A Study on the Cobb-Douglas Production Function - with an Application to the Rice Prduction in Hokkaido

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Since the 1940's, agricultural economists in Japan have become acquainted with production function analysis of the Cobb-Douglas type and have applied it mainly to paddy farms. The first work "On Productivity of Lador" by prof. K. Kamiya of Tokyo University appeared in 1941 in the "Journal of Rural Economics" vol. 17, No. 3.

In 1945, Prof. I. Okawa of Hitotsubashi College of Commerce and Economics came out with "The Theory and Measurement of Food Economy". He was concerned mainly about distributive shares of resource services and the degree of coincidence of production coefficients with them.

A former Professor of Hokkaido University, Dr. T. Watanabe, a strong advocate of marginal analysis came clearly to recognize that a linear homogeneous production function, especially of the Cobb-Douglas type, shows that cost equals total value of the product, assuming that factor prices equal their respective marginal productivities.* Thus the cost consideration enters the picture. After his work, problems of imputation have been taken up widely. Referring to Prof. E. O. Heady, "The problems of valuation are first those of marginal productivity analysis, and only second those of placing a price tag on specific factors".** Although problems of imputation are theoretically clear, before going on with problems of valuation, statistically significant and economically meaningful production function must be obtained.

Mr. Y. Yuwata of the Food Bureau made a contribution with his study on production functions of rice and barley in the "Quarterly Journal of Agricultural Economy" vol. 7, No. 2, in 1953.

The same type of production function analysis was made by Mr. Kiryu

* Journal of Rural Economics. vol. 21, No. 3.

** Economics of Agricultural Production and Resource Use. P. 403

of Shizuoka University in a monthly Journal of "Agriculture and Economy', in July, 1954.

This was followed by a study on the same subject by Mr. K. Tsuchiya of Tokyo University in the "Quarterly Journal of Agricultural Economy" vol. 9, No. 1, in Jannary, 1955. He also made a further contribution to the development of analytical tool for measurement of marginal efficiency of capital using marginal productivity of capital.

Many studies in this field have been made by the above-mentioned agricultural economists and at the Department of Agricultural Economics of Hokkaido University in recent years.

The first part of this paper deals with the studies on the Cobb-Douglas productiou function made by the above-mentioned researchers in chronological order with their evaluations. Further studeis on the subject by us are expounded, and questions are raised and discussed to some extent in the latter part of this paper.

Chronological Review on Production Function Analyses

First we take up the study on production function by Prof. K. Kamiya of Tokyo University.

In his study he discusses the main results of Dr. Douglas' work,* especially on the distributive aspect of it. Then he goes on to a regression analysis to obtain the relation among yields per tan, temperature, rate of sun-shine per month, and precipitation. Thus he gets the following relation among them.

$$X_1 - 2478 = 104\ (X_2 - 25.3) + 13.1(X_3 - 45.8) - 0.687\ (X_4 - 190) \quad (I.\ 1)$$

Here X_1, X_2, X_3, and X_4 denote yields in go, temperature of August in centigrade, rate of sun-shine of September in percentage, and precipitation of September in centimetre respectively.

This analysis gives us relationships among yields and natural conditions, but does not tell the productivity of other important production factors such as land, labor, and capital.

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He, them, turns to the Cobb-Douglas production function to get insights into productivities of land and labor.

To this purpose he employs the method of least squares, and gets the following functions for Tohoku District, and Seinan District the data for

* Douglas, P. H; The Theory of Wage, 1934.

which are from "The Survey on the Management of Paddy Farms in 1939" by the Imperial Agricultural Society.

For Tohoku District :

$$P = 924\ T^{.73}\ L^{-.07} \qquad (I.\ 2)$$

and for Seinan District :

$$P = 1101\ T^{1.3}\ L^{-.53} \qquad (I.\ 2)$$

Here, P denotes receipts from rice after deduction of expenses such as fertilizers, seeds, and other materials in yen, T denotes area under cultivation in cho (ha.), and L denotes labor days.

From (I. 2) and (I. 3) above marginal labor productivity for both Districts is estimated to be minus. On this point Prof. Kamiya explains; "…we obtain a result that an increase in labor days brings about a decrease in yield, other things being equal. The conclusion that labor days should be decreased in order to get higher yield is brought about through the following logic. Fixity of labor, as I have pointed out previously, is a special character of family farms. This makes the variance of labor days of individual farms less than that of farming areas, together with the special combination, both social and economic, of land and farmers keep the relation among receipts, land, and labor in a special form. This family and land system and the self-sufficing system of farming gives a special character to farm management, thus bringing about the relation that farmers themselves purchase a high proportion of their own farm prodpct by their own labor, and it is understood that this causes negative marginal labor productivity". Further, he says : "…taking this (self demand for his own products) into account other production functions are worked out. From them it is concluded that the positive marginal productivity of labor is obtained for the farms of more than four cho (ha.) in Tohoku District and for those of more than three cho (ha.) in Seinan District. … That is, productivity of labor becomes meaningful at those size levels mentioned above."

