# On the Logical Origin of the Laws Governing the Fundamental Forces of Nature: A New Axiomatic Matrix Approach

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The main idea of this article is based on my previous publications (Refs. [1], [2], [3], [4], 1997-1998). In this article we present a new axiomatic matrix approach (and subsequently constructing a linearization theory) based on the ring theory and the generalized Clifford algebra. On the basis of this (primary) mathematical approach and also the assumption of discreteness of the relativistic energy-momentum (D-momentum), by linearization (and simultaneous parameterization, as necessary algebraic conditions), followed by "first" quantization of the relativistic energy-momentum relation, a unique and original set of the general relativistic single-particle wave equations are derived directly. These equations are shown to correspond to certain massive forms of the laws governing the fundamental forces of nature, including the Gravitational, Electromagnetic and Nuclear field equations (which based on this approach are solely formulable in (1+3) dimensional space-time), in addition to the (half-integer spin) single-particle wave equations such as the Dirac equation (formulated solely in (1+2) dimensional space-time). Each derived single-particle field equation is in a complex tensor form, where in matrix representation (i.e. in the geometric algebra formulation) it could be written in the form of two coupled symmetric equations – which assumedly have chiral symmetry if the particle wave equation be source-free. We show that the massless cases of the complex relativistic wave equations so obtained correspond to the classical fields including the Einstein, Maxwell and Yang-Mills field equations. In particular, a unique massive form of the general theory of relativity – with a definite complex torsion – is shown to be obtained solely by first quantization of a special relativistic algebraic matrix relation. Moreover, it is shown that the massive Lagrangian density of the obtained Maxwell and Yang-Mills fields could be also locally gauge invariant – where these fields are formally re-presented on a background space-time with certain (coupled) complex torsion which is generated by the invariant mass of the gauge field carrier particle. Subsequently, in agreement with certain experimental data, the invariant mass of a particle (that would be identified as massive photon) has been specified ($m_0 \approx 1.4070696 \times 10^{-41}$ kg), which is coupled with background space-time geometry. Assuming our approach is the unique and principal way for deriving (all) the laws governing the fundamental forces of nature, then based on the unique structure of general relativistic single-particle wave equations derived and also the assumption of chiral symmetry as a basic discrete symmetry of the source-free cases of these fields, it is shown that the universe cannot have more than four space-time dimensions. In addition, a mathematical argument for the asymmetry of left and right handed (interacting) particles is presented. Furthermore, on the basis of definite mathematical structure of the field equations derived, we also conclude that magnetic monopoles (in contrast with electric monopoles) could not exist in nature.[1]

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## 1. Introduction and Summary

**W**hy do the fundamental forces acting on the Universe (i.e., the forces that appear to cause all the movements and interactions) manifest in the way, shape, and form they do? This is one of the greatest ontological questions that science can investigate. In this article, we are going to consider this question by a mathematical axiomatic (*matrix*) approach.

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**E**ugene Wigner's foundational paper, “On the Unreasonable Effectiveness of Mathematics in the Natural Sciences”, famously observed that purely mathematical structures and relations often lead to deep physical insights, in turn serving as the basis of highly successful physical theories [50]. Referring to the Oxford English dictionary, a law of physics (or a scientific law) is: "A theoretical principle deduced from particular facts, applicable to a defined group or class of phenomena, and expressible by the statement that a particular phenomenon always occurs if certain conditions be present [55]. In actual fact, laws of physics (including the fundamental laws) are typically conclusions based on repeated scientific experiments and observations over many years and which have become accepted universally within the scientific communities, and one of the most fundamental and operational aims of the human race has been to acknowledge (as truths) and formulate a summary description of the natural world in the form of such laws [56, 57]. In this article, based on our axiomatic approach, we show that one of the most fundamental logical blocks of the universe's structure are certain "matrices", which their components are the quantized (discrete) basic physical quantities (i.e. are "integer" multiples of the quantum of action "$h$"). Subsequently, we provide a unique logical foundation to the most acknowledged fundamental (empirical) laws of nature, i.e. the laws governing the fundamental forces of nature.

**T**his article is based on my previous publications (Refs. [1], [2], [3], 1997-1998), and also my thesis work [4] (but in a new generalized and axiomatized framework). We present a new axiomatic matrix approach based on the algebraic structure of ring theory (including the integral domains [5]) and the generalized Clifford algebra [40 - 47], and subsequently, we construct a linearization theory. On the basis of this (primary) mathematical approach and the assumption of discreteness of the relativistic energy-momentum (D-momentum), **by linearization (and simultaneous parameterization, as necessary algebraic conditions), followed by first quantization of the special relativistic energy-momentum relation (defined algebraically for a single particle with invariant mass $\boldsymbol{m_0}$)**, we derive a unique and original set of the general relativistic (single-particle) wave equations directly. These equations are shown to correspond uniquely to certain massive forms of the laws governing the fundamental forces of nature, including the Gravitational, Electromagnetic and Nuclear field equations (which are solely formulable in (1+3) dimensional space-time), in addition to the (half-integer spin) single-particle wave equations (formulated solely in (1+2) dimensional space-time).

Each derived relativistic wave equation is in a complex tensor form, that in the matrix representation (i.e. in the geometric algebra formulation) it could be written in the form of two coupled symmetric equations – which assumedly have chiral symmetry if the particle wave equation be source-free. In fact, the complex relativistic (single-particle) wave equations so uniquely obtained, correspond to certain massive form of classical fields including the Einstein, Maxwell and Yang-Mills field equations, in addition to the (half-integer spin) single-particle wave equations such as the Dirac equation (where the Dirac spinor field is isomorphically re-presented solely by a tensor field in three dimensional space-time [29, 31]).

In particular, a unique massive form of the general theory of relativity – with a definite complex torsion – is shown to be obtained solely by first quantization of a special relativistic algebraic matrix relation. Moreover, it is shown that the massive Lagrangian density of the obtained Maxwell and Yang-Mills fields could be also locally gauge invariant – where these fields are formally re-presented on a background space-time with certain (coupled) complex torsion, which is generated by the invariant mass of the gauge field carrier particle. Subsequently, in agreement with certain experimental data, the invariant mass of a particle (that actually would be identified as massive photon) has been specified ($m_0 \approx 1.4070696 \times 10^{-41}$ kg), which is coupled with background space-time geometry (see Section 3-4-1).

Assuming our approach is the unique and principal way for deriving (all) the laws governing the fundamental forces of nature, then based on the unique structure of general relativistic single-particle fields derived and also the assumption of chiral symmetry as a basic discrete symmetry of the source-free cases of these fields, it has been shown that the universe cannot have more than four space-time dimensions. In addition, on the basis of definite structure of the field equations derived, we also conclude that magnetic monopoles - in contrast with electric monopoles - could not exist in nature. Furthermore, a basic argument for the asymmetry of left and right handed (interacting) particles is presented.

**1-1. T**he main arguments and consequences presented in this article (particularly in connection with the logical origin of the laws governing the fundamental forces) follow from these three basic and primary assumptions:

**(1)- "Generalization of the algebraic axiom of nonzero divisors for integer elements (based on the ring theory and the matrix representation of generalized Clifford algebra, and subsequently, constructing a definite algebraic linearization theory);"**

This is one of the new and principal concepts presented in this article (see Section 2-1, formula (23)).

**(2)- "Discreteness of the relativistic energy-momentum (D-momentum);"**

This is a basic and original quantum mechanical assumption. Quantum theory, particularly, tells us that energy and momentum are only transferred in discrete quantities, i.e., as integer multiples of the quantum of action (Planck constant) $h$.

**(3)- "The general relativistic massive forms of the laws governing the fundamental forces of nature, including the gravitational, electromagnetic and nuclear field equations, in addition to the relativistic (half-integer spin) single-particle wave equations, are derived solely by first quantization (as a postulate) of linearized (and simultaneously parameterized, as necessary algebraic conditions) unique forms of the relativistic energy-momentum relation – which are defined algebraically for a single particle with invariant mass $m_0$)."**

We also assume that the source-free cases of these fields have "chiral symmetry".

**F**ollowing is a summary description of some notable consequences of the axiomatic matrix approach presented in this article (note that the geometrized units, metric signature (+ − ... −) and the sign conventions (97) will be used):

**1-2.** Two categories of the general relativistic (single-particle) wave equations are derived directly by linearization (and simultaneous parameterization, as necessary algebraic conditions), followed by first quantization (as a postulate) of the special relativistic energy-momentum relation (defined for a particle with rest mass $m_0$), as follows:

$$\breve{\nabla}_\lambda R_{\mu\nu\rho\sigma} + \breve{\nabla}_\mu R_{\nu\lambda\rho\sigma} + \breve{\nabla}_\nu R_{\lambda\mu\rho\sigma} = T^\tau{}_{\lambda\mu} R_{\tau\nu\rho\sigma} + T^\tau{}_{\mu\nu} R_{\tau\lambda\rho\sigma} + T^\tau{}_{\nu\lambda} R_{\tau\mu\rho\sigma} \,, \tag{1-1}$$

$$\breve{\nabla}_\mu R^\mu{}_{\nu\rho\sigma} - \frac{im_0^{(G)}}{\hbar} k_\mu R^\mu{}_{\nu\rho\sigma} = -J^{(G)}_{\nu\rho\sigma} \tag{1-2}$$

$$R_{\mu\nu\rho\sigma} = (\partial_\nu \Gamma_{\rho\sigma\mu} - \Gamma^\lambda{}_{\rho\nu} \Gamma_{\lambda\sigma\mu}) - (\partial_\mu \Gamma_{\rho\sigma\nu} - \Gamma^\lambda{}_{\rho\mu} \Gamma_{\lambda\sigma\nu}) \;; \tag{1-3}$$

$$\breve{\nabla}_\lambda F_{\mu\nu} + \breve{\nabla}_\mu F_{\nu\lambda} + \breve{\nabla}_\nu F_{\lambda\mu} = Z^\tau{}_{\lambda\mu} F_{\tau\nu} + Z^\tau{}_{\mu\nu} F_{\tau\lambda} + Z^\tau{}_{\nu\lambda} F_{\tau\mu} \,, \tag{2-1}$$

$$\breve{\nabla}_\mu F^\mu{}_\nu - \frac{im_0^{(E)}}{\hbar} k_\mu F^\mu{}_\nu = -J^{(E)}_\nu \,, \tag{2-2}$$

$$F_{\mu\nu} = (\breve{\nabla}_\nu - \frac{im_0^{(E)}}{2\hbar} k_\nu) A_\mu - (\breve{\nabla}_\mu - \frac{im_0^{(E)}}{2\hbar} k_\mu) A_\nu \,. \tag{2-3}$$

where $\Gamma^\rho{}_{\sigma\mu}$ is the affine connection, $T_{\tau\mu\nu}$ is a definite complex torsion tensor given by

$$T_{\tau\mu\nu} = \frac{im_0^{(G)}}{2\hbar} (g_{\tau\mu} k_\nu - g_{\tau\nu} k_\mu) \,, \quad T_\nu = T^\mu{}_{\mu\nu} = (D-1) \frac{im_0^{(G)}}{2\hbar} k_\nu \tag{3}$$

D is the number of dimensions; and the sources of the fields $R_{\mu\nu\rho\sigma}$ and $F_{\mu\nu}$ are defined by

$$J^{(G)}_{\nu\rho\sigma} = -(\breve{\nabla}_\nu + \frac{im_0^{(G)}}{\hbar} k_\nu) \varphi^{(G)}_{\rho\sigma} \,, \quad J^{(E)}_\nu = -(\breve{\nabla}_\nu + \frac{im_0^{(E)}}{\hbar} k_\nu) \varphi^{(E)} \tag{4}$$

Tensor $Z_{\tau\mu\nu}$ is also given by:
$$Z_{\tau\mu\nu} = \frac{im_0^{(E)}}{2\hbar} (g_{\tau\mu} k_\nu - g_{\tau\nu} k_\mu) \tag{5}$$

where (based on the assumption (3) in Section 1-1) these field equations are formulable solely in D ≤ 4 dimensional space-time. Moreover, in the above equations $m_0^{(G)}$ and $m_0^{(E)}$ are the invariant masses of the (free and interacting) fields carrier single-particles, $i\hbar\breve{\nabla}_\mu$ is the covariant kinetic energy-momentum operator (generally defined on a background space-time with the complex torsion $T_{\tau\mu\nu}$ generated by the invariant mass (of the field carrier particle) and given by formula (3)), $k^\mu = (c/\sqrt{g_{00}}, 0, ..., 0)$ is the general relativistic velocity of a static observer (that is a time-like contravariant vector), and $J^{(G)}_{\nu\rho\sigma}$ and $J^{(E)}_\nu$ are the sources for these fields.

**A**s an additional principal requirement, we show that we may also assume that the Lagrangian density for the obtained general relativistic "massive" single-particle wave equations (2-1) – (2-3), be locally gauge invariant as well – where these fields be still massive, and $\breve{\nabla}_\mu$ be equivalent to the general relativistic (with torsion (3), which is compatible with the local gauge invariance condition [9, 58, 60 - 63]) form of the (local) gauge-covariant derivative [67]. On this basis, in (1+3) dimensional space-time, equations (2-1) – (2-3) not only would describe a certain massive form of Maxwell's (single-photon) field based on the Abelian gauge group $U(1)$, but also would present a certain massive form of Yang–Mills (single-particle) fields based on the (non-Abelian) gauge groups $SU(N)$. For the latter case, the field strength tensor, vector gauge potential and the current in equations (2-1) - (2-3), are also written in component notation as: $F^a_{\ \mu\nu}$, $A^a_{\ \mu}$, $J^{a(E)}_{\ \mu}$, where the Latin index $a$ = 1, 2, 3, ..., $N^2$ -1, and $N^2$ -1 is the number of linearly independent generators of the group $SU(N)$ (as a real manifold) [58]. Hence, by requiring the local gauge invariance for general relativistic massive particle field equations (2-1) – (2-3), in Section 3-4, we show that the massive Lagrangian density specified for these fields could be locally gauge invariant – if these fields be formally re-presented on a background space-time with certain complex torsion which is generated by the invariant mass of the gauge field carrier particle. Subsequently, in agreement with certain experimental data, the invariant mass of a particle (that would be identified as massive photon) has been specified ($m_0^{(E)} \approx 1.4070696\times10^{-41}$ kg), which is coupled with background space-time geometry.

It is noteworthy to recall that gauge symmetries can be viewed as analogues of the principle of general covariance of general relativity in which the coordinate system can be chosen freely under arbitrary diffeomorphism of space-time. Both gauge invariance and diffeomorphism invariance reflect a redundancy in the description of the system. In point of fact, a global symmetry is just a local symmetry whose group's parameters are fixed in space-time. The requirement of local symmetry, the cornerstone of gauge theories, is a stricter constraint [58]. However, our approach could be also considered in the framework of the theories that lie beyond the Standard Model [71], as it also includes new consequences such as a certain formulation for the gravitational particle field.

**I**n addition, we should note that the field equations (2-1) – (2-3) would correspond to two different particle fields: they describe the spin-1/2 single-particle fields formulated solely in (1+2) dimensional space-time – where we necessary have $F_{20} = F_{02} = 0$ [29, 31]; these equations also describe the spin-1 single-particle fields formulated solely in (1+3) dimensional space-time [68, 69]. In the precisely same manner, the field equations (1-1) – (1-2) describe the spin-3/2 single-particle field (gravitational) formulated solely in (1+2) dimensional space-time – where $R_{\mu\nu\rho\sigma}$ is the Riemann curvature tensor, and we necessary have $R_{20\rho\sigma} = R_{02\rho\sigma} = 0$. The field equations (1-1) – (1-2) also describe the spin-2 single-particle (gravitational) field formulated solely in (1+3) dimensional space-time. However, it should be emphasize here that for single-particle field equations (1-1) – (2-3), the (quantum mechanical) solutions are taken *to be complex* [29, 30, 31, 68, 69].

In particular, for massless cases (i.e. $m_0^{(G)} = 0$, $m_0^{(E)} = 0$), the field equations (1-1) – (2-3) turn into the classical fields including the Einstein (with a cosmological constant), Maxwell and Yang-Mills field equations, and only these fields. In the context of relativistic quantum mechanics, equations (1-1) – (2-3) are subject to a process of $2^{nd}$ quantization anyhow; then these equations would describe the bosonic fields in (1+3) dimensional space-time, and the fermionic fields in (1+2) dimensional space-time.

**A**s it was mentioned above, in this article the geometrized units, metric signature (+ − ... −) and the sign conventions (97) will be used. So particularly, we assume the speed of light $c = 1$. However, for clarity and emphasis, in some essential relativistic relations "$c$" as a constant be restored and indicated formally.

**1-3.** **T**he field equations (1-1) – (2-3) that are obtained straightforwardly by first quantization of linearized (and simultaneously parameterized, as necessary algebraic conditions) unique forms of the relativistic energy-momentum (matrix) relation, i.e. formulas (92) – (96), could be also written in matrix representation, (or in the geometric algebra formulation, as we show in Section 3-4), as follows:

$$(i\hbar\alpha^{\mu}\breve{\nabla}_{\mu} - m_0\tilde{\alpha}^{\mu}k_{\mu})\Psi_R = 0, \tag{1-A}$$

$$(i\hbar\alpha^{\mu}\breve{\nabla}_{\mu} - m_0\tilde{\alpha}^{\mu}k_{\mu})\Psi_F = 0 \tag{2-A}$$

where
$$\alpha^{\mu} = \beta^{\mu} + \beta'^{\mu}, \ \tilde{\alpha}^{\mu} = \beta^{\mu} - \beta'^{\mu} \tag{6}$$

$\Psi_R$, $\Psi_E$ are column matrices, $\beta^{\mu}$ and $\beta'^{\mu}$ are contravariant square matrices *(corresponding to the generalized Clifford algebra, see Sections 2-4, 3-3, 3-6, 3-7 and Appendix B);* these matrices in (1+2) and (1+3) dimensional space-time given by, respectively

$$\beta^0 = \begin{bmatrix} 0 & 0 \\ 0 & -(\sigma^0+\sigma^1) \end{bmatrix}, \ \beta'_0 = \begin{bmatrix} \sigma^0+\sigma^1 & 0 \\ 0 & 0 \end{bmatrix}, \ \beta^1 = \begin{bmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{bmatrix}, \ \beta'_1 = \begin{bmatrix} 0 & \sigma^3 \\ -\sigma^3 & 0 \end{bmatrix},$$
$$\beta^2 = \begin{bmatrix} 0 & -\sigma^1 \\ -\sigma^0 & 0 \end{bmatrix}, \ \beta'_2 = \begin{bmatrix} 0 & -\sigma^0 \\ -\sigma^1 & 0 \end{bmatrix};$$

$$\Psi_R = \begin{bmatrix} R_{10\rho\sigma} \\ 0 \\ R_{21\rho\sigma} \\ \varphi^{(R)}_{\rho\sigma} \end{bmatrix}, \ \Psi_F = \begin{bmatrix} F_{10} \\ 0 \\ F_{21} \\ \varphi^{(E)} \end{bmatrix}, \ J^{(G)}_{\nu\rho\sigma} = -(\breve{\nabla}_{\nu} + \frac{im_0^{(G)}}{\hbar}k_{\nu})\varphi^{(G)}_{\rho\sigma}, \ J^{(E)}_{\nu} = -(\breve{\nabla}_{\nu} + \frac{im_0^{(E)}}{\hbar}k_{\nu})\varphi^{(E)} \tag{7}$$

where

$$\sigma^0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \ \sigma^1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \ \sigma^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \ \sigma^3 = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} \tag{7-1}$$

In relations (7) it is assumed that $R_{02\rho\sigma} = R_{20\rho\sigma} = 0, \ F_{02} = F_{20} = 0$; in Section 3-7 we show that a certain symmetric assumption yields these conditions.

For (1+3) dimensional space-time we have:

$$\beta^0=\begin{bmatrix}0&0\\0&-(\gamma^0+\gamma^1)\end{bmatrix},\ \beta_0'=\begin{bmatrix}(\gamma^0+\gamma^1)&0\\0&0\end{bmatrix},\ \beta^1=\begin{bmatrix}0&\gamma^2\\-\gamma^3&0\end{bmatrix},\ \beta_1'=\begin{bmatrix}0&\gamma^3\\-\gamma^2&0\end{bmatrix},$$
$$\beta^2=\begin{bmatrix}0&\gamma^4\\\gamma^5&0\end{bmatrix},\ \beta_2'=\begin{bmatrix}0&-\gamma^5\\-\gamma^4&0\end{bmatrix},\ \beta^3=\begin{bmatrix}0&\gamma^6\\-\gamma^7&0\end{bmatrix},\ \beta_3'=\begin{bmatrix}0&\gamma^7\\-\gamma^6&0\end{bmatrix},$$

$$\Psi_R=\begin{bmatrix}R_{10\rho\sigma}\\R_{20\rho\sigma}\\R_{30\rho\sigma}\\0\\R_{23\rho\sigma}\\R_{31\rho\sigma}\\R_{12\rho\sigma}\\\varphi^{(G)}_{\rho\sigma}\end{bmatrix},\ \ \Psi_F=\begin{bmatrix}F_{10}\\F_{20}\\F_{30}\\0\\F_{23}\\F_{31}\\F_{12}\\\varphi^{(E)}\end{bmatrix},\ \ J^{(G)}_{\nu\rho\sigma}=-(\breve{\nabla}_\nu+\frac{im_0^{(G)}}{\hbar}k_\nu)\varphi^{(G)}_{\rho\sigma},\ \ J^{(E)}_{\nu}=-(\breve{\nabla}_\nu+\frac{im_0^{(E)}}{\hbar}k_\nu)\varphi^{(E)} \tag{8}$$

where

$$\gamma^0=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix},\ \gamma^1=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix},\ \gamma^2=\begin{bmatrix}0&0&0&1\\0&0&0&0\\0&0&0&0\\-1&0&0&0\end{bmatrix},\ \gamma^3=\begin{bmatrix}0&0&0&0\\0&0&-1&0\\0&1&0&0\\0&0&0&0\end{bmatrix}.$$
$$\gamma^4=\begin{bmatrix}0&0&0&0\\0&0&0&1\\0&0&0&0\\0&-1&0&0\end{bmatrix},\ \gamma^5=\begin{bmatrix}0&0&-1&0\\0&0&0&0\\1&0&0&0\\0&0&0&0\end{bmatrix},\ \gamma^6=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&1\\0&0&-1&0\end{bmatrix},\ \gamma^7=\begin{bmatrix}0&-1&0&0\\1&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}. \tag{8-1}$$

**1-4. I**n Section 3, we show that from the gravitational field equations (1-1) – (1-2), or their equivalent matrix formulation, i.e. equation (1-A) (for $m_0^{(G)}=0$), the Einstein field equations (including the cosmological constant $\Lambda$, which emerges naturally via derivation process) are derived straightforwardly as follows:

$$R_{\mu\nu}=-8\pi(T_{\mu\nu}-BTg_{\mu\nu})-\Lambda g_{\mu\nu} \tag{9}$$

where $J^{(G)}_{\nu\rho\sigma}=-8\pi(\nabla_\sigma T_{\nu\rho}-\nabla_\rho T_{\nu\sigma})+8\pi B(\nabla_\sigma Tg_{\nu\rho}-\nabla_\rho Tg_{\nu\sigma})$, $T_{\mu\nu}$ is the stress-energy tensor($T=T^{\mu}{}_{\mu}$), $\Lambda$ is a cosmological constant, and $B=0,\ 1\ ,\frac{1}{2}$ for two, three and four dimensional space-time, respectively.

**1-5. B**ased on the unique structure of general relativistic single-particle fields derived and also the assumption of chiral symmetry as a basic discrete symmetry of the source-free cases of these fields, we conclude (in Section 3-7) that the universe cannot have more than (1+3) space-time dimensions. In addition, a basic argument for the asymmetry of left and right handed (interacting) particles is presented (in Section 3-8). Furthermore, on this basis, we may conclude that the miscellaneous and various relativistic wave equations (which has been written on the basis of experiments and so on) such as the Majorana, Breit, Proca, Rarita–Schwinger, Bargmann–Wigner equations, etc. [12 – 14], should be modified and/or replaced by the uniquely derived general relativistic (single-particle) wave equations (1-A) – (2-A) (or their equivalent formulations, i.e. equations (1-1) – (2-3)).

**1-6. A**ccording to the definite mathematical structure of derived field equations (2-1) – (2-3) that correspond to the *Maxwell's equations (and also the Yang-Mills equations as the generalization of these equations, see Sections 3-4 and 3-4-1)*, in Section 3-4-2 we conclude that magnetic monopoles (in contrast with electric monopoles) could not exist in nature.

**Let we emphasize again that the above noted results are direct outcomes of a new primary and axiomatic mathematical approach**[1]**. In this article we try generally to present the main schemes of applications of this new axiomatic approach in mathematics and particularly in fundamental physics.** Hence, in Section 2, we present and develop this new mathematical approach, where we formulate a theory of linearization based on the ring theory (including the integral domains) and the generalized Clifford algebra. In Section 3, we show one of the main applications of this (primary) mathematical approach in the foundations of physics, where particularly in connection with the laws governing the fundamental forces and their logical origin, we'll focus on the direct "mathematical derivation" of a definite set of the general relativistic (single-particle) wave equations which uniquely represent these laws as the most fundamental laws of nature.

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1. Besides, we may argue that our presented axiomatic matrix approach (for a direct derivation and formulating the fundamental laws of nature uniquely) is not subject to the Gödel's incompleteness theorems [51]. As in our axiomatic approach, firstly, we've basically changed (i.e. replaced and generalized) one of the main Peano axioms (when these axioms algebraically are augmented with the operations of addition and multiplication [52, 53, 54]) for integers, which is the algebraic axiom of nonzero divisors.

Secondly, based on our approach, one of the axiomatic properties of integers (i.e. axiom of nonzero divisors) could be accomplished solely by the arbitrary square matrices (with integer components). This axiomatic reformulation of algebraic properties of integers thoroughly has been presented in Section 2 of this article.

## 2. The Theory of Linearization: A New Axiomatic Matrix Approach

*"Based on the Ring Theory (Including the Integral Domains) and the Matrix Representation of Generalized Clifford Algebra"*

**M**athematical models of physical processes include certain classes of mathematical objects and relations between these objects. The models of this type, which are most commonly used, are groups, rings, vector spaces, and linear algebras. A group is a set $G$ with a single operation (multiplication) $a\times b=c$; $a,b,c\in G$ which obeys the known conditions [5]. A ring is a set of elements $R$, where two binary operations, namely, addition and multiplication, are defined.

With respect to addition this set is a group, and multiplication is connected with addition by the distributivity laws $a\times(b+c)=(a\times b)+(a\times c), (b+c)\times a=(b\times a)+(c\times a)$; $a,b,c\in R$. The rings reflect the structural properties of the set $R$. As distinct from the group models, those connected with rings are not frequently applied, although in physics various algebras of matrices, algebras of hypercomplex numbers, Grassman and Clifford algebras are widely used. This is due to the intricacy of finding a connection between the binary relations of addition and multiplication and the element of the rings [5, 2].

**T**his article is devoted to the development of a rather simple approach of establishing such a connection and an analysis of concrete problems on this basis.

I find out that if we axiomatically generalize the set of single-integer elements $\mathrm{Z}$ to the set of $n\times n$ square matrix elements (as axioms' elements with single-integer components, which we show it by $\mathrm{Z}_{n\times n}$), fruitful new relations and results hold. Thus in this Section, we present a matrix generalization of the algebraic axiom of nonzero divisors for the ring of integers. Then, we axiomatically formulate a matrix model for constructing a linearization theory over this ring. On this basis, we introduce the necessary and sufficient conditions for transforming the homogeneous non-linear equations (of any order) to their equivalent systems of linear equations (or matrix equations). These matrix equations definitely correspond to the matrix representation of the generalized Clifford algebras. In Section 2-4, quadratic forms (and relevant equations) are studied and analyzed on this basis explicitly.

**2-1.** **T**he algebraic axioms of the domain of integers $\mathrm{Z}$ with binary operations $(+,\times)$, usually are defined as follows [5]:

- $a_1,a_2,a_3,...\in\mathrm{Z}$,

- Closure: $$a_k+a_l\in\mathrm{Z},\quad a_k\times a_l\in\mathrm{Z} \tag{10}$$

- Associativity: $$a_k+(a_l+a_p)=(a_k+a_l)+a_p,\quad a_k\times(a_l\times a_p)=(a_k\times a_l)\times a_p \tag{11}$$

- Commutativity: $$a_k+a_l=a_l+a_k,\quad a_k\times a_l=a_l\times a_k \tag{12}$$

- Existence of an identity element: $$a_k+0=a_k,\quad a_k\times 1=a_k \tag{13}$$

- Existence of inverse element (for addition): $$a_k+(-a_k)=0 \tag{14}$$

- Distributivity: $a_k \times (a_l + a_p) = (a_k \times a_l) + (a_k \times a_p),\ (a_k + a_l) \times a_p = (a_k \times a_p) + (a_l \times a_p)$ (15)

- No zero divisors: $$(a_k = 0 \vee a_l = 0) \Leftrightarrow a_k \times a_l = 0 \qquad (16)$$

It is easy to show that the axiom (16), equivalently, could be also presented as follows

$$(a_k \times m_l = 0, m_l \neq 0) \Leftrightarrow a_k = 0 \qquad (16\text{-}1)$$

In axioms (10) – (15), we may simply suppose that the single elements $a_k \in Z$ are $1 \times 1$ matrices with integer components: $[a_1]_{1\times 1}(\equiv a_1), [a_2]_{1\times 1}(\equiv a_2), [a_3]_{1\times 1}(\equiv a_3), \ldots \in Z_{1\times 1}(\equiv Z),$ then equivalently, the axioms (10) – (15) could also be written by square matrices (with integer components) as follows:

- $M_k = [m_{k_{ij}}]$ , $m_{k_{ij}} \in Z$, $\exists n \in N:\ i, j = 1,2,3,\ldots,n$ , $M_1, M_2, M_3, \ldots \in Z_{n\times n}$,

- Closure: $$M_k + M_l \in Z_{n\times n}, \quad M_k \times M_l \in Z_{n\times n} \qquad (17)$$

- Associativity: $M_k + (M_l + M_p) = (M_k + M_l) + M_p,\ M_k \times (M_l \times M_p) = (M_k \times M_l) \times M_p$ (18)

- Commutativity (for addition): $$M_k + M_l = M_l + M_k \qquad (19\text{-}1)$$

- Property of the transpose for matrix multiplication:

$$(M_k \times M_l)^T = M_l^{\ T} \times M_k^{\ T} \qquad (19\text{-}2)$$

where $M_k^{\ T}$ is the transpose of matrix $M_k$ .

