendix A, with a slight exception: the marginal revenue product of capital is $(1-\gamma-\mu)\underline{y/k}$, but in order not to avoid further clatter, we here simplify it to $(1-\gamma)y/k$.

Assumption 1: $0 < \theta < 0.5$.

Using $\chi^m \equiv k^m/k$, and with information derived above, (7) may be re-expressed as:

$$c^m/k = \gamma A\left(1-\frac{\varphi(\gamma-\lambda)^2}{2}\right)+\rho\chi^m\ . \tag{7'}$$

Aggregate consumption is:

$$c = k\left[\frac{\gamma y}{k}+\rho\theta\chi^i+\rho(1-\theta)\chi^j\right] = k\left[A\left(1-\frac{\varphi(\gamma-\lambda)^2}{2}\right)+\rho\right]. \tag{7''}$$

Instantaneous utility of individual m (4) is therefore given by (with time introduced explicitly):

$$u^m(t) = \log\left[(1-\alpha)\gamma k(t)A\left(1-\frac{\varphi(\gamma-\lambda)^2}{2}\right)+\rho(\chi^m-\alpha)k(t)\right]. \tag{4'}$$

Aggregating over time as instructed by (3) and (4'), we have:

$$U_0^m = \int_0^\infty \exp\{-\rho t\}\left\{\log\left[(1-\alpha)\gamma A\left(1-\frac{\varphi(\gamma-\lambda)^2}{2}\right)+\rho(\chi^m-\alpha)\right]+log k(t)\right\} dt\ . \tag{8}$$

Considering the constant growth rate g, we have $k(t) = \exp\{gt\}k_0$; normalising initial aggregate capital to $k_0=1$, (8) becomes:

$$U_0^m = \int_0^\infty \exp\{-\rho t\}\left\{\log\left[(1-\alpha)\gamma A\left(1-\frac{\varphi(\gamma-\lambda)^2}{2}\right)+\rho(\chi^m-\alpha)\right]+gt\right\} dt\ .$$

Integration with (5') gives:

$$U_0^m = \frac{\log\left[(1-\alpha)\gamma A\left(1-\frac{\varphi(\gamma-\lambda)^2}{2}\right)+\rho(\chi^m-\alpha)\right]}{\rho}+\frac{r-\rho}{\rho^2}\ . \tag{8'}$$

We also require:

Assumption 2

$$\alpha < \chi^j \leq 1 \leq \chi^i.$$

The equality (to unity) would apply in the special case of equality across classes. Assumption 2 and the restriction $\varphi<2$ in (1) ensure that the log in (4') and (8') can be taken. Finally, using (6), (8') becomes:

$$W_0^m = \underbrace{\frac{\log\left[(1-\alpha)\gamma A\left(1-\frac{\varphi(\gamma-\lambda)^2}{2}\right)+\rho(\chi^m-\alpha)\right]}{\rho}}_{\log consumption\ with\ status}+\underbrace{\frac{(1-\gamma)A\left(1-\frac{\varphi(\gamma-\lambda)^2}{2}\right)-\rho}{\rho^2}}_{cumulative\ growth}\ . \tag{9}$$

There are two important remarks in relation to intertemporal welfare (9). Firstly, it makes clear that factor shares have three types of effect, static-efficiency implications, growth and distributional effects. We have commented on static efficiency; any movement of the labour share away from its competitive rate (λ) reduces the output-capital ratio, an effect introduced by φ. Regarding the latter two effects, a rise in γ shifts income to labour earnings, thereby increasing consumption (7), but also reducing the interest rate and growth (the second term on the right-hand side of (9)).[11] With some abuse of language, we may refer to the growth effects as 'dynamic efficiency' considerations. The balance between the three effects determines the optimal labour share for each individual. Since the contribution of capital income to consumption is ρk^m, individuals with less capital rely relative more on labour income would therefore prefer more of that; those with more capital, would like more capital income because of its growth effects.

Secondly, (9) shows the welfare of the whole group, or class, rather than of the typical individual within the class, hence the change to W-notation. The tradition in economics is that of methodological individualism; recently, however, and in view of the salience of groups, there has been increasing attention on the effects of group decision-making: Charness and Sutter (2012) survey a number of contributions which show that group decision-making may be less prone to cognitive biases and limitations or problems of self-control. Two of the main corollaries of Charness and Sutter (2012) are that 'that groups are at least an element in most decisions' (p. 158) and that 'it might make sense to have groups making decisions' (p. 159). The implications of group decision-making are far-reaching, as emphasised e.g., in politics by Gennaioli and Tabellini (2019). In view of this, although the labour share is exogenous to the individual, it is amenable to group or class action, and its distributional and efficiency implications are internalised in the group calculus. We capture this effect in one of our positive criteria of justice (Nash bargaining solution), by making the group or class (workers-capitalists) the decision-making unit. In the other criteria, positive or normative, we refer to classes, but those are collections of identical agents without any decision-making power at class level.

In the next Section, we formally show how the labour share affects the welfare of each class. Intuitively, as labour income is equally shared whereas capital income is unequal, the worker (capitalist) group prefers a higher (lower) labour share. However, even the capitalist class may like a labour share above the minimum (0) because they also earn a labour income; symmetrically, the worker class desires less-than-maximum labour share (1) as they also care about growth.

[11] Thus, a growth-equality trade-off is built into the model. Both the theory and evidence on this is mixed; see Aghion, Caroli and Garcia-Penalosa (1999) and Ostry, Berg and Tsangarides (2014) for reviews. Here, the key trade-off is between higher consumption and higher investment. Because consumption is facilitated by the labour share which is egalitarian, the consumption-investment trade-off translates into an equality-growth one.

3. Distributive conflict and the Utility Possibility Frontier

3.1: Partisan solutions

For future reference, it is useful to establish the solutions that each class would select unilaterally; we call these the 'partisan' solutions, to be denoted by an asterisk. They arise from maximising (9) with respect to γ, for m=i,j. The resulting first-order condition for the worker class (m=j), ignoring quadratic $(\gamma-\lambda)^2$ terms, are:

$$\rho\frac{\partial W_0^m}{\partial\gamma} = (1-\alpha)\frac{1-\gamma_m^*\varphi(\gamma_m^*-\lambda)}{\gamma_m^* A(1-\alpha)+\rho(\chi^m-\alpha)} - \frac{1+(1-\gamma_m^*)\varphi(\gamma_m^*-\lambda)}{\rho} = 0, \qquad (10)$$

where γ_m^* is the optimal for class m, defined by this condition. This implies an optimal share for individual m such that:

$$\rho\frac{1-\gamma_m^*\varphi(\gamma_m^*-\lambda)}{1+(1-\gamma_m^*)\varphi(\gamma_m^*-\lambda)} = \gamma_m^* A + \frac{\rho(\chi^m-\alpha)}{1-\alpha}. \qquad (10')$$

It is easy to check that the left (right)-hand side is monotonically decreasing (increasing) in $0\leq\gamma_m\leq 1$. In order for a unique positive equilibrium to be guaranteed, the following must hold:

Condition C1:

$$\chi^i < \frac{1-\alpha}{1-\lambda\varphi} + \alpha.$$

Condition C1 places an upper limit to inequality; if, instead, inequality is extreme such that for the capitalist class (*m*=*i*) we have: $1 < \frac{1-\alpha}{1-\lambda\varphi} + \alpha < \chi^i$, then this class prefers a negative ideal labour share. In order to incorporate a nonnegativity restriction, the partisan preference of class *i* is postulated to be $\gamma^0 \equiv \max\{\gamma_i^*, 0\}$.