This study was really a stepping stone to further study on this subject. There are many points to be refined, and more statistical significance and more economic meaning than is pursued in his study should be asked for the development of the production function analysis. The next study on this subject was made by Prof. Okawa, and we shall review it briefly.

Professor Okawa's work on production function was really epoch-making. He used the Cobb-Douglas production function in many articles of the book. However, in this short paper, the production function used only with respect to production and distribution will be discussed. Thus, chapter 8 of the book, "Marginal Productivity and Income Formation of Summer Crop…Paddy Farm Rent and Labor Income…," is taken up.

We shall first follow his statement.

"Of the agricultural economy of our country distributive problems such as a high farm rent, low labor wage and so on have strongly attracted the attention of so many people since agricultural crisis after the World War I. Recently the problem of productivity has put a heavy weight because of the strategic request by the whole nation for an increased production. This is the natural academic response to the needs of the times. However, from the view point of economic theory, the problem of productivity and that of income formation are to be understood in unity"

He argues that the unified method of grasping two prodlems of production and distribution in unity is the product of modern economics. Many works on the historical basis have been done, but he thinks that the tendency of the theory and the explanation has been to give too much importance and emphais to the uneconomic factors. And he argues that the proponents of the so-called "Small Farms Theory" have made clear the various characteristics of Japanese agriculture, but tney have been narrow-minded in applying modern economics to their studies. Then, he explains an obligation of applied scientists: "One of the direct obligations of practical studies in empirical science is to extend the applicable range of the general law as wide as possible." And as a tool of the analysis he makes it clear that he uses the Cobb-Douglas production function. However, he does not attempt to discuss the theory of marginal productivity with respect to the national economy as a whole, thus confining the analysis in agriculture.

He starts with the Walras' or Wicksteed formula such as;

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$$X_1 = F\ (X_2, X_3, X_4, \cdots\cdots) = X_2\frac{\delta F}{\delta X_2} + X_3\frac{\delta F}{\delta X_3} + X_4\frac{\delta F}{\delta X_4} + \cdots\cdots \quad (I.\ 4)$$

He remarks that the Cobb-Douglas production function is very convenient in the measurement of marginal productivity and in the comparison of the rates of distribution of resource services in production.

Following Dr. Okawa we enter the problem of moeasurement.

He uses X_1, X_2, X_3 and X_4 as total yields in brown rice, total labor days, total paddy areas, and liquid capital per farm respectively. Sample number, in this case, is 339 out of vast area of Eastern Japan, as of 1937. He carried out same measurement for 1938, 1939, and 1940. Using the method of least squares, he odtained the following values of b_2, b_3 and b_4 for each year's production.

Table 1

	b_2	b_3	b_4	Sum
1937	.237±.017	.555±.024	.205±.017	.997
1938	.280±.021	.510±.024	.155±.018	.945
1939	.185±.017	.662±.020	.196±.014	.997
(1940)	(.222)	(.613)	(.075)	(.910)

Figures of 1940 are in brackets. This year was a very poor crop year, and it was considered inappropriate to take the figures as representative. The sum of the above three coefficients is very close to unity. And this fact is looked upon that the conditions of constant returns to scale are nearly fulfilled. Next comes the comparison of the coefficients with the rates of distribution of each resource servsce.

Table below shows the proportion of various cost items in the total costs.

Table 2

	year	rent %	wages %	capital %	(a) %	(b) %	Total %
Owner	1937	38.7	31.0	30.3	20.8	9.5	100.0
	1938	37.2	32.1	30.7	21.1	9.6	100.0
Operator	1939	29.1	38.5	32.4	21.2	11.2	100.0
	1940	26.3	38.5	35.2	24.2	11.0	10000
Tenant	1937	43.4	29.4	27.2	18.1	9.1	100.0
	1938	42.6	30.1	27.3	18.6	8.7	100.0
Farmer	1939	40.9	32.7	26.3	17.3	9.0	100.0
	1940	35.8	33.9	30.3	20.7	9.6	100.0

In this table rent consists of interest on capital invested in land and taxes and other public expenses for owner operator, and of farm rent in kind and taxes and other public expenses for tenant farmer. Included in (a) are expenses for unrsery bed, and in (b) are expenses for draft animals, land improvement, farm machineries and buildings. Wages for family labor are the estimated value equivalent to farm day laborer's wages.