- Existence of an identity element: $$M_k + 0 = M_k, \quad M_k \times I_{n\times n} = M_k \qquad (20)$$

- Existence of the inverse element (for addition):

$$M_k + (-M_k) = 0 \qquad (21)$$

- Distributivity: $$M_k \times (M_l + M_p) = (M_k \times M_l) + (M_k \times M_p),$$

$$(M_k + M_l) \times M_p = (M_k \times M_p) + (M_l \times M_p); \qquad (22)$$

**Note that from the axioms (10) – (15), we can obtain the matrix-formulation axioms (17) – (22) and vice versa.**

In this article, we principally take into account the set of matrix-formulation axioms (17) – (22) for integers, and present the following new additional axiom (formulated "solely" via square matrices) as a new algebraic property of the ring (or domain) of integers – which is a generalized matrix-formulation of axiom (16-1), and be replaced by (16-1):

**Axiom 2-1.** **" Let $F(b_1, b_2, b_3, ..., b_s)$ be a homogeneous polynomial of degree $r \geq 2$ over the integer elements $b_p \in Z \ (\equiv Z_{1\times 1})$, we have the following axiom:**

$\exists A, M \in Z_{n\times n}$,

$$[(A \times M = 0,\ M \neq 0) \wedge (A^r = F(b_1, b_2, b_3, ..., b_s) I_n)] \Leftrightarrow F(b_1, b_2, b_3, ..., b_s) = 0 \qquad \textbf{(23)}$$

**where** $A = [a_{ij}]$**,** $a_{ij} = \sum_{p=1}^{s} H_{ijp} b_p$, **and** $b_p \in Z\ (\equiv Z_{1\times 1})$, $\exists n, s:\ i, j = 1,2,3,...,n$ , $p = 1,2,3,...,s$ , $H_{ijp}$ **are coefficients,** $M\ (M \neq 0)$ **is a parametric arbitrary matrix, and** $I_n$ **is the identity matrix."**

In fact, axiom (16) (or its equivalent, i.e. axiom (16-1)) can be obtained from Axiom 2-1, *but definitely not vice versa*. Only for special case $n = 1$, the set of axioms (17) – (23) (formulated by square matrices) become equivalent to the set of ordinary axioms (10) – (16-1) for integer elements. Axiom 2-1 is formulated "solely" by square matrices (with integer components), and definitely ***is a new axiom for integers***. In this Section and Section 3 we will demonstrate its main and direct consequences and fruitful applications.

**Remark 2-1.** Note that in Axiom 2-1, according to the arbitrariness of all the parametric components of $n \times n$ matrix $M$ ($M \neq 0$), without loss of generality, we may replace matrix $M$ with a $n \times 1$ matrix $M_{n\times 1}$ in equations $AM = 0$ (with the same condition $M \neq 0$, but only with "*n*" number of arbitrary parametric components).

**Remark 2-2.** Algebraic axiomatic relation (23) presents a new fundamental matrix structure and framework for constructing a basic linearization theory in the ring theory (including the integral domains), which will be described below. Also note that the elements $a_{ij}$ are the "**linear**" homogeneous forms of integer elements $b_{pq}$. Furthermore, (using the definition $A = [a_{ij}]$, $a_{ij} = \sum_{p=1}^{s} H_{ijp} b_p$ ), we may simply represent matrix $A$ by this linear formula: $A = \sum_{p=1}^{s} b_p E_p$ , where $E_p$ are square matrices; then the relation $A^r = F(b_1, b_2, b_3, ..., b_s) I_n$ will definitely express the standard definition of the generalized Clifford algebra associated with homogeneous form $F(b_1, b_2, b_3, ..., b_s)$, and generated by matrices $E_p$ [43- 47]. **Thus based on the definition of Axiom 2-1 (formula (23)) over the integer elements, the Clifford algebra (and its matrix representation) as a very well known and studied mathematical theory, is the main and central characteristic of the axiomatic approach presented in this article.** We use this essential property in Section 2-4, for a unique determination of square matrices $A$ that generate a generalized Clifford algebra associated with quadratic homogeneous forms of the type (25) (defined below).

**2-2.** *Generally, there are standard and certain methods for solving homogeneous linear equations over the ring of integers [7]. Moreover, on the basis of the* **Axiom 2-1**, *the necessary and sufficient condition for solving an homogeneous equation of the $r^{th}$ order such as $F(b_1, b_2, b_3, ..., b_s) = 0$, is the transforming or converting it (by "linearization" and simultaneous parameterization, as necessary algebraic conditions) into an equivalent system of linear equations of type $AM = 0$ (where $M \neq 0$, $M$: $n \times 1$ matrix with parametric components).* On this basis, below we'll obtain merely the systems of linear equations that equivalently correspond to the various homogeneous quadratic equations, and also systems of linear equations corresponding to some of the higher order equations.

On the methodological standpoint, firstly, for obtaining and specifying a system of linear equations that corresponds to a given equation $F(b_1, b_2, b_3, ..., b_s) = 0$, we definitely should assume and take the minimum value for $n$ (i.e. the size of $n \times n$ matrix $A$ defined in (23)). Secondly, by replacing the components of matrix $A$ with the linear forms $a_{ij} = \sum_{p=1}^{s} H_{ijp} b_p$ (defined in (23)), in the equation $A^r = F(b_1, b_2, b_3, ..., b_s) I_n$, we basically should calculate the coefficients $H_{ijp}$ (which are independent of elements $b_p$), by ordinary and standard methods of solving certain equations over the ring of integers.

**2-3.** In particular, an arbitrary homogeneous quadratic form such as

$$F(c_1, c_2, c_3, ..., c_s) = \sum_{i_1, i_2 = 1}^{s} B_{i_1 i_2} \prod_{p=1}^{2} c_{i_p} \tag{24}$$

could be transformed into a simpler quadratic form of the type

$$Q(b_{11}, b_{21}, b_{12}, b_{22}, ..., b_{2s}) = \sum_{q=1}^{s} \prod_{p=1}^{2} b_{pq} \tag{25}$$

by the following linear isomorphic transformations:

$$b_{11} = c_1,\ \ b_{21} = \sum_{i_2=1}^{s} B_{1 i_2} c_{i_2},\ \ b_{12} = c_2,\ \ b_{22} = \sum_{i_2=1}^{s} B_{2 i_2} c_{i_2},\ \ ...,\ \ b_{1s} = c_s,\ \ b_{2s} = \sum_{i_2=1}^{s} B_{s i_2} c_{i_2} \tag{24-1}$$

Furthermore, as we'll also show later, the following quadratic form could be transformed into quadratic forms of the type (25) as well (by a similar isomorphic transformations):

$$Q(c_1, c_2, c_3, ..., c_s, d_1, d_2, d_3, ..., d_s) = \sum_{i_1, i_2 = 1}^{s} B_{i_1 i_2} \prod_{p=1}^{2} c_{i_p} - \sum_{i_1, i_2 = 1}^{s} B_{i_1 i_2} \prod_{p=1}^{2} d_{i_p} \tag{26}$$

**Remark 2-3.** In addition, concerning the formula (25), the following general relations could be proven easily:

$$(\sum_{q=1}^{s}\prod_{p=1}^{r} b_{pq}c_{(r+1)q}=0,\ \ \sum_{q=1}^{s}\prod_{p=1}^{r} b_{pq}d_{(r+1)q}=0)\ \Rightarrow\ \sum_{q=1}^{s}\prod_{p=1}^{r} b_{pq}(c_{(r+1)q}\pm d_{(r+1)q})=0\ , \qquad (25\text{-}1)$$

$$\sum_{q=1}^{s}\prod_{p=1}^{r} b_{pq}c_{(r+1)q}=0\ \Leftrightarrow\ \sum_{q=1}^{s}\prod_{p=1}^{r} b_{pq}(tc_{(r+1)q})=0 \qquad (25\text{-}2)$$

where the parameter $t$ is an arbitrary non-zero integer.

**2-4.** According to the general forms of homogeneous quadratic equations (24) – (26), and the isomorphic linear transformations (24-1), below we merely write the systems of linear equations that correspond to the following quadratic equation (moreover, as we mentioned in Section 2-2, according to the Axiom 2-1 (formula (23)), only one system of linear equations – with the minimum number of equations – for each particular case is sufficient):

$$Q(b_{11},b_{21},b_{12},b_{22},...,b_{2s})=\sum_{q=1}^{s}\prod_{p=1}^{2} b_{pq}=0 \qquad (25\text{-}3)$$

Now according to Axiom 2-1 (formula (23)), the following relation should be specified for equation (25-3), which by to formulate its equivalent system of linear equations $AM=0$,

$$A^2=(\sum_{p=1}^{2}\sum_{q=1}^{s} b_{pq}E_{pq})^2=Q(b_{11},b_{21},b_{12},b_{22},...,b_{2s})I_n \qquad (25\text{-}4)$$

where $n=2^{s-1}$, and matrices $E_{pq}$ generate a generalized Clifford algebra associated with form $Q(b_{11},b_{21},b_{12},b_{22},...,b_{2s})$.

**W**e should note here that the matrix equations, which will be obtained on this basis for the above homogeneous quadratic equations, also could simply be modified to be hermitian (and we will do it for these equations). The hermiticity is a necessary condition for these matrix equations (i.e. matrix equations (32), (34), (36), (37), (38), corresponding quadratic equation (24)), which we will use to formulate the relativistic wave equations of physics, in Section 3 (see Sections 3-3, 3-6, 3-7 and Appendix B).

**T**hus let we expand the equation (25-3) as follows, respectively:

$$\sum_{q=1}^{1}\prod_{p=1}^{2} b_{pq} = b_{11}b_{21} = 0, \tag{27}$$

$$\sum_{q=1}^{2}\prod_{p=1}^{2} b_{pq} = b_{11}b_{21} + b_{12}b_{22} = 0, \tag{28}$$

$$\sum_{q=1}^{3}\prod_{p=1}^{2} b_{pq} = b_{11}b_{21} + b_{12}b_{22} + b_{13}b_{23} = 0, \tag{29}$$

$$\sum_{q=1}^{4}\prod_{p=1}^{2} b_{pq} = b_{11}b_{21} + b_{12}b_{22} + b_{13}b_{23} + b_{14}b_{24} = 0, \tag{30}$$

$$\sum_{q=1}^{5}\prod_{p=1}^{2} b_{pq} = b_{11}b_{21} + b_{12}b_{22} + b_{13}b_{23} + b_{14}b_{24} + b_{15}b_{25} = 0; \tag{31}$$

…

Hence, based on the Axiom 2-1 (formula (23)), Remark 2-2 and our methodology mentioned in Section 2-2, the equivalent matrix equations (here we mean a system of linear equations) corresponding (uniquely) to the quadratic equations (27) – (31) are, respectively, as follows:

First, the equivalent matrix equation corresponding to quadratic equation (27) is given by

$$A \times M = \begin{bmatrix} 0 & e_0 \\ f_0 & 0 \end{bmatrix}\begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = 0 \tag{32}$$

where $e_0 = b_{11}, f_0 = b_{21}$, and we have

$$A^2 = \begin{bmatrix} 0 & e_0 \\ f_0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & e_0 \\ f_0 & 0 \end{bmatrix} = (e_0 f_0) I_2 \tag{32-1}$$

For (28) we have the following equivalent matrix equation

$$A \times M = \begin{bmatrix} 0 & 0 & e_0 & f_1 \\ 0 & 0 & -e_1 & f_0 \\ f_0 & f_1 & 0 & 0 \\ -e_1 & e_0 & 0 & 0 \end{bmatrix}\begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix} = 0 \tag{33}$$

where $e_0 = b_{11}, f_0 = b_{21}, e_1 = b_{12}, f_1 = b_{22}$, and we have

$$A^2 = \begin{bmatrix} 0 & 0 & e_0 & f_1 \\ 0 & 0 & -e_1 & f_0 \\ f_0 & -f_1 & 0 & 0 \\ e_1 & e_0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & e_0 & f_1 \\ 0 & 0 & -e_1 & f_0 \\ f_0 & -f_1 & 0 & 0 \\ e_1 & e_0 & 0 & 0 \end{bmatrix} = (e_0 f_0 + e_1 f_1) I_4 \qquad (33\text{-}1)$$

Notice that using (33) we get the following two separate and fully equivalent matrix equations for (28):

$$\begin{bmatrix} e_0 & f_1 \\ -e_1 & f_0 \end{bmatrix} \begin{bmatrix} m_3 \\ m_4 \end{bmatrix} = 0, \qquad (33\text{-}2)$$

$$\begin{bmatrix} f_0 & f_1 \\ -e_1 & e_0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = 0 \qquad (33\text{-}3)$$

As the matrix equations (33-2) and (33-3) are equivalent, we can choose the equation (33-2) as the system of linear equations corresponding to quadratic equation (28) – where for simplicity we may also replace the parameters $m_1$ and $m_2$ by parameters $m_3$ and $m_4$, as follows

$$\begin{bmatrix} e_0 & f_1 \\ -e_1 & f_0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = 0 \qquad (34)$$

The following system of linear equations corresponds to quadratic equation (29):

$$A \times M = \begin{bmatrix} 0 & 0 & 0 & 0 & e_0 & 0 & -e_2 & f_1 \\ 0 & 0 & 0 & 0 & 0 & e_0 & -e_1 & -f_2 \\ 0 & 0 & 0 & 0 & f_2 & f_1 & f_0 & 0 \\ 0 & 0 & 0 & 0 & -e_1 & e_2 & 0 & f_0 \\ f_0 & 0 & e_2 & -f_1 & 0 & 0 & 0 & 0 \\ 0 & f_0 & e_1 & f_2 & 0 & 0 & 0 & 0 \\ -f_2 & -f_1 & e_0 & 0 & 0 & 0 & 0 & 0 \\ e_1 & -e_2 & 0 & e_0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \\ m_6 \\ m_7 \\ m_8 \end{bmatrix} = 0 \qquad (35)$$

where $e_0 = b_{11}, f_0 = b_{21}, e_1 = b_{12}, f_1 = b_{22}, e_2 = b_{13}, f_2 = b_{23},$ and we have

$A^2 =$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & e_0 & 0 & -e_2 & f_1 \\ 0 & 0 & 0 & 0 & 0 & e_0 & -e_1 & -f_2 \\ 0 & 0 & 0 & 0 & f_2 & f_1 & f_0 & 0 \\ 0 & 0 & 0 & 0 & -e_1 & e_2 & 0 & f_0 \\ f_0 & 0 & e_2 & -f_1 & 0 & 0 & 0 & 0 \\ 0 & f_0 & e_1 & f_2 & 0 & 0 & 0 & 0 \\ -f_2 & -f_1 & e_0 & 0 & 0 & 0 & 0 & 0 \\ e_1 & -e_2 & 0 & e_0 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 & 0 & e_0 & 0 & -e_2 & f_1 \\ 0 & 0 & 0 & 0 & 0 & e_0 & -e_1 & -f_2 \\ 0 & 0 & 0 & 0 & f_2 & f_1 & f_0 & 0 \\ 0 & 0 & 0 & 0 & -e_1 & e_2 & 0 & f_0 \\ f_0 & 0 & e_2 & -f_1 & 0 & 0 & 0 & 0 \\ 0 & f_0 & e_1 & f_2 & 0 & 0 & 0 & 0 \\ -f_2 & -f_1 & e_0 & 0 & 0 & 0 & 0 & 0 \\ e_1 & -e_2 & 0 & e_0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= (e_0 f_0 + e_1 f_1 + e_2 f_2) I_8$$

(35-1)

From (35) (similar to the (33)) the following two equivalent matrix equations are obtained,

$$\begin{bmatrix} e_0 & 0 & -e_2 & f_1 \\ 0 & e_0 & -e_1 & -f_2 \\ f_2 & f_1 & f_0 & 0 \\ -e_1 & e_2 & 0 & f_0 \end{bmatrix} \begin{bmatrix} m_5 \\ m_6 \\ m_7 \\ m_8 \end{bmatrix} = 0, \qquad (35\text{-}2)$$

$$\begin{bmatrix} f_0 & 0 & e_2 & -f_1 \\ 0 & f_0 & e_1 & f_2 \\ -f_2 & -f_1 & e_0 & 0 \\ e_1 & -e_2 & 0 & e_0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix} = 0 \qquad (35\text{-}3)$$

So, we can choose equation (35-2), as the system of linear equations corresponding to the quadratic equation (29) (where for simplicity here we also replace the parameters $m_1, m_2$, $m_3$ and $m_4$ by parameters $m_5, m_6$, $m_7$ and $m_8$):

$$\begin{bmatrix} e_0 & 0 & -e_2 & f_1 \\ 0 & e_0 & -e_1 & -f_2 \\ f_2 & f_1 & f_0 & 0 \\ -e_1 & e_2 & 0 & f_0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix} = 0 \qquad (36)$$

Similar to the matrix equations (34) and (36) that correspond to the quadratic equations (28) and (29), the unique and equivalent matrix equations corresponding to the quadratic equations (30) and (31) are also obtained as follows, respectively,

$$\begin{bmatrix} e_0 & 0 & 0 & 0 & 0 & -e_3 & e_2 & f_1 \\ 0 & e_0 & 0 & 0 & e_3 & 0 & -e_1 & f_2 \\ 0 & 0 & e_0 & 0 & -e_2 & e_1 & 0 & f_3 \\ 0 & 0 & 0 & e_0 & -f_1 & -f_2 & -f_3 & 0 \\ 0 & -f_3 & f_2 & e_1 & f_0 & 0 & 0 & 0 \\ f_3 & 0 & -f_1 & e_2 & 0 & f_0 & 0 & 0 \\ -f_2 & f_1 & 0 & e_3 & 0 & 0 & f_0 & 0 \\ -e_1 & -e_2 & -e_3 & 0 & 0 & 0 & 0 & f_0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \\ m_6 \\ m_7 \\ m_8 \end{bmatrix} = 0 \qquad (37)$$

where $e_0 = b_{11},\ f_0 = b_{21}, e_1 = b_{12},\ f_1 = b_{22}, e_2 = b_{13},\ f_2 = b_{23}, e_3 = b_{14},\ f_3 = b_{24};$

and for (31) we obtain

$$\begin{bmatrix} e_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -e_4 & 0 & -e_3 & -e_2 & f_1 \\ 0 & e_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_4 & 0 & e_3 & 0 & -e_1 & -f_2 \\ 0 & 0 & e_0 & 0 & 0 & 0 & 0 & 0 & 0 & -e_4 & 0 & 0 & e_2 & e_1 & 0 & f_3 \\ 0 & 0 & 0 & e_0 & 0 & 0 & 0 & 0 & e_4 & 0 & 0 & 0 & -f_1 & f_2 & -f_3 & 0 \\ 0 & 0 & 0 & 0 & e_0 & 0 & 0 & 0 & 0 & -e_3 & -e_2 & -e_1 & 0 & 0 & 0 & -f_4 \\ 0 & 0 & 0 & 0 & 0 & e_0 & 0 & 0 & e_3 & 0 & f_1 & -f_2 & 0 & 0 & f_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & e_0 & 0 & e_2 & -f_1 & 0 & f_3 & 0 & -f_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_0 & e_1 & f_2 & -f_3 & 0 & f_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & -f_4 & 0 & -f_3 & -f_2 & -f_1 & f_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & f_4 & 0 & f_3 & 0 & e_1 & -e_2 & 0 & f_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -f_4 & 0 & 0 & f_2 & -e_1 & 0 & e_3 & 0 & 0 & f_0 & 0 & 0 & 0 & 0 & 0 \\ f_4 & 0 & 0 & 0 & f_1 & e_2 & -e_3 & 0 & 0 & 0 & 0 & f_0 & 0 & 0 & 0 & 0 \\ 0 & -f_3 & -f_2 & e_1 & 0 & 0 & 0 & -e_4 & 0 & 0 & 0 & 0 & f_0 & 0 & 0 & 0 \\ f_3 & 0 & -f_1 & -e_2 & 0 & 0 & e_4 & 0 & 0 & 0 & 0 & 0 & 0 & f_0 & 0 & 0 \\ f_2 & f_1 & 0 & e_3 & 0 & -e_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & f_0 & 0 \\ -e_1 & e_2 & -e_3 & 0 & e_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & f_0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \\ m_6 \\ m_7 \\ m_8 \\ m_9 \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{15} \\ m_{16} \end{bmatrix} = 0$$

(38)

where $e_0 = b_{11},\ f_0 = b_{21}, e_1 = b_{12},\ f_1 = b_{22}, e_2 = b_{13},\ f_2 = b_{23}, e_3 = b_{14},\ f_3 = b_{24}, e_4 = b_{15},\ f_4 = b_{25}.$

**I**n a similar manner, the systems of linear equations with larger sizes could be obtained for special cases of general quadratic equation (25-3), where $s = 1,2,3,...$ . The size of the square matrices of these matrix equations is $2^s \times 2^s$ . But, this size is reducible to $2^{s-1} \times 2^{s-1}$ for the quadratic forms (as we had these sizes for matrix equations (34), (36), (37) and (38) which correspond to the quadratic equations (29) – (31)). In general, the size of the square matrices $A$ that correspond to the homogeneous $r^{th}$ order form $F(b_1, b_2, b_3,..., b_s)$ defined in (23), is $r^s \times r^s$, which for particular cases this size could be reduced. Moreover, based on Axiom 1-2 (formula (23)), by obtaining (and solving) a system of linear equations which corresponds to a $r^{th}$ order equation, we may systematically show (or decide) whether this equation has the integral solution.

**W**e present below the systems of linear equations (defined by (23)) corresponding to some particular higher (3rd, 4th and 5th) order homogeneous equations. For a third order equation of the type

$$F(e_0, f_0, e_1, f_1, e_2, f_2) = e_0^2 f_0 - e_0 f_0^2 + e_2^2 f_2 - e_2 f_2^2 + e_1 f_1 g_1 = 0 \qquad (39)$$

the following matrix equation is obtained

$$AM = \begin{bmatrix} 0 & 0 & A_1 \\ A_2 & 0 & 0 \\ 0 & A_3 & 0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ :::: \\ m_{27} \end{bmatrix} = 0, \qquad (40)$$

where $A$ is a $27 \times 27$ square matrix and we have

$$A_1 = \begin{bmatrix} -e_2 + f_2 & 0 & 0 & 0 & 0 & 0 & -e_0 + f_0 & e_1 & 0 \\ 0 & -e_2 + f_2 & 0 & 0 & 0 & 0 & 0 & e_0 & g_1 \\ 0 & 0 & -e_2 + f_2 & 0 & 0 & 0 & f_1 & 0 & -f_0 \\ -f_0 & e_1 & 0 & e_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -e_0 + f_0 & g_1 & 0 & e_2 & 0 & 0 & 0 & 0 \\ f_1 & 0 & e_0 & 0 & 0 & e_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & e_0 & e_1 & 0 & -f_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -f_0 & g_1 & 0 & -f_2 & 0 \\ 0 & 0 & 0 & f_1 & 0 & -e_0 + f_0 & 0 & 0 & -f_2 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -f_2 & 0 & 0 & 0 & 0 & 0 & -e_0 + f_0 & e_1 & 0 \\ 0 & -f_2 & 0 & 0 & 0 & 0 & 0 & e_0 & g_1 \\ 0 & 0 & -f_2 & 0 & 0 & 0 & f_1 & 0 & -f_0 \\ -f_0 & e_1 & 0 & -e_2 + f_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -e_0 + f_0 & g_1 & 0 & -e_2 + f_2 & 0 & 0 & 0 & 0 \\ f_1 & 0 & e_0 & 0 & 0 & -e_2 + f_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & e_0 & e_1 & 0 & e_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -f_0 & g_1 & 0 & e_2 & 0 \\ 0 & 0 & 0 & f_1 & 0 & -e_0 + f_0 & 0 & 0 & e_2 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} e_2 & 0 & 0 & 0 & 0 & 0 & -e_0+f_0 & e_1 & 0 \\ 0 & e_2 & 0 & 0 & 0 & 0 & 0 & e_0 & g_1 \\ 0 & 0 & e_2 & 0 & 0 & 0 & f_1 & 0 & -f_0 \\ -f_0 & e_1 & 0 & -f_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -e_0+f_0 & g_1 & 0 & -f_2 & 0 & 0 & 0 & 0 \\ f_1 & 0 & e_0 & 0 & 0 & -f_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & e_0 & e_1 & 0 & -e_2+f_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -f_0 & g_1 & 0 & -e_2+f_2 & 0 \\ 0 & 0 & 0 & f_1 & 0 & -e_0+f_0 & 0 & 0 & -e_2+f_2 \end{bmatrix}. \qquad (41)$$

Based on (23), the system of linear equations corresponding to the 3rd order equation

$$F(a,b,c) = 2(a^3 - c^3 + Bb^3) = 0, \qquad (42)$$

is given by

$$AM = \begin{bmatrix} 0 & 0 & A_1 \\ A_2 & 0 & 0 \\ 0 & A_3 & 0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ :::: \\ m_{27} \end{bmatrix} = 0, \qquad (43)$$

where $A$ is a 27×27 square matrix and we have:

$$A_1 = \begin{bmatrix} -a & 0 & 0 & 0 & 0 & 0 & -2c & b & 0 \\ 0 & -a & 0 & 0 & 0 & 0 & 0 & c & 2b \\ 0 & 0 & -a & 0 & 0 & 0 & b & 0 & c \\ c & b & 0 & -a & 0 & 0 & 0 & 0 & 0 \\ 0 & -2c & 2b & 0 & -a & 0 & 0 & 0 & 0 \\ b & 0 & c & 0 & 0 & -a & 0 & 0 & 0 \\ 0 & 0 & 0 & c & b & 0 & 2a & 0 & 0 \\ 0 & 0 & 0 & 0 & c & 2b & 0 & 2a & 0 \\ 0 & 0 & 0 & b & 0 & -2c & 0 & 0 & 2a \end{bmatrix}, \quad A_2 = \begin{bmatrix} 2a & 0 & 0 & 0 & 0 & 0 & -2c & b & 0 \\ 0 & 2a & 0 & 0 & 0 & 0 & 0 & c & 2b \\ 0 & 0 & 2a & 0 & 0 & 0 & b & 0 & c \\ c & b & 0 & -a & 0 & 0 & 0 & 0 & 0 \\ 0 & -2c & 2b & 0 & -a & 0 & 0 & 0 & 0 \\ b & 0 & c & 0 & 0 & -a & 0 & 0 & 0 \\ 0 & 0 & 0 & c & b & 0 & -a & 0 & 0 \\ 0 & 0 & 0 & 0 & c & 2b & 0 & -a & 0 \\ 0 & 0 & 0 & b & 0 & -2c & 0 & 0 & -a \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -a & 0 & 0 & 0 & 0 & 0 & -2c & b & 0 \\ 0 & -a & 0 & 0 & 0 & 0 & 0 & c & 2b \\ 0 & 0 & -a & 0 & 0 & 0 & b & 0 & c \\ c & b & 0 & 2a & 0 & 0 & 0 & 0 & 0 \\ 0 & -2c & 2b & 0 & 2a & 0 & 0 & 0 & 0 \\ b & 0 & c & 0 & 0 & 2a & 0 & 0 & 0 \\ 0 & 0 & 0 & c & b & 0 & -a & 0 & 0 \\ 0 & 0 & 0 & 0 & c & 2b & 0 & -a & 0 \\ 0 & 0 & 0 & b & 0 & -2c & 0 & 0 & -a \end{bmatrix}. \tag{44}$$

For the 4th order homogeneous equation,

$$F(e_1, e_2, f_1, f_2, f_3, f_4) = -e_1 e_2^3 + e_1^3 e_2 + f_1\ f_2\ f_3\ f_4\ = 0 \tag{45}$$

the equivalent system of linear equations, that follow from Axiom 1-2, is

$$AM = \begin{bmatrix} 0 & 0 & 0 & A_1 \\ -A_2 & 0 & 0 & 0 \\ 0 & A_3 & 0 & 0 \\ 0 & 0 & A_4 & 0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ \dots \\ \dots \\ m_{16} \end{bmatrix} = 0, \tag{46}$$

where $A$ is a 16×16 square matrix and matrices $A_1, A_2, A_3, A_4$ are

$$A_1 = \begin{bmatrix} e_1+e_2 & 0 & 0 & f_1 \\ -f_2 & e_1 & 0 & 0 \\ 0 & f_3 & -e_1+e_2 & 0 \\ 0 & 0 & f_4 & -e_2 \end{bmatrix}, A_2 = \begin{bmatrix} -e_2 & 0 & 0 & f_1 \\ f_2 & e_1+e_2 & 0 & 0 \\ 0 & -f_3 & e_1 & 0 \\ 0 & 0 & f_4 & -e_1+e_2 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -e_1+e_2 & 0 & 0 & f_1 \\ f_2 & -e_2 & 0 & 0 \\ 0 & f_3 & e_1+e_2 & 0 \\ 0 & 0 & -f_4 & e_1 \end{bmatrix}, A_4 = \begin{bmatrix} e_1 & 0 & 0 & -f_1 \\ f_2 & -e_1+e_2 & 0 & 0 \\ 0 & f_3 & -e_2 & 0 \\ 0 & 0 & f_4 & e_1+e_2 \end{bmatrix}. \tag{47}$$

In addition, the obtained system of linear equations which corresponds to the 5th order homogeneous equation,

$$F(e_1, e_2, f_1, f_2, f_3, f_4, f_5) = e_1^4 e_2 - e_1^3 e_2^2 - e_1^2 e_2^3 + e_1 e_2^4 + f_1\ f_2\ f_3\ f_4\ f_5\ = 0 \tag{48}$$

is:

$$AM = \begin{bmatrix} 0 & 0 & 0 & 0 & A_1 \\ A_2 & 0 & 0 & 0 & 0 \\ 0 & A_3 & 0 & 0 & 0 \\ 0 & 0 & A_4 & 0 & 0 \\ 0 & 0 & 0 & A_5 & 0 \end{bmatrix} . \begin{bmatrix} m_1 \\ m_2 \\ \ldots \\ \ldots \\ m_{25} \end{bmatrix} = 0, \tag{49}$$

where $A$ is a 25×25 square matrix and matrices $A_1, A_2, A_3, A_4, A_5$ are given by

$$A_1 = \begin{bmatrix} e_1 - e_2 & 0 & 0 & 0 & f_1 \\ f_2 & e_1 & 0 & 0 & 0 \\ 0 & f_3 & e_2 & 0 & 0 \\ 0 & 0 & f_4 & -e_1 + e_2 & 0 \\ 0 & 0 & 0 & f_5 & -e_1 - e_2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -e_1 - e_2 & 0 & 0 & 0 & f_1 \\ f_2 & e_1 - e_2 & 0 & 0 & 0 \\ 0 & f_3 & e_1 & 0 & 0 \\ 0 & 0 & f_4 & e_2 & 0 \\ 0 & 0 & 0 & f_5 & -e_1 + e_2 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -e_1 + e_2 & 0 & 0 & 0 & f_1 \\ f_2 & -e_1 - e_2 & 0 & 0 & 0 \\ 0 & f_3 & e_1 - e_2 & 0 & 0 \\ 0 & 0 & f_4 & e_1 & 0 \\ 0 & 0 & 0 & f_5 & e_2 \end{bmatrix}, \quad A_4 = \begin{bmatrix} e_2 & 0 & 0 & 0 & f_1 \\ f_2 & -e_1 + e_2 & 0 & 0 & 0 \\ 0 & f_3 & -e_1 - e_2 & 0 & 0 \\ 0 & 0 & f_4 & e_1 - e_2 & 0 \\ 0 & 0 & 0 & f_5 & e_1 \end{bmatrix},$$

$$A_5 = \begin{bmatrix} e_1 & 0 & 0 & 0 & f_1 \\ f_2 & e_2 & 0 & 0 & 0 \\ 0 & f_3 & -e_1 + e_2 & 0 & 0 \\ 0 & 0 & f_4 & -e_1 - e_2 & 0 \\ 0 & 0 & 0 & f_5 & e_1 - e_2 \end{bmatrix}. \tag{50}$$

**2-5.** *Because of particular applications of the obtained systems of linear equations corresponding to the general quadratic equation (25-3)in Section 3 (concerning the derivation of the relativistic single-particle field equations of physics and Lorentz transformations and so on), in this section we analyze and solve the matrix equations (34), (36), (37), (38). However, it should be noted again that these matrix equations (that are structurally unique) have been obtained not only on the basis of the algebraic* **Axiom 2-1** (formula (23)), *but also have been modified to be hermitian.*

**F**irst, let we consider the following more general homogeneous quadratic equation

$$\sum_{i,j=0}^{n} B_{ij}(c_i c_j - d_i d_j) = 0 \qquad (51)$$

where $B = \lfloor B_{ij} \rfloor$ is a symmetric matrix $B_{ij} = B_{ji}$, and $\det\lfloor B_{ij} \rfloor \neq 0$.