Turning to specific i and j now, and using $\theta\chi^i + (1-\theta)\chi^j = 1$, from (10'), we get an expression for the partisan labour share (superscripted *) for each class:

$$\gamma_i^* A(1-\alpha) = -(1-\theta)\rho(\chi^i-\chi^j) - \rho(1-\alpha)\frac{\varphi(\gamma_i^*-\lambda)}{1+(1-\gamma_i^*)\varphi(\gamma_i^*-\lambda)}, \qquad (11a)$$

$$\gamma_j^* A(1-\alpha) = \theta\rho(\chi^i-\chi^j) - \rho(1-\alpha)\frac{\varphi(\gamma_j^*-\lambda)}{1+(1-\gamma_j^*)\varphi(\gamma_j^*-\lambda)}. \qquad (11b)$$

Of course, these partisan (i.e. unilaterally optimal) solutions will never be implemented; nonetheless, they are informative, not least in giving precise meaning to 'social conflict' below. We can obtain more succinct expressions by linearising around $\gamma_m^*=\lambda$:

$$\gamma_i^* = -(1-\Phi)\frac{(1-\theta)\rho(\chi^i-\chi^j)}{A(1-\alpha)} + \Phi\lambda, \qquad (12a)$$

$$\gamma_j^* = (1-\Phi)\frac{\theta\rho(\chi^i-\chi^j)}{A(1-\alpha)} + \Phi\lambda \quad \text{with} \qquad 0 \le \Phi \equiv \frac{\rho\varphi}{A+\rho\varphi} < 1. \tag{12b}$$

Where $0 \le \Phi \equiv \frac{\rho\varphi}{A+\rho\varphi} < 1$ reflects the importance of efficiency effects (static effects of distortions combined with 'dynamic' effects on growth) of the labour share, relative to their distributive effects. Put slightly differently, the partisan labour share is a weighted average of distributional (weight of 1-Φ) and efficiency (weight Φ) considerations.

Though it is not formally allowed (as φ<2), it is informative to consider the limiting case of $\lim_{\varphi\to\infty}\Phi = 1$: As static efficiency considerations become infinitely important (output is very responsive to deviations from the competitive-equilibrium benchmark), then the partisan labour shares are simply the competitive one by both groups; in that case, distributional considerations would be entirely sidelined. It can also be seen that stronger dynamic efficiency effects of γ on growth (captured by *A*) unambiguously decrease both partisan shares; both classes prefer a lower labour share, in order to boost growth. One key result is that social conflict (SC) is proportional to 1-Φ, with the effect that a more prominent efficiency impact of distortions (and perception of those), i.e. a higher Φ, decreases SC: There is more agreement and consensus around λ. Other special cases include the case of no static efficiency distortions of the labour share (φ→0), in which case distributive considerations are paramount: In this case, $\gamma_j^* = \frac{\theta\rho(\chi^i-\chi^j)}{A(1-\alpha)} > 0$ but $\gamma_i^* < 0$ (hence the replacement of the latter by $\gamma^0 \equiv \max\{\gamma_i^*, 0\}$). Furthermore, in the case of equal capital endowments, $\gamma_j^* = \gamma_i^* = \Phi\lambda \le \lambda$: Both classes agree on the labour share that is potentially less than the competitive one so as to foster growth rather than current consumption; note that the growth effects are captured by *A* which decreases Φ.

A couple of further remarks are in order. Firstly, comparison of (12a, b) reveals that $\gamma_i^* < \Phi\lambda < \gamma_j^*$: As mentioned, the partisan labour shares rank in reverse order to capital holdings of the two classes. Intuitively, the capitalist class enjoys higher consumption due to higher capital income, so they rely less on labour income and their partisan preference is correspondingly less.

Second, it is worth emphasising that the partisan solutions (12a, b) are points at which the expression in (10) changes sign; for $\gamma < \gamma_i^*$, we have $\frac{\partial W_0^m}{\partial \gamma} > 0$ and for $\gamma > \gamma_j^*$, we have $\frac{\partial W_0^m}{\partial \gamma} < 0$ for *m=i,j*. It is only in the range $\gamma_i^* < \gamma < \gamma_j^*$ that we have $\frac{\partial W_0^i}{\partial \gamma} < 0 < \frac{\partial W_0^j}{\partial \gamma}$. In other words, incorporating the non-negativity restriction, everyone's preferences are for the actually chosen labour share to be in the range $\gamma \in [\gamma^0, \gamma_j^*]$. These represent the bounds of effective social conflict (no-one wishes to be outside those) and we therefore restrict the distributive justice criteria to select a labour share in that range. Both partisan solutions (12a, b) are firstly anchored on the weighted Walrasian outcome (Φλ), which captures deep fundamentals such as technology. We also see that various factors affect the partisan shares in a symmetrically opposite way: (a) the rate of time preference, ρ; (b) productivity, A; and (c) the capital holdings of the typical individual in each class. Asset inequality, $\chi^i - \chi^j$, widens the gap between the partisan labour shares of the two classes and therefore intensifies social conflict. Finally, the effect of status (α) is to widen the gap between the partisan solutions, as the consumption differential due to heterogeneous asset holdings matters more. Formally, these effects are stated as follows:

Proposition 1: Properties of the partisan solutions:

a) $0 \leq \gamma^0 < \Phi\lambda < \gamma_j^*$, $\quad (\gamma^0 \equiv \max\{\gamma_i^*, 0\})$,

b) $\frac{\partial \gamma_j^*}{\partial \lambda} = \Phi$, $\quad \frac{\partial \gamma^0}{\partial \lambda} = \begin{cases} \Phi \\ 0 \end{cases} if \begin{cases} \gamma_i^* > 0 \\ \gamma_i^* < 0 \end{cases}$,

c) $\partial \gamma_j^* / \partial \rho > 0$, $\quad \partial \gamma^0 / \partial \rho \leq 0$,

d) $\partial \gamma_j^* / \partial A < 0$, $\quad \partial \gamma^0 / \partial A \geq 0$,

e) $\partial \gamma_j^* / \partial \alpha > 0$, $\quad \partial \gamma^0 / \partial \alpha \leq 0$,

f) $\partial \gamma_j^* / \partial (\chi^i - \chi^j) > 0$, $\quad \partial \gamma^0 / \partial (\chi^i - \chi^j) \leq 0$.

In the second part of Propositions 1(c-f), the equalities apply when $\gamma^0 = 0$.

Proofs are omitted as all clauses follow readily from (12a, b). Part a) ranks the partisan optima; part b) suggests that the Walrasian labour share (λ), as a natural benchmark, increases both partisan shares. The rate of time preference (ρ) increases the gap between the partisan solutions; this is due to a dual effect: the relevance of the common growth declines (note the g/ρ^2 term in (7')), while the component of consumption financed by the return to capital, which varies across classes, increases; in other words, the rise in ρ weakens the force that unites classes while it strengthens the force that divides them. An intensified consumption externality (status, α) changes the marginal utility of instantaneous consumption for both classes - corresponding to the second component of the middle term of (8') - but in opposite ways; for the worker (capitalist) class, the marginal utility increases (decreases). This change alters the equality of that term with the marginal cost of foregone growth and leads to altered partisan labour shares. In a similar vein, a greater asset inequality ($\chi^i - \chi^j$) changes marginal utilities and therefore the partisan shares in opposite ways; here, the key is that the asset differential alters the balance between the capital-financed own consumption and the average consumption which causes the status-related externality. Finally, productivity (A) changes the marginal utility of instantaneous consumption asymmetrically; in essence, a rise in productivity increases both the immediate consumption effect of the labour share but also enhances growth through (5'); for workers (capitalists), the former (latter) effect is of greater significance, which leads them to demand a lower (higher) labour share.

The distance between the partisan solutions, $\gamma_j^* - \gamma^0 > 0$, is a natural measure of social conflict:

Definition D1: The degree of social conflict is defined as $0 < SC \equiv \gamma_j^* - \gamma^0 < 1$.