Now he takes (b) into account, and argues that in the production function using 4 variables (b) must be included mostly in land. And,thus, the approximate values of three coefficients become .2 to .3, .4 to .5, and less than .3 for b_2, b_3, and b_4 respectively. Further he argues about the indivisibility of production factors, and reject the fact of existence of indivisibility. Constant returns to scale plays its full role. The fact that constant returns to scale is verified statistically means that there is no clear difference in technology of production among different sizes of farms. And this affirms, again, the existence and prevalence of small-sized family farms in Japan.

He argues that these findings give the assurance of validity of the theory of marginal productivity with respect to resource allocation and resource pricing. But his argument is very short and not clear. Also, he did not show how high is the marginal value product of each resource service and its market price.

From these findings and arguments it is not difficult to see that he was concerned more about the distributive aspects of production function analysis than about the operational or managemental aspects of it. Even though he points out the importance of the study on the relationships between marginal products of production factors and the market prices of the factors, in other words, substitution relationships among factors of production, he did not take up this problem explicily. This point is apparent from the sampling method. The sample areas cover large areas of Tohoku, Kanto, Hokuriku (Shimane, and Tottori prefectures inclusive), and Tosan Districts. Among these there are considered to be large differences in natural conditions and in the technology of rice cnltures. Further, we are not sure if homogeneity of samples is retained. It will not appropriate to use production fnuctions derived from these samples in the direction of helping improve farm management in a specific farming area.

However, this study ought to be highly appreciated in that it set a firm foundation for the establishment of modern applied eccnomics in agricultural economics in Japan.

Former Professor of Economics of Hokkaido University, Dr. Tadashi Watanabe, discussed production functions mathematically and theoretically in his papar mentioned previously. We skip mathematical part of the paper since we assume that most of readers are well acquainted with the subject.

To quote: "Most easily understandable equation among various ones is that of wicksteed's and Moore's,* which is rightly to be called the equation of costs of production or of rates of costs of production. Because each resource of production should so employed that the marginal product becomes equal to the marginal costs (based on the law of maximum profits or economic law), and it in turn is rewarded with the marginal product. Multiply it by the quantity employed, and the partial costs of one resource in the production is given. The percentage of the costs in the total costs is the rate of costs of production.······An important fact is that the rate of costs of production itself is the production coefficient of the Cobb-Douglas production function. This is pointed out, though rather vaguely, by I. Okawa. He applied Douglas' equation to the survey of costs data of production of agricultural products of our country, and concluded, like Douglas, that constant returns to scale exist, and thus called the relation as a relation of constant physical returns. Then he imagined that this expresses the rate of costs of production However, it turned out that this laborious round-about computation is nothing but the computation of the rate of costs of production."

He summarized his findings as follows:

1. The so-called law of production is the law of constant rate of costs of production.
2. The law of production, that is the law of constant rate of costs of production is static.
3. Resource with a high rate of costs of production does not necessary mean the efficient resource.

Under the restrictions of the Cobb-Douglas production function the finding 1 is legitimate, and the finding 2 is also legitimate. However, the third finding is not always clear because efficiency depends on efficiency criterion to be employed and there may be many efficiency criteria. He also seems to nulify the utility of the Cobb-Donglas production function,

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* Wicksteed's formula is obtained by using the Euler's Theorem:
$P = F(L, M, \cdots)$, thus $P = (\delta P/\delta L).L + (\delta y/\delta M).M + \cdots\cdots$
Moore's formula is given in the following way:
$I = (\delta P/\delta L)\ L/P + (\delta P/\delta M)\ M/P + \cdots\cdots\cdots$ and $I = \Sigma$ coefficients of prodaction
see H. L. Moore: Synthetic Economics, 1929
R. D. Allen: Mathematical Analysis for Economists, 1938

as is clear from the previous quotation. There have been many discussions on this point, and this problem seems to be of the nature that may be solved practically and not theoretically.

After the World war II economic conditions of Japan were complicated and chaotic. However, in less than ten years the economy recovered to some extent, and theoretical and applied studies on the economy began to attract economists' attention.

Mr. Yuwata's study was mentioned in the introductory part of this paper. He made the same study on rice and barley of Japan as Dr. Ohkawa. The results are tabulated in Table 111.

Table 111

	Rice			Barley		
	b_3	b_4	$b_3 + b_4$	b_3	b_4	$b_3 + b_4$
1949	.58	.41	.99	.453	.592	1.045
1950	.520	.449	.969	–	–	–
1951	–	–	–	.494	.655	1.149
Average	.56	.42	.98	.4–.6	.5–.6	1.05–1.10

Here b_3 denotes coefficient of labor input per <u>tan</u>, b_4 denotes coefficient of capital input per <u>tan</u>, b_2 is coefficient of land input in the study by Dr. I. Okawa.