For obtaining the systems of linear equations corresponding to (51) (for $n = 0,1,2,3,...;$ ), we define the following isomorphic linear transformations

$$e_i = \sum_{j=0}^{n} B_{ij}(c_j + d_j), \;\; f_i = c_i - d_i, \qquad (52)$$

which could also be represent as

$$\begin{bmatrix} f_0 \\ f_1 \\ f_3 \\ . \\ . \\ . \\ f_n \end{bmatrix} = \begin{bmatrix} c_0 - d_0 \\ c_1 - d_1 \\ c_2 - d_2 \\ . \\ . \\ . \\ c_n - d_n \end{bmatrix}, \qquad (52\text{-}1)$$

$$\begin{bmatrix} e_0 \\ e_1 \\ e_3 \\ . \\ . \\ . \\ e_n \end{bmatrix} = \begin{bmatrix} B_{00} & B_{01} & B_{02} & . & . & . & B_{0n} \\ B_{10} & B_{11} & B_{12} & . & . & . & B_{1n} \\ B_{20} & B_{21} & B_{22} & . & . & . & B_{2n} \\ . & & & & & & \\ . & & & & & & \\ . & & & & & & \\ B_{n0} & B_{n1} & B_{n2} & . & . & . & B_{nn} \end{bmatrix} \begin{bmatrix} c_0 + d_0 \\ c_1 + d_1 \\ c_2 + d_2 \\ . \\ . \\ . \\ c_n + d_n \end{bmatrix}; \qquad (52\text{-}2)$$

where $\det B \neq 0$, and matrix $B$ be invertible. Thus, by the above transformations, the systems of linear equations corresponding to (51) are (34) (for $n=1$), (36) (for $n=2$), (37) (for $n=3$) and (38) (for $n=4$). Hence, using (52-1) and (52-2), these matrix equations are represented as follows

$$\left[B_{00}(c_0+d_0)\right]\left[m_1\right]=0, \tag{53}$$

$$\begin{bmatrix} \sum_{j=0}^{1} B_{0j}(c_j+d_j) & c_1-d_1 \\ -\sum_{j=0}^{1} B_{1j}(c_j+d_j) & c_0-d_0 \end{bmatrix}\begin{bmatrix} m_1 \\ m_2 \end{bmatrix}=0, \tag{54}$$

$$\begin{bmatrix} \sum_{j=0}^{2} B_{0j}(c_j+d_j) & 0 & -\sum_{j=0}^{2} B_{2j}(c_j+d_j) & c_1-d_1 \\ 0 & \sum_{j=0}^{2} B_{0j}(c_j+d_j) & -\sum_{j=0}^{2} B_{1j}(c_j+d_j) & -(c_2-d_2) \\ c_2-d_2 & c_1-d_1 & c_0-d_0 & 0 \\ -\sum_{j=0}^{2} B_{1j}(c_j+d_j) & \sum_{j=0}^{2} B_{2j}(c_j+d_j) & 0 & c_0-d_0 \end{bmatrix}\begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix}=0, \tag{55}$$

$$\begin{bmatrix} e_0 & 0 & 0 & 0 & 0 & -e_3 & e_2 & f_1 \\ 0 & e_0 & 0 & 0 & e_3 & 0 & -e_1 & f_2 \\ 0 & 0 & e_0 & 0 & -e_2 & e_1 & 0 & f_3 \\ 0 & 0 & 0 & e_0 & -f_1 & -f_2 & -f_3 & 0 \\ 0 & -f_3 & f_2 & e_1 & f_0 & 0 & 0 & 0 \\ f_3 & 0 & -f_1 & e_2 & 0 & f_0 & 0 & 0 \\ -f_2 & f_1 & 0 & e_3 & 0 & 0 & f_0 & 0 \\ -e_1 & -e_2 & -e_3 & 0 & 0 & 0 & 0 & f_0 \end{bmatrix}\begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \\ m_6 \\ m_7 \\ m_8 \end{bmatrix}=0, \tag{56}$$

where

$$\begin{aligned} e_0 &= \sum_{j=0}^{3} B_{0j}(c_j+d_j), \quad f_0=c_0-d_0, \\ e_1 &= \sum_{j=0}^{3} B_{1j}(c_j+d_j), \quad f_1=c_1-d_1, \\ e_2 &= \sum_{j=0}^{3} B_{2j}(c_j+d_j), \quad f_2=c_2-d_2, \\ e_3 &= \sum_{j=0}^{3} B_{3j}(c_j+d_j), \quad f_3=c_3-d_3. \end{aligned} \tag{56-1}$$

$$
\begin{bmatrix}
e_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -e_4 & 0 & -e_3 & -e_2 & f_1 \\
0 & e_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_4 & 0 & e_3 & 0 & -e_1 & -f_2 \\
0 & 0 & e_0 & 0 & 0 & 0 & 0 & 0 & 0 & -e_4 & 0 & 0 & e_2 & e_1 & 0 & f_3 \\
0 & 0 & 0 & e_0 & 0 & 0 & 0 & 0 & e_4 & 0 & 0 & 0 & -f_1 & f_2 & -f_3 & 0 \\
0 & 0 & 0 & 0 & e_0 & 0 & 0 & 0 & 0 & -e_3 & -e_2 & -e_1 & 0 & 0 & 0 & -f_4 \\
0 & 0 & 0 & 0 & 0 & e_0 & 0 & 0 & e_3 & 0 & f_1 & -f_2 & 0 & 0 & f_4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & e_0 & 0 & e_2 & -f_1 & 0 & f_3 & 0 & -f_4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & e_0 & e_1 & f_2 & -f_3 & 0 & f_4 & 0 & 0 & 0 \\
0 & 0 & 0 & -f_4 & 0 & -f_3 & -f_2 & -f_1 & f_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & f_4 & 0 & f_3 & 0 & e_1 & -e_2 & 0 & f_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -f_4 & 0 & 0 & f_2 & -e_1 & 0 & e_3 & 0 & 0 & f_0 & 0 & 0 & 0 & 0 & 0 \\
f_4 & 0 & 0 & 0 & f_1 & e_2 & -e_3 & 0 & 0 & 0 & 0 & f_0 & 0 & 0 & 0 & 0 \\
0 & -f_3 & -f_2 & e_1 & 0 & 0 & 0 & -e_4 & 0 & 0 & 0 & 0 & f_0 & 0 & 0 & 0 \\
f_3 & 0 & -f_1 & -e_2 & 0 & 0 & e_4 & 0 & 0 & 0 & 0 & 0 & 0 & f_0 & 0 & 0 \\
f_2 & f_1 & 0 & e_3 & 0 & -e_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & f_0 & 0 \\
-e_1 & e_2 & -e_3 & 0 & e_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & f_0
\end{bmatrix}
\begin{bmatrix}
m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \\ m_6 \\ m_7 \\ m_8 \\ m_9 \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{15} \\ m_{16}
\end{bmatrix}
= 0 \tag{57}
$$

where

$$
\begin{aligned}
e_0 &= \sum_{j=0}^{4} B_{0j}(c_j + d_j), \quad f_0 = c_0 - d_0, \\
e_1 &= \sum_{j=0}^{4} B_{1j}(c_j + d_j), \quad f_1 = c_1 - d_1, \\
e_2 &= \sum_{j=0}^{4} B_{2j}(c_j + d_j), \quad f_2 = c_2 - d_2, \\
e_3 &= \sum_{j=0}^{4} B_{3j}(c_j + d_j), \quad f_3 = c_3 - d_3, \\
e_4 &= \sum_{j=0}^{4} B_{4j}(c_j + d_j), \quad f_4 = c_4 - d_4.
\end{aligned} \tag{57-1}
$$

**I**t is noteworthy here that there are not the similar isomorphic linear transformations such as (51-1) - (51-2) (that were definable for quadratic equation (51)) for the third and the higher order equations of the form:

$$
\sum_{i,j,k=0}^{n} B_{ijk}(c_i c_j c_k - d_i d_j d_k) = 0 \ , \quad \sum_{i,j,k,l=0}^{n} B_{ijkl}(c_i c_j c_k c_l - d_i d_j d_k d_l) = 0 \ , \ \ldots \, . \tag{58}
$$

**M**oreover, by the following choices

$$B = \begin{bmatrix} B_{00} & B_{01} & B_{02} & . & . & . & B_{0n} \\ B_{10} & B_{11} & B_{12} & . & . & . & B_{1n} \\ B_{20} & B_{21} & B_{22} & . & . & . & B_{2n} \\ . & & & & & & \\ . & & & & & & \\ . & & & & & & \\ B_{n0} & B_{n1} & B_{n2} & . & . & . & B_{nn} \end{bmatrix},$$

$$C = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ . \\ . \\ . \\ c_n \end{bmatrix}, \quad D = \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ . \\ . \\ . \\ d_n \end{bmatrix}, \quad E = \begin{bmatrix} e_0 \\ e_1 \\ e_3 \\ . \\ . \\ . \\ e_n \end{bmatrix}, \quad F = \begin{bmatrix} f_0 \\ f_1 \\ f_3 \\ . \\ . \\ . \\ f_n \end{bmatrix}; \tag{59}$$

we can rewrite the transformations (52-1) and (52-2) as follows

$$E = B(C + D), \quad F = C - D, \tag{60}$$

From (60) we also get

$$C = \tfrac{1}{2}(B^{-1}E + F), \quad D = \tfrac{1}{2}(B^{-1}E - F) \tag{61}$$

In (61) $B^{-1}$ is the inverse of matrix $B$. In Section 2-6, using the relations (60) and (61) and also the solutions of matrix equations (34), (36), (37) and (38), we directly will determine the general solutions of systems of linear equations (54) – (57), which will be the general solutions of quadratic equation (51) as well, for $n = 0,1,2,3,...$ .

**2-6.** *Now utilizing the standard and specific methods of solving the systems of homogeneous linear equations over the ring of integers [7], below we obtain the general parametric solutions for the systems of homogeneous linear equations (34), (36), (37) and (38) for unknowns $e_i$ and $f_i$.*

**F**irst, let we write a definite parametric solution of general homogeneous linear equation of the type

$$\sum_{i=1}^{n} a_i x_i = 0 \tag{62}$$

for unknowns $x_i$, over the ring of integer elements [7, 8]:

$$x_j = a_n k_j, \ \ (j = 1,2,3,...,n-1), \ \ x_n = -\sum_{j=1}^{n-1} a_j k_j \tag{63}$$

where $k_i$ are arbitrary integer parameters, and $a_n \neq 0$. Furthermore, if $x_i'$ and $x_i''$ $(i = 1,2,3,...,n)$ be two solutions of equation (62), then $x_i' \pm x_i''$ and $tx_i'$ (where $t$ is a non-zero integer) also are the solutions of (62), such that

$$(\sum_{i=1}^{n} a_i x_i' = 0, \ \sum_{i=1}^{n} a_i x_i'' = 0) \ \Rightarrow \sum_{i=1}^{n} a_i (x_i' \pm x_i'') = 0, \ \ \sum_{i=1}^{n} a_i (tx_i') = 0 \Leftrightarrow \sum_{i=1}^{n} a_i x_i' = 0 \tag{63-1}$$

On this basis, for equations (34) we directly get the following general symmetric solutions

$$e_0 = k_2 m_2, \ \ f_0 = k_1 m_1, \ \ e_1 = k_1 m_2, \ \ f_1 = -k_2 m_1 \tag{64}$$

where $k_1, k_2; m_1, m_2$ ($m_2 \neq 0$) are arbitrary integer parameters.

For system of linear equations (36) we get

$$e_0 = k_3 m_4, \ \ f_0 = k_2 m_1 - k_1 m_2, \ \ e_1 = k_2 m_4, \ \ f_1 = k_1 m_3 - k_3 m_1, \ \ e_2 = k_1 m_4, \ \ f_2 = k_3 m_2 - k_2 m_3 \tag{65}$$

where $k_1, k_2, k_3; m_1, m_2, m_3, m_4$ $(m_4 \neq 0)$ are arbitrary integer parameters. *Particularly for matrix equation* (36), using (25-1), (25-2), and (65) we may also obtain the following type of the general solution (which includes the solution (65) as well):

$$\begin{gathered} e_0 = k_3 m_4 - k_4 m_3, \ \ f_0 = k_2 m_1 - k_1 m_2, \ \ e_1 = k_2 m_4 - k_4 m_2, \\ f_1 = k_1 m_3 - k_3 m_1, \ \ e_2 = k_1 m_4 - k_4 m_1, \ \ f_2 = k_3 m_2 - k_2 m_3. \end{gathered} \tag{66}$$

where $k_1, k_2, k_3, k_4; m_1, m_2, m_3, m_4$ $(m_4 \neq 0 \ \ or \ \ k_4 \neq 0)$ are arbitrary integer parameters.

**F**or the system of equations (37), similarly, the following general parametric solutions are obtained (where we suppose $m_8 \neq 0$):

$$e_0 = k_4 m_8, \quad f_0 = k_3 m_1 + k_2 m_2 + k_1 m_3, \quad e_1 = k_3 m_8, \quad f_1 = -k_4 m_1 + k_1 m_6 - k_2 m_7,$$
$$e_2 = k_2 m_8, \quad f_2 = -k_4 m_2 - k_1 m_5 + k_3 m_7, \quad e_3 = k_1 m_8, \quad f_3 = -k_4 m_3 + k_2 m_5 - k_3 m_6. \tag{67}$$

where here $k_1, k_2, k_3, k_4$ are arbitrary integer parameters. Meanwhile, from the matrix equation (37) a necessary additional condition also appears for parameters $m_i$ (that appears in the course of obtaining solution (67) from the equation (37)) as follows

$$m_4 m_8 + m_1 m_5 + m_2 m_6 + m_3 m_7 = 0 \tag{68}$$

Condition (68) is also a homogeneous quadratic equation that corresponds to matrix equation (37), and should be separately solved. On this basis, using the matrix equation (37) for (68), and since the parameter $m_4$ does not appear in the solutions (67), the following unique relations and two types of general solutions (which are algebraically equivalent) for condition (68) are obtained:

$$m_4 = 0, \tag{69}$$

$$m_8: \ \textit{an arbitrary Integer parameter} \ (m_8 \neq 0), \tag{70}$$

$$m_1 m_5 + m_2 m_6 + m_3 m_7 = 0 \tag{71}$$

where one type of solution is

$$m_1 = u_3 v_4, \quad m_2 = u_2 v_4, \quad m_3 = u_1 v_4,$$
$$m_5 = u_2 v_1 - u_1 v_2, \quad m_6 = u_1 v_3 - u_3 v_1, \quad m_7 = u_3 v_2 - u_2 v_3, \tag{72}$$
$$m_4 = 0, \quad m_8: \ \textit{an arbitrary Integer parameter} \ (m_8 \neq 0) \ ;$$

and another type of solution (which includes the first type as well) is as follows

$$m_1 = u_3 v_4 - u_4 v_3, \quad m_2 = u_2 v_4 - u_4 v_2, \quad m_3 = u_1 v_4 - u_4 v_1,$$
$$m_5 = u_2 v_1 - u_1 v_2, \quad m_6 = u_1 v_3 - u_3 v_1, \quad m_7 = u_3 v_2 - u_2 v_3, \quad m_4 = 0, \tag{73}$$
$$m_4 = 0, \quad m_8: \ \textit{an arbitrary Integer parameter} \ (m_8 \neq 0).$$

where $u_1, u_2, u_3, u_4$ and $v_1, v_2, v_3, v_4$ are arbitrary integer parameters. By replacing the values of $m_i$, (from the relations (72) or (73)) in formulas (67), the general parametric solution for matrix equation (37) is obtained. We should note here that since the relation (71) correspond to the matrix equation (36), two sets of (algebraically equivalent) solutions (72) and (73) follow from two basic parametric solutions (65) and (66). *It is noteworthy that, in particular, the relations (73) in terms of parameters* $m_1, m_2, m_3, m_5, m_6, m_7$ *have a certain appropriate symmetric structure which is compatible with a symmetric requirement in the course of the application of these results in physics, presented in Section 3.*

**F**or the system of equations (38), the following general parametric solutions are obtained as well (where we suppose $m_{16} \neq 0)$,

$$e_0 = k_5 m_{16}, \quad f_0 = k_4 m_1 - k_3 m_2 + k_2 m_3 - k_1 m_5, \quad e_1 = k_4 m_{16}, \quad f_1 = -k_5 m_1 + k_1 m_{12} + k_2 m_{14} + k_3 m_{15},$$

$$e_2 = k_3 m_{16}, \quad f_2 = k_5 m_2 + k_1 m_{11} + k_2 m_{13} - k_4 m_{15}, \quad e_3 = k_2 m_{16}, \quad f_3 = -k_5 m_3 + k_1 m_{10} - k_3 m_{13} - k_4 m_{14}, \quad (74)$$

$$e_4 = k_1 m_{16}, \quad f_4 = k_5 m_5 - k_2 m_{10} - k_3 m_{11} - k_4 m_{12}.$$

$k_1, k_2, k_3, k_4, k_5$ are arbitrary integer parameters. Moreover, the parameters $m_i$ should satisfy the following conditions (which in the course of obtaining the solution (74) from matrix equation (38)):

$$\begin{aligned} m_4 m_{16} &= -m_1 m_{13} - m_2 m_{14} - m_3 m_{15}, \\ m_6 m_{16} &= m_1 m_{11} + m_2 m_{12} - m_5 m_{15}, \\ m_7 m_{16} &= m_1 m_{10} - m_3 m_{12} - m_5 m_{14}, \\ m_8 m_{16} &= m_2 m_{10} + m_3 m_{11} + m_5 m_{13}, \\ m_9 m_{16} &= m_{10} m_{15} - m_{11} m_{14} + m_{12} m_{13}. \end{aligned} \quad (75)$$

Conditions (75) are a set of homogeneous quadratic equations that generally correspond to the matrix equation (37), and should be solved as a separate set of quadratic equations. On this basis, using the matrix equation (37) for conditions (75), and since the parameters $m_4, m_6, m_7, m_8, m_9$ don't appear in the solutions (74), the following unique relations and two types of general solutions (which are algebraically equivalent) for conditions (75) are obtained:

$$m_4 = m_6 = m_7 = m_8 = m_9 = 0, \quad (76)$$

$$m_{16}: \ \textit{an arbitrary Integer parameter} \ (m_{16} \neq 0), \quad (77)$$

$$-m_1 m_{10} + m_3 m_{12} + m_5 m_{14} = 0, \quad (78)$$
$$m_1 m_{11} + m_2 m_{12} - m_5 m_{15} = 0, \quad (79)$$
$$m_1 m_{13} + m_2 m_{14} + m_3 m_{15} = 0, \quad (80)$$
$$m_2 m_{10} + m_5 m_{13} + m_3 m_{11} = 0, \quad (81)$$
$$m_{10} m_{15} + m_{12} m_{13} - m_{11} m_{14} = 0 \ ; \quad (82)$$

where one type of solution reads

$$m_1 = u_4 v_5, \quad m_2 = u_3 v_5,$$

$$m_3 = -u_2 v_5, \quad m_4 = 0,$$

$$m_5 = -u_1 v_5, \quad m_6 = 0,$$

$$m_7 = 0, \quad m_8 = 0, \quad m_9 = 0,$$

$$m_{10} = u_2 v_1 - u_1 v_2, \quad m_{11} = u_3 v_1 - u_1 v_3,$$

$$m_{12} = u_1 v_4 - u_4 v_1, \quad m_{13} = u_2 v_3 - u_3 v_2,$$

$$m_{14} = u_4 v_2 - u_2 v_4, \quad m_{15} = u_4 v_3 - u_3 v_4,$$

$$m_{16}: \ a \ \ free \ \ Integer \ \ parameter \ (m_{16} \neq 0). \tag{83}$$

and another type of solution (which includes the first type as well) is derived as follows:

$$m_1 = u_4 v_5 - u_5 v_4, \quad m_2 = u_3 v_5 - u_5 v_3,$$

$$m_3 = u_5 v_2 - u_2 v_5, \quad m_4 = 0,$$

$$m_5 = u_5 v_1 - u_1 v_5, \quad m_6 = 0,$$

$$m_7 = 0, \quad m_8 = 0, \quad m_9 = 0,$$

$$m_{10} = u_2 v_1 - u_1 v_2, \quad m_{11} = u_3 v_1 - u_1 v_3,$$

$$m_{12} = u_1 v_4 - u_4 v_1, \quad m_{13} = u_2 v_3 - u_3 v_2,$$

$$m_{14} = u_4 v_2 - u_2 v_4, \quad m_{15} = u_4 v_3 - u_3 v_4,$$

$$m_{16}: \ a \ \ free \ \ Integer \ \ parameter \ (m_{16} \neq 0). \tag{84}$$

where $u_1, u_2, u_3, u_4, u_5; v_1, v_2, v_3, v_4, v_5$ are arbitrary integer parameters. By replacing the parametric values of $m_i$ (83) or (84) in relations (74), we get the general parametric symmetric solution for matrix equation (38).

We should note here that since the relations of the types (78) – (82) correspond generally to the matrix equation (36), two sets of (algebraically equivalent) solutions (83) and (84) follow from two basic parametric solutions (65) and (66). *Similar to relations (73), it is noteworthy again that, in particular, the relations (84) in terms of parameters* $m_1, m_2, m_3, m_5, m_{10}, m_{11}, m_{12}, m_{13}, m_{14}, m_{15}$ *has a certain appropriate symmetric structure which is compatible with a symmetric requirement in the course of the application of these results in physics, presented in Section 3.*

**A**pplying the axiomatic linearization (and simultaneous parameterization, as necessary algebraic conditions) approach based on the new Axiom 2-1 (formula (23)), the quadratic homogeneous equations with more unknowns are solved in the same manner. We use the above obtained results for the homogeneous quadratic relations, in Section 3, where we assume that the components of the relativistic energy-momentum relation (as a definite quadratic relation) are discrete quantities.

# 3. A Direct Logical Derivation of the Laws Governing the Fundamental Forces of Nature

"*Including a Unique Set of the General Relativistic (Single-Particle) Wave Equations – Formulated Solely in* D $\leq$ *4 Dimensional Space-Time*"

**I**n this Section on the basis of the mathematical axiomatic approach presented in Section 2 (particularly, the A, by linearization (and simultaneous parameterization, as necessary algebraic conditions), followed by first quantization (as a postulate) of linearized (and simultaneously parameterized, as necessary algebraic conditions) unique forms of the special relativistic energy-momentum relation (which are defined algebraically for a single particle with invariant mass $m_0$), we derive a unique and original set of the general relativistic (single-particle) wave equations directly. These equations are shown to correspond uniquely to certain massive forms of the laws governing the fundamental forces of nature, including the Gravitational, Electromagnetic and Nuclear field equations (which based on our approach are solely formulable in (1+3) dimensional space-time), in addition to the (half-integer spin) single-particle wave equations (formulated solely in (1+2) dimensional space-time). **Notably, these results are primarily mathematical, assuming the relativistic energy-momentum is a discrete quantity**– that is a basic quantum mechanical assumption.

Each derived relativistic wave equation is in a complex tensor form, that in the matrix representation (i.e. in the geometric algebra formulation, see equations (120) – (121)) it could be written in the form of two coupled symmetric equations – which assumedly have chiral symmetry if the particle wave equation be source-free. In fact, the complex relativistic (single-particle) wave equations so uniquely obtained, correspond to certain massive form of classical fields including the Einstein, Maxwell and Yang-Mills field equations, in addition to the (half-integer spin) single-particle wave equations such as the Dirac equation (where the Dirac spinor field is isomorphically re-presented solely by a tensor field in three dimensional space-time [29, 31]).

In particular, a unique massive form of the general theory of relativity – with a definite complex torsion – is shown to be obtained solely by first quantization of a special relativistic algebraic matrix relation. Moreover, it is shown that the massive Lagrangian density of the obtained Maxwell and Yang-Mills fields could be also locally gauge invariant – where these fields are formally re-presented on a background space-time with certain (coupled) complex torsion which is generated by the invariant mass of the gauge field carrier particle. Subsequently, in agreement with certain experimental data, the invariant mass of a particle (that actually would be identified as massive photon) has been specified ($m_0^{(E)} \approx 1.4070696 \times 10^{-41}$ kg), which is coupled with background space-time geometry (see Section 3-4-1). Assuming our approach is the unique and principal way for deriving (all) the laws governing the fundamental forces of nature, then based on the unique structure of general relativistic single-particle fields derived and also the assumption of chiral symmetry as a basic discrete symmetry of the source-free cases of these fields, it has been shown that the universe cannot have more than four space-time dimensions. Furthermore, a basic argument for the asymmetry of left and right handed (interacting) particles is presented. In addition, on the basis of definite structure of the field equations derived, we also conclude that magnetic monopoles – in contrast with electric monopoles – could not exist in nature.

**A**s it was mentioned in Section 1-1, the main arguments and consequences presented in this article (particularly in this section) follow from these three basic and primary assumptions:

**(1)- "Generalization of the algebraic axiom of nonzero divisors for integer elements (based on the ring theory and the matrix representation of generalized Clifford algebra, and subsequently, constructing a definite algebraic linearization theory);"**

This is one of the new and principal concepts presented in Section 2 (see formula (23).

**(2)- "Discreteness of the relativistic energy-momentum (D-momentum);"**

This is a basic quantum mechanical assumption. As the quantum theory, particularly, tells us that energy and momentum are only transferred in discrete quantities, i.e., as integer multiples of the quantum of action (Planck constant) $h$.