The following Corollary of Proposition 1 analyses the determinants of social conflict:

Corollary 1: On the determinants of social conflict:

a) $\frac{\partial SC}{\partial \lambda} = 0$,

b) $\frac{\partial SC}{\partial \rho} > 0$,

c) $\frac{\partial SC}{\partial A} < 0$,

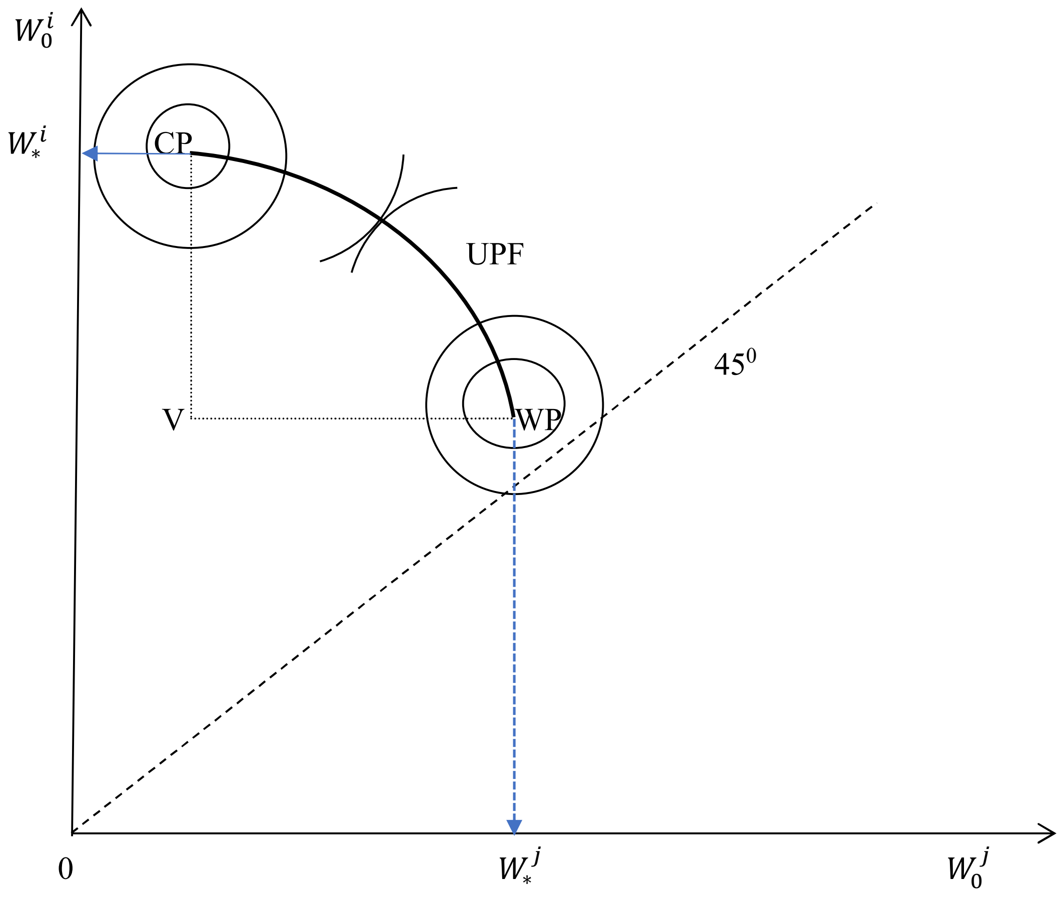
d) $\frac{\partial SC}{\partial \alpha} > 0$,

e) $\frac{\partial SC}{\partial (\chi^i - \chi^j)} > 0$.

Thus, conflict is invariant to the Walrasian benchmark. A rise in productivity (A) alleviates it: Good times moderate social conflict, thereby promoting social cohesion. More impatient societies (higher ρ) are more prone to conflict. There is a parallel here with well-known results from game theory: A greater horizon introduces repeated games and facilitates cooperate outcomes; here, a shortening of the horizon due to greater impatience gives rise to more conflict, which is akin to less coordination (see also Tornell and Lane, 1999). The intensity of the status motive (α) and a rise in asset inequality ($\chi^i - \chi^j$) also exacerbate social conflict. These results are intuitive and novel; as noted, they can shed useful light on the nature of recent real-world developments. For instance, a rise in wealth inequality (exogenous here) intensifies social conflict (part d) of Corollary 1) and may thus be a contributing factor to the rise of the 'anti-systemic' vote. So do short-term (impatient) societies and stagnating economies. We next turn to the Utility Possibility Frontier that connects these partisan solutions, along which a compromise may be sought.

3.2: Welfare and the Utility Possibility Frontier (UPF)

Figure 1: The two partisan solutions W_*^i and W_*^j, the social indifference map and the UPF



As mentioned, social conflict exists only in the range $\gamma \in [\gamma^0, \gamma_j^*]$, to which we restrict attention. A diagrammatic exposition is given in Figure 1 in (W_0^i, W_0^j) space. The maximal permissible welfare levels for the two classes are $W_*^j \equiv W_0^j|_{\gamma=\gamma_j^*}$ and $W_*^i \equiv W_0^i|_{\gamma=\gamma^0}$. Point CP is the capitalists' partisan optimal while WP that of workers'. Indicative indifference curves are also shown. Point V is defined as $V \equiv \left(W_{\gamma_j^*}^i, W_{\gamma^0}^j\right)$, i.e. a threshold point, where $W_{\gamma_j^*}^i$ and $W_{\gamma^0}^j$ are the welfares of the two respective classes at the labour share that is the partisan optimal for the other class. The restriction $\gamma^0 < \gamma < \gamma_j^*$ implies that we only consider the region northeast of the dotted lines.

In the region of interest, there is a trade-off between the welfares of the two classes – the Utility Possibility Frontier (UPF), shown by the bold line. The UPF is a downward-sloping concave curve. The welfare of the capitalist class is on the vertical axis, hence the higher up we are on the UPF, the lower the labour share. The entire area of interest is above the 45^0, reflecting the fact that by definition the welfare of the capitalist class (i) is higher than that of the worker class (j). Hence, even with their partisan labour share, workers will be worse off than capitalists; at point R (Rawlsian – see Figure 2 below), corresponding to $\gamma = \gamma_j^*$, they will have narrowed the welfare gap as much as possible. It follows from (10') for *m=i,j* that the slope of the UPF is:

$$\frac{dW_0^i}{dW_0^j} = \frac{\frac{\partial W_0^i}{\partial \gamma}}{\frac{\partial W_0^j}{\partial \gamma}} = \frac{-\frac{1}{\rho}+\frac{(1-\alpha)\,f(\gamma)}{A\gamma(1-\alpha)+\rho(\chi^i-\alpha)}}{-\frac{1}{\rho}+\frac{(1-\alpha)\,f(\gamma)}{A\gamma(1-\alpha)+\rho(\chi^j-\alpha)}} < 0 \quad \text{with } f(\gamma) \equiv \frac{1-\gamma\varphi(\gamma-\lambda)}{1+(1-\gamma)\varphi(\gamma-\lambda)} > 0, \qquad (13)$$

$$\text{for } \gamma \in [\gamma^0, \gamma_j^*], \qquad \gamma^0 \equiv \max\{\gamma_i^*, 0\}.$$

Proposition 2 dwells on the slope of the UPF as that will be important in gaining intuition on more substantive results below:

Proposition 2: On the slope of the UPF (13): For $\gamma \in [\gamma^0, \gamma_j^*]$:

a) $\frac{dW_0^i}{dW_0^j} < 0,$

b) $d\frac{dW_0^i}{dW_0^j}/d\gamma < 0,$

c) $\frac{\partial\left(\frac{dW_0^i}{dW_0^j}\right)}{\partial A} < 0,$

d) $\frac{\partial\left(\frac{dW_0^i}{dW_0^j}\right)}{\partial \alpha} > 0$, at least for (sufficient condition) $\gamma \geq (1-\theta)\gamma_j^* + \theta\gamma_i^* = \Phi\lambda,$

e) $\partial\left(\frac{dW_0^i}{dW_0^j}\right)/\partial(\chi^i - \chi^j) > 0.$

The concavity of UPF is important as several of the solutions below intuitively pick up the point of tangency between the UPF and an indifference curve of the criterion in question. All clauses follow from straightforward comparative statics analysis of (13) using (12a, b) and Assumptions 1-2. Part a) suggests that there is conflict between the classes over the labour (equivalently: capital) share in the region of interest. Part b) shows that the

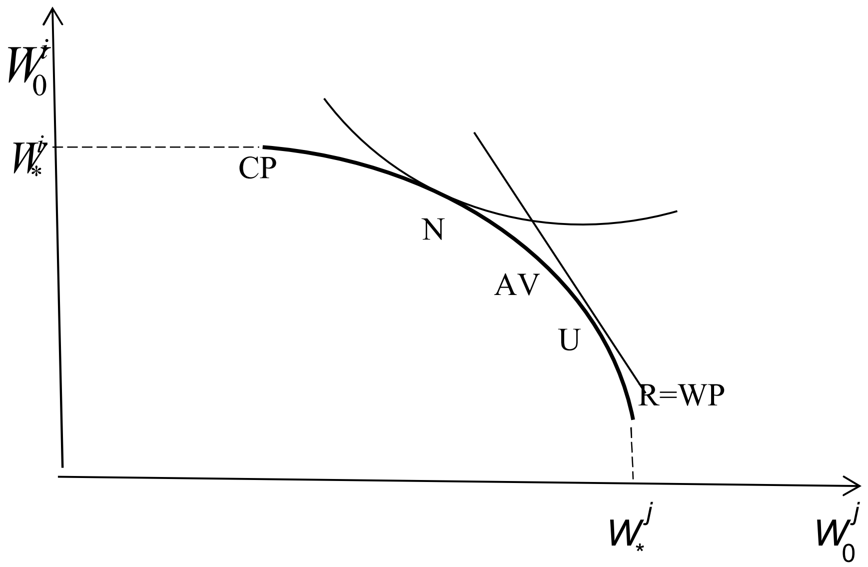
UPF is concave – it becomes steeper as γ increases, therefore as we move downwards. Part c) suggests that a rise in productivity makes the UPF steeper. This is because a rise in A raises growth but lowers marginal utility from instantaneous consumption. So, marginal utility falls for both classes but relatively more (less) for the workers whose consumption is smaller. Hence, $\left|\frac{dW_0^i}{dW_0^j}\right|$ rises. A rise in the intensity of the status motive makes the UPF flatter at least in its lower part (high labour share); finally, a rise in the inequality of asset endowments also makes the UPF flatter. Part (d), on the effects of the consumption externality, arises under the proviso of (sufficient condition) $\gamma \geq (1-\theta)\gamma_j^* + \theta\gamma_i^*$; this is always satisfied in the criteria below.

The main question of the paper is what point on the UPF is picked up by the various criteria of distributive justice. We next turn to these criteria.

4. Positive and normative criteria

In this Section, we analyse the various criteria of justice, positive and normative. In examining the labour share that they yield, recall that the labour share increases as we go down along the UPF. The situation is depicted in Figure 2 under the assumption underlying Proposition (5d).

Figure 2: Comparison of different outcomes and criteria



4.1: Two positive outcomes: Electoral competition and the Nash bargaining solution

A basic model of electoral competition is based on the median voter, but this analysis would be trivial here. As there are two classes of identical individuals, the more numerous class (here, workers) would always win the political contest; the median-voter model trivially predicts $\gamma = \gamma_j^*$. In contrast, we postulate an 'average voter' (AV) electoral model which delivers an outcome that is a weighted average of the two partisan optima, when the preferences of the classes are heterogeneous:[12]

$$\gamma^{AV} = \theta\gamma_i^* + (1-\theta)\gamma_j^* \, . \qquad (14)$$

This outcome is denoted 'AV'. Using (12a, b), we first establish:

$$\gamma^{AV} = \theta\gamma_i^* + (1-\theta)\gamma_j^* = \Phi\lambda \, . \qquad (14')$$

The AV electoral model predicts that the labour share, γ^{AV}, may or may not be greater than the Walrasian benchmark, because the proportion Φ is determined by the importance of efficiency concerns (Φ) (which also depends on the degree of preference heterogeneity across the classes).

We next turn attention to the Nash bargaining solution (see e.g. Roemer, 1996, Chapter 2). We take this solution to capture a direct bargain between the two groups, which are the decision-making units in this case: Apart from class size, the outcome of the bargain is also based on broad ideological influence (e.g. through the media, think-tanks, education, culture), political power (through affine parties and trade-unions or employer organisations) and organisational strength (cohesion, capacity for lobbying or for funding campaigns).[13] In other words, this positive criterion argues that electoral competition based on pure numerical strength is a poor predictor of factor shares; one needs to take into account additional indicators of class strength. It is this combined strength that determines policies (e.g. redistributive taxation), laws (such as minimum wages and anti-monopoly regulation) and institutions (such as those in the labour market) that in turn determine outcomes.

[12] To be more specific, this model can be justified on the ground of the 'probabilistic-voting' model of two-party electoral competition. Persson and Tabellini (2000) explains such a 'probabilistic-voting' model when two partisan parties maximize their probability of winning (alternatively, vote share) and when the degree of preference heterogeneity within each class is uncertain to these two parties. In equilibrium, income transfers are chosen to equalize voter's marginal utility of post-transfer income across different classes. This solution can alternatively be derived from maximizing a Bentahamite social welfare function which is weighted by the degree of preference heterogeneity. Hence, the maximization of the above Bentahamite social welfare function amounts to maximizing the utility of the average voter (14) (see Theorem 22.6, Acemoglu, 2009).

[13] Much of this influence may in fact be endogenous, depending on the capital endowment, but we ignore this point here.

We capture the per capita relative strength of the capitalist class by 0<ω<1: ω→1 (0) signifies that capitalists (workers) are omnipotent, while the case of ω=0.5 signifies equal per capita strength.

Analytically, the Nash solution involves the maximisation of the product of welfare improvements over a disagreement point (or 'outside option'). We hypothesise this disagreement point to be the amended Walrasian $\bar{\lambda} \equiv \Phi\lambda$ on which there is agreement between the social partners. Hence, welfare in the 'outside option' (with overbar), is:

$$\bar{W}_0^m \equiv W_0^m|_{\gamma=\bar{\lambda}}, \qquad m=i,j.$$

Taking a Taylor approximation of welfare (9) around $\gamma=\bar{\lambda} \equiv \Phi\lambda$, it is written as:

$$W_0^m = \bar{W}_0^m + \frac{\partial W_0^m}{\partial \gamma}\bigg|_{\gamma=\bar{\lambda}} (\gamma - \bar{\lambda}) , \tag{15a}$$

with

$$\frac{\partial W_0^m}{\partial \gamma}\bigg|_{\gamma=\bar{\lambda}} = \frac{f(\bar{\lambda})}{\bar{\lambda}A+\rho(\chi^m-\alpha)/(1-\alpha)} - \frac{1}{\rho} , \tag{15b}$$

where we recall the definition of *f(.)* from (13). Therefore, the improvement over the 'outside option' becomes:

$$W_0^m - \bar{W}_0^m = \frac{f(\bar{\lambda})(\gamma-\bar{\lambda})}{\bar{\lambda}A+\rho(\chi^m-\alpha)/(1-\alpha)} - \frac{(\gamma-\bar{\lambda})}{\rho} . \tag{15'}$$

Note that $\frac{\partial W_0^j}{\partial \gamma}\Big|_{\gamma=\bar{\lambda}} < 0 < \frac{\partial W_0^i}{\partial \gamma}\Big|_{\gamma=\bar{\lambda}}$ as $\gamma_i^* < \bar{\lambda} < \gamma_j^*$. Hence, we need to take absolute values.

Accordingly, the Nash bargaining criterion is given by:

$$\text{Max}_{\gamma} \quad |W_0^i - \bar{W}_0^i|^{\omega\theta} |W_0^j - \bar{W}_0^j|^{(1-\omega\theta)}, \qquad 0<\omega<1 ,$$

s.t. the UPF (13).

The capitalists' overall power is θω, the product of numerical and socio-political strength, while that of the workers is (1-ωθ). It is easily shown that the limits of the labour share determined by this solution (denoted by N) are the partisan solutions: $lim_{\omega\to 0}\, \gamma^N = \gamma_j^*$ and $lim_{\omega\to 1}\, \gamma^N = \gamma_0$. We call the iso-quant of the Nash product $|W_0^i - \bar{W}_0^i|^{\omega\theta} |W_0^j - \bar{W}_0^j|^{(1-\omega\theta)}$ the 'Nash indifference curve'; it is given by:

$$\omega\theta \frac{dW_0^i}{|W_0^i-\bar{W}_0^i|} + (1-\omega\theta)\frac{dW_0^j}{|W_0^j-\bar{W}_0^j|} = 0. \tag{16}$$

With (15') and (16), the slope of the (linearised) Nash indifference curve is:

$$\frac{dW_0^i}{dW_0^j} = -\frac{(1-\omega\theta)}{\omega\theta}\frac{\left|\frac{f(\bar{\lambda})}{\left[\bar{\lambda}A+\frac{\rho(\chi^i-\alpha)}{1-\alpha}\right]}^{-\frac{1}{\rho}}\right|}{\left|\frac{f(\bar{\lambda})}{\left[\bar{\lambda}A+\frac{\rho(\chi^j-\alpha)}{1-\alpha}\right]}^{-\frac{1}{\rho}}\right|} < 0\,. \qquad (16')$$

At the point of tangency, the slope of this curve (16') will be equated to the slope of the UPF, (13); this allows us to solve for γ^N. Tedious manipulations, relegated to Online Appendix, suggest that, to a first approximation, the Nash solution is:

$$\gamma^N = \frac{\omega\theta^2\gamma_i^* + (1-\omega\theta)(1-\theta)\gamma_j^*}{\omega\theta^2 + (1-\omega\theta)(1-\theta)} = \Phi\lambda + \frac{(1-2\omega\theta)(1-\theta)\theta}{\omega\theta^2 + (1-\omega\theta)(1-\theta)}\frac{\rho(\chi^i-\chi^j)}{A(1-\alpha)}\,. \qquad (17)$$

From the middle expression, the Nash labour share may be interpreted as a weighted average of the partisan shares, with the weights depending on both the size and socio-political influence of each class,. The second term on the right-hand side of (17) introduces a critical threshold: The properties of this solution, formalised in Proposition 3 below, critically depend on the sign of $1 - 2\omega\theta$. If $2\omega\theta > (<)1$, the overall strength of the capitalist class is more (less) than half; the opposite is the case for the workers.