**(3)- "The general relativistic massive forms of the laws governing the fundamental forces of nature, including the gravitational, electromagnetic and nuclear field equations, in addition to the relativistic (half-integer spin) single-particle wave equations, are derived solely by first quantization (as a postulate) of linearized (and simultaneously parameterized, as necessary algebraic conditions) unique forms of the relativistic energy-momentum relation – which are defined algebraically for a single particle with invariant mass $m_0$)."**

We also assume that the source-free cases of these fields have "**chiral symmetry**".

**N**ote that the geometrized units, metric signature (+ − ... −) and the sign conventions (97) will be used. So particularly, we assume the speed of light $c = 1$. However, for clarity and emphasis, in some relativistic relations "$c$" as a constant be restored and indicated formally.

**3-1.** **A**ssuming the components of the energy-momentum vector are discrete quantities (as a basic and original quantum mechanical assumption)[1], then definitely the invariant and relation of the energy-momentum for a massive particle in the special relativistic conditions, i.e.

$$g^{\mu\nu} p_\mu p_\nu = g^{\mu\nu} p'_\mu p'_\nu \ , \tag{85}$$

$$g^{\mu\nu} p_\mu p_\nu = p^\nu p_\nu = (\mp m_0 c)^2 = g^{00} \left(\frac{\mp m_0 c}{\sqrt{g^{00}}}\right)^2 \tag{86}$$

are the special cases of the general algebraic quadratic relation (51). Where $g^{\mu\nu}$ are *constant* symmetric coefficients, $m_0$ is the rest mass of a single particle, and $p_\mu, p'_\mu$ are the components of the relativistic energy-momentum vector in two reference frames. Note that based on our chosen sign conventions, the minus sign in relation (86) is for the particle, and the plus sign is connected to its anti-particle. The next formulas and results are formulated and obtained only for particles (for anti-particles the mass sign should be changed).As a direct consequence of the axiomatic approach presented in Section 2, the relations (85) and (86), necessarily, should be *linearized* (and simultaneously *parameterized, as necessary algebraic conditions*).

---

1. This is a primary quantum mechanical assumption. However, for general and expanded cases of the discreteness, and concerning discrete physics, it is noteworthy that in many modern and standard quantum theories (Lorentz invariance), it is assumed that certain physical quantities are discrete. These theories include the lattice field gauge theories such as lattice QCD, quantum gravity theories, etc. [15 – 22]. However, the discrete axiomatic approach presented in this article directly yields to parametric linear transformations corresponding directly to the Lorentz transformations.

**H**ence, using the matrix relations (53) – (57) (which are also hermitian), we get the following unique set of systems of linear equations that correspond equivalently to the relations (85) and (86):

First for relation (85), the equivalent set of matrix relations for various space-time dimensions are given as follows, respectively (where $s_i$ are parameters similar to parameters $m_i$ in the matrix relations (53) – (57)),

$$\left[g^{00}(p_0+p_0')\right]\left[s_1\right]=0 \tag{87}$$

$$\begin{bmatrix} g^{0\nu}(p_\nu+p_\nu') & p_1-p_1' \\ -g^{1\nu}(p_\nu+p_\nu') & p_0-p_0' \end{bmatrix}\begin{bmatrix} s_1 \\ s_2 \end{bmatrix}=0 \tag{88}$$

where $\nu=0,1$;

$$\begin{bmatrix} g^{0\nu}(p_\nu+p_\nu') & 0 & -g^{2\nu}(p_\nu+p_\nu') & p_1-p_1' \\ 0 & g^{0\nu}(p_\nu+p_\nu') & -g^{1\nu}(p_\nu+p_\nu') & -(p_2-p_2') \\ p_2-p_2' & p_1-p_1' & p_0-p_0' & 0 \\ -g^{1\nu}(p_\nu+p_\nu') & g^{2\nu}(p_\nu+p_\nu') & 0 & p_0-p_0' \end{bmatrix}\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix}=0 \tag{89}$$

where $\nu=0,1,2$;

$$\begin{bmatrix} e_0 & 0 & 0 & 0 & 0 & -e_3 & e_2 & f_1 \\ 0 & e_0 & 0 & 0 & e_3 & 0 & -e_1 & f_2 \\ 0 & 0 & e_0 & 0 & -e_2 & e_1 & 0 & f_3 \\ 0 & 0 & 0 & e_0 & -f_1 & -f_2 & -f_3 & 0 \\ 0 & -f_3 & f_2 & e_1 & f_0 & 0 & 0 & 0 \\ f_3 & 0 & -f_1 & e_2 & 0 & f_0 & 0 & 0 \\ -f_2 & f_1 & 0 & e_3 & 0 & 0 & f_0 & 0 \\ -e_1 & -e_2 & -e_3 & 0 & 0 & 0 & 0 & f_0 \end{bmatrix}\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \\ s_8 \end{bmatrix}=0 \tag{90}$$

where $\nu = 0,1,2,3$ and

$$s_4 s_8 + s_1 s_5 + s_2 s_6 + s_3 s_7 = 0, \tag{90-1}$$

$$\begin{aligned}
e_0 &= g^{0\nu}(p_\nu + p'_\nu), \quad f_0 = p_0 - p'_0, \\
e_1 &= g^{1\nu}(p_\nu + p'_\nu), \quad f_1 = p_1 - p'_1, \\
e_2 &= g^{2\nu}(p_\nu + p'_\nu), \quad f_2 = p_2 - p'_2, \\
e_3 &= g^{3\nu}(p_\nu + p'_\nu), \quad f_3 = p_3 - p'_3.
\end{aligned} \tag{90-2}$$

(notice that the condition (90-1) is equivalent to the algebraic condition (68));

$$\begin{bmatrix}
e_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -e_4 & 0 & -e_3 & -e_2 & f_1 \\
0 & e_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_4 & 0 & e_3 & 0 & -e_1 & -f_2 \\
0 & 0 & e_0 & 0 & 0 & 0 & 0 & 0 & 0 & -e_4 & 0 & 0 & e_2 & e_1 & 0 & f_3 \\
0 & 0 & 0 & e_0 & 0 & 0 & 0 & 0 & e_4 & 0 & 0 & 0 & -f_1 & f_2 & -f_3 & 0 \\
0 & 0 & 0 & 0 & e_0 & 0 & 0 & 0 & 0 & -e_3 & -e_2 & -e_1 & 0 & 0 & 0 & -f_4 \\
0 & 0 & 0 & 0 & 0 & e_0 & 0 & 0 & e_3 & 0 & f_1 & -f_2 & 0 & 0 & f_4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & e_0 & 0 & e_2 & -f_1 & 0 & f_3 & 0 & -f_4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & e_0 & e_1 & f_2 & -f_3 & 0 & f_4 & 0 & 0 & 0 \\
0 & 0 & 0 & -f_4 & 0 & -f_3 & -f_2 & -f_1 & f_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & f_4 & 0 & f_3 & 0 & e_1 & -e_2 & 0 & f_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -f_4 & 0 & 0 & f_2 & -e_1 & 0 & e_3 & 0 & 0 & f_0 & 0 & 0 & 0 & 0 & 0 \\
f_4 & 0 & 0 & 0 & f_1 & e_2 & -e_3 & 0 & 0 & 0 & 0 & f_0 & 0 & 0 & 0 & 0 \\
0 & -f_3 & -f_2 & e_1 & 0 & 0 & 0 & -e_4 & 0 & 0 & 0 & 0 & f_0 & 0 & 0 & 0 \\
f_3 & 0 & -f_1 & -e_2 & 0 & 0 & e_4 & 0 & 0 & 0 & 0 & 0 & 0 & f_0 & 0 & 0 \\
f_2 & f_1 & 0 & e_3 & 0 & -e_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & f_0 & 0 \\
-e_1 & e_2 & -e_3 & 0 & e_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & f_0
\end{bmatrix}
\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \\ s_8 \\ s_9 \\ s_{10} \\ s_{11} \\ s_{12} \\ s_{13} \\ s_{14} \\ s_{15} \\ s_{16} \end{bmatrix} = 0 \tag{91}$$

where we have

$$s_4 s_{16} = -s_1 s_{13} - s_2 s_{14} - s_3 s_{15}, \qquad (91\text{-}1)$$

$$s_6 s_{16} = s_1 s_{11} + s_2 s_{12} - s_5 s_{15}, \qquad (91\text{-}2)$$

$$s_7 s_{16} = s_1 s_{10} - s_3 s_{12} - s_5 s_{14}, \qquad (91\text{-}3)$$

$$s_8 s_{16} = s_2 s_{10} + s_3 s_{11} + s_5 s_{13}, \qquad (91\text{-}4)$$

$$s_9 s_{16} = s_{10} s_{15} - s_{11} s_{14} + s_{12} s_{13}; \qquad (91\text{-}5)$$

and

$$\begin{aligned} e_0 &= g^{0\nu}(p_\nu + p'_\nu), \quad f_0 = p_0 - p'_0, \\ e_1 &= g^{1\nu}(p_\nu + p'_\nu), \quad f_1 = p_1 - p'_1, \\ e_2 &= g^{2\nu}(p_\nu + p'_\nu), \quad f_2 = p_2 - p'_2, \\ e_3 &= g^{3\nu}(p_\nu + p'_\nu), \quad f_3 = p_3 - p'_3, \\ e_4 &= g^{4\nu}(p_\nu + p'_\nu), \quad f_4 = p_4 - p'_4. \end{aligned} \qquad (91\text{-}6)$$

and $\nu = 0,1,2,3,4$. Notice that the algebraic conditions (91-1) – (91-5) are equivalent to the conditions (75).

The systems of linear equations that correspond to the relation (86) are also obtained as follows, respectively, for various space-time dimensions:

(where $p'_0 = -\frac{m_0 c}{\sqrt{g^{00}}}$, $if \quad \mu \neq 0 : p'_\mu = 0$)

$$\left[ g^{00}(p_0 - \frac{m_0 c}{\sqrt{g^{00}}}) \right] [s_1] = 0 \tag{92}$$

$$\begin{bmatrix} g^{0\nu} p_\nu - g^{00}(\frac{m_0 c}{\sqrt{g^{00}}}) & p_1 \\ -g^{1\nu} p_\nu + g^{10}(\frac{m_0 c}{\sqrt{g^{00}}}) & p_0 + (\frac{m_0 c}{\sqrt{g^{00}}}) \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = 0 \tag{93}$$

where $\nu = 0,1$;

$$\begin{bmatrix} g^{0\nu} p_\nu - g^{00}(\frac{m_0 c}{\sqrt{g^{00}}}) & 0 & -g^{2\nu} p_\nu + g^{20}(\frac{m_0 c}{\sqrt{g^{00}}}) & p_1 \\ 0 & g^{0\nu} p_\nu - g^{00}(\frac{m_0 c}{\sqrt{g^{00}}}) & -g^{1\nu} p_\nu + g^{10}(\frac{m_0 c}{\sqrt{g^{00}}}) & -p_2 \\ p_2 & p_1 & p_0 + (\frac{m_0 c}{\sqrt{g^{00}}}) & 0 \\ -g^{1\nu} p_\nu + g^{10}(\frac{m_0 c}{\sqrt{g^{00}}}) & g^{2\nu} p_\nu - g^{20}(\frac{m_0 c}{\sqrt{g^{00}}}) & 0 & p_0 + (\frac{m_0 c}{\sqrt{g^{00}}}) \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} = 0 \tag{94}$$

where $\nu = 0,1,2$;

$$\begin{bmatrix} e_0 & 0 & 0 & 0 & 0 & -e_3 & e_2 & f_1 \\ 0 & e_0 & 0 & 0 & e_3 & 0 & -e_1 & f_2 \\ 0 & 0 & e_0 & 0 & -e_2 & e_1 & 0 & f_3 \\ 0 & 0 & 0 & e_0 & -f_1 & -f_2 & -f_3 & 0 \\ 0 & -f_3 & f_2 & e_1 & f_0 & 0 & 0 & 0 \\ f_3 & 0 & -f_1 & e_2 & 0 & f_0 & 0 & 0 \\ -f_2 & f_1 & 0 & e_3 & 0 & 0 & f_0 & 0 \\ -e_1 & -e_2 & -e_3 & 0 & 0 & 0 & 0 & f_0 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \\ s_8 \end{bmatrix} = 0 \tag{95}$$

where $\nu = 0,1,2,3$ and

$$s_4 s_8 + s_1 s_5 + s_2 s_6 + s_3 s_7 = 0, \tag{95-1}$$

$$\begin{aligned}
e_0 &= g^{0\nu} p_\nu - g^{00}\left(\frac{m_0 c}{\sqrt{g^{00}}}\right), \quad f_0 = p_0 + \left(\frac{m_0 c}{\sqrt{g^{00}}}\right), \\
e_1 &= g^{1\nu} p_\nu - g^{10}\left(\frac{m_0 c}{\sqrt{g^{00}}}\right), \quad f_1 = p_1, \\
e_2 &= g^{2\nu} p_\nu - g^{20}\left(\frac{m_0 c}{\sqrt{g^{00}}}\right), \quad f_2 = p_2, \\
e_3 &= g^{3\nu} p_\nu - g^{30}\left(\frac{m_0 c}{\sqrt{g^{00}}}\right), \quad f_3 = p_3.
\end{aligned} \tag{95-2}$$

Notice that here also the condition (95-1) is equivalent to the algebraic condition (68).

For (1+4) dimensional case we obtain:

$$\begin{bmatrix}
e_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -e_4 & 0 & -e_3 & -e_2 & f_1 \\
0 & e_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_4 & 0 & e_3 & 0 & -e_1 & -f_2 \\
0 & 0 & e_0 & 0 & 0 & 0 & 0 & 0 & 0 & -e_4 & 0 & 0 & e_2 & e_1 & 0 & f_3 \\
0 & 0 & 0 & e_0 & 0 & 0 & 0 & 0 & e_4 & 0 & 0 & 0 & -f_1 & f_2 & -f_3 & 0 \\
0 & 0 & 0 & 0 & e_0 & 0 & 0 & 0 & 0 & -e_3 & -e_2 & -e_1 & 0 & 0 & 0 & -f_4 \\
0 & 0 & 0 & 0 & 0 & e_0 & 0 & 0 & e_3 & 0 & f_1 & -f_2 & 0 & 0 & f_4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & e_0 & 0 & e_2 & -f_1 & 0 & f_3 & 0 & -f_4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & e_0 & e_1 & f_2 & -f_3 & 0 & f_4 & 0 & 0 & 0 \\
0 & 0 & 0 & -f_4 & 0 & -f_3 & -f_2 & -f_1 & f_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & f_4 & 0 & f_3 & 0 & e_1 & -e_2 & 0 & f_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -f_4 & 0 & 0 & f_2 & -e_1 & 0 & e_3 & 0 & 0 & f_0 & 0 & 0 & 0 & 0 & 0 \\
f_4 & 0 & 0 & 0 & f_1 & e_2 & -e_3 & 0 & 0 & 0 & 0 & f_0 & 0 & 0 & 0 & 0 \\
0 & -f_3 & -f_2 & e_1 & 0 & 0 & 0 & -e_4 & 0 & 0 & 0 & 0 & f_0 & 0 & 0 & 0 \\
f_3 & 0 & -f_1 & -e_2 & 0 & 0 & e_4 & 0 & 0 & 0 & 0 & 0 & 0 & f_0 & 0 & 0 \\
f_2 & f_1 & 0 & e_3 & 0 & -e_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & f_0 & 0 \\
-e_1 & e_2 & -e_3 & 0 & e_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & f_0
\end{bmatrix}
\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \\ s_8 \\ s_9 \\ s_{10} \\ s_{11} \\ s_{12} \\ s_{13} \\ s_{14} \\ s_{15} \\ s_{16} \end{bmatrix} = 0 \tag{96}$$

where we have

$$s_4 s_{16} = -s_1 s_{13} - s_2 s_{14} - s_3 s_{15}, \tag{96-1}$$
$$s_6 s_{16} = s_1 s_{11} + s_2 s_{12} - s_5 s_{15}, \tag{96-2}$$
$$s_7 s_{16} = s_1 s_{10} - s_3 s_{12} - s_5 s_{14}, \tag{96-3}$$
$$s_8 s_{16} = s_2 s_{10} + s_3 s_{11} + s_5 s_{13}, \tag{96-4}$$
$$s_9 s_{16} = s_{10} s_{15} - s_{11} s_{14} + s_{12} s_{13}; \tag{96-5}$$

$$\begin{aligned}
e_0 &= g^{0\nu} p_\nu - g^{00}(\frac{m_0 c}{\sqrt{g^{00}}}), \quad f_0 = p_0 + (\frac{m_0 c}{\sqrt{g^{00}}}), \\
e_1 &= g^{1\nu} p_\nu - g^{10}(\frac{m_0 c}{\sqrt{g^{00}}}), \quad f_1 = p_1, \\
e_2 &= g^{2\nu} p_\nu - g^{20}(\frac{m_0 c}{\sqrt{g^{00}}}), \quad f_2 = p_2, \\
e_3 &= g^{3\nu} p_\nu - g^{30}(\frac{m_0 c}{\sqrt{g^{00}}}), \quad f_3 = p_3, \\
e_4 &= g^{4\nu} p_\nu - g^{40}(\frac{m_0 c}{\sqrt{g^{00}}}), \quad f_4 = p_4
\end{aligned} \tag{96-6}$$

and $\nu = 0,1,2,3,4$ . Notice that the conditions (96-1) – (96-5) are equivalent to the conditions (75).

## 3-2. A Direct Derivation of the Lorentz Transformations for "Discrete" Momentums

From the matrix equations (87) – (91) obtained by our axiomatic discrete approach (based on the Axiom 1-2, i.e. formula (23)), by linearization and simultaneous parameterization (as necessary algebraic conditions) of quadratic relation (85)) and the assumption of discreteness of the relativistic quantities $p_\mu$ and $p'_\mu$, and also the relations (90-2), (91-6), (95-2) and (96-6), and the parametric solutions corresponding to the matrix equations (87) – (91, i.e. solutions (64), (66), (67) and (74), the parametric linear transformations between two reference frames, that correspond to the Lorentz invariance, are derived directly. *In fact, the general parametric form of linear transformations (corresponding to the Lorentz transformations) between two reference frames are determined directly from the matrix equations (87) – (91) and their parametric solutions. Notably via this approach, the discreteness of the relativistic energy-momentum (*D*-momentum) merely implies the linearity of these obtained parametric transformations.*

For instance, we show that how these transformations for (1+2) dimensional space-time are derived here.[1] Hence from the matrix equation (89) and the integer-parametric solution (65) (for $m_4 = 1$, or equivalently for $s_4 = 1$ in (89)) and assuming the Minkowski metric, we get the following isomorphic linear transformations:

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1. Lorentz transformations for higher space-time dimensions are derived by the same method for discrete momentums, and definitely the derived transformations could be extended and applied for other physical quantities which supposedly would take discrete values, such as space-time coordinates.

$$\begin{bmatrix} \frac{-1-s_3^2-s_1^2-s_2^2}{s_2^2+s_1^2-1-s_3^2} & \frac{-2\,s_1-2\,s_2\,s_3}{s_2^2+s_1^2-1-s_3^2} & \frac{-2\,s_1\,s_3+2\,s_2}{s_2^2+s_1^2-1-s_3^2} \\ \frac{-2\,s_1+2\,s_2\,s_3}{s_2^2+s_1^2-1-s_3^2} & \frac{-s_1^2-1+s_2^2+s_3^2}{s_2^2+s_1^2-1-s_3^2} & \frac{2\,s_2\,s_1-2\,s_3}{s_2^2+s_1^2-1-s_3^2} \\ \frac{2\,s_2+2\,s_1\,s_3}{s_2^2+s_1^2-1-s_3^2} & \frac{2\,s_2\,s_1+2\,s_3}{s_2^2+s_1^2-1-s_3^2} & \frac{-s_2^2+s_3^2-1+s_1^2}{s_2^2+s_1^2-1-s_3^2} \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} p'_0 \\ p'_1 \\ p'_2 \end{bmatrix}, \qquad (89\text{-}1)$$

where $s_i$ are arbitrary integer parameters, and the inverse of (89-1) is given by

$$\begin{bmatrix} -\frac{1+s_3^2+s_1^2+s_2^2}{s_2^2+s_1^2-1-s_3^2} & \frac{2\,(s_1-s_2\,s_3)}{s_2^2+s_1^2-1-s_3^2} & -\frac{2\,(s_2+s_1\,s_3)}{s_2^2+s_1^2-1-s_3^2} \\ \frac{2\,(s_1+s_2\,s_3)}{s_2^2+s_1^2-1-s_3^2} & -\frac{s_1^2+1-s_2^2-s_3^2}{s_2^2+s_1^2-1-s_3^2} & \frac{2\,(s_2\,s_1+s_3)}{s_2^2+s_1^2-1-s_3^2} \\ \frac{2\,(s_1\,s_3-s_2)}{s_2^2+s_1^2-1-s_3^2} & \frac{2\,(s_2\,s_1-s_3)}{s_2^2+s_1^2-1-s_3^2} & \frac{-s_2^2+s_3^2-1+s_1^2}{s_2^2+s_1^2-1-s_3^2} \end{bmatrix} \begin{bmatrix} p'_0 \\ p'_1 \\ p'_2 \end{bmatrix} = \begin{bmatrix} p_0 \\ p_1 \\ p_2 \end{bmatrix} \qquad (89\text{-}2)$$

and for transformations (89-1) we also have:

$$det \begin{vmatrix} \frac{-1-s_3^2-s_1^2-s_2^2}{s_2^2+s_1^2-1-s_3^2} & \frac{-2\,s_1-2\,s_2\,s_3}{s_2^2+s_1^2-1-s_3^2} & \frac{-2\,s_1\,s_3+2\,s_2}{s_2^2+s_1^2-1-s_3^2} \\ \frac{-2\,s_1+2\,s_2\,s_3}{s_2^2+s_1^2-1-s_3^2} & \frac{-s_1^2-1+s_2^2+s_3^2}{s_2^2+s_1^2-1-s_3^2} & \frac{2\,s_2\,s_1-2\,s_3}{s_2^2+s_1^2-1-s_3^2} \\ \frac{2\,s_2+2\,s_1\,s_3}{s_2^2+s_1^2-1-s_3^2} & \frac{2\,s_2\,s_1+2\,s_3}{s_2^2+s_1^2-1-s_3^2} & \frac{-s_2^2+s_3^2-1+s_1^2}{s_2^2+s_1^2-1-s_3^2} \end{vmatrix} =1 \qquad (89\text{-}3)$$

The linear transformations (89-1) are equivalent to the Lorentz transformations, for certain values of parameters $s_i$ obtaining from the initial conditions and given values (such as the relative velocity between the frames). As an example, from (89-1) and the following values specified for parameters $s_i$, the Lorentz transformations for momentums in standard configuration [59] are directly obtained in the x-direction:

$s_2=s_3=0$ , $s_1=-\frac{\beta}{1+\gamma}, \gamma=\frac{1}{\sqrt{1-\beta^2}}, \beta=\frac{v}{c}$ :

$$\begin{bmatrix} \frac{-1-s_1^2}{s_1^2-1} & \frac{-2\,s_1}{s_1^2-1} \\ \frac{-2\,s_1}{s_1^2-1} & \frac{-s_1^2-1}{s_1^2-1} \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \end{bmatrix} = \begin{bmatrix} p'_0 \\ p'_1 \end{bmatrix} \qquad (89\text{-}4)$$

**3-3.** **I**n this Section the geometrized units [9], the Einstein notation, and the following **sign** conventions will be used (So, we would assume that the speed of light $c=1$; however, for clarity and emphasis, in some essential relativistic relations $"c"$ as a constant be restored and indicated formally):

- The Metric sign convention $(+--...-)$,

- The Riemann curvature and Ricci tensors:

$$R^{\rho}{}_{\sigma\mu\nu}=\partial_{\nu}\Gamma^{\rho}{}_{\mu\sigma}+\Gamma^{\rho}{}_{\nu\lambda}\Gamma^{\lambda}{}_{\mu\sigma}-\partial_{\mu}\Gamma^{\rho}{}_{\nu\sigma}-\Gamma^{\rho}{}_{\mu\lambda}\Gamma^{\lambda}{}_{\nu\sigma}\ ,R_{\sigma\mu}=-R^{\nu}{}_{\sigma\mu\nu},$$

- The Einstein tensor (sign): $G_{\mu\nu}=-8\pi\mathrm{T}_{\mu\nu}+...$ . (97)

**B**y "first" quantization of the linearized (and simultaneous parameterized, as necessary algebraic conditions) unique forms of the relativistic energy-momentum relation (86) (i.e. matrix relations (92) – (96) formulated for various space-time dimensions), we may directly derive and formulate a unique and original set of the general relativistic (single-particle) wave equations directly. As we show below, these equations correspond uniquely to certain massive forms of the laws governing the fundamental forces of nature, including the Gravitational, Electromagnetic and Nuclear field equations (which based on our axiomatic approach are solely formulable in (1+3) dimensional space-time), and also the (half-integer spin) single-particle wave equations (that could be formulated solely in (1+2) dimensional space-time). **I**n addition, when first quantization procedure (as a postulate) is applied to the linearized (and simultaneously parameterized) unique forms of the energy-momentum relation, i.e. formulas (92) – (96), as the principal substitution rule, we assume the following general covariant quantities and (quantum-mechanical) operator to be substituted by their corresponding quantities in algebraic matrix relations (92) – (96):

- General covariant kinetic energy-momentum operator:

$$\hat{p}_{\mu}=i\hbar\breve{\nabla}_{\mu} \tag{98}$$

- The metric tensor:

$$\hat{g}^{\mu\nu}=g^{\mu\nu} \tag{99}$$

- There is a one-to-one correspondence between "the parameters $s_i$" and "the components of field strength tensor $X_{\mu\nu\rho..\zeta}$ and the components of covariant quantities $\varphi^{(X)}_{\rho..\zeta}$ (which by formula (101-3) defines a covariant current as the source of field $X_{\mu\nu\rho..\zeta}$)". We may show this correspondence by the following formula:

$$\hat{s}_i=X_{\mu\nu\rho..\zeta}\ ,\ \varphi^{(X)}_{\nu\rho..\zeta} \tag{100}$$

where $\hat{s}_i$ are formally the quantities converted by first quantization of arbitrary parameters $s_i$. Thus the quantities (98) – (100) will be substituted by their corresponding quantities in relations (92) – (96), that respectively, are $p_{\mu}$ (general relativistic kinetic energy-momentum vector), $g^{\mu\nu}$ (as constant values in these algebraic relations) and $s_i$ (arbitrary algebraic parametric quantities). Hence, diffeomorphism invariance and substitutions (98) – (100) corresponding to the quantities in the relativistic energy-momentum (matrix) relations (92) – (96), directly yield a unique general relativistic (single-particle) wave equation, which will be written in Section 3-4 explicitly. In addition, in Appendix A we show that the field strength tensor $X_{\mu\nu\rho..\zeta}$ (as a general tensor form) could solely two (separate) values $F_{\mu\nu}$ , $R_{\mu\nu\rho\sigma}$. In the same manner, $\varphi^{(F)}$, $\varphi^{(R)}_{\mu\nu}$ (corresponding to these two particular fields) are also covariant quantities which will appear in the derived equations and their derivatives be equal to the sources of fields $F_{\mu\nu}$ , $R_{\mu\nu\rho\sigma}$ (see below).