Proposition 3: Determinants of the Nash bargaining solution, γ^N:

a) $sgn\left\{\frac{\partial\gamma^N}{\partial A}\right\} = sgn\{2\omega\theta - 1\}$,

b) $sgn\left\{\frac{\partial\gamma^N}{\partial\alpha}\right\} = -sgn\{2\omega\theta - 1\}$,

c) $sgn\left\{\frac{\partial\gamma^N}{\partial(\chi^i-\chi^j)}\right\} = -sgn\{2\omega\theta - 1\}$,

d) $\frac{\partial\gamma^N}{\partial\omega} < 0$.

All results follow readily from the Nash solution (17). The interpretation is that with a relatively high (low) worker power ($1-\omega\theta$), more inequality and higher implicit social competition in the form of status-seeking are insured against by means of a higher labour share. But the opposite is the case with a relatively high capitalist power. Thus, one reason why we may be seeing a lower labour share today is the rise in inequality, or social competition, coupled with high capitalist influence. And this is made clearer in the last result, which shows that the labour share determined by the Nash solution declines with capitalist power.

4.2: Two normative criteria: 'Rawlsian' justice and the Utilitarian outcome

Rawls's (1971) celebrated maximin criterion suggests that, behind the 'veil of ignorance', all individuals agree to adopt policies that maximise the welfare of the poorer, worker class (j); the rationale is that in that 'original position', no individual knows what class they will belong to when the die is cast and they take their position in society. Therefore, the criterion entails:

$$\text{Max}_{\gamma}\ W_0^j, \qquad \text{s.t. the UPF,}$$

and hence, the Rawlsian outcome (indicated by R) coincides with the partisan choice of group *j* (12a):

$$\gamma^R = \gamma_j^* \ . \tag{18}$$

Under the Utilitarian outcome, on the other hand, a weighted average of welfares is maximised, with the weights being the group or class sizes. This may be rationalised as the choice made by a social planner caring about the 'public interest'. The criterion entails:

$$\text{Max}_{\gamma}\ \ \theta W_0^i + (1-\theta) W_0^j, \qquad \text{s.t. the UPF.}$$

The iso-welfare line (called the 'Utilitarian line') is given by:

$$\theta \frac{\partial W_0^i}{\partial \gamma} + (1-\theta)\frac{\partial W_0^j}{\partial \gamma} = 0. \tag{19}$$

Therefore, the indifference curve is a straight line with slope:

$$\frac{dW_0^i}{dW_0^j} = -\frac{1-\theta}{\theta}\ . \tag{19'}$$

The first-order condition requires that the Utilitarian line be tangent to the UPF and that the slopes (13) and (19') be equalised. The Utilitarian outcome is indicated as 'U' in superscripts and in Figure 2 above. Application of Proposition 2, with the indifference curve (19'), leads to:

Proposition 4: On the Utilitarian labour share, γ^U:

a) $\frac{\partial \gamma^U}{\partial A} < 0,$

b) $\frac{\partial \gamma^U}{\partial \alpha} > 0,$ if (sufficient condition) $\gamma^U \geq (1-\theta)\gamma_j^* + \theta\gamma_i^*,$

c) $\frac{\partial \gamma^U}{\partial (\chi^i - \chi^j)} > 0.$

All these results mirror the effects of parameters on the slope of the UPF (13); see also Proposition 2. The key point is that the parameters that increase (algebraically) the slope of the UPF and make it flatter also increase γ^U. The rise in productivity produces an increase in the level of labour earnings, which is egalitarian; therefore it allows a decrease of its share in national income, so as to facilitate growth. The rise in the consumption externality (status) and inequality cause an increase in the labour share as a form of protection for the economically disadvantaged. Mirroring Proposition (2d), part (b) shows that the effect of the consumption externality arises under the proviso of (sufficient condition) $\gamma^U \geq (1-\theta)\gamma_j^* + \theta\gamma_i^*$; but as we note in Proposition 5, this is unambiguously satisfied.

4.3: Comparison

The first comparison will be between the electoral (AV) and Utilitarian outcomes. As online Appendix OA shows, we have:

$$\gamma^U > \gamma^{AV} = \Phi\lambda .$$

The second comparison is between the electoral outcome (AV) and the Nash bargaining solution. Straightforward comparison of (14') and (17) reveals that iff (necessary and sufficient condition) the overall strength of the capitalist class, including numbers and influence, is more (less) than half (i.e., $2\omega\theta > (<)1$), then $\gamma^N < (>)\gamma^{AV}$. Finally, we compare the Nash and Utilitarian solutions. It turns out that, at least (sufficient condition) under a high capitalist power, i.e. $2\omega\theta > 1$, hence $\frac{(1-\omega\theta)}{\omega\theta} < 1$, we get $\left|\frac{dW_0^i}{dW_0^j}\Big|_{Nash}\right| < \frac{(1-\theta)}{\theta}$: In this case, the slope of the N-indifference curve is less than the slope of U, therefore $\gamma^N<\gamma^U$; again, Online Appendix has the details.[14] The condition $2\omega\theta > 1$ is in fact quite plausible, so it may be built into the comparisons with some confidence. We bring all these results together below:

Proposition 5: Relation between labour shares under different criteria:

a) $\gamma_i^* < \gamma^{AV} = (1-\theta)\gamma_j^* + \theta\gamma_i^* = \Phi\lambda < \gamma^U < \gamma^R = \gamma_j^*$,

b) $sgn\{\gamma^N - \gamma^{AV}\} = sgn\{1 - 2\omega\theta\}$,

c) At least (sufficient condition) under $2\omega\theta > 1$, $\gamma^N < \gamma^U$,

[14] This seems to contradict Binmore (2005, Ch. 11, p. 180): 'People in need get more because their desperation leads them to bargain harder.'

d) Thus, for $2\omega\theta > 1$, we have a full ranking:

$$\gamma_i^* < \gamma^N < \gamma^{AV} = (1-\theta)\gamma_j^* + \theta\gamma_i^* = \Phi\lambda < \gamma^U < \gamma^R = \gamma_j^*.$$

The ranking shown in Proposition (5d) is depicted graphically in Figure 2.

5. Justice as minimal social friction

We finally introduce a third normative criterion and concept of justice, based on minimising 'social friction', defined below. To our knowledge, this concept has not been proposed in existing literature. We define 'social friction' to be the welfare difference between classes, divided by the degree of synergy in society. The welfare difference is group size-weighted; while synergy is the sum of welfares over and above a benchmark ($\bar{V}$ below). In other words, synergy is the aggregate welfare above a threshold. Thus, social friction may be defined as:

Definition D2: Social friction F:

$$F \equiv \frac{\left(\theta W_0^i - (1-\theta) W_0^j\right)^2}{\left(\theta W_0^i + (1-\theta) W_0^j - \bar{V}\right)^2}. \qquad (20)$$

The numerator gives the difference in group welfares, while the denominator gives the improvement in aggregate welfare over the reference point: $\bar{V}$>0 is a benchmark aggregate welfare, defined as:

$$\bar{V} \equiv \theta W_{\gamma_j^*}^i + (1-\theta) W_{\gamma^0}^j. \qquad (21)$$

The benchmark $\bar{V}$ aggregates $W_{\gamma_j^*}^i$ and $W_{\gamma^0}^j$, the per capita welfare in each group if the *other* partisan labour share prevails. The square of the numerator in 18) is taken in order to rule out negative quantities; and the denominator is squared for symmetry.

Based on this, distributive justice as minimal social friction may be defined as,

Min $_\gamma$ F, s.t. the UPF ,

i.e., as the point on the UPF that minimises social friction. This criterion combines both the maximisation of joint welfare implicit in the Utilitarian criterion and the minimisation of difference implicit in the Rawlsian criterion. It is motivated by the aim to minimise welfare differences (egalitarianism) whilst taking into account the effects on growth and welfare improvement (efficiency implications). So, there are two ways to reduce social friction: One is to reduce inequality, while the other to increase the average. Policies that increase inequality may be tolerated if they deliver a sufficiently strong increase in the total 'pie'. These issues can be meaningfully analysed in the present production economy.

Setting a labour share that minimises F yields a point of the UPF where the following holds:

$$\frac{dW_0^i}{dW_0^j} = \frac{(1+\sqrt{F})(1-\theta)}{(1-\sqrt{F})\theta} \,. \qquad (22)$$

Appendix B shows the indifference map described by (20); it is shown that the indifference curves are downward-sloping and convex in the quadrant northeast of point *V'* of Figure B1. We have $dW_0^i/dW_0^j < 0$ in the area of interest. Importantly also, F falls as we move outwards in the northeast region of the diagram, so the usual tangency-point geometry applies. Thus, in that region, F is high enough that 1-√F <0. Since the Utilitarian solution involves $\frac{dW_0^i}{dW_0^j} = -\frac{1-\theta}{\theta}$ and since (1+√F)/(1-√F)<-1, it follows that the 'minimal social friction' criterion gives a point (denoted by F) with a steeper slope along the UPF than the Utilitarian criterion. Therefore the labour share thus determined will be between the Utilitarian and Rawlsian criteria. In other words, the 'minimal friction' criterion is more egalitarian than the Utilitarian criterion, as it also pays attention to the distribution of the pie as well as its size; but it is less egalitarian than the Rawlsian one exactly because it also pays more attention to the production implications of egalitarianism. Thus, this criterion may be seen as a half-way house between the mildly egalitarian Utilitarian criterion and the extremely inequality-averse Rawlsian criterion. We summarise:

Proposition 6: The labour share according to the 'minimal social friction' criterion obeys:

$$\gamma^U < \gamma^F < \gamma^R.$$

As a corollary, we can extend Proposition 5 part (c) to argue that all our normative criteria require a higher labour share, and more equality, than all the positive criteria at least for a relatively high capitalist influence ($2\omega\theta > 1$).

Proposition 7: Relation between positive and normative outcomes

At least (sufficient condition) under $2\omega\theta > 1$, we get: $\max\{\gamma^{AV}, \gamma^N\} < \min\{\gamma^U, \gamma^F, \gamma^R\}$.

The significance of this result is that, under the stated plausible proviso, both positive criteria are less-than-optimally egalitarian if measured up against our three normative criteria. If the model and our positive criteria capture key aspects of reality, then there is a sub-optimal (judged against plausible benchmarks of justice) bias against egalitarianism. This important result, not highlighted before, is worth further investigation in more realistic setups. We leave this to future work.

6. Conclusions

At the interface between economics, philosophy and other social disciplines, distributive justice is gaining attention as inequality is arguably rising in most advanced economies. Our contribution is to examine it in a simple aggregate production economy which captures the growth implications of any distributive arrangements. In particular, we focus on the 'functional' division of income into capital and labour shares as a key aspect of income distribution; considered as fixed at about one to three, the ratio of factor shares is now reportedly changing. In this paper, we treat the labour share as endogenous and the choice variable in the distributive justice game. Our analysis casts light on factor shares and their development but the paper is primarily about wider issues related to distributive justice. We take a social conflict approach to all these issues: There is a multi-faceted social conflict among the beneficiaries of the capital and labour shares. The main thrust of the paper is to explore the criteria, positive and normative, that resolve the conflict. The normative criteria are informed by notions of distributive justice, while the positive ones capture key features of political institutions. We emphasise the interaction between macroeconomic outcomes (growth, welfare) and such possible resolutions. Despite the importance of these topics, and important individual advances, there is as yet only limited integration of distributive justice with standard macroeconomic theory; this paper's overall contribution is in this direction.

More specifically, we introduce conflict about factor shares into a simple production and growth setup, an AK model. There are two groups of otherwise identical capital-rich ('capitalists') and capital-poor ('workers') individuals; all supply one unit of labour inelastically. For concreteness, we assume that 'workers' are more numerous. As is standard in the AK model with fixed labour, all individuals consume the labour share plus a fraction of their capital wealth; thus, capitalists consume more and have a higher period utility. All agents are subject to a status (or 'keeping up with the Joneses') consumption externality. The key point, and the main result of the capital heterogeneity, is the conflict in preferences of the two classes concerning the fraction of income that is awarded to labour. A higher labour share fuels current consumption but also harms growth as it reduces the marginal return to capital. As the capitalists' consumption is based relatively more on wealth, they are relatively more keen to safeguard growth, therefore they prefer a lower labour share; while the workers are keen to have a higher labour share (though not unity, as they are aware of the growth consequences of that). Thus, both classes have non-trivial partisan preferences over the labour share, with the workers preferring more of it. Social conflict is defined as the difference between the partisan preferences on labour shares between the two groups.

We develop the utility possibilities frontier (UPF) that shows the trade-off between the intertemporal welfare of the two groups and the effects of the labour share on 'static efficiency', 'dynamic efficiency' (i.e. growth) and labour income. We consider two positive criteria for resolving the distributive conflict and determining the labour share, namely an 'average voter' model and the Nash bargaining solution that is determined by a socio-political bargain between the two groups. In normative terms, we consider the Rawlsian minimax criterion of maximising the welfare of the poorest, and the Utilitarian criterion. Another contribution of this paper is to propose the criterion of minimising 'social friction', defined as the welfare difference among the groups divided by aggregate welfare gains from a benchmark (as a measure of social synergy or cooperation). Thus, both distributive and growth considerations are incorporated in this criterion, which may be further argued to be a hybrid between the extremely equality-minded Rawlsian criterion and the 'common good' Utilitarian approach. We explore its nature in Appendix B.

Our key results are stated in a series of Propositions. Social conflict is exacerbated by greater impatience (the rate of time preference), intensified status comparisons and negative consumption externalities, greater wealth inequality and a decline in productivity. These are interesting results in their own right and, arguably, highly topical as they may shed light on the 'anti-systemic' vote and sentiment. We also find that status comparisons and wealth inequality tend to raise the labour share under all positive and normative criteria. We also provide a ranking of most of those criteria: We show that the highest labour share is proposed by the Rawlsian criterion, which is equal to the partisan workers' preference – almost by definition. The second highest labour share is given by our proposed 'minimum social friction' criterion, followed by the Utilitarian one. The lowest labour share is given by the 'average voter' solution which equals the modified Walrasian outcome ($\Phi\lambda$). Comparing the Nash bargaining solution with the Utilitarian criterion, the former involves a lower labour share at least under the assumption that the capitalists' overall power is greater than that of workers. If so, both positive criteria deliver a lower labour share, and are therefore less egalitarian, than all our three normative criteria. In that sense, it may be argued that there is an in-built anti-egalitarian bias in key economic and political institutions that is sub-optimal from the point of view of established norms of justice; this has not been highlighted before.

Appendix A: Derivation of the aggregative production function (1)

We first incorporate monopolistic competition into our model so as to capture product market power. We postulate a continuum of firms $n \in [0,1]$, each of which produces a differentiated good and therefore is monopolistically competitive; a Dixit-Stiglitz (1977) function aggregates all good varieties into a composite good:

$$y = \left(\int_0^1 y^{n\frac{\theta-1}{\theta}} dn \right)^{\frac{\theta}{\theta-1}}. \qquad \text{(A.1)}$$

A superscript indicates the variety produced by the n-th firm and y (without a subscript) indicates aggregate output. As is standard, each monopolistic firm faces a demand for its variety:

$$y^n = (p^n/p)^{-\theta} y, \qquad \text{(A.2)}$$

where p^n is the nominal price of variety -n-, p the aggregate price level, defined as: $p = \left(\int_0^1 p^{n1-\theta} dn \right)^{\frac{1}{1-\theta}}$, and θ the elasticity of substitution between varieties.