**F**urthermore, based on the assumption (3) in Section 1-1, and taking into account a notification presented in Appendix A, we show that there are only two kinds of definable and acceptable field strength tensors whose components could be substituted by the parametric quantities $s_i$, and they convert (by first quantization procedure, i.e. by applying the basic formal substitutions (98) – (100)) the matrix energy-momentum relations (92) – (96) into a unique set of the general relativistic wave equations. These are a 2nd and a 4th rank anti-symmetric field strength tensors; where the 4th rank tensor is (supposedly) equal to the Riemann curvature tensor $R_{\mu\nu\rho\sigma}$, and another tensor be represented by $F_{\mu\nu}$. Assuming the local gauge symmetry, we show that this 2nd rank anti-symmetric tensor corresponds to certain massive Electromagnetic and Yang-Mills (single-particle) field strength tensors.[1]

**3-4. O**n this basis, the tensor representation of the general relativistic single-particle wave equations that are uniquely obtained and formulated by the (direct) logical derivation procedure described in Section 3-3, is as follows (see Section 3-6, formulas (120) – (121)) for the original matrix representation, i.e. the geometric algebra formulation of these equations):

$$\breve{\nabla}_{\lambda} X_{\mu\nu\rho..\zeta} + \breve{\nabla}_{\mu} X_{\nu\lambda\rho..\zeta} + \breve{\nabla}_{\nu} X_{\lambda\mu\rho..\zeta} = M^{\tau}{}_{\lambda\mu} X_{\tau\nu\rho..\zeta} + M^{\tau}{}_{\mu\nu} X_{\tau\lambda\rho..\zeta} + M^{\tau}{}_{\nu\lambda} X_{\tau\mu\rho..\zeta} \quad , \qquad (101\text{-}1)$$

$$\breve{\nabla}_{\mu} X^{\mu}{}_{\nu\rho..\zeta} - \frac{im_0^{(X)}}{\hbar} k_{\mu} X^{\mu}{}_{\nu\rho..\zeta} = -J^{(X)}_{\nu\rho..\zeta} \ . \qquad (101\text{-}2)$$

where
$$J^{(X)}_{\nu\rho..\zeta} = -(\breve{\nabla}_{\nu} + \frac{im_0^{(X)}}{\hbar} k_{\nu}) \varphi^{(X)}_{\rho..\zeta} \ , \qquad (101\text{-}3)$$

$$M_{\tau\mu\nu} = \frac{im_0^{(X)}}{2\hbar} (g_{\tau\mu} k_{\nu} - g_{\tau\nu} k_{\mu}) \ , \quad M_{\nu} = M^{\mu}{}_{\mu\nu} = \frac{(D-1) im_0^{(X)}}{2\hbar} k_{\nu} \ . \qquad (101\text{-}4)$$

and where (according to our derivation approach) these field equations will be formulated solely in one in D ≤ 4 dimensional space-time. Furthermore, in the above field equations $X_{\mu\nu\rho,\zeta}$ is the field strength tensor, $m_0^{(X)}$ is the invariant mass of the (free and interacting) field carrier single-particle, $i\hbar\breve{\nabla}_{\mu}$ is the covariant kinetic energy-momentum operator (generally defined on a background space-time with the complex torsion $T_{\tau\mu\nu}$ generated by the invariant mass and given by formula (110), and $k^{\mu} = (c/\sqrt{g_{00}}, 0, ..., 0)$ is the general relativistic velocity of a static observer (that is a time-like contravariant vector). $J^{(X)}_{\nu\rho..\zeta}$ is the source of the field strength tensor $X_{\mu\nu\rho..\zeta}$. Moreover, as it was notes in Section 3-3, field strength tensor $X_{\mu\nu\rho..\zeta}$ could take solely two separate tensor values, $X_{\mu\nu\rho..\zeta} = F_{\mu\nu}$ , $R_{\mu\nu\rho\sigma}$ , where $R_{\mu\nu\rho\sigma}$be the Riemann curvature tensor, and $F_{\mu\nu}$ be a rank two anti-symmetric field strength tensor corresponding to the known single-particle fields (including the Maxwell and Yang-Mills fields for (1+3) dimensional space-time, and Dirac fields for (1+2) dimensional space-time). Moreover, in Appendix A we show that in (1+3) and higher space-time dimensions, the field strength tensor $X_{\mu\nu\rho..\zeta}$ should be presentable by a formula of the type: $X_{\mu\nu\rho..\zeta} = \hat{A}_{\nu} \hat{B}_{\mu\rho..\zeta} - \hat{A}_{\mu} \hat{B}_{\nu\rho..\zeta}$ , for some quantities $\hat{A}_{\mu}$ , $\hat{B}_{\nu\rho..\zeta}$ , where this formula should be also “derivable” from the general relativistic single-particle field equations (101-1) – (101-4).

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1. We will show in Section 3-7, that the field equations (101-1) – (101-4) in five dimensional space-time, which correspond to the matrix relations (96), and also for the higher space-time dimensions, are incompatible with some certain supposed essential symmetry – where we'll conclude that the universe cannot have more than four space-time dimensions.

Hence in various space-time dimensions, the field equations (101-1) – (101-2) (for two separate field strength tensors $F_{\mu\nu}$ , $R_{\mu\nu\rho\sigma}$ , and taking into account a necessary condition mentioned above that be presented in Appendix A) be formulated as follows, respectively:

Relation (92) $\xrightarrow{\text{first quantization}}$ $$\breve{\nabla}_{\mu}\widehat{F}^{\mu} - (im_0^{(E)}/\hbar)k_{\mu}\widehat{F}^{\mu} = 0 \qquad (102)$$

where $g^{00} = 1, \quad \mu = 0$ , and $s_1 \mapsto \hat{s}_1 = \widehat{F}_0$ .[1]

Relation (93) $\xrightarrow{\text{first quantization}}$

$$\breve{\nabla}_{\lambda}F_{\mu\nu} + \breve{\nabla}_{\mu}F_{\nu\lambda} + \breve{\nabla}_{\nu}F_{\lambda\mu} = Z^{\tau}{}_{\lambda\mu}F_{\tau\nu} + Z^{\tau}{}_{\mu\nu}F_{\tau\lambda} + Z^{\tau}{}_{\nu\lambda}F_{\tau\mu} \ , \qquad (103\text{-}1)$$

$$\breve{\nabla}_{\mu}F^{\mu}{}_{\nu} - \frac{im_0^{(E)}}{\hbar}k_{\mu}F^{\mu}{}_{\nu} = -J^{(E)}_{\nu} \qquad (103\text{-}2)$$

where $\lambda, \rho, \sigma, \mu, \nu, \tau = 0, 1$ , and

$$s_1 \mapsto \hat{s}_1 = F_{10}, \quad s_2 \mapsto \hat{s}_2 = \varphi^{(E)}, \quad J^{(E)}_{\nu} = -(\breve{\nabla}_{\nu} + \frac{im_0^{(E)}}{\hbar}k_{\nu})\varphi^{(E)}, \quad Z_{\tau\mu\nu} = \frac{im_0^{(E)}}{2\hbar}(g_{\tau\mu}k_{\nu} - g_{\tau\nu}k_{\mu}).$$

Relation (93) $\xrightarrow{\text{first quantization}}$

$$\breve{\nabla}_{\lambda}R_{\mu\nu\rho\sigma} + \breve{\nabla}_{\mu}R_{\nu\lambda\rho\sigma} + \breve{\nabla}_{\nu}R_{\lambda\mu\rho\sigma} = T^{\tau}{}_{\lambda\mu}R_{\tau\nu\rho\sigma} + T^{\tau}{}_{\mu\nu}R_{\tau\lambda\rho\sigma} + T^{\tau}{}_{\nu\lambda}R_{\tau\mu\rho\sigma} \ , \qquad (103\text{-}3)$$

$$\breve{\nabla}_{\mu}R^{\mu}{}_{\nu\rho\sigma} - \frac{im_0^{(G)}}{\hbar}k_{\mu}R^{\mu}{}_{\nu\rho\sigma} = -J^{(G)}_{\nu\rho\sigma} \qquad (103\text{-}4)$$

where $\lambda, \rho, \sigma, \mu, \nu, \tau = 0, 1$ , and

$$s_1 \mapsto \hat{s}_1 = R_{10\rho\sigma}, \quad s_2 \mapsto \hat{s}_2 = \varphi^{(G)}_{\rho\sigma}, \quad J^{(G)}_{\nu\rho\sigma} = -(\breve{\nabla}_{\nu} + \frac{im_0^{(G)}}{\hbar}k_{\nu})\varphi^{(G)}_{\rho\sigma}, \quad T_{\tau\mu\nu} = \frac{im_0^{(G)}}{2\hbar}(g_{\tau\mu}k_{\nu} - g_{\tau\nu}k_{\mu}).$$

Relation (94) $\xrightarrow{\text{first quantization}}$

$$\breve{\nabla}_{\lambda}F_{\mu\nu} + \breve{\nabla}_{\mu}F_{\nu\lambda} + \breve{\nabla}_{\nu}F_{\lambda\mu} = Z^{\tau}{}_{\lambda\mu}F_{\tau\nu} + Z^{\tau}{}_{\mu\nu}F_{\tau\lambda} + Z^{\tau}{}_{\nu\lambda}F_{\tau\mu} \ , \qquad (104\text{-}1)$$

$$\breve{\nabla}_{\mu}F^{\mu}{}_{\nu} - \frac{im_0^{(E)}}{\hbar}k_{\mu}F^{\mu}{}_{\nu} = -J^{(E)}_{\nu} \qquad (104\text{-}2)$$

where $\lambda, \rho, \sigma, \mu, \nu, \tau = 0, 1, 2$ , and

$$s_1 \mapsto \hat{s}_1 = F_{10}, \quad s_2 \mapsto \hat{s}_2 = F_{02}, \quad s_3 \mapsto \hat{s}_3 = F_{21}, \quad s_4 \mapsto \hat{s}_4 = \varphi^{(E)}, \quad J^{(E)}_{\nu} = -(\breve{\nabla}_{\nu} + \frac{im_0^{(E)}}{\hbar}k_{\nu})\varphi^{(E)},$$

$$Z_{\tau\mu\nu} = (im_0^{(E)}/2\hbar)(g_{\tau\mu}k_{\nu} - g_{\tau\nu}k_{\mu}).$$

---

1. Exceptionally, the tensor equation corresponding to relation (92) (for one dimensional space-time, i.e. where only the time dimension exists) is a special and trivial case, where the Riemann tensor vanishes; so for this case, formally, we just assume a tensor such as $\widehat{F}_{\mu}$ that is substituted by the only parameter $s_1$ in (92)).

Relation (94) $\xrightarrow{\text{first quantization}}$

$$\breve{\nabla}_{\lambda} R_{\mu\nu\rho\sigma} + \breve{\nabla}_{\mu} R_{\nu\lambda\rho\sigma} + \breve{\nabla}_{\nu} R_{\lambda\mu\rho\sigma} = T^{\tau}{}_{\lambda\mu} R_{\tau\nu\rho\sigma} + T^{\tau}{}_{\mu\nu} R_{\tau\lambda\rho\sigma} + T^{\tau}{}_{\nu\lambda} R_{\tau\mu\rho\sigma} \ , \qquad (104\text{-}3)$$

$$\breve{\nabla}_{\mu} R^{\mu}{}_{\nu\rho\sigma} - \frac{im_0^{(G)}}{\hbar} k_{\mu} R^{\mu}{}_{\nu\rho\sigma} = -J^{(G)}_{\nu\rho\sigma} \qquad (104\text{-}4)$$

where $\lambda, \rho, \sigma, \mu, \nu, \tau = 0, 1, 2$ , and

$$s_1 \mapsto \hat{s}_1 = R_{10\rho\sigma}\,, \ \ s_2 \mapsto \hat{s}_2 = R_{02\rho\sigma}\,, \ \ s_3 \mapsto \hat{s}_3 = R_{21\rho\sigma}\,, \ \ s_4 \mapsto \hat{s}_4 = \varphi^{(G)}_{\rho\sigma}\,, \ \ J^{(G)}_{\nu\rho\sigma} = -(\breve{\nabla}_{\nu} + \frac{im_0^{(G)}}{\hbar} k_{\nu})\varphi^{(G)}_{\rho\sigma} \ ,$$

$$T_{\tau\mu\nu} = \frac{im_0^{(G)}}{2\hbar}(g_{\tau\mu}k_{\nu} - g_{\tau\nu}k_{\mu}).$$

Relation (95) $\xrightarrow{\text{first quantization}}$

$$\breve{\nabla}_{\lambda} F_{\mu\nu} + \breve{\nabla}_{\mu} F_{\nu\lambda} + \breve{\nabla}_{\nu} F_{\lambda\mu} = Z^{\tau}{}_{\lambda\mu} F_{\tau\nu} + Z^{\tau}{}_{\mu\nu} F_{\tau\lambda} + Z^{\tau}{}_{\nu\lambda} F_{\tau\mu} \ , \qquad (105\text{-}1)$$

$$\breve{\nabla}_{\mu} F^{\mu}{}_{\nu} - \frac{im_0^{(E)}}{\hbar} k_{\mu} F^{\mu}{}_{\nu} = -J^{(E)}_{\nu} \qquad (105\text{-}2)$$

where $\lambda, \rho, \sigma, \mu, \nu, \tau = 0, 1, 2, 3$ , and

$$s_1 \mapsto \hat{s}_1 = F_{10} = \hat{A}_0\hat{B}_1 - \hat{A}_1\hat{B}_0, \ \ s_2 \mapsto \hat{s}_2 = F_{20} = \hat{A}_0\hat{B}_2 - \hat{A}_2\hat{B}_0,$$

$$s_3 \mapsto \hat{s}_3 = F_{30} = \hat{A}_0\hat{B}_3 - \hat{A}_3\hat{B}_0, \ \ s_4 \mapsto \hat{s}_4 = 0, \ \ s_5 \mapsto \hat{s}_5 = F_{23} = \hat{A}_3\hat{B}_2 - \hat{A}_2\hat{B}_3,$$

$$s_6 \mapsto \hat{s}_6 = F_{31} = \hat{A}_1\hat{B}_3 - \hat{A}_3\hat{B}_1, \ \ s_7 \mapsto \hat{s}_7 = F_{12} = \hat{A}_2\hat{B}_1 - \hat{A}_1\hat{B}_2, \ \ s_8 \mapsto \hat{s}_8 = \varphi^{(E)} \ ,$$

$$J^{(E)}_{\nu} = -(\breve{\nabla}_{\nu} + \frac{im_0^{(E)}}{\hbar} k_{\nu})\varphi^{(E)} \ , \ \ Z_{\tau\mu\nu} = \frac{im_0^{(E)}}{2\hbar}(g_{\tau\mu}k_{\nu} - g_{\tau\nu}k_{\mu}) \ ;$$

$$s_1 = A_0B_1 - A_1B_0, \ \ s_2 = A_0B_2 - A_2B_0, \ \ s_3 = A_0B_3 - A_3B_0, \ \ s_4 = 0,$$

$$s_5 = A_3B_2 - A_2B_3, \ \ s_6 = A_1B_3 - A_3B_1, \ \ s_7 = A_2B_1 - A_1B_2,$$

$s_8$ : *an arbitrary parameter* ;

*where*

$$A_0 = v_4, \ \ B_0 = u_4, \ \ A_1 = v_3, \ \ B_1 = u_3,$$

$$A_2 = v_2, \ \ B_2 = u_2, \ \ A_3 = v_1, \ \ B_3 = u_1.$$

Relation (95) $\xrightarrow{first\ quantization}$

$$\breve{\nabla}_{\lambda} R_{\mu\nu\rho\sigma} + \breve{\nabla}_{\mu} R_{\nu\lambda\rho\sigma} + \breve{\nabla}_{\nu} R_{\lambda\mu\rho\sigma} = T^{\tau}{}_{\lambda\mu} R_{\tau\nu\rho\sigma} + T^{\tau}{}_{\mu\nu} R_{\tau\lambda\rho\sigma} + T^{\tau}{}_{\nu\lambda} R_{\tau\mu\rho\sigma} \ , \quad (105\text{-}3)$$

$$\breve{\nabla}_{\mu} R^{\mu}{}_{\nu\rho\sigma} - \frac{im_0^{(G)}}{\hbar} k_{\mu} R^{\mu}{}_{\nu\rho\sigma} = -J^{(G)}_{\nu\rho\sigma} \qquad (105\text{-}4)$$

where $\lambda, \rho, \sigma, \mu, \nu, \tau = 0, 1, 2, 3$ , and

$$s_1 \mapsto \hat{s}_1 = R_{10\rho\sigma} = \hat{A}_0 \hat{B}_{\rho\sigma 1} - \hat{A}_1 \hat{B}_{\rho\sigma 0}, \quad s_2 \mapsto \hat{s}_2 = R_{20\rho\sigma} = \hat{A}_0 \hat{B}_{\rho\sigma 2} - \hat{A}_2 \hat{B}_{\rho\sigma 0},$$

$$s_3 \mapsto \hat{s}_3 = R_{30\rho\sigma} = \hat{A}_0 \hat{B}_{\rho\sigma 3} - \hat{A}_3 \hat{B}_{\rho\sigma 0}, \quad s_4 \mapsto \hat{s}_4 = 0, \quad s_5 \mapsto \hat{s}_5 = R_{23\rho\sigma} = \hat{A}_3 \hat{B}_{\rho\sigma 2} - \hat{A}_2 \hat{B}_{\rho\sigma 3},$$

$$s_6 \mapsto \hat{s}_6 = R_{31\rho\sigma} = \hat{A}_1 \hat{B}_{\rho\sigma 3} - \hat{A}_3 \hat{B}_{\rho\sigma 1}, \quad s_7 \mapsto \hat{s}_7 = R_{12\rho\sigma} = \hat{A}_2 \hat{B}_{\rho\sigma 1} - \hat{A}_1 \hat{B}_{\rho\sigma 2}, \quad s_8 \mapsto \hat{s}_8 = \varphi^{(N)}_{\rho\sigma} \quad ,$$

$$J^{(G)}_{\nu\rho\sigma} = -(\breve{\nabla}_{\nu} + \frac{im_0^{(G)}}{\hbar} k_{\nu}) \varphi^{(G)}_{\rho\sigma} \ , \ T_{\tau\mu\nu} = \frac{im_0^{(G)}}{2\hbar} (g_{\tau\mu} k_{\nu} - g_{\tau\nu} k_{\mu}) \ ;$$

$$s_1 = A_0 B_{\rho\sigma 1} - A_1 B_{\rho\sigma 0}, \quad s_2 = A_0 B_{\rho\sigma 2} - A_2 B_{\rho\sigma 0}, \quad s_3 = A_0 B_{\rho\sigma 3} - A_3 B_{\rho\sigma 0}, \quad s_4 = 0,$$

$$s_5 = A_3 B_{\rho\sigma 2} - A_2 B_{\rho\sigma 3}, \quad s_6 = A_1 B_{\rho\sigma 3} - A_3 B_{\rho\sigma 1}, \quad s_7 = A_2 B_{\rho\sigma 1} - A_1 B_{\rho\sigma 2},$$

$s_8$ : *an arbitrary parameter* ;

*where*

$$A_0 = v_4, \quad B_{\rho\sigma 0} = u_4, \quad A_1 = v_3, \quad B_{\rho\sigma 1} = u_3,$$

$$A_2 = v_2, \quad B_{\rho\sigma 2} = u_2, \quad A_3 = v_1, \quad B_{\rho\sigma 3} = u_1.$$

Relation (96) $\xrightarrow{\text{first quantization}}$

$$\breve{\nabla}_{\lambda} F_{\mu\nu} + \breve{\nabla}_{\mu} F_{\nu\lambda} + \breve{\nabla}_{\nu} F_{\lambda\mu} = Z^{\tau}{}_{\lambda\mu} F_{\tau\nu} + Z^{\tau}{}_{\mu\nu} F_{\tau\lambda} + Z^{\tau}{}_{\nu\lambda} F_{\tau\mu} \ , \tag{106-1}$$

$$\breve{\nabla}_{\mu} F^{\mu}{}_{\nu} - \frac{i m_0^{(E)}}{\hbar} k_{\mu} F^{\mu}{}_{\nu} = -J^{(E)}_{\nu} \tag{106-2}$$

where $\lambda, \rho, \sigma, \mu, \nu, \tau = 0, 1, 2, 3, 4$ , and

$$s_1 \mapsto \hat{s}_1 = F_{10} = \hat{A}_0 \hat{B}_1 - \hat{A}_1 \hat{B}_0, \ \ s_2 \mapsto \hat{s}_2 = F_{02} = \hat{A}_2 \hat{B}_0 - \hat{A}_0 \hat{B}_2,$$

$$s_3 \mapsto \hat{s}_3 = F_{30\rho} = \hat{A}_0 \hat{B}_3 - \hat{A}_3 \hat{B}_0, \ \ s_4 \mapsto \hat{s}_4 = 0, \ \ s_5 \mapsto \hat{s}_5 = F_{04} = \hat{A}_4 \hat{B}_0 - \hat{A}_0 \hat{B}_4,$$

$$s_6 \mapsto \hat{s}_6 = 0, \ \ s_7 \mapsto \hat{s}_7 = 0, \ \ s_8 \mapsto \hat{s}_8 = 0 \ \ s_9 \mapsto \hat{s}_9 = 0, \ \ s_{10} \mapsto \hat{s}_{10} = F_{43} = \hat{A}_3 \hat{B}_4 - \hat{A}_4 \hat{B}_3,$$

$$s_{11} \mapsto \hat{s}_{11} = F_{42} = \hat{A}_2 \hat{B}_4 - \hat{A}_4 \hat{B}_2, \ \ s_{12} \mapsto \hat{s}_{12} = F_{41} = \hat{A}_1 \hat{B}_4 - \hat{A}_4 \hat{B}_1,$$

$$s_{13} \mapsto \hat{s}_{13} = F_{32} = \hat{A}_2 \hat{B}_3 - \hat{A}_3 \hat{B}_2, \ \ s_{14} \mapsto \hat{s}_{14} = F_{31} = \hat{A}_1 \hat{B}_3 - \hat{A}_3 \hat{B}_1,$$

$$s_{15} \mapsto \hat{s}_{15} = F_{21} = \hat{A}_1 \hat{B}_2 - \hat{A}_2 \hat{B}_1, \ \ s_{16} \mapsto \hat{s}_{16} = \varphi^{(N)},$$

$$J^{(E)}_{\nu} = -(\breve{\nabla}_{\nu} + \frac{i m_0^{(E)}}{\hbar} k_{\nu}) \varphi^{(E)} \ , \ Z_{\tau\mu\nu} = \frac{i m_0^{(E)}}{2\hbar} (g_{\tau\mu} k_{\nu} - g_{\tau\nu} k_{\mu}) \ ;$$

$$s_1 = A_0 B_1 - A_1 B_0, \ \ s_2 = A_2 B_0 - A_0 B_2, \ \ s_3 = A_0 B_3 - A_3 B_0,$$

$$s_4 = 0, \ \ s_5 = A_4 B_0 - A_0 B_4, \ \ s_6 = 0, \ \ s_7 = 0, \ \ s_8 = 0, \ \ s_9 = 0,$$

$$s_{10} = A_3 B_4 - A_4 B_3, \ \ s_{11} = A_2 B_4 - A_4 B_2, \ \ s_{12} = A_1 B_4 - A_4 B_1,$$

$$s_{13} = A_2 B_3 - A_3 B_2, \ \ s_{14} = A_1 B_3 - A_3 B_1, \ \ s_{15} = A_1 B_2 - A_2 B_1,$$

$s_{16}$ : *an arbitrary parameter* ;

*where*

$A_0 = v_5, \ B_0 = u_5, \ A_1 = v_4, \ B_1 = u_4, \ A_2 = -v_3,$
$B_2 = -u_3, \ A_3 = -v_2, \ B_3 = -u_2, \ A_4 = v_1, \ B_4 = u_1.$

Relation (96) $\xrightarrow{\text{first quantization}}$

$$\breve{\nabla}_{\lambda} R_{\mu\nu\rho\sigma} + \breve{\nabla}_{\mu} R_{\nu\lambda\rho\sigma} + \breve{\nabla}_{\nu} R_{\lambda\mu\rho\sigma} = T^{\tau}{}_{\lambda\mu} R_{\tau\nu\rho\sigma} + T^{\tau}{}_{\mu\nu} R_{\tau\lambda\rho\sigma} + T^{\tau}{}_{\nu\lambda} R_{\tau\mu\rho\sigma} \ , \qquad (106\text{-}3)$$

$$\breve{\nabla}_{\mu} R^{\mu}{}_{\nu\rho\sigma} - \frac{im_0^{(G)}}{\hbar} k_{\mu} R^{\mu}{}_{\nu\rho\sigma} = -J^{(G)}_{\nu\rho\sigma} \qquad (106\text{-}4)$$

where $\lambda, \rho, \sigma, \mu, \nu, \tau = 0, 1, 2, 3, 4$ , and

$$s_1 \mapsto \hat{s}_1 = R_{10\rho\sigma} = \hat{A}_0 \hat{B}_{\rho\sigma 1} - \hat{A}_1 \hat{B}_{\rho\sigma 0}, \quad s_2 \mapsto \hat{s}_2 = R_{02\rho\sigma} = \hat{A}_2 \hat{B}_{\rho\sigma 0} - \hat{A}_0 \hat{B}_{\rho\sigma 2},$$

$$s_3 \mapsto \hat{s}_3 = R_{30\rho\sigma} = \hat{A}_0 \hat{B}_{\rho\sigma 3} - \hat{A}_3 \hat{B}_{\rho\sigma 0}, \quad s_4 \mapsto \hat{s}_4 = 0, \quad s_5 \mapsto \hat{s}_5 = R_{04\rho\sigma} = \hat{A}_4 \hat{B}_{\rho\sigma 0} - \hat{A}_0 \hat{B}_{\rho\sigma 4},$$

$$s_6 \mapsto \hat{s}_6 = 0, \quad s_7 \mapsto \hat{s}_7 = 0, \quad s_8 \mapsto \hat{s}_8 = 0 \quad s_9 \mapsto \hat{s}_9 = 0, \quad s_{10} \mapsto \hat{s}_{10} = R_{43\rho\sigma} = \hat{A}_3 \hat{B}_{\rho\sigma 4} - \hat{A}_4 \hat{B}_{\rho\sigma 3},$$

$$s_{11} \mapsto \hat{s}_{11} = R_{42\rho\sigma} = \hat{A}_2 \hat{B}_{\rho\sigma 4} - \hat{A}_4 \hat{B}_{\rho\sigma 2}, \quad s_{12} \mapsto \hat{s}_{12} = R_{41\rho\sigma} = \hat{A}_1 \hat{B}_{\rho\sigma 4} - \hat{A}_4 \hat{B}_{\rho\sigma 1},$$

$$s_{13} \mapsto \hat{s}_{13} = R_{32\rho\sigma} = \hat{A}_2 \hat{B}_{\rho\sigma 3} - \hat{A}_3 \hat{B}_{\rho\sigma 2}, \quad s_{14} \mapsto \hat{s}_{14} = R_{31\rho\sigma} = \hat{A}_1 \hat{B}_{\rho\sigma 3} - \hat{A}_3 \hat{B}_{\rho\sigma 1},$$

$$s_{15} \mapsto \hat{s}_{15} = R_{21\rho\sigma} = \hat{A}_1 \hat{B}_{\rho\sigma 2} - \hat{A}_2 \hat{B}_{\rho\sigma 1}, \quad s_{16} \mapsto \hat{s}_{16} = \varphi^{(G)}_{\rho\sigma},$$

$$J^{(G)}_{\nu\rho\sigma} = -(\breve{\nabla}_{\nu} + \frac{im_0^{(G)}}{\hbar} k_{\nu}) \varphi^{(G)}_{\rho\sigma} \ , \ T_{\tau\mu\nu} = \frac{im_0^{(G)}}{2\hbar} (g_{\tau\mu} k_{\nu} - g_{\tau\nu} k_{\mu}) \ ;$$

$$s_1 = A_0 B_{\rho\sigma 1} - A_1 B_{\rho\sigma 0}, \quad s_2 = A_2 B_{\rho\sigma 0} - A_0 B_{\rho\sigma 2}, \quad s_3 = A_0 B_{\rho\sigma 3} - A_3 B_{\rho\sigma 0},$$

$$s_4 = 0, \quad s_5 = A_4 B_{\rho\sigma 0} - A_0 B_{\rho\sigma 4}, \quad s_6 = 0, \quad s_7 = 0, \quad s_8 = 0, \quad s_9 = 0,$$

$$s_{10} = A_3 B_{\rho\sigma 4} - A_4 B_{\rho\sigma 3}, \quad s_{11} = A_2 B_{\rho\sigma 4} - A_4 B_{\rho\sigma 2}, \quad s_{12} = A_1 B_{\rho\sigma 4} - A_4 B_{\rho\sigma 1},$$
$$s_{13} = A_2 B_{\rho\sigma 3} - A_3 B_{\rho\sigma 2}, \quad s_{14} = A_1 B_{\rho\sigma 3} - A_3 B_{\rho\sigma 1}, \quad s_{15} = A_1 B_{\rho\sigma 2} - A_2 B_{\rho\sigma 1},$$

$s_{16}$ : *an arbitrary parameter* ;

*where*

$$A_0 = v_5, \quad B_{\rho\sigma 0} = u_5, \quad A_1 = v_4, \quad B_{\rho\sigma 1} = u_4, \quad A_2 = -v_3,$$
$$B_{\rho\sigma 2} = -u_3, \quad A_3 = -v_2, \quad B_{\rho\sigma 3} = -u_2, \quad A_4 = v_1, \quad B_{\rho\sigma 4} = u_1.$$

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1. We will show in Section 3-7, that the field equations (106-1) – (106-4), which correspond to the matrix relations (96) (i.e. for five dimensional space-time), as well as for higher space-time dimensions, are incompatible with some certain symmetry, where we'll also conclude that the universe cannot have more than four space-time dimensions.