The firm's production function, this is postulated to be of a constant elasticity of substitution (CES) form so as to allow a meaningful discussion of factor shares:

$$y^n = a[(1 - \Lambda)(k^n)^b + \Lambda(kl^n)^b]^{1/b}, \qquad b \neq 0,\ b < 1, \qquad 0 < \Lambda < 1. \qquad \text{(A.3)}$$

The presence of aggregate capital proxies the labour-augmenting productivity due to externalities arising from 'learning-by-doing' (Romer, 1986) or from productive public services (Barro, 1990). $\varepsilon = 1/(1-b)$ is the elasticity of substitution between factors of production (i.e., k^n and l^n). Parameters ε and b are related by: $b = \frac{\varepsilon-1}{\varepsilon} < 1$ with $\varepsilon < \infty$. There is no government or external sector.

Due to the unit mass of firms and because capital is homogeneous (made up of the aggregate good, which is common to all firms), each monopolistic firm is perfectly competitive in the capital rental as well as labour markets, taking the economy-wide rental price of capital (r) and the real wage (w) as given. Setting up the profit maximisation problem for firm -n- facing their product demand (A.2) and production function (A.3), we have:

$$\text{Max}_{k^n, l^n} \quad \frac{p^n y^n}{p} - rk^n - wl^n,$$

$$\text{s.t.} \quad y^n = (p^n/p)^{-\theta} y \quad \text{and} \quad \text{(A.3)}.$$

The first-order conditions with respect to capital (k^n) and labour (l^n) are respectively given by :

$$a^b \left(\frac{y^n}{y}\right)^{-1/\theta} \left(\frac{1}{1+\mu}\right)(1-\Lambda)\left(\frac{y^n}{k^n}\right)^{1-b} = r, \tag{A.4}$$

$$a^b \left(\frac{y^n}{y}\right)^{-1/\theta} \left(\frac{1}{1+\mu}\right)\Lambda k^b \left(\frac{y^n}{l^n}\right)^{1-b} = w, \tag{A.5}$$

where $1+\mu \equiv \frac{\theta}{\theta-1}$, therefore $\mu \geq 0$ is an index of the (symmetric) monopoly power of each firm; $\mu=0$ is the polar case of perfect competition. Dividing (A.4) by (A.5) yields the relative demands for the two factors:

$$\frac{l^n}{k^n} = k^{\frac{b}{1-b}} \left(\frac{r}{w}\frac{\Lambda}{1-\Lambda}\right)^{\frac{1}{1-b}}. \tag{A.6}$$

Aggregating over a continuum of firms, we obtain the *aggregate* demand for labour:

$$l = k^{\frac{1}{1-b}} \left(\frac{r}{w}\frac{\Lambda}{1-\Lambda}\right)^{\frac{1}{1-b}}. \tag{A.7}$$

The wage will be assumed to be fixed by 'labour market institutions (or distortions)'. For (A.6) and (A.7) to have a steady state in a growing economy, the wage should grow in line with the economy's capacity, proxied by k. *This allows the wage bill to be a constant fraction of output in the steady state with a constant labour*. Hence, we can postulate $w \equiv \Omega(1+\omega)k$, where $\omega \geq 0$ indicates labour market distortions and Ω is a scaling factor that will be defined later.

By substituting (A.6) into l^n in (A.3) and rearranging, we can get the individual and aggregate production functions:

$$y^n = ak^n \left[(1-\Lambda) + \Lambda k^{\frac{b}{1-b}} \left(\frac{r}{\Omega(1+\omega)}\frac{\Lambda}{1-\Lambda}\right)^{\frac{b}{1-b}}\right]^{1/b} = ak^n[(1-\Lambda)+\Lambda l^b]^{1/b}, \quad b \neq 0. \tag{A.8}$$

Aggregating yields

$$y = ak[(1-\Lambda)+\Lambda l^b]^{1/b}, \qquad b \neq 0. \tag{A.9}$$

Using (A.6), (A.7), (A.8) and (A.9) we can show that the individual-aggregate output and factor ratios are equalised;

$$\frac{y^n}{y} = \frac{k^n}{k} = \frac{l^n}{l}. \tag{A.10}$$

From (A.7) with the $w \equiv \Omega(1+\omega)k$ rule, the aggregate demand for labour is:

$$l = \left(\frac{\Lambda r}{(1-\Lambda)\Omega(1+\omega)}\right)^{\frac{1}{1-b}}. \tag{A.11}$$

Combining (A.2) with (A.4) and aggregating, the interest rate is:

$$r = a^b \frac{1-\Lambda}{1+\mu}\left(\frac{y}{k}\right)^{1-b}. \tag{A.12}$$

The interest rate equals the marginal revenue product of capital. Note that setting μ=0 implies perfect competition in the product market, so that the equality between the interest rate and the marginal product holds. The product and labour market 'institutions' (μ, ω ≥ 0, respectively), are also variously called 'distortions', 'frictions' or 'rigidities', as they imply deviations from the competitive benchmark.

Individuals supply any quantity of their homogenous labour that is demanded by firms when the wage is set according to the $w \equiv \Omega(1+\omega)k$ rule. From the aggregate demand for labour (A.11), the interest rate (A.12) and the aggregate production function (A.9), we obtain labour demand:

$$l = D \equiv \frac{\delta(1-\Lambda)}{1-\Lambda\delta} > 0, \qquad 0<\Lambda\delta<1, \tag{A.13}$$

where

$$\delta \equiv \left(\frac{\Lambda a}{(1+\mu)\Omega(1+\omega)}\right)^{\frac{b}{1-b}} > 0 \tag{A.14}$$

represents a composite index of product market monopoly power (μ) and labour market institutions (ω). Appropriate choice of units ensures that $0<\Lambda\delta<1$, as we discuss below.

We shall find it useful to express everything in terms of D (the inverse of labour demand). Solving from (A.13):

$$\Lambda\delta = \frac{\Lambda D}{[(1-\Lambda)+\Lambda D]}. \tag{A.15}$$

We may define a value of δ without any imperfections in the capital and labour markets (i.e., setting $\mu = \omega = 0$)

$$\delta_{comp} \equiv \left(\frac{\Lambda a}{\Omega}\right)^{\frac{b}{1-b}} > 0 \tag{A.16}$$

Setting labour units such that $l_{comp} = 1$ implies from (A.13) $\delta_{comp} = 1$, and this allows us to define $\Omega \equiv \Lambda a$. From (A.15) $\delta_{comp} = 1$ also implies $D_{comp} = 1$. Therefore, it follows from (A.14) that

$$\delta = \delta_{comp}\left(\frac{1}{(1+\mu)(1+\omega)}\right)^{\frac{b}{1-b}} = \left(\frac{1}{(1+\mu)(1+\omega)}\right)^{\frac{b}{1-b}}. \tag{A.17}$$

Using labour demand (A.5) and factor symmetry (A10) and aggregating, we obtain:

$$a^b \frac{\Lambda\left(\frac{y}{kl}\right)^{1-b} k}{1+\mu} = w. \tag{A.18}$$

The labour share is $\gamma \equiv wl/y$, therefore using (A.18), (A.9), (A.13), (A.15), and $\Omega \equiv \Lambda a$ as appropriate it becomes:

$$\gamma = \frac{l}{y} a^b \frac{\Lambda\left(\frac{y}{kl}\right)^{1-b} k}{1+\mu} = l^b a^b \frac{\Lambda\left(\frac{y}{k}\right)^{-b}}{1+\mu} = \frac{1}{1+\mu}\frac{\Lambda D}{(1-\Lambda)+\Lambda D} = \frac{\Lambda\delta}{1+\mu} = \frac{\Lambda}{1+\mu}\left(\frac{1}{(1+\mu)(1+\omega)}\right)^{\frac{b}{1-b}}, \tag{A.19}$$

Re-writing equation (A.9):

$$\frac{y}{k} = a[(1-\Lambda)+\Lambda D]^{\frac{1}{b}} = a\left[\frac{1-\Lambda}{1-\Lambda\left(\frac{1}{(1+\mu)(1+\omega)}\right)^{\frac{b}{1-b}}}\right]^{\frac{1}{b}} = a\left[\frac{1-\Lambda}{1-\gamma(1+\mu)}\right]^{\frac{1}{b}}, \tag{A.20}$$