Where in field equations (103-1) – (106-4) we have (see also Appendix A):

$$R_{\mu\nu\rho\sigma} = (\partial_\nu \Gamma_{\rho\sigma\mu} - \Gamma^\lambda{}_{\rho\nu}\Gamma_{\lambda\sigma\mu}) - (\partial_\mu \Gamma_{\rho\sigma\nu} - \Gamma^\lambda{}_{\rho\mu}\Gamma_{\lambda\sigma\nu}) \ , \qquad (107\text{-}1)$$

$$F_{\mu\nu} = (\breve{\nabla}_\nu + \frac{im_0^{(E)}}{2\hbar}k_\nu)A_\mu - (\breve{\nabla}_\mu + \frac{im_0^{(E)}}{2\hbar}k_\mu)A_\nu \qquad (107\text{-}2)$$

$\Gamma^\rho{}_{\sigma\mu}$ is the affine connection: $\Gamma^\rho{}_{\sigma\mu} = \overline{\Gamma}^\rho{}_{\sigma\mu} - K^\rho{}_{\sigma\mu}$, $\overline{\Gamma}^\rho{}_{\sigma\mu}$ is the Christoffel symbol (or the torsion-free connection), $K^\rho{}_{\sigma\mu}$ is a definite complex contorsion tensor generated by the invariant mass (of the field carrier particle), given by: $K^\rho{}_{\sigma\mu} = \frac{im_0^{(G)}}{2\hbar} g^\rho{}_\mu k_\sigma$ (which is anti-symmetric in the first and last indices), $A_\mu$ is the vector potential, and $T_{\tau\mu\nu}$ is the torsion tensor defined by

$$T_{\tau\mu\nu} = -K_{\tau\mu\nu} + K_{\tau\nu\mu} = \frac{im_0^{(G)}}{2\hbar}(g_{\tau\mu}k_\nu - g_{\tau\nu}k_\mu),\ T_\nu = T^\mu{}_{\mu\nu} = \frac{(D-1)im_0^{(G)}}{2\hbar}k_\nu \qquad (107\text{-}3)$$

D is the number of space-time dimensions;[1]

Tensor $Z_{\tau\mu\nu}$ is also given by: $$Z_{\tau\mu\nu} = \frac{im_0^{(E)}}{2\hbar}(g_{\tau\mu}k_\nu - g_{\tau\nu}k_\mu) \qquad (107\text{-}4)$$

Moreover, according to our derivation approach (see Section 3-7), the above field equations are solely definable in $D \le 4$ dimensional space-time. So, only the equations (104-1) – (105-4) would be acceptable. In addition, in the above field equations, $m_0^{(G)}$ and $m_0^{(E)}$ are the invariant mass of the (free and interacting) fields carrier single-particles, $i\hbar\breve{\nabla}_\mu$ is the covariant kinetic energy-momentum operator (generally defined on a background space-time with the complex torsion $T_{\tau\mu\nu}$ generated by the invariant mass and given by formula (3)), $k^\mu = (c/\sqrt{g_{00}}, 0, ..., 0)$ is the general relativistic velocity of a static observer (that is a time-like contravariant vector), and the covariant currents $J^{(G)}_{\nu\rho\sigma}$ and $J^{(E)}_\nu$ are the sources of the above fields. In particular, for massless cases (i.e. $m_0^{(G)} = 0$, $m_0^{(E)} = 0$), the field equations (105-1) – (105-4) turn into the Maxwell, Yang-Mills (if assuming the local gauge invariance, see below) and the gravitational single-particle fields [68, 69] (in fact, in the context of relativistic quantum mechanics, these relativistic fields could precisely describe single-particles, where the solutions are taken to be complex).

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1. In the general relativistic wave equations (102) – (106-4), quantities $\hat{B}_\mu$, $\hat{B}_{\rho\mu\nu}$, and $\hat{A}_\mu$ (as a differential operator) are defined as (See Appendix A):

$$\hat{A}_\mu Y_{\alpha_1\alpha_2...\alpha_n} = \partial_\mu Y_{\alpha_1\alpha_2...\alpha_n} - \Gamma^\lambda_{\alpha_1\mu} Y_{\lambda\alpha_2...\alpha_n} \ , \ \hat{A}_\mu Y^{\alpha_1}{}_{\alpha_2\alpha_3...\alpha_n} = \partial_\mu Y^{\alpha_1}{}_{\alpha_2\alpha_3...\alpha_n} + \Gamma^{\alpha_1}_{\mu\lambda} Y^\lambda{}_{\alpha_2\alpha_3...\alpha_n} \ ,$$

$\hat{B}_\mu = A_\mu$, $\hat{B}_{\rho\mu\nu} = \Gamma_{\rho\mu\nu}$. The conservation laws would be $(\breve{\nabla}_\nu - \frac{im_0^{(E)}}{2\hbar}k_\nu)J^{(E)\nu} = 0$, $(\breve{\nabla}_\nu - \frac{im_0^{(G)}}{\hbar}k_\nu)J^{(G)\nu}{}_{\rho\sigma} = 0.$

Moreover, the commutator of covariant derivative with torsion is [25, 27]: $(\nabla_\rho\nabla_\sigma - \nabla_\sigma\nabla_\rho)V_\nu = V_\mu R_{\rho\sigma}{}^\mu{}_\nu - T^\mu{}_{\rho\sigma}\nabla_\mu V_\nu.$

**I**n Section 3-5 we show that the Einstein field equations (that necessarily include a cosmological constant) are derived directly from field equations (103-3) – (103-4), (104-3) – (104-4) and (105-3) – (105-4) and so on, for various space-time dimensions.

**A**s an additional principal requirement, we show below (in Section 3-4-1) that we may also assume that the "massive" Lagrangian density for the obtained general relativistic single-particle wave equations (2-1) – (2-3), be locally gauge invariant as well – where these fields be still massive, and $\breve{\nabla}_\mu$ be equivalent to the general relativistic (with torsion (107-3), which is compatible with the local gauge invariance condition [9, 58, 60 - 63]) form of the (local) gauge-covariant derivative [67]. On this basis, in (1+3) dimensional space-time, equations (105-1) – (105-2) not only would describe a certain massive form of Maxwell's (single-photon) field based on the Abelian gauge group $U(1)$, but also would present a certain massive form of Yang–Mills (single-particle) fields based on the (non-Abelian) gauge groups $SU(N)$. For the latter case, the field strength tensor, vector gauge potential and the current in equations (105-1) – (105-2), are also written in component notation as: $F^a_{\ \mu\nu}$, $A^a_{\ \mu}$, $J^{a(E)}_{\ \ \mu}$, where the Latin index $a = 1, 2, 3, \ldots, N^2 -1$, and $N^2 -1$ is the number of linearly independent generators of the group $SU(N)$ (as a real manifold) [58]. Hence, by requiring the local gauge invariance for general relativistic massive particle field equations (105-1) – (105-2), in Section 3-4-1, we show that the massive Lagrangian density specified for these fields could be locally gauge invariant – if these fields be formally re-presented on a background space-time with certain complex torsion which is generated by the invariant mass of the gauge field carrier particle. Subsequently, in agreement with certain experimental data, the invariant mass of a particle (that would actually be identified as massive photon) has been specified ($m_0^{(E)} \approx 1.4070696 \times 10^{-41}$ kg), which is coupled with background space-time geometry. Note that the gauge theoretic approach for the field equations (104-1) - (104-2) could be applied, too.

It is noteworthy to recall that gauge symmetries can be viewed as analogues of the principle of general covariance of general relativity in which the coordinate system can be chosen freely under arbitrary diffeomorphism of space-time. Both gauge invariance and diffeomorphism invariance reflect a redundancy in the description of the system. In point of fact, a global symmetry is just a local symmetry whose group's parameters are fixed in space-time. The requirement of local symmetry, the cornerstone of gauge theories, is a stricter constraint [58]. However, our approach could be also considered in the framework of the theories that lie beyond the Standard Model [71], as it also includes new consequences such as a certain formulation for the gravitational particle field.

**I**n addition, we should note that the field equations (104-1) – (104-4) and (105-1) – (105-4) would correspond to two different single-particle fields: (104-1) – (104-2) correspond to the tensor representation of the spin-1/2 single-particle fields formulated solely in (1+2) dimensional space-time [29, 31] – where we necessary have $F_{20} = F_{02} = 0$ [29, 31]; and (105-1) – (105-2) describe the spin-1 single-particle fields formulated solely in (1+3) dimensional space-time [68, 69]. In the precisely same manner, the field equations (104-3) – (104-4) describe the spin-3/2 single-particle field (gravitational) formulated solely in (1+2) dimensional space-time – where $R_{\mu\nu\rho\sigma}$ is the Riemann curvature tensor, and we necessary have $R_{20\rho\sigma} = R_{02\rho\sigma} = 0$. The field equations (105-3) – (105-4) also describe the spin-2 single-particle (gravitational) field formulated solely in (1+3) dimensional space-time. However, it should be emphasized here (as we noted above) that, in general, for single-particle field equations (102) – (105-4), the (quantum mechanical) solutions are taken **to be complex** [29, 30, 31, 68, 69]. However, in the context of relativistic quantum mechanics, the field equations (104-1) – (105-4) are subject to a process of 2nd quantization anyhow; then these equations would describe solely the bosonic fields in (1+3) dimensional space-time, and the fermionic fields in (1+2) dimensional space-time.[1]

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1. We will show in Section 3-7, that the field equations (106-1) – (106-4), corresponding to the matrix relations (96) (i.e. for five space-time dimensions), as well as for higher space-time dimensions, are incompatible with some certain symmetry and are not definable, where we'll also conclude that the universe cannot have more than four space-time dimensions.

## 3-4-1. The Local Gauge-Invariance of “Massive” Lagrangian Density of the (unique) Obtained Massive Forms of Maxwell and Yang-Mills Fields

In this section we present (for the first time) a complex torsion approach to the massive gauge field theory. Hence, corresponding to the general relativistic massive single-particle wave equations (103-1) – (103-2), (104-1) – (104-2), (105-1) – (105-2), and so on – as special cases of the generally derived field equations (101-1) – (101-4) in various space-time dimensions, which particularly represent the Maxwell and Yang-mills fields in 1+3 dimensions – we may write formally the following their equivalent field equations (formulable solely for a “**rank two anti-symmetric**” field strength tensor):

$$\breve{\vec{\bar{\nabla}}}_{\lambda}F_{\mu\nu}+\breve{\vec{\bar{\nabla}}}_{\mu}F_{\nu\lambda}+\breve{\vec{\bar{\nabla}}}_{\nu}F_{\lambda\mu}=0\ , \tag{108-1}$$

$$\breve{\vec{\bar{\nabla}}}_{\mu}F^{\mu}{}_{\nu}=-J^{(E)}_{\nu} \tag{108-2}$$

where we suppose the general covariant derivative $\breve{\vec{\bar{\nabla}}}_{\lambda}$ is defined with the following complex torsion tensor (similar to the torsion tensor (107-3)) generated by both invariant masses $m_0^{(E)}$ and $m_0^{(G)}$:

$$\breve{\vec{\bar{\nabla}}}_{\lambda}F_{\mu\nu}=\partial_{\lambda}F_{\mu\nu}-(\bar{\Gamma}^{\tau}{}_{\lambda\mu}-\vec{K}^{\tau}{}_{\lambda\mu})F_{\tau\nu}-(\bar{\Gamma}^{\tau}{}_{\lambda\nu}-\vec{K}^{\tau}{}_{\lambda\nu})F_{\mu\tau}\ , \tag{109}$$

$$\vec{K}_{\tau\mu\nu}=\frac{i(m_0^{(G)}+m_0^{(E)})}{2\hbar}g_{\tau\nu}k_{\mu}\ ; \tag{109-1}$$

where $\bar{\Gamma}^{\rho}{}_{\sigma\mu}$ is the Christoffel symbol (or the torsion-free connection), $\vec{K}_{\tau\mu\nu}$ is contorsion tensor, and the torsion tensor read

$$\vec{T}_{\tau\mu\nu}=T_{\tau\mu\nu}+Z_{\tau\mu\nu}=\frac{im_0^{(G)}}{2\hbar}(g_{\tau\mu}k_{\nu}-g_{\tau\nu}k_{\mu})+\frac{im_0^{(E)}}{2\hbar}(g_{\tau\mu}k_{\nu}-g_{\tau\nu}k_{\mu}) \tag{110}$$

Using formula (110) in the field equations (108-1) – (108-2), we may get a general relativistic wave equation formally similar to massive field (103-1) – (103-2), … , (105-1) – (105-2), as follows

$$\breve{\nabla}_{\lambda}F_{\mu\nu}+\breve{\nabla}_{\mu}F_{\nu\lambda}+\breve{\nabla}_{\nu}F_{\lambda\mu}=Z^{\tau}{}_{\lambda\mu}F_{\tau\nu}+Z^{\tau}{}_{\mu\nu}F_{\tau\lambda}+Z^{\tau}{}_{\nu\lambda}F_{\tau\mu}\ , \tag{108-3}$$

$$\breve{\nabla}_{\mu}F^{\mu}{}_{\nu}-\frac{im_0^{(E)}}{\hbar}k_{\mu}F^{\mu}{}_{\nu}=-J^{(E)}_{\nu} \tag{108-4}$$

however, here the background space-time is defined with torsion $\vec{T}_{\tau\mu\nu}$, and the covariant derivative $\breve{\vec{\bar{\nabla}}}_{\lambda}$ is defined by (109).

The Lagrangian density for source-free ( $J^{(E)}_{\nu}=0$ ) case of the fields of the type (108-1) – (108-2), is given by [58],

$$L^{(E)}=-\frac{1}{4}\sqrt{-g}F^{\mu\nu}F_{\mu\nu} \tag{111}$$

where $g$ is the metric's determinant.

Assuming the local gauge invariance for field (108-1) – (108-2), in component notation formula (111) is also written as

$$L_{gf}^{(E)} = -\frac{1}{4}\sqrt{-g}\,F^{\alpha}{}_{\mu\nu}F_{\alpha}{}^{\mu\nu} \tag{112}$$

Now as for the trace part of torsion tensor $\vec{T}_{\tau\mu\nu}$ (110) we have

$$\begin{aligned}\vec{T}^{\mu}{}_{\mu\nu} = \vec{T}_{\nu} &= (D-1)\frac{i(m_0^{(G)}+m_0^{(E)})}{2\hbar}k_{\nu} \\ &= (D-1)\alpha\partial_{\nu}\varphi\end{aligned} \tag{113}$$

where D is the number of space-time dimensions, and

$$\exists\varphi:\ k_{\nu} = \breve{\nabla}_{\mu}\varphi = \partial_{\nu}\varphi,\quad \alpha = \frac{i(m_0^{(G)}+m_0^{(E)})}{2\hbar} \tag{114}$$

we may simply conclude that the conditions (113) – (114) are sufficient for the general relativistic field equations (108-1) – (108-2) and the Lagrangian density (112) to be locally gauge-invariant [9, 58, 60-63].

Now, using the torsion tensor geometrical properties, in the above gauge fields if we assume that the background space-time be re-defined with torsion $T_{\tau\mu\nu} = (im_0^{(G)}/2\hbar)(g_{\tau\mu}k_{\nu} - g_{\tau\nu}k_{\mu})$, as a geometrical abject, and $Z_{\tau\mu\nu} = (im_0^{(E)}/2\hbar)(g_{\tau\mu}k_{\nu} - g_{\tau\nu}k_{\mu})$ as an independent tensor field generated by $m_0^{(E)}$ (as the invariant mass of the gauge field carrier particle), which its trace part, i.e.

$$Z^{\mu}{}_{\mu\nu} = Z_{\nu} = (D-1)(im_0^{(E)}/2\hbar)k_{\nu} = (D-1)\gamma\partial_{\nu}\varphi\ ,\ \ \gamma = im_0^{(E)}/2\hbar \tag{115-1}$$

couples to the gauge fields (108-1) – (108-2), then these fields be formally equivalent to certain massive Maxwell and Yang-Mills single-particle fields. Hence the field strength tensor $F^{\alpha}{}_{\mu\nu}$ and the locally gauge invariant massive Lagrangian density (112) could be equivalently re-written as follows, respectively:

$$F^{\alpha}{}_{\mu\nu} = \breve{\nabla}_{\nu}A^{a}{}_{\mu} - \breve{\nabla}_{\mu}A^{a}{}_{\nu} + \frac{im_0^{(E)}}{2\hbar}(k_{\nu}A^{a}{}_{\mu} - k_{\mu}A^{a}{}_{\nu}) \tag{115-2}$$

$$L_{gf}^{(E)} = -\frac{1}{4}\sqrt{-g}[(\breve{\nabla}^{\nu}A^{\mu}{}_{\alpha} - \breve{\nabla}^{\mu}A^{\nu}{}_{\alpha} + \frac{im_0^{(E)}}{2\hbar}(k^{\nu}A^{\mu}{}_{\alpha} - k^{\mu}A^{\nu}{}_{\alpha})][(\breve{\nabla}_{\nu}A^{a}{}_{\mu} - \breve{\nabla}_{\mu}A^{a}{}_{\nu} + \frac{im_0^{(E)}}{2\hbar}(k_{\nu}A^{a}{}_{\mu} - k_{\mu}A^{a}{}_{\nu})] \tag{115-3}$$

where $A^{a}{}_{\mu}$ is the gauge field vector (potential) and $\breve{\nabla}_{\mu}$ denotes the general relativistic form of the local gauge-covariant derivative [67]. In addition, it is noteworthy that since $(im_0^{(E)}/2\hbar)k^{\mu}$ is a contravarint "time-like" vector, the obtained unique massive Lagrangian density (115-3) for Maxwell and Yang-Mills (single-particle) fields could be also considered in the framework of both three and four dimensional cases of the Chern-Simons gauge theory [64, 65].

**I**t is noteworthy that for the case of electromagnetic field, according to [60 – 63], based on certain experimental data we can establish a lower bound for $\gamma$ (as a constant) in (115-1): $|\gamma| \geq 20$. Thus, in agreement with these experimental results, the invariant mass of a particle (that actually would be identified as massive photon), which is coupled with background space-time geometry, can be approximately specified as follows:

$$m_0^{(E)} \approx 1.4070696 \times 10^{-41} \text{ kg} \tag{115-4}$$

The above torsion (generally defined by formulas (110) and (115-1)) approach could be also applied for massive neutrino which is coupled with background space-time geometry. Such massive particle fields which are coupled to background space-time geometry (with complex torsion defined above) could be fully responsible for the dark energy and dark matter as well [75].

**W**e should note that the above particular approach for formulating the (local) gauge-invariance formulation of massive Maxwell and Yang-Mills field equations (as well as the half-integer relativistic single-particle wave equations such as the Dirac equation), by using a certain (coupled) complex torsion tensor field (generated by the invariant mass of the gauge field carrier particle), may be also considered via the teleparallel geometry – where for example the gravitation whether requires a curved or a torsionned space-time, is a matter of convention [72 - 74]. **In fact on this basis, the torsion tensor can always be treated as an independent tensor field, or equivalently, as part of the space-time geometry.** That's it.

**F**urthermore, the mass gap in quantum Yang-Mills theory may be connected with the background space-time geometry with the above complex torsion. As professors A. Jaffe and E. Witten in the conclusion of their famous article [70] concerning the mass gap problem in Yang-Mills theory, in particular, have mentioned: "… **One view of the mass gap in Yang–Mills theory suggests that it may be tied to curvature in the space of connections**".

## 3-4-2. A Direct Proving of the Absence of Magnetic Monopoles in Nature

As a direct consequence of the assumption (3) in Section 1-1, and also the unique structure of derived general relativistic single-particle field equations (101-1) – (101-4), as well as one of their particular case for four dimensional space-time, i.e. equations (105-1) – (105-2) corresponding to the Electromagnetic field (and also Yang-Mills field equations, see Section 3-4) we may conclude that magnetic monopoles – in contrast with electric charges – cannot exist in nature. This conclusion is based solely on certain algebraic properties of linearized (and simultaneously parameterized, as necessary algebraic conditions) unique forms of the relativistic energy-momentum relation (95).

## 3-5. Deriving the Einstein Field Equations with a Cosmological Constant

**T**he massless cases of general relativistic wave equations (103-3) – (103-4), (104-3) – (104-4) and (105-3) – (105-4), are given by ( $m_0^{(G)} = 0$ ):

$$\breve{\nabla}_\lambda R_{\mu\nu\rho\sigma} + \breve{\nabla}_\mu R_{\nu\lambda\rho\sigma} + \breve{\nabla}_\nu R_{\lambda\mu\rho\sigma} = 0 \ , \tag{116-1}$$

$$\breve{\nabla}_\mu R^\mu{}_{\nu\rho\sigma} = -J^{(G)}_{\nu\rho\sigma} \tag{116-2}$$

Hence, by contraction the 2nd Bianchi identity (116-1) and assuming the sign conventions (97), we get

$$\nabla_\sigma R_{\mu\nu\rho}{}^\sigma = \nabla_\nu R_{\mu\rho} - \nabla_\mu R_{\nu\rho} \tag{117}$$

Then from (116-2) and (117) and the following definition

$$J^{(G)}_{\nu\rho\sigma} = -8\pi(\nabla_\sigma T_{\nu\rho} - \nabla_\rho T_{\nu\sigma}) + 8\pi B(\nabla_\sigma T g_{\nu\rho} - \nabla_\rho T g_{\nu\sigma}) \ , \tag{118}$$

where $T_{\mu\nu}$ is the stress-energy tensor ($T = T^{\mu}{}_{\mu}$), $g_{\mu\nu}$ is the metric and '$B$' is a constant (which be specified for each space-time dimension), we easily obtain **the Einstein field equations** as follows:

$$R_{\mu\nu} = -8\pi(T_{\mu\nu} - BTg_{\mu\nu}) - \Lambda g_{\mu\nu} \tag{119}$$

where $\Lambda$ is a cosmological constant (emerged naturally in the course of obtaining (119)). Hence for (1+1) dimensional case from (119) we get

$$R_{\mu\nu} = -4\pi T g_{\mu\nu} + \tfrac{1}{2}\Lambda g_{\mu\nu} \tag{119-1}$$

where $B = 0$. For (1+2) dimensional case we have (however, for this case as we show in Section 3-7, we have $R_{02\rho\sigma} = 0$):

$$R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} = -8\pi T_{\mu\nu} - 2\Lambda g_{\mu\nu} \tag{119-2}$$

where $B = 1$. For (1+3) dimensional space-time, we obtain

$$R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} = -8\pi T_{\mu\nu} - \Lambda g_{\mu\nu} \tag{119-3}$$

where $B = 1/2$.

## 3-6. Geometric Algebra Representation of the Derived General Relativistic (Single-Particle) Wave Equations (103-1) – (106-4)

**I**t should be mentioned that the general relativistic single-particle wave equations (103-1) – (106-4) derived by first quantization (as a postulate) of the energy-momentum matrix relations (92) – (96) (including matrices that are hermitian and generate a Clifford algebra ), in matrix representation, i.e. in the geometric algebra formulation, are written as follows:

$$(i\hbar\alpha^{\mu}\breve{\nabla}_{\mu} - m_0\tilde{\alpha}^{\mu}k_{\mu})\Psi_F = 0, \tag{120}$$

$$(i\hbar\alpha^{\mu}\breve{\nabla}_{\mu} - m_0\tilde{\alpha}^{\mu}k_{\mu})\Psi_R = 0 \tag{121}$$

where
$$\alpha^{\mu} = \beta^{\mu} + \beta'^{\mu}\ ,\ \ \tilde{\alpha}^{\mu} = \beta^{\mu} - \beta'^{\mu} \tag{122}$$

In matrix equations (120) – (121) – which each equation could be written in the form of two coupled equations that assumedly have chiral symmetry if the single-particle field equation be source-free (see Section 3-7) – $\Psi_E$ and $\Psi_R$ are column matrices (representing the single-particle wave functions) and matrices $\beta^{\mu}$ and $\beta'^{\mu}$ are the contravariant square matrices *(corresponding to a Clifford algebra, see Appendix B for special relativistic cases)*, are as follow[1] (for various space-time dimensions). Hence for (1+1) dimensional case (corresponding to equations (103-1) – (103-4)) we have

$$\begin{gathered}
\beta^0 = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix},\ \beta'_0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\ \beta^1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\ \beta'_1 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \\
\Psi_F = \begin{bmatrix} F_{10} \\ \varphi^{(E)} \end{bmatrix},\ \Psi_R = \begin{bmatrix} R_{10\rho\sigma} \\ \varphi^{(G)}_{\rho\sigma} \end{bmatrix}, \\
J^{(G)}_{\nu\rho\sigma} = -[\breve{\nabla}_{\nu} + (im^{(G)}_0/2\hbar)k_{\nu}]\varphi^{(G)}_{\rho\sigma},\ \ J^{(E)}_{\nu} = -[\breve{\nabla}_{\nu} + (im^{(E)}_0/2\hbar)k_{\nu}]\varphi^{(E)}.
\end{gathered} \tag{123}$$

---

1. The covariant matrices $\beta'_{\mu}$ has been written instead of $\beta'^{\mu}$.

Using (123), the special relativistic cases of equations (120) – (121) in (1+1) dimensions (corresponding to equations (103-1) – (103-4)) are given by, respectively:

$$\begin{bmatrix} i\hbar\breve{\nabla}_0 - m_0^{(E)} & i\hbar\breve{\nabla}_1 \\ -i\hbar\breve{\nabla}_1 & -i\hbar\breve{\nabla}_0 - m_0^{(E)} \end{bmatrix} \begin{bmatrix} \psi_{(F)L} \\ \psi_{(F)R} \end{bmatrix} = 0 \qquad (120\text{-}1)$$

where

$$\Psi_F = \begin{bmatrix} \psi_{(F)L} \\ \psi_{(F)R} \end{bmatrix} = \begin{bmatrix} F_{10} \\ \varphi^{(E)} \end{bmatrix},\ \psi_{(F)L} = [F_{10}]\ ,\ \psi_{(F)R} = [\varphi^{(E)}]\ ;$$

$$\begin{bmatrix} i\hbar\breve{\nabla}_0 - m_0^{(G)} & i\hbar\breve{\nabla}_1 \\ -i\hbar\breve{\nabla}_1 & -i\hbar\breve{\nabla}_0 - m_0^{(G)} \end{bmatrix} \begin{bmatrix} \psi_{(R)L} \\ \psi_{(R)R} \end{bmatrix} = 0 \qquad (121\text{-}1)$$

where (for this special case we may suppose: $m_0 \neq 0 \Rightarrow T_{\tau\mu\nu} \neq 0$)

$$\Psi_R = \begin{bmatrix} \psi_{(R)L} \\ \psi_{(R)R} \end{bmatrix} = \begin{bmatrix} R_{10\rho\sigma} \\ \varphi_{\rho\sigma}^{(G)} \end{bmatrix},\ \psi_{(R)L} = [R_{10\rho\sigma}]\ ,\ \psi_{(R)R} = [\varphi_{\rho\sigma}^{(G)}]\ .$$

**F**or (1+2) dimensional case (corresponding to equations (104-1) – (104-4)) we get

$$\beta^0 = \begin{bmatrix} 0 & 0 \\ 0 & -(\sigma^0 + \sigma^1) \end{bmatrix},\ \beta_0' = \begin{bmatrix} \sigma^0 + \sigma^1 & 0 \\ 0 & 0 \end{bmatrix},\ \beta^1 = \begin{bmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{bmatrix},\ \beta_1' = \begin{bmatrix} 0 & \sigma^3 \\ -\sigma^3 & 0 \end{bmatrix},$$

$$\beta^2 = \begin{bmatrix} 0 & -\sigma^1 \\ -\sigma^0 & 0 \end{bmatrix},\ \beta_2' = \begin{bmatrix} 0 & -\sigma^0 \\ -\sigma^1 & 0 \end{bmatrix};$$

$$\Psi_R = \begin{bmatrix} R_{10\rho\sigma} \\ R_{02\rho\sigma} \\ R_{21\rho\sigma} \\ \varphi_{\rho\sigma}^{(G)} \end{bmatrix},\ \Psi_F = \begin{bmatrix} F_{10} \\ F_{02} \\ F_{21} \\ \varphi^{(E)} \end{bmatrix},\ J_{\nu\rho\sigma}^{(G)} = -(\breve{\nabla}_\nu + \frac{im_0^{(G)}}{\hbar} k_\nu)\varphi_{\rho\sigma}^{(G)},\ J_\nu^{(E)} = -(\breve{\nabla}_\nu + \frac{im_0^{(E)}}{\hbar} k_\nu)\varphi^{(E)} \qquad (124)$$

where

$$\sigma^0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\ \sigma^1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},\ \sigma^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\ \sigma^3 = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}. \qquad (124\text{-}1)$$

Using (124) and (124-1), the special relativistic cases of equations (120) – (121) in (1+2) dimensions, are given by, respectively:

$$\begin{bmatrix} i\hbar\breve{\nabla}_0(\sigma^0+\sigma^1)-m_0^{(E)} & i\hbar[\breve{\nabla}_1(\sigma^2-\sigma^3)+\breve{\nabla}_2(-\sigma^1+\sigma^0)] \\ -i\hbar[\breve{\nabla}_1(\sigma^2-\sigma^3)+\breve{\nabla}_2(-\sigma^1+\sigma^0)] & -i\hbar\breve{\nabla}_0(\sigma^0+\sigma^1)-m_0^{(E)} \end{bmatrix}\begin{bmatrix}\psi_{(F)L}\\ \psi_{(F)R}\end{bmatrix}=0 \qquad (120\text{-}2)$$

where

$$\Psi_F=\begin{bmatrix}\psi_{(F)L}\\ \psi_{(F)R}\end{bmatrix}=\begin{bmatrix}F_{10}\\ F_{02}\\ F_{21}\\ \varphi^{(E)}\end{bmatrix},\ \psi_{(F)L}=\begin{bmatrix}F_{10}\\ F_{02}\end{bmatrix},\ \psi_{(F)R}=\begin{bmatrix}F_{21}\\ \varphi^{(E)}\end{bmatrix};$$

$$\begin{bmatrix} i\hbar\breve{\nabla}_0(\sigma^0+\sigma^1)-m_0^{(G)} & i\hbar[\breve{\nabla}_1(\sigma^2-\sigma^3)+\breve{\nabla}_2(-\sigma^1+\sigma^0)] \\ -i\hbar[\breve{\nabla}_1(\sigma^2-\sigma^3)+\breve{\nabla}_2(-\sigma^1+\sigma^0)] & -i\hbar\breve{\nabla}_0(\sigma^0+\sigma^1)-m_0^{(G)} \end{bmatrix}\begin{bmatrix}\psi_{(R)L}\\ \psi_{(R)R}\end{bmatrix}=0 \qquad (121\text{-}2)$$

where (for this special case we may suppose: $m_0 \neq 0 \Rightarrow T_{\tau\mu\nu} \neq 0$)

$$\Psi_R=\begin{bmatrix}\psi_{(R)L}\\ \psi_{(R)L}\end{bmatrix}=\begin{bmatrix}R_{10\rho\sigma}\\ R_{02\rho\sigma}\\ R_{21\rho\sigma}\\ \varphi^{(G)}_{\rho\sigma}\end{bmatrix},\ \psi_{(R)L}=\begin{bmatrix}R_{10\rho\sigma}\\ R_{02\rho\sigma}\end{bmatrix},\ \psi_{(R)R}=\begin{bmatrix}R_{21\rho\sigma}\\ \varphi^{(G)}_{\rho\sigma}\end{bmatrix}.$$