Next, in order to render the production function into the standard '*AK*' form (where *l*=1 trivially), we set $A \equiv a\left(\frac{1}{(1+\mu)(1+\omega)}\right)^{\Lambda/(1-\Lambda)}$. With this, the production function (A.20) becomes:

$$\frac{y}{k} = A\left[\frac{(1-\Lambda)}{\left(\frac{1}{(1+\mu)(1+\omega)}\right)^{\frac{b\Lambda}{1-\Lambda}} - \Lambda\left(\frac{1}{(1+\mu)(1+\omega)}\right)^{\frac{b}{1-b}+\frac{b\Lambda}{1-\Lambda}}}\right]^{\frac{1}{b}}. \tag{A.21}$$

Using the expression for the labour share (A.19), we may re-write (A.21) as:

$$\frac{y}{k} = A\left[\frac{1-\Lambda}{\gamma^{\frac{(1-b)\Lambda}{1-\Lambda}}\left(\frac{1+\mu}{\Lambda}\right)^{\frac{(1-b)\Lambda}{1-\Lambda}}(1-\gamma(1+\mu))}\right]^{\frac{1}{b}}. \tag{A.22}$$

Following Harberger (1954), the dead welfare loss (*DWL*) in the product market, which is caused by distortions in the labour market, is approximated by

$$DWL \equiv \frac{1}{2}(\gamma-\lambda)(y(\gamma)-y) \cong \frac{1}{2}\frac{dy}{d\gamma}\bigg|_{\gamma=\lambda}(\gamma-\lambda)^2, \tag{A.23}$$

where

$$y(\gamma) \equiv ak\left[\frac{1-\Lambda}{1-\gamma(1+\mu)}\right]^{\frac{1}{b}} \quad \text{and} \quad \frac{d(y/k)}{d\gamma} = -\frac{1}{b}\frac{y}{k}\left[(1-b)\frac{\Lambda}{(1-\Lambda)\gamma} - \frac{1+\mu}{(1-\gamma(1+\mu))}\right],$$

the second equation of which implies

$$\left.\frac{d(y/k)}{d\gamma}\right|_{\gamma=\lambda} = A\frac{1+\mu}{1-\Lambda}. \qquad (A.24)$$

Using (A.24), the dead welfare loss can be rewritten as follows:

$$DWL = \frac{1}{2}(\gamma-\lambda)\left.\frac{dy}{d\gamma}\right|_{\gamma=\lambda}(\gamma-\lambda) = Ak\frac{1}{2}\frac{1+\mu}{1-\Lambda}(\gamma-\lambda)^2. \qquad (A.25)$$

Using (A.25), the net production function is given by

$$\frac{y}{k} = y(\gamma)|_{\gamma=\lambda} - DWL{=}Ak\left(1 - \frac{1}{2}\frac{1+\mu}{1-\Lambda}(\gamma-\lambda)^2\right),$$

which can be expressed by

$$y = Ak\left(1-\frac{\varphi(\gamma-\lambda)^2}{2}\right), \qquad 0<\varphi\equiv\frac{1+\mu}{1-\Lambda}<2.$$

The restriction φ<2 is imposed so as for the parenthesis to be positive. Finally, using (A.20), the interest rate (A.12) becomes:

$$r = a^b\frac{1-\Lambda}{1+\mu}\left(\frac{y}{k}\right)^{1-b} = \frac{1-\Lambda}{1+\mu}\frac{y}{k}\left(\frac{1-\Lambda}{1-\gamma(1+\mu)}\right)^{-1} = \frac{1-\gamma(1+\mu)}{1+\mu}\frac{y}{k} \cong (1-\gamma-\mu)\frac{y}{k}.$$

The key point is that *r* essentially inherits the properties of *y/k* times (1-γ). To avoid further clatter without losing anything essential, we simplify this to equation (6) of the main text.

Appendix B: The 'iso-F' indifference map

$$dW_0^i / dW_0^j = \frac{(1+\sqrt{F})(1-\theta)}{(1-\sqrt{F})\theta}. \qquad (22)$$

It is straightforward to check that:

$$\frac{1+\sqrt{F}}{1-\sqrt{F}} = \frac{2\theta W_0^i - \overline{V}}{-2(1-\theta)W_0^j + \overline{V}}.$$

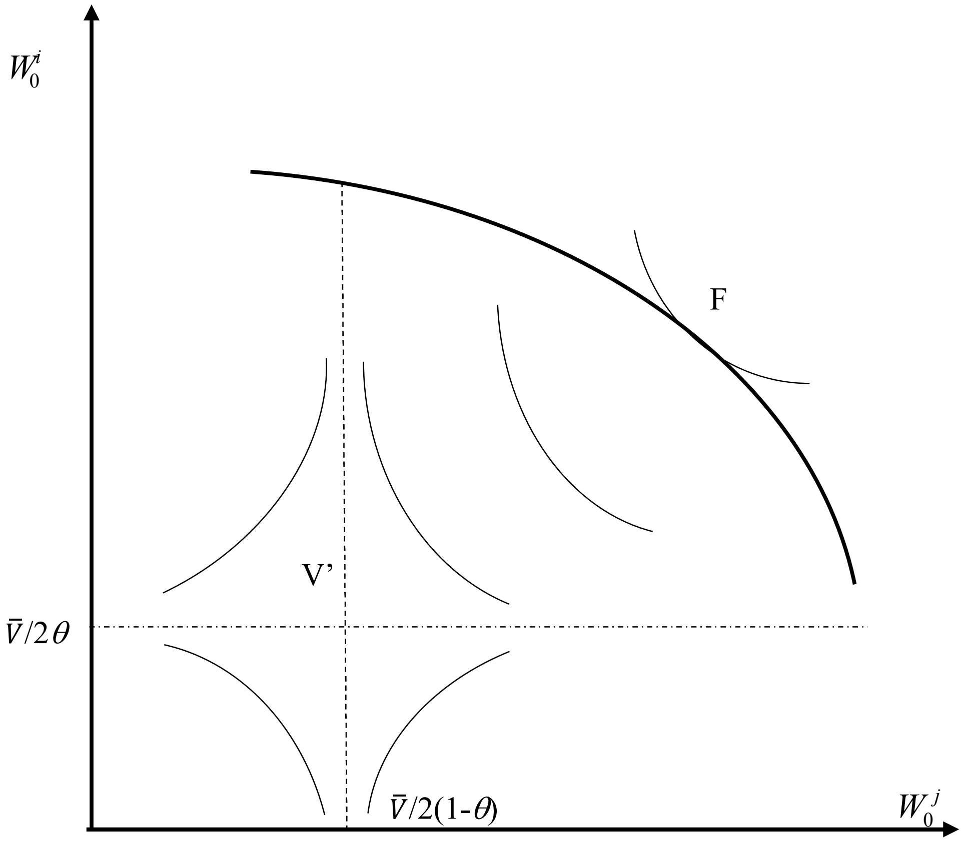
The indifference map is shown in Figure B.1; this is the same as Figure 1 of the main text with the exception that the reference point is $V' \equiv \left(\frac{\overline{V}}{2\theta}, \frac{\overline{V}}{2(1-\theta)}\right)$. Note that the region northeast of *V'* does not correspond exactly to the boundaries of the UPF (the boundaries of the northeast quadrant can be either higher or lower, left or right, than the boundaries of the area under the UPF). It is readily checked that these indifference curves are convex against the origin in the quadrant northeast of *V'*.

Importantly, F falls as we move outwards in the northeast region. Consider any ray that extends northeastwards from the V' point, i.e. such that the ratio of welfares remains constant; alongside any such ray, $\sqrt{F}$ equals

$$\sqrt{F} = \frac{W_0^i(\theta+(1-\theta)\xi)}{W_0^i(\theta+(1-\theta)\xi)-\bar{V}}.$$

Where the ratio of utilities $\xi \equiv W_0^j/W_0^i$ is constant along the ray. Then, it is easy to check that $\sqrt{F}$ falls with W_0^i so that, as the iso-curves do not cross, an outer curve involves lower friction than an inner one. It follows that a tangency point (F) minimises F.

Figure B1: The iso-F indifference map



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