**F**or (1+3) dimensional casees (corresponding to the field equations (105-1) – (105-4)) of equations (120) – (121), we have

$$\beta^0=\begin{bmatrix}0 & 0\\ 0 & -(\gamma^0+\gamma^1)\end{bmatrix},\ \beta'_0=\begin{bmatrix}(\gamma^0+\gamma^1) & 0\\ 0 & 0\end{bmatrix},\ \beta^1=\begin{bmatrix}0 & \gamma^2\\ -\gamma^3 & 0\end{bmatrix},\ \beta'_1=\begin{bmatrix}0 & \gamma^3\\ -\gamma^2 & 0\end{bmatrix},$$

$$\beta^2=\begin{bmatrix}0 & \gamma^4\\ \gamma^5 & 0\end{bmatrix},\ \beta'_2=\begin{bmatrix}0 & -\gamma^5\\ -\gamma^4 & 0\end{bmatrix},\ \beta^3=\begin{bmatrix}0 & \gamma^6\\ -\gamma^7 & 0\end{bmatrix},\ \beta'_3=\begin{bmatrix}0 & \gamma^7\\ -\gamma^6 & 0\end{bmatrix},$$

$$\Psi_R=\begin{bmatrix}R_{10\rho\sigma}\\ R_{20\rho\sigma}\\ R_{30\rho\sigma}\\ 0\\ R_{23\rho\sigma}\\ R_{31\rho\sigma}\\ R_{12\rho\sigma}\\ \varphi^{(G)}_{\rho\sigma}\end{bmatrix},\ \ \Psi_F=\begin{bmatrix}F_{10}\\ F_{20}\\ F_{30}\\ 0\\ F_{23}\\ F_{31}\\ F_{12}\\ \varphi^{(E)}\end{bmatrix},\ \ J^{(G)}_{\nu\rho\sigma}=-(\breve{\nabla}_\nu+\frac{im_0^{(G)}}{\hbar}k_\nu)\varphi^{(G)}_{\rho\sigma},\ \ J^{(E)}_{\nu}=-(\breve{\nabla}_\nu+\frac{im_0^{(E)}}{\hbar}k_\nu)\varphi^{(E)} \qquad (125)$$

where we have

$$\gamma^0=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix},\ \gamma^1=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix},\ \gamma^2=\begin{bmatrix}0&0&0&1\\0&0&0&0\\0&0&0&0\\-1&0&0&0\end{bmatrix},\ \gamma^3=\begin{bmatrix}0&0&0&0\\0&0&-1&0\\0&1&0&0\\0&0&0&0\end{bmatrix}.$$

$$\gamma^4=\begin{bmatrix}0&0&0&0\\0&0&0&1\\0&0&0&0\\0&-1&0&0\end{bmatrix},\ \gamma^5=\begin{bmatrix}0&0&-1&0\\0&0&0&0\\1&0&0&0\\0&0&0&0\end{bmatrix},\ \gamma^6=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&1\\0&0&-1&0\end{bmatrix},\ \gamma^7=\begin{bmatrix}0&-1&0&0\\1&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}. \tag{125-1}$$

Using (125) and (125-1), the special relativistic cases of equations (120) – (121) in (1+3) dimensions, are given by, respectively:

$$\begin{bmatrix} i\hbar\breve{\nabla}_0(\gamma^0+\gamma^1)-m_0^{(E)} & i\hbar[\breve{\nabla}_1(\gamma^2-\gamma^3)+\breve{\nabla}_2(\gamma^4+\gamma^5)+\breve{\nabla}_3(\gamma^6-\gamma^7)] \\ i\hbar[\breve{\nabla}_1(\gamma^2-\gamma^3)+\breve{\nabla}_2(\gamma^4+\gamma^5)+\breve{\nabla}_3(\gamma^6-\gamma^7)] & -i\hbar\breve{\nabla}_0(\gamma^0+\gamma^1)-m_0^{(E)}\end{bmatrix}\begin{bmatrix}\psi_{(F)L}\\ \psi_{(F)R}\end{bmatrix}=0 \tag{120-3}$$

where

$$\Psi_F=\begin{bmatrix}\psi_{(F)L}\\ \psi_{(F)R}\end{bmatrix}=\begin{bmatrix}F_{10}\\F_{20}\\F_{30}\\0\\F_{23}\\F_{31}\\F_{12}\\ \varphi^{(E)}\end{bmatrix},\ \psi_{(F)L}=\begin{bmatrix}F_{10}\\F_{20}\\F_{30}\\0\end{bmatrix},\ \psi_{(F)R}=\begin{bmatrix}F_{23}\\F_{31}\\F_{12}\\ \varphi^{(E)}\end{bmatrix};$$

$$\begin{bmatrix} i\hbar\breve{\nabla}_0(\gamma^0+\gamma^1)-m_0^{(G)} & i\hbar[\breve{\nabla}_1(\gamma^2-\gamma^3)+\breve{\nabla}_2(\gamma^4+\gamma^5)+\breve{\nabla}_3(\gamma^6-\gamma^7)] \\ i\hbar[\breve{\nabla}_1(\gamma^2-\gamma^3)+\breve{\nabla}_2(\gamma^4+\gamma^5)+\breve{\nabla}_3(\gamma^6-\gamma^7)] & -i\hbar\breve{\nabla}_0(\gamma^0+\gamma^1)-m_0^{(G)} \end{bmatrix}\begin{bmatrix}\psi_{(R)L}\\ \psi_{(R)R}\end{bmatrix}=0$$

(121-3)

where (for this special case we may suppose: $m_0 \neq 0 \Rightarrow T_{\tau\mu\nu} \neq 0$ )

$$\Psi_R = \begin{bmatrix}\psi_{(R)L}\\ \psi_{(R)R}\end{bmatrix} = \begin{bmatrix} R_{10\rho\sigma} \\ R_{20\rho\sigma} \\ R_{30\rho\sigma} \\ 0 \\ R_{23\rho\sigma} \\ R_{31\rho\sigma} \\ R_{12\rho\sigma} \\ \varphi_{\rho\sigma}^{(G)} \end{bmatrix},\ \psi_{(R)L} = \begin{bmatrix} R_{10\rho\sigma} \\ R_{20\rho\sigma} \\ R_{30\rho\sigma} \\ 0 \end{bmatrix},\ \psi_{(R)R} = \begin{bmatrix} R_{23\rho\sigma} \\ R_{31\rho\sigma} \\ R_{12\rho\sigma} \\ \varphi_{\rho\sigma}^{(G)} \end{bmatrix}.$$

And for (1+4) dimensional case of (120) – (121), corresponding to the field equations (106-1) – (106-4), we obtain:[1]

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1. For this case (just for clarity) the square matrices $\beta^{\mu}$ and $\beta'_{\mu}$ have been written in detail, however, these square matrices, definitely, could be written by formulations similar to relations (123) and (12-1) (including the special relativistic cases (120-1) – (120-3) and (121-1) –(121-3)).

$$\beta^0 = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1 \end{bmatrix},$$

$$\beta_0' = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix},$$

$$\beta^1 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},$$

$$\beta'_1 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},$$

$$\beta^2 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},$$

$$\beta_2' = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},$$

$$\beta^3 = \begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix},$$

$$\beta_3' = \begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix},$$

$$\beta^4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\beta_4' = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Psi_F = \begin{bmatrix} F_{10} \\ F_{02} \\ F_{30} \\ 0 \\ F_{04} \\ 0 \\ 0 \\ 0 \\ 0 \\ F_{43} \\ F_{42} \\ F_{41} \\ F_{32} \\ F_{31} \\ F_{21} \\ \varphi^{(E)} \end{bmatrix}, \quad \Psi_R = \begin{bmatrix} R_{10\rho\sigma} \\ R_{02\rho\sigma} \\ R_{30\rho\sigma} \\ 0 \\ R_{04\rho\sigma} \\ 0 \\ 0 \\ 0 \\ 0 \\ R_{43\rho\sigma} \\ R_{42\rho\sigma} \\ R_{41\rho\sigma} \\ R_{32\rho\sigma} \\ R_{31\rho\sigma} \\ R_{21\rho\sigma} \\ \varphi^{(G)}_{\rho\sigma} \end{bmatrix}, \quad J^{(G)}_{\nu\rho\sigma} = -(\breve{\nabla}_\nu + \frac{im_0^{(G)}}{\hbar} k_\nu)\varphi^{(G)}_{\rho\sigma}, \quad J^{(E)}_{\nu} = -(\breve{\nabla}_\nu + \frac{im_0^{(E)}}{\hbar} k_\nu)\varphi^{(E)}. \tag{126}$$

The size of matrices $\alpha_\mu$ and $\tilde{\alpha}_\mu$ in field equations (120) – (121) for (1+5) dimensional space-time is 32×32, and the column matrices $\Psi_F$ and $\Psi_R$ are defined as follows:

$$\Psi_F = \begin{bmatrix} F_{10} \\ F_{20} \\ F_{30} \\ 0 \\ F_{40} \\ 0 \\ 0 \\ 0 \\ F_{50} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ F_{45} \\ 0 \\ F_{53} \\ F_{25} \\ F_{51} \\ 0 \\ F_{34} \\ F_{42} \\ F_{14} \\ F_{32} \\ F_{13} \\ F_{21} \\ \varphi^{(E)} \end{bmatrix}, \quad \Psi_R = \begin{bmatrix} R_{10\rho\sigma} \\ R_{20\rho\sigma} \\ R_{30\rho\sigma} \\ 0 \\ R_{40\rho\sigma} \\ 0 \\ 0 \\ 0 \\ R_{50\rho\sigma} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ R_{45\rho\sigma} \\ 0 \\ R_{53\rho\sigma} \\ R_{25\rho\sigma} \\ R_{51\rho\sigma} \\ 0 \\ R_{34\rho\sigma} \\ R_{42\rho\sigma} \\ R_{14\rho\sigma} \\ R_{32\rho\sigma} \\ R_{13\rho\sigma} \\ R_{21\rho\sigma} \\ \varphi^{(G)}_{\rho\sigma} \end{bmatrix}, \quad J^{(G)}_{\nu\rho\sigma} = -(\breve{\nabla}_\nu + \frac{im_0^{(G)}}{\hbar} k_\nu)\varphi^{(G)}_{\rho\sigma}, \quad J^{(E)}_{\nu} = -(\breve{\nabla}_\nu + \frac{im_0^{(E)}}{\hbar} k_\nu)\varphi^{(E)} \tag{127}$$

**3-7.** **I**n this Section, as another direct consequence of the assumption (3) in Section 1-1, and also the unique structure of the derived general relativistic wave equations (101-1) – (101-4) (including equations (102) – (105-4) and so on as particular cases of these equations for various space-time dimensions), we conclude that the universe cannon have more than four space-time dimensions.

As it was mentioned in Section 3-4, in the context of relativistic quantum mechanics, the relativistic (single-particle) wave equations (120) – (121) (equivalent to equations (101-1) – (101-4), including equations (102) – (106-4) and so on for various dimensions) give an equivalent tensor representation of the half-integer spin single-particle fields (defined solely in (1+2) dimensional space-time by the matrices (124-1), or matrices (B-2) for the special relativistic case in Appendix B), as well as a tensor representation of the integer spin single-particle fields (defined solely in (1+3) dimensional space-time by the matrices (125-1), or matrices (B-3) for the special relativistic case in Appendix B).

Meanwhile, it is noteworthy that the half-integer spin particle fields derived solely by this approach, is compatible with various experimental data related to two (spatial) dimensional property for electrons. These experiments include two-dimensional electron gas (2DEG), that is a gas of electrons free to move in two dimensions, but tightly confined in the third [38, 39]. For the latter case, it has been shown in several works that the Dirac spinor field in (1+2) dimensional space-time, could be also isomorphically represented by (anti-symmetric two-index) tensor representation of the Lorentz group – which is equivalent to three dimensional case of the single-particle tensor field (120) defined by matrices (124-1) (or matrices (B-2) for the special relativistic case in Appendix B) [29, 31]. In fact, it has been shown that in three dimensional space-time all the basic effects attributed to spinors can be also explained using the tensor formulation of the relativistic wave equation for particles of spin-½ (as well as any particle of half-integer spin exhibit Fermi–Dirac statistics). Moreover, for a particle of spin-1 (as well as any particle of integer spin exhibit Bose–Einstein statistics) it has also been shown that its spinor representation could be equivalent to an anti-symmetric two-index tensor representation of the Lorentz group [30, 31].

**T**he relativistic wave equations could be also represented by the left and right handed components of the wave-functions $\psi_{\mu\nu\rho\sigma}$, $\psi_{\mu\nu}$ (defined by formulas (123) – (127)). *We can basically show that by the assumption that the source-free particle fields preserve chiral symmetry* [32 - 37], *some or all the components of the wave functions $\psi_{\mu\nu\rho\sigma}$, $\psi_{\mu\nu}$ in relativistic single-particle wave equations (120) – (121) should vanish in certain space-time dimensions (for dimensions D >5 all the wave-functions' components will vanish).*

Hence, if the components of tensor fields $R_{\mu\nu\rho\sigma}$, $F_{\mu\nu}$ equivalently be represented by the wave-functional components $\psi_{\mu\nu\rho\sigma}$, $\psi_{\mu\nu}$, using the formula (123) for two dimensional case of equations (120) – (121) (corresponding to the field equations (103-1) – (103-4), where $\mu$, $\nu$, $\rho$ ,$\sigma$ = 0,1; and $\psi_{\mu\nu} = -\psi_{\nu\mu}$, $\psi_{\mu\nu\rho\sigma} = -\psi_{\nu\mu\rho\sigma}$), we have:

$$\begin{bmatrix} i\hbar\breve{\nabla}_0 - m_0^{(E)} & i\hbar\breve{\nabla}_1 \\ -i\hbar\breve{\nabla}_1 & -i\hbar\breve{\nabla}_0 - m_0^{(E)} \end{bmatrix}\begin{bmatrix} \psi_{(F)L} \\ \psi_{(F)R} \end{bmatrix} = 0 \ , \ \begin{bmatrix} i\hbar\breve{\nabla}_0 - m_0^{(G)} & i\hbar\breve{\nabla}_1 \\ -i\hbar\breve{\nabla}_1 & -i\hbar\breve{\nabla}_0 - m_0^{(G)} \end{bmatrix}\begin{bmatrix} \psi_{(R)L} \\ \psi_{(R)R} \end{bmatrix} = 0 \quad (128)$$

and

$$
\begin{aligned}
&\Psi_F = \begin{bmatrix} \psi_{(F)L} \\ \psi_{(F)R} \end{bmatrix} = \begin{bmatrix} \psi_{10} \\ 0 \end{bmatrix}, \ \psi_{(F)L} = [\psi_{10}] \ , \ \psi_{(F)R} = [0] \ , \\
&\Rightarrow \psi_{(F)L} = [0] \ ; \\
&\Psi_R = \begin{bmatrix} \psi_{(R)L} \\ \psi_{(R)R} \end{bmatrix} = \begin{bmatrix} \psi_{10\rho\sigma} \\ 0 \end{bmatrix}, \ \psi_{(R)L} = [\psi_{10\rho\sigma}] \ , \ \psi_{(R)R} = [0] \ , \\
&\Rightarrow \psi_{(R)L} = [0] \ .
\end{aligned}
\tag{128-1}
$$

So, in (1+1) dimensional cases of the relativistic wave equations (120) – (121), chiral symmetry implies that the wave-function vanishes.

For three dimensional space-time case of equations (120) – (121), i.e. equations (120-2) – (121-2) (corresponding to the field equations (104-1) – (104-4), where $\mu, \nu, \rho, \sigma = 0,1,2$; $\psi_{\mu\nu} = -\psi_{\nu\mu}$, $\psi_{\mu\nu\rho\sigma} = -\psi_{\nu\mu\rho\sigma}$), we get

$$
\begin{aligned}
&\Psi_F = \begin{bmatrix} \psi_{(F)L} \\ \psi_{(F)R} \\ 0 \end{bmatrix} = \begin{bmatrix} \psi_{10} \\ \psi_{02} \\ \psi_{21} \\ 0 \end{bmatrix}, \ \psi_{(F)L} = \begin{bmatrix} \psi_{10} \\ \psi_{02} \end{bmatrix}, \ \psi_{(F)R} = \begin{bmatrix} \psi_{21} \\ 0 \end{bmatrix} \\
&\Rightarrow \psi_{02} = 0 \ ; \\
&\Psi_R = \begin{bmatrix} \psi_{(R)L} \\ \psi_{(R)R} \\ 0 \end{bmatrix} = \begin{bmatrix} \psi_{10\rho\sigma} \\ \psi_{02\rho\sigma} \\ \psi_{21\rho\sigma} \\ 0 \end{bmatrix}, \ \psi_{(R)L} = \begin{bmatrix} \psi_{10\rho\sigma} \\ \psi_{02\rho\sigma} \end{bmatrix}, \ \psi_{(R)R} = \begin{bmatrix} \psi_{21\rho\sigma} \\ 0 \end{bmatrix} \\
&\Rightarrow \psi_{02\rho\sigma} = 0
\end{aligned}
\tag{129}
$$

Thus, the components $\psi_{02}$, $\psi_{02\rho\sigma}$ in (129) vanish, due to assumption of the chiral symmetry, and the particle fields be represented by two components wave-functions $\Psi_F$ and $\Psi_R$. This result (for example) for $\Psi_F$ is fully compatible with the tensor representation of the Dirac spinor field in (1+2) dimensional space-time [29, 48].

Subsequently, for four dimensional cases of equations (120) – (121), i.e. equations (120-3) – (121-3) (corresponding to the field equations (105-1) – (105-4), where $\mu$, $\nu$, $\rho$ ,$\sigma$ = 0,1,2,3, and $\psi_{\mu\nu} = -\psi_{\nu\mu}$, $\psi_{\mu\nu\rho} = -\psi_{\nu\mu\rho}$ , $\psi_{\mu\nu\rho\sigma} = -\psi_{\nu\mu\rho\sigma}$) we obtain, respectively:

$$\Psi_F = \begin{bmatrix} \psi_{(F)L} \\ \psi_{(F)R} \\ 0 \end{bmatrix} = \begin{bmatrix} \psi_{10} \\ \psi_{20} \\ \psi_{30} \\ 0 \\ \psi_{23} \\ \psi_{31} \\ \psi_{12} \\ 0 \end{bmatrix}, \ \psi_{(F)L} = \begin{bmatrix} \psi_{10} \\ \psi_{20} \\ \psi_{30} \\ 0 \end{bmatrix}, \ \psi_{(F)R} = \begin{bmatrix} \psi_{23} \\ \psi_{31} \\ \psi_{12} \\ 0 \end{bmatrix} \ ;$$

$$\Psi_R = \begin{bmatrix} \psi_{(R)L} \\ \psi_{(R)R} \\ 0 \end{bmatrix} = \begin{bmatrix} \psi_{10\rho\sigma} \\ \psi_{20\rho\sigma} \\ \psi_{30\rho\sigma} \\ 0 \\ \psi_{23\rho\sigma} \\ \psi_{31\rho\sigma} \\ \psi_{12\rho\sigma} \\ 0 \end{bmatrix}, \ \psi_{(R)L} = \begin{bmatrix} \psi_{10\rho\sigma} \\ \psi_{20\rho\sigma} \\ \psi_{30\rho\sigma} \\ 0 \end{bmatrix}, \ \psi_{(R)R} = \begin{bmatrix} \psi_{23\rho\sigma} \\ \psi_{31\rho\sigma} \\ \psi_{12\rho\sigma} \\ 0 \end{bmatrix} . \tag{130}$$

This result for $\psi_F$ and $\psi_R$ is also fully compatible with the integer spin particle fields describing solely by six components wave-functions [35, 49].

For relativistic wave equations (120) – (121) in five dimensional space-time (corresponding to the single-particle field equations (106-1) – (106-4), where $\mu, \nu, \rho, \sigma$ = 0,1,2,3,4; $\psi_{\mu\nu} = -\psi_{\nu\mu}$, $\psi_{\mu\nu\rho\sigma} = -\psi_{\nu\mu\rho\sigma}$), we obtain

$$\Psi_F = \begin{bmatrix} \psi_{(F)L} \\ \psi_{(F)R} \\ 0 \end{bmatrix} = \begin{bmatrix} \psi_{10} \\ \psi_{02} \\ \psi_{30} \\ 0 \\ \psi_{04} \\ 0 \\ 0 \\ 0 \\ 0 \\ \psi_{43} \\ \psi_{42} \\ \psi_{41} \\ \psi_{32} \\ \psi_{31} \\ \psi_{21} \\ 0 \end{bmatrix}, \ \psi_{(F)L} = \begin{bmatrix} \psi_{10} \\ \psi_{02} \\ \psi_{30} \\ 0 \\ \psi_{04} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ \psi_{(F)R} = \begin{bmatrix} 0 \\ \psi_{43} \\ \psi_{42} \\ \psi_{41} \\ \psi_{32} \\ \psi_{31} \\ \psi_{21} \\ 0 \end{bmatrix} \quad \Rightarrow \psi_{10} = \psi_{41} = \psi_{31} = \psi_{21} = 0$$

$$\Rightarrow \ \psi_{(F)L} = \begin{bmatrix} 0 \\ \psi_{02} \\ \psi_{30} \\ 0 \\ \psi_{04} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ \psi_{(F)R} = \begin{bmatrix} 0 \\ \psi_{43} \\ \psi_{42} \\ 0 \\ \psi_{32} \\ 0 \\ 0 \\ 0 \end{bmatrix} ;$$

$$\Psi_R = \begin{bmatrix} \psi_{(R)L} \\ \psi_{(R)R} \\ 0 \end{bmatrix} = \begin{bmatrix} \psi_{10\rho\sigma} \\ \psi_{02\rho\sigma} \\ \psi_{30\rho\sigma} \\ 0 \\ \psi_{04\rho\sigma} \\ 0 \\ 0 \\ 0 \\ 0 \\ \psi_{43\rho\sigma} \\ \psi_{42\rho\sigma} \\ \psi_{41\rho\sigma} \\ \psi_{32\rho\sigma} \\ \psi_{31\rho\sigma} \\ \psi_{21\rho\sigma} \\ 0 \end{bmatrix}, \ \psi_{(R)L} = \begin{bmatrix} \psi_{10\rho\sigma} \\ \psi_{02\rho\sigma} \\ \psi_{30\rho\sigma} \\ 0 \\ \psi_{04\rho\sigma} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ \psi_{(R)R} = \begin{bmatrix} 0 \\ \psi_{43\rho\sigma} \\ \psi_{42\rho\sigma} \\ \psi_{41\rho\sigma} \\ \psi_{32\rho\sigma} \\ \psi_{31\rho\sigma} \\ \psi_{21\rho\sigma} \\ 0 \end{bmatrix} \ \Rightarrow \psi_{10\rho\sigma} = \psi_{41\rho\sigma} = \psi_{31\rho\sigma} = \psi_{21\rho\sigma} = 0$$

$$\Rightarrow \ \psi_{(R)L} = \begin{bmatrix} 0 \\ \psi_{02\rho\sigma} \\ \psi_{30\rho\sigma} \\ 0 \\ \psi_{04\rho\sigma} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ \psi_{(R)R} = \begin{bmatrix} 0 \\ \psi_{43\rho\sigma} \\ \psi_{42\rho\sigma} \\ 0 \\ \psi_{32\rho\sigma} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{131}$$

Thus, assuming that the source-free cases of the relativistic (single-particle) wave equations (120) – (121) in five dimensional space-time (specified by (131)) must preserve chiral symmetry, implies the wave function's components $\psi_{10}, \psi_{41}, \psi_{31}, \psi_{21}$, $\psi_{10\rho}, \psi_{41\rho}, \psi_{31\rho}, \psi_{21\rho}$ and $\psi_{10\rho\sigma}, \psi_{41\rho\sigma}, \psi_{31\rho\sigma}, \psi_{21\rho\sigma}$ vanish, where, consequently, one of the spatial components (that is $\hat{p}_1$) of the covariant derivative vanishes as well. Thus the equations (120) – (121) are not formulable in five dimensional space-time, and reduced to (be equivalent to) their four dimensional cases, i.e. (130). **Therefore, based on the assumption (3) in Section 1-1, and also the unique structure of general relativistic single-particle wave equations derived, the universe is not definable in (1+4) dimensional space-time.**

**I**n addition, the size of matrices $\alpha_\mu$ and $\tilde{\alpha}_\mu$ in equations (120) – (121) in six dimensional space-time is 32×32, and the wave functions $\Psi_F$ and $\Psi_R$ representing by their left and right handed components, are given by (where $\mu, \nu, \rho, \sigma$ = 0,1,2,3,4,5; $\psi_{\mu\nu} = -\psi_{\nu\mu}$, $\psi_{\mu\nu\rho\sigma} = -\psi_{\nu\mu\rho\sigma}$):

$$\Psi_F = \begin{bmatrix} \psi_{(F)L} \\ \psi_{(F)R} \\ 0 \end{bmatrix} = \begin{bmatrix} \psi_{10} \\ \psi_{20} \\ \psi_{30} \\ 0 \\ \psi_{40} \\ 0 \\ 0 \\ 0 \\ \psi_{50} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \psi_{45} \\ 0 \\ \psi_{53} \\ \psi_{25} \\ \psi_{51} \\ 0 \\ \psi_{34} \\ \psi_{42} \\ \psi_{14} \\ \psi_{32} \\ \psi_{13} \\ \psi_{21} \\ 0 \end{bmatrix}, \quad \psi_{(F)L} = \begin{bmatrix} \psi_{10} \\ \psi_{20} \\ \psi_{30} \\ 0 \\ \psi_{40} \\ 0 \\ 0 \\ 0 \\ \psi_{50} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \psi_{(F)R} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \psi_{45} \\ 0 \\ \psi_{53} \\ \psi_{25} \\ \psi_{51} \\ 0 \\ \psi_{34} \\ \psi_{42} \\ \psi_{14} \\ \psi_{32} \\ \psi_{13} \\ \psi_{21} \\ 0 \end{bmatrix} \qquad \Rightarrow$$

$$\Rightarrow \psi_{10} = \psi_{20} = \psi_{30} = \psi_{40} = \psi_{50} = \psi_{45} = \psi_{53} = \psi_{25} = \psi_{51} = \psi_{34} = \psi_{42} = \psi_{14} =$$
$$= \psi_{32} = \psi_{13} = \psi_{21} = 0 \;\; \Rightarrow \;\; \psi_{(F)L} = \psi_{(F)R} = [0] \;;$$

$$\Psi_R = \begin{bmatrix} \psi_{(R)L} \\ \psi_{(R)R} \\ 0 \end{bmatrix} = \begin{bmatrix} \psi_{10\rho\sigma} \\ \psi_{20\rho\sigma} \\ \psi_{30\rho\sigma} \\ 0 \\ \psi_{40\rho\sigma} \\ 0 \\ 0 \\ 0 \\ \psi_{50\rho\sigma} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \psi_{45\rho\sigma} \\ 0 \\ \psi_{53\rho\sigma} \\ \psi_{25\rho\sigma} \\ \psi_{51\rho\sigma} \\ 0 \\ \psi_{34\rho\sigma} \\ \psi_{42\rho\sigma} \\ \psi_{14\rho\sigma} \\ \psi_{32\rho\sigma} \\ \psi_{13\rho\sigma} \\ \psi_{21\rho\sigma} \\ 0 \end{bmatrix}, \quad \psi_{(R)L} = \begin{bmatrix} \psi_{10\rho\sigma} \\ \psi_{20\rho\sigma} \\ \psi_{30\rho\sigma} \\ 0 \\ \psi_{40\rho\sigma} \\ 0 \\ 0 \\ 0 \\ \psi_{50\rho\sigma} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \psi_{(R)R} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \psi_{45\rho\sigma} \\ 0 \\ \psi_{53\rho\sigma} \\ \psi_{25\rho\sigma} \\ \psi_{51\rho\sigma} \\ 0 \\ \psi_{34\rho\sigma} \\ \psi_{42\rho\sigma} \\ \psi_{14\rho\sigma} \\ \psi_{32\rho\sigma} \\ \psi_{13\rho\sigma} \\ \psi_{21\rho\sigma} \\ 0 \end{bmatrix} \quad \Rightarrow \tag{132}$$

$$\Rightarrow \psi_{10\rho\sigma} = \psi_{20\rho\sigma} = \psi_{30\rho\sigma} = \psi_{40\rho\sigma} = \psi_{50\rho\sigma} = \psi_{45\rho\sigma} = \psi_{53\rho\sigma} = \psi_{25\rho\sigma} = \psi_{51\rho\sigma} =$$

$$= \psi_{34\rho\sigma} = \psi_{42\rho\sigma} = \psi_{14\rho\sigma} = \psi_{32\rho\sigma} = \psi_{13\rho\sigma} = \psi_{21\rho\sigma} = 0 \;\Rightarrow\; \psi_{(R)L} = \psi_{(R)R} = [0] \ .$$

Thus due to the chiral symmetry, in six dimensional space-time all the components of wave-functions $\Psi_F$ and $\Psi_R$ should vanish. Consequently, the equations (120) – (121) are not formulable in (1+5) dimensional space-time. Hence based on the assumption (3) in Section 1-1 and also the unique structure of the general relativistic particle wave equations derived, the universe is not definable in (1+5) dimensional space-time as well. There is precisely the same result for the higher dimensional cases of relativistic wave equations (120) – (121). Consequently, based on our axiomatic approach (including the basic assumptions (1) – (3) in Section 1-1, the universe cannot have more than four space-time dimensions.

## 3-8. On the Asymmetry of Left and Right Handed (interacting) Particles

**I**n Section 3-7, we concluded that the general relativistic (single-particle) wave equations (120) – (121) are definable solely in $\mathrm{D} \leq 4$ dimensional space-time. In particular, three and four dimensional cases of equations (120) – (121), i.e. equations (120-2) – (121-2) and (120-3) – (121-3), each equation particularly includes a set of equations that merely contain ***the divergences*** of wave-functions $\psi_{\mu\nu}$ and $\psi_{\rho\sigma\mu\nu}$. These sets of massive equations for the non source-free cases (i.e. for $J_{\nu}^{(E)} \neq 0$ , $J_{\nu\rho\sigma}^{(G)} \neq 0$) of equations (120) – (121), respectively are as follows:

For three dimensional case we obtain,

$$(120\text{-}2) \quad \Rightarrow \quad \left\{ (\breve{\nabla}_1 - \frac{im_0^{(E)}}{\hbar} k_1)\psi^1{}_0 = (\breve{\nabla}_0 + \frac{im_0^{(E)}}{\hbar} k_0)\varphi^{(E)} \right. \tag{133-1}$$

$$(121\text{-}2) \quad \Rightarrow \quad \left\{ (\breve{\nabla}_1 - \frac{im_0^{(G)}}{\hbar} k_1)\psi^1{}_{0\rho\sigma} = (\breve{\nabla}_0 + \frac{im_0^{(G)}}{\hbar} k_0)\varphi_{\rho\sigma}^{(G)} \right. \tag{133-2}$$

where we have (using (129)):

$$\Psi_F = \begin{bmatrix} \psi_{(F)L} \\ \psi_{(F)R} \end{bmatrix} = \begin{bmatrix} \psi_{10} \\ 0 \\ \psi_{21} \\ \varphi^{(E)} \end{bmatrix}, \ \psi_{(F)L} = \begin{bmatrix} \psi_{10} \\ 0 \end{bmatrix}, \ \psi_{(F)R} = \begin{bmatrix} \psi_{21} \\ \varphi^{(E)} \end{bmatrix},$$

$$\Psi_R = \begin{bmatrix} \psi_{(F)L} \\ \psi_{(F)R} \end{bmatrix} = \begin{bmatrix} \psi_{10\rho\sigma} \\ 0 \\ \psi_{21\rho\sigma} \\ \varphi_{\rho\sigma}^{(G)} \end{bmatrix}, \ \psi_{(F)L} = \begin{bmatrix} \psi_{10\rho\sigma} \\ 0 \end{bmatrix}, \ \psi_{(F)R} = \begin{bmatrix} \psi_{21\rho\sigma} \\ \varphi_{\rho\sigma}^{(G)} \end{bmatrix};$$

$$J^{(E)}_{\nu} = -(\breve{\nabla}_{\nu} + \frac{im_0^{(E)}}{\hbar} k_{\nu})\varphi^{(E)} \ , \ J_{\nu\rho\sigma}^{(G)} = -(\breve{\nabla}_{\nu} + \frac{im_0^{(G)}}{\hbar} k_{\nu})\varphi_{\rho\sigma}^{(G)} \ .$$

And for four dimensional cases, the mentioned sets of equations read,

$$(120-3) \ \Rightarrow$$

$$\begin{cases} (\breve{\nabla}_1 - \frac{im_0^{(E)}}{\hbar} k_1)\psi^1{}_0 + (\breve{\nabla}_2 - \frac{im_0^{(E)}}{\hbar} k_2)\psi^2{}_0 + (\breve{\nabla}_3 - \frac{im_0^{(E)}}{\hbar} k_3)\psi^3{}_0 = (\breve{\nabla}_0 + \frac{im_0^{(E)}}{\hbar} k_0)\varphi^{(E)} \ , \\ (\breve{\nabla}_1 + \frac{im_0^{(E)}}{\hbar} k_1)\psi_{23} + (\breve{\nabla}_2 + \frac{im_0^{(E)}}{\hbar} k_2)\psi_{31} + (\breve{\nabla}_3 + \frac{im_0^{(E)}}{\hbar} k_3)\psi_{12} = 0 \ ; \end{cases} \tag{134-1}$$

$$(121-3)\ \Rightarrow$$

$$\begin{cases}(\breve{\nabla}_1 - \frac{im_0^{(G)}}{\hbar}k_1)\psi^1{}_{0\rho\sigma} + (\breve{\nabla}_2 - \frac{im_0^{(G)}}{\hbar}k_2)\psi^2{}_{0\rho\sigma} + (\breve{\nabla}_3 - \frac{im_0^{(G)}}{\hbar}k_3)\psi^3{}_{0\rho\sigma} = (\breve{\nabla}_0 + \frac{im_0^{(G)}}{\hbar}k_0)\varphi_{\rho\sigma}^{(G)}\ , \\ (\breve{\nabla}_1 + \frac{im_0^{(G)}}{\hbar}k_1)\psi_{23\rho\sigma} + (\breve{\nabla}_2 + \frac{im_0^{(G)}}{\hbar}k_2)\psi_{31\rho\sigma} + (\breve{\nabla}_3 + \frac{im_0^{(G)}}{\hbar}k_3)\psi_{12\rho\sigma} = 0\end{cases} \quad (134\text{-}2)$$

where we have (using (130)):

$$\Psi_F = \begin{bmatrix}\psi_{(F)L}\\ \psi_{(F)R}\end{bmatrix} = \begin{bmatrix}\psi_{10}\\ \psi_{20}\\ \psi_{30}\\ 0\\ \psi_{23}\\ \psi_{31}\\ \psi_{12}\\ \varphi^{(E)}\end{bmatrix},\ \psi_{(F)L} = \begin{bmatrix}\psi_{10}\\ \psi_{20}\\ \psi_{30}\\ 0\end{bmatrix},\ \psi_{(F)R} = \begin{bmatrix}\psi_{23}\\ \psi_{31}\\ \psi_{12}\\ \varphi^{(E)}\end{bmatrix}\ ,$$

$$\Psi_R = \begin{bmatrix}\psi_{(R)L}\\ \psi_{(R)R}\end{bmatrix} == \begin{bmatrix}\psi_{10\rho\sigma}\\ \psi_{20\rho\sigma}\\ \psi_{30\rho\sigma}\\ 0\\ \psi_{23\rho\sigma}\\ \psi_{31\rho\sigma}\\ \psi_{12\rho\sigma}\\ \varphi_{\rho\sigma}^{(G)}\end{bmatrix},\ \psi_{(R)L} = \begin{bmatrix}\psi_{10\rho\sigma}\\ \psi_{20\rho\sigma}\\ \psi_{30\rho\sigma}\\ 0\end{bmatrix},\ \psi_{(R)R} = \begin{bmatrix}\psi_{23\rho\sigma}\\ \psi_{31\rho\sigma}\\ \psi_{12\rho\sigma}\\ \varphi_{\rho\sigma}^{(G)}\end{bmatrix};$$

$$J_\nu^{(E)} = -(\breve{\nabla}_\nu + \frac{im_0^{(E)}}{\hbar}k_\nu)\varphi^{(E)}\ ,\ J_{\nu\rho\sigma}^{(G)} = -(\breve{\nabla}_\nu + \frac{im_0^{(G)}}{\hbar}k_\nu)\varphi_{\rho\sigma}^{(G)}\ .$$

Equations (133-1) – (133-2) and (134-1) – (134-2) describe the relationship between the static fields (corresponding solely to the left handed components of fields $\psi_{\mu\nu}$, $\psi_{\rho\sigma\mu\nu}$) and the charged (or massive in case of gravitational field) particles as the sources of these fields. On this basis, massive and non source-free cases of the relativistic particle field equations (120) – (121) (including equations (133-1) – (133-2) and (134-1) – (134-2)) definitely, violate chiral symmetry of these relativistic (single-particle) wave equations. Moreover, this also expressly means that the sources of all the interacting massive fields should be made solely by the left-handed particles.

# 4. Conclusion

This article is based on my previous publications (Refs. [1], [2], [3], 1997-1998), and also my thesis work [4] (but in a new generalized and axiomatized framework). As it was mentioned in Sections 1 and 2, the main arguments and consequences presented in this article (particularly in connection with the mathematical structure of the laws governing the fundamental forces of nature) followed from these three basic assumptions:

**(1)- "Generalization of the algebraic axiom of nonzero divisors for integer elements (based on the ring theory and the matrix representation of generalized Clifford algebra, and subsequently, constructing a definite algebraic linearization theory);"**

This is one of the new and axiomatic concepts presented in this article (see Section 2-1, formula (23)).

**(2)- "Discreteness of the relativistic energy-momentum (D-momentum);"**

This is a basic quantum mechanical assumption. Quantum theory, particularly, tells us that energy and momentum are only transferred in discrete quantities, i.e., as *integer* multiples of the quantum of action (Planck constant) $h$.

**(3)- "The general relativistic massive forms of the laws governing the fundamental forces of nature, including the gravitational, electromagnetic and nuclear field equations, in addition to the relativistic (half-integer spin) single-particle wave equations, are derived solely by first quantization (as a postulate) of linearized (and simultaneously parameterized, as necessary algebraic conditions) unique forms of the relativistic energy-momentum relation – which are defined algebraically for a single particle with invariant mass $m_0$)."** We also assume that the source-free cases of these fields have "chiral symmetry".

**I**n Section 2, we presented a new axiomatic matrix approach based on the algebraic structure of ring theory (including the integral domains [5]) and the generalized Clifford algebra [40 - 47], and subsequently, we constructed a linearization theory. In Section 3, on the basis of this (primary) mathematical approach and also the assumption of discreteness of the relativistic energy-momentum (D-momentum), by linearization (and simultaneous parameterization, as necessary algebraic conditions), followed by first quantization of the special relativistic energy-momentum relation (defined algebraically for a single particle with invariant mass $m_0$), we derived a unique and original set of the general relativistic (single-particle) wave equations directly. These equations were shown to correspond uniquely to certain massive forms of the laws governing the fundamental forces of nature, including the Gravitational, Electromagnetic and Nuclear field equations (which based on our approach were solely formulable in (1+3) dimensional space-time), in addition to the (half-integer spin) single-particle wave equations (that were formulated solely in (1+2) dimensional space-time). Each derived relativistic wave equation is in a complex tensor form, that in the matrix representation (i.e. in the geometric algebra formulation, including equations (120) and (121)) it could was written in the form of two coupled symmetric equations – which assumedly have chiral symmetry if the particle wave equation be source-free. In fact, the complex relativistic (single-particle) wave equations so uniquely obtained, corresponded to certain massive forms of classical fields including the Einstein, Maxwell and Yang-Mills field equations, in addition to the (half-integer spin) single-particle wave equations such as the Dirac equation (where the Dirac spinor field is isomorphically re-presented solely by a tensor field in three dimensional space-time [29, 31]). We should note that the unique set of general relativistic wave equations derived by our approach doesn't include the Klein-Gordon equation - and so on - which describes the spinless elementary particles; thus based on our assumption (3) of Section 1-1, we may conclude that any spinless particle should be a composite particle (this includes the Higgs particle as well, however the Higgs mechanism could be formulated by a composite Higgs particle as well). In particular, in Section 3-5, a unique massive form of the general theory of relativity – with a definite complex torsion – was shown to be obtained solely by first quantization of a special relativistic algebraic matrix relation. Moreover, in Section 3-4-1, it was shown that the "massive" Lagrangian density of the obtained Maxwell and Yang-Mills fields could be also locally gauge invariant – where these fields were formally re-presented on a background space-time with certain (coupled) complex torsion which is generated by the invariant mass of the gauge field carrier particle. Subsequently, in agreement with certain experimental data, the invariant mass of a particle (that actually would be identified as massive photon) was specified (formula (115-4)), which is coupled with background space-time geometry. In Section 3-7, based on the unique mathematical structure of the general relativistic single-particle fields derived (i.e. equations (120) - (121)) and also the assumption of chiral symmetry as a basic discrete symmetry of the source-free cases of these fields, we showed that the universe cannot have more than four space-time dimensions. Furthermore, in Section 3-8, a basic argument for asymmetry of the left and right handed (interacting) particles was presented. In addition, in Section 3-4-2, on the basis of definite mathematical structure of the field equations derived, we also concluded that magnetic monopoles – in contrast with electric monopoles – could not exist in nature.

**T**he results obtained in this article, demonstrate the efficiency of linearization theory as a new mathematical axiomatic approach formulated for certain algebraic structures (presented in Section 2) and a wide range of its possible applications in mathematics and fundamental physics.

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# Appendix A.

**A**s it was mentioned in Sections 3-3 and 3-4, the field strength tensor $X_{\mu\nu\rho..\zeta}$ in the derived general relativistic single-particle field equations (101-1) – (101-4), for (1+3) and higher space-time dimensions should be presentable by a formula of the type: $X_{\mu\nu\rho..\zeta} = \hat{A}_\nu \hat{B}_{\mu\rho..\zeta} - \hat{A}_\mu \hat{B}_{\nu\rho..\zeta}$ , for some quantities $\hat{A}_\mu$ , $\hat{B}_{\nu\rho..\zeta}$ , where this formula should be also "derivable" from the equations (101-1) – (101-4). As we show below, this condition follows from the conditions that appeared for integer parameters $s_i$ in relativistic matrix energy-momentum relations (95) – (96) (formulated for four and five space-time dimensions), i.e. conditions (95-1) and (96-1) – (96-5), and also taking into account the substitution rule (100) as part of the first quantization (as a postulate) procedure and shows that there is one-to-one correspondence between "the parameters $s_i$ " and "the components of field strength tensor $X_{\mu\nu\rho..\zeta}$ and the components of covariant quantities $\varphi^{(X)}_{\rho..\zeta}$ (which by formula (101-3) defines a covariant current as the source of field $X_{\mu\nu\rho..\zeta}$ )". We should note that in the same manner, for the higher dimensional cases of matrix energy-momentum relations (obtained in Section 3-1), there are the similar conditions.

First, using the solutions (73) and (84) obtained in Section 2-6 for quadratic equations (68) and (75), the general solutions of conditions (95-1) and (96-1) – (96-5) can be written as follows, respectively:

$$
\begin{aligned}
& s_1 = u_3 v_4 - u_4 v_3, \quad s_2 = u_2 v_4 - u_4 v_2, \\
& s_3 = u_1 v_4 - u_4 v_1, \quad s_5 = u_2 v_1 - u_1 v_2, \\
& s_6 = u_1 v_3 - u_3 v_1, \quad s_7 = u_3 v_2 - u_2 v_3, \quad s_4 = 0, \\
& s_8 : \ \textit{arbitrary parameter}
\end{aligned}
\tag{A-1}
$$

and

$$
\begin{aligned}
& s_1 = u_4 v_5 - u_5 v_4, \quad s_2 = u_3 v_5 - u_5 v_3, \quad s_3 = u_5 v_2 - u_2 v_5, \\
& s_4 = 0, \quad s_5 = u_5 v_1 - u_1 v_5, \quad s_6 = 0, \quad s_7 = 0, \quad s_8 = 0, \quad s_9 = 0, \\
& s_{10} = u_2 v_1 - u_1 v_2, \quad s_{11} = u_3 v_1 - u_1 v_3, \quad s_{12} = u_1 v_4 - u_4 v_1, \\
& s_{13} = u_2 v_3 - u_3 v_2, \quad s_{14} = u_4 v_2 - u_2 v_4, \quad s_{15} = u_4 v_3 - u_3 v_4, \\
& s_{16} : \ \textit{arbitrary parameter}
\end{aligned}
\tag{A-2}
$$

The substitution rule (100) as part of the first quantization procedure of matrix relations (95) and (96) (along with the obtained field equations (101-1) – (101-4) by first quantization of relations (92) – (96) and so on), and also the general solutions (A-1) and (A-2) imply, respectively:

$$s_1 \mapsto \hat{s}_1 = X_{10\rho..,\zeta} = \hat{A}_0 \hat{B}_{1\rho..,\zeta} - \hat{A}_1 \hat{B}_{0\rho..,\zeta}, \quad s_2 \mapsto \hat{s}_2 = X_{20\rho..,\zeta} = \hat{A}_0 \hat{B}_{2\rho..,\zeta} - \hat{A}_2 \hat{B}_{0\rho..,\zeta},$$

$$s_3 \mapsto \hat{s}_3 = X_{30\rho..,\zeta} = \hat{A}_0 \hat{B}_{3\rho..,\zeta} - \hat{A}_3 \hat{B}_{0\rho..,\zeta}, \quad s_4 \mapsto \hat{s}_4 = 0, \quad s_5 \mapsto \hat{s}_5 = X_{23\rho..,\zeta} = \hat{A}_3 \hat{B}_{2\rho..,\zeta} - \hat{A}_2 \hat{B}_{3\rho..,\zeta},$$

$$s_6 \mapsto \hat{s}_6 = X_{31\rho..,\zeta} = \hat{A}_1 \hat{B}_{3\rho..,\zeta} - \hat{A}_3 \hat{B}_{1\rho..,\zeta}, \quad s_7 \mapsto \hat{s}_7 = X_{12\rho..,\zeta} = \hat{A}_2 \hat{B}_{1\rho..,\zeta} - \hat{A}_1 \hat{B}_{2\rho..,\zeta}, \quad s_8 \mapsto \hat{s}_8 = \varphi^{(X)}_{\rho..,\zeta} \quad ,$$

*where*

$$s_1 = A_0 B_1 - A_1 B_0, \quad s_2 = A_0 B_2 - A_2 B_0, \quad s_3 = A_0 B_3 - A_3 B_0, \quad s_4 = 0,$$

$$s_5 = A_3 B_2 - A_2 B_3, \quad s_6 = A_1 B_3 - A_3 B_1, \quad s_7 = A_2 B_1 - A_1 B_2,$$

$s_8$ : *an arbitrary parameter* ;

*and*

$$A_0 = v_4, \quad B_0 = u_4, \quad A_1 = v_3, \quad B_1 = u_3,$$

$$A_2 = v_2, \quad B_2 = u_2, \quad A_3 = v_1, \quad B_3 = u_1. \tag{A-3}$$

$$s_1 \mapsto \hat{s}_1 = X_{10\rho..\zeta} = \hat{A}_0\hat{B}_{1\rho..\zeta} - \hat{A}_1\hat{B}_{0\rho..\zeta}, \quad s_2 \mapsto \hat{s}_2 = X_{02\rho..\zeta} = \hat{A}_2\hat{B}_{0\rho..\zeta} - \hat{A}_0\hat{B}_{2\rho..\zeta},$$

$$s_3 \mapsto \hat{s}_3 = X_{30\rho..\zeta} = \hat{A}_0\hat{B}_{3\rho..\zeta} - \hat{A}_3\hat{B}_{0\rho..\zeta}, \quad s_4 \mapsto \hat{s}_4 = 0, \quad s_5 \mapsto \hat{s}_5 = X_{04\rho..\zeta} = \hat{A}_4\hat{B}_{0\rho..\zeta} - \hat{A}_0\hat{B}_{4\rho..\zeta},$$

$$s_6 \mapsto \hat{s}_6 = 0, \quad s_7 \mapsto \hat{s}_7 = 0, \quad s_8 \mapsto \hat{s}_8 = 0 \quad s_9 \mapsto \hat{s}_9 = 0, \quad s_{10} \mapsto \hat{s}_{10} = X_{43\rho..\zeta} = \hat{A}_3\hat{B}_{4\rho..\zeta} - \hat{A}_4\hat{B}_{3\rho..\zeta},$$

$$s_{11} \mapsto \hat{s}_{11} = X_{42\rho..\zeta} = \hat{A}_2\hat{B}_{4\rho..\zeta} - \hat{A}_4\hat{B}_{2\rho..\zeta}, \quad s_{12} \mapsto \hat{s}_{12} = X_{41\rho..\zeta} = \hat{A}_1\hat{B}_{4\rho..\zeta} - \hat{A}_4\hat{B}_{1\rho..\zeta},$$

$$s_{13} \mapsto \hat{s}_{13} = X_{32\rho..\zeta} = \hat{A}_2\hat{B}_{3\rho..\zeta} - \hat{A}_3\hat{B}_{2\rho..\zeta}, \quad s_{14} \mapsto \hat{s}_{14} = X_{31\rho..\zeta} = \hat{A}_1\hat{B}_{3\rho..\zeta} - \hat{A}_3\hat{B}_{1\rho..\zeta},$$

$$s_{15} \mapsto \hat{s}_{15} = X_{21\rho..\zeta} = \hat{A}_1\hat{B}_{2\rho..\zeta} - \hat{A}_2\hat{B}_{1\rho..\zeta}, \quad s_{16} \mapsto \hat{s}_{16} = \varphi^{(X)}_{\rho..\zeta},$$

*where*

$$s_1 = A_0B_1 - A_1B_0, \quad s_2 = A_2B_0 - A_0B_2, \quad s_3 = A_0B_3 - A_3B_0,$$

$$s_4 = 0, \quad s_5 = A_4B_0 - A_0B_4, \quad s_6 = 0, \quad s_7 = 0, \quad s_8 = 0, \quad s_9 = 0,$$

$$s_{10} = A_3B_4 - A_4B_3, \quad s_{11} = A_2B_4 - A_4B_2, \quad s_{12} = A_1B_4 - A_4B_1,$$

$$s_{13} = A_2B_3 - A_3B_2, \quad s_{14} = A_1B_3 - A_3B_1, \quad s_{15} = A_1B_2 - A_2B_1,$$

$s_{16}$ : *an arbitrary parameter* ;

*and*

$$A_0 = v_5, \; B_0 = u_5, \; A_1 = v_4, \; B_1 = u_4, \; A_2 = -v_3,$$
$$B_2 = -u_3, \; A_3 = -v_2, \; B_3 = -u_2, \; A_4 = v_1, \; B_4 = u_1. \tag{A-4}$$

Thus, we should try to derive and specify the quantities $\hat{A}_\mu$ and $\hat{B}_{\nu\rho..\zeta}$ from the field equations (101-1) – (101-4), for various rank (such as $F_{\mu\nu}, H_{\mu\nu\rho}, R_{\mu\nu\rho\sigma},...$) of the general field strength tensor $X_{\mu\nu\rho..\zeta}$. As it was mentioned in Section 3-4, the case of rank four of tensor $X_{\mu\nu\rho..\zeta}$ is compatible with Riemann curvature tensor, and we principally assume that these are equivalent. So, let we start with the Riemann curvature tensor (as a mathematical tensor with a definite structure), we have

$$R^{\rho}{}_{\sigma\mu\nu} = (\partial_\nu \Gamma^{\rho}{}_{\mu\sigma} + \Gamma^{\rho}{}_{\nu\lambda}\Gamma^{\lambda}{}_{\mu\sigma}) - (\partial_\mu \Gamma^{\rho}{}_{\nu\sigma} + \Gamma^{\rho}{}_{\mu\lambda}\Gamma^{\lambda}{}_{\nu\sigma}) \tag{A-5}$$

$$\Rightarrow \qquad R_{\rho\sigma\mu\nu} = (\partial_\nu \Gamma_{\rho\mu\sigma} - \Gamma^{\lambda}{}_{\nu\rho}\Gamma_{\lambda\mu\sigma}) - (\partial_\mu \Gamma_{\rho\nu\sigma} - \Gamma^{\lambda}{}_{\mu\rho}\Gamma_{\lambda\nu\sigma}) \tag{A-6}$$

From (A-5) and (A-6) we get

$$R_{\rho\sigma\mu\nu} = (\hat{A}_\nu \hat{B}_{\rho\mu\sigma} - \hat{A}_\mu \hat{B}_{\rho\nu\sigma}) \qquad \text{(A-7)}$$

where
$$\hat{A}_\mu = \widehat{D}_\mu, \quad \hat{B}_{\rho\mu\sigma} = \Gamma_{\rho\mu\sigma}. \qquad \text{(A-8)}$$

$$R_{\rho\sigma\mu\nu} = \widehat{D}_\nu \Gamma_{\rho\mu\sigma} - \widehat{D}_\mu \Gamma_{\rho\nu\sigma} \qquad \text{(A-9)}$$

or
$$R^\rho{}_{\sigma\mu\nu} = (\hat{A}_\nu \hat{B}^\rho{}_{\mu\sigma} - \hat{A}_\mu \hat{B}^\rho{}_{\nu\sigma}) \qquad \text{(A- 10)}$$

where
$$\hat{A}_\mu = \widehat{D}_\mu, \quad \hat{B}^\rho{}_{\mu\sigma} = \Gamma^\rho{}_{\mu\sigma}. \qquad \text{(A-11)}$$

$$R^\rho{}_{\sigma\mu\nu} = \widehat{D}_\nu \Gamma^\rho{}_{\mu\sigma} - \widehat{D}_\mu \Gamma^\rho{}_{\nu\sigma} \qquad \text{(A-12)}$$

and $\widehat{D}_\mu$ is a differential operator $\widehat{D}_\mu$ given by

$$\widehat{D}_\mu A^{\alpha_1}{}_{\alpha_2\alpha_3 ...\alpha_n} = \partial_\mu A^{\alpha_1}{}_{\alpha_2\alpha_3 ...\alpha_n} + \Gamma^{\alpha_1}_{\mu\lambda} A^\lambda{}_{\alpha_2\alpha_3 ...\alpha_n} \qquad \text{(A-13)}$$

$$\widehat{D}_\mu A_{\alpha_1\alpha_2\alpha_3 ...\alpha_n} = \partial_\mu A_{\alpha_1\alpha_2\alpha_3 ...\alpha_n} - \Gamma^\lambda{}_{\mu\alpha_1} A_{\lambda\alpha_2\alpha_3 ...\alpha_n} \qquad \text{(A-14)}$$

As the next step, let we consider a rank two field strength tensor $F_{\mu\nu}$ (as a particular case of general field strength tensor $X_{\mu\nu\rho..\zeta}$, with a minimum rank). Hence from the field equations (101-1) – (101-4) we can obtain

$$F_{\mu\nu} = \breve{\nabla}_\nu A_\mu - \breve{\nabla}_\mu A_\nu, \qquad \text{(A-15)}$$

Where we can also re-write formula (A-15) as follows,

$$F_{\mu\nu} = \hat{A}_\nu \hat{B}_\mu - \hat{A}_\mu \hat{B}_\nu \qquad \text{(A-16)}$$

where

$$\hat{A}_\mu = \breve{\nabla}_\mu, \quad \hat{B}_\mu = A_\mu. \qquad \text{(A-17)}$$

For a rank three field strength tensor $H_{\mu\nu\rho}$ (as a special case of general field strength tensor $X_{\mu\nu\rho..\zeta}$) and higher rank field strength tensors, from the field equations (101-1) – (101-4) the formulas similar to (A-7) and (A-15) (which are determinable for rank two and rank four field strength tensors $F_{\mu\nu}, R_{\mu\nu\rho\sigma}$) could not be obtained. Therefore, we may conclude that $F_{\mu\nu}, R_{\mu\nu\rho\sigma}$ are the only field strength tensors which are compatible with formula (100) and the general field equations (101-1) – (101-4).

# Appendix B.

**I**n this Appendix we write (explicitly) the special relativistic cases of contravariant real matrices $\alpha_\mu$ and $\tilde{\alpha}_\mu$ – that are hermitian and generate Clifford algebras $C\ell_{1,1}$, $C\ell_{1,2}$, $C\ell_{1,3}$, $C\ell_{1,4}$ and so on – indicated in the general covariant single-particle wave equations (120) – (121). Note that for special relativistic cases of these equations, matrices $\tilde{\alpha}_\mu$ are simply given by relation $m_0 \tilde{\alpha}^\mu k_\mu = m_0 I$, where $I$ is the identity matrix. So, below only the matrices $\alpha_\mu$ are written (where the signature is (+ – –...–), see the sign conventions (97)):

Hence for (1+1) dimensional space-time we have

$$\alpha^0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \alpha^1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \tag{B-1}$$

For (1+2) dimensional space-time we get

$$\alpha^0 = \begin{bmatrix} \sigma^0 + \sigma^1 & 0 \\ 0 & -(\sigma^0 + \sigma^1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \quad \alpha^1 = \begin{bmatrix} 0 & \sigma^2 - \sigma^3 \\ -\sigma^2 + \sigma^3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix},$$

$$\alpha^2 = \begin{bmatrix} 0 & -\sigma^1 + \sigma^0 \\ \sigma^1 - \sigma^0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \tag{B-2}$$

Subsequently, for (1+3) dimensional cases of equations (120) – (121) (corresponding to particle field equations (105-1) – (105-4)) we have, respectively:

$$\alpha^0 = \begin{bmatrix} \gamma^0 + \gamma^1 & 0 \\ 0 & -(\gamma^0 + \gamma^1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix},$$

$$\alpha^1 = \begin{bmatrix} 0 & \gamma^2 - \gamma^3 \\ \gamma^2 - \gamma^3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\alpha^2 = \begin{bmatrix} 0 & \gamma^4 + \gamma^5 \\ \gamma^4 + \gamma^5 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\alpha^3 = \begin{bmatrix} 0 & \gamma^6 - \gamma^7 \\ \gamma^6 - \gamma^7 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \qquad \text{(B-3)}$$

In addition, matrices $\alpha^{\mu}$ for five dimensional cases of equations (120) – (121) (corresponding to equations (106-1) – (106-4)) are given by, respectively

$$\alpha_0 = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1
\end{bmatrix},$$

$$\alpha_1 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},$$

$$\alpha_2 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},$$

$$\alpha_3 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},$$

$$\alpha_4 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}, \tag{B-4}$$