Following is summary of his findings:

1. Consistent conclusions are not attained for barley production because of large errors in the data.
2. It is observed that production structure of rice production in Japan is relatively stable.
3. Marginal productivity of resources of rice production and barley production is not equal. And, even at present, paddy farms are more advantageous than the other.

It might be instructive to put a table of his work which shows the comparison of coefficients before and after the war.

Table Ⅳ

		Rie				Barley			
		b_3	b_4	b_2	$b_2+b_3+b_4$	b_3	b_4	b_2	$b_2+b_3+b_4$
Before	1937	.237	.205	.555	.997				
		(.531)	(.466)		(.997)				
	1938	.280	.155	.510	.945				
		(.609)	(.336)		(.945)				

	1939	.185	.190	.622	.997	.201	.372	.407	.980
		(.490)	(.507)		(.997)	(.350)	(.630)		(.980)
	Avg	.545	.436		.981	.350	.630		.980
After	1949	.58	.41		.99	.96	.126	.823	1.045
						(.453)	(.592)		
	1950	.52	.45		.97	.494	.655		1.149
	Avg.	.56	.42		.98	.473	.524		. 1.097

The same evaluation as that of the study by Dr. I. Okawa will be given to this study. And we shall skip it.

On these theoretical and applied economic foundations economists tried to make an evaluation of resources of production using the Douglas production function.

Mr. Kiryu of Shizuoka University made a study on the subject. He discusses that there are three ways of evaluating rent One is subtracting production expenses including expenses on land from gross farm income. However, he doubts if the price of rice is normal,and, moreover, if family labor expenses are a good estimation of labor expenses. If not then there will be a pure profit of rent quite different from the theoretical one. The second method is to multiply paddy field purchase price by rate of interest on investment in land. This method has disadvantages because there is scarcely a free market for land even though control on the purchase price of land is taken off the picture. Further, there is a control on farm rent by government regulation.

The last method is to use the theory of marginal productivity. He uses the following formula.

$$\text{price of land} = \frac{\text{Marginal productivity of land}}{\text{Marginal productivity of labor (capital)}} \times \text{price of labor (capital)} \qquad (I.\ 5)$$

Thus, he employs the Douglas production function. He works out the function as follows:

$$X_1 = 1.343\ X_2^{.542}\ X_3^{.113}\ X_4^{.314} \qquad (I.\ 6)$$

Here X_1, X_2, X_3 and X_4 denote products in koku, area of land in tan, labor hours, and capital in 1,000 yen, respectively. Here $R_{1.234} = .995$, sample number is 11, and the correlation is significant at 1% level.

Using an equation $P_2 = (b_2/b_4)(X_4/X_2)\ P_4$, he obtains the rent of 7,440 yen per tan. In this computation $X_4/X_2 =$ (4,164 + interest on the liquid capital), and $P_4 =$ unity. According to this computation farm rent accounts for 59.9% of the total costs per tan.

Mr Kiryu′s fundamental equations are a modified expression of the following equation system.

$$\frac{\delta X_1/\delta X_2}{P_2} = \frac{\delta X_1/\delta X_2}{P_3} = \cdots\cdots\cdots = \frac{\delta X_1/\delta_2 X_n}{P_n} \qquad (I.\ 7)$$

Here P_i is the unit price of the i-th resource service The equation system (I. 7) is a necessary condition for minimization of costs. And the purpose of (I. 7) is to secure correct factor allocation. In using this system of equations a P_i is assumed set at the market price, and other prices are obtained accordingly. However, it will be made clear that this mathod of pricing resource services has almost nothing to do with the exhaustion of product problem at all. Only when we take merginal value products of resource services at the arithmetic means of resource inputs we get to the exhaustion prodlem as well as residnal valuation of a factor problem. Thus Mr. Kiryu′s procedure is one-sided and erroneous.

However, this study made a step forward to the valuation problem. And this was followed by Mr. K. Tsuchiya of Tokyo University.

Mr. Tsuchiya′s paper "Production Function of Agriculture in Japan" dealt with the same Douglas production function, and traced the same logic as most of the above-mentioned researchers.

He criticizes Dr. Watanabe in that he almost nullified the utility of measurement of a production function of the Douglas type.

To quote: "…The case we are interestad in is the one in which an assumption of a static equiriblium is not fulfilled as the prices of resources of agricultural production do not coincide with the marginal products influenced by political and subjective factors such as legislated farm rent after the Land Reform and subjective estimation of family labor by the farmer etc. This situation is fairly different from what Dr. Watanabe had in mind. In such a cach as this, it is possible to measure marginal products only by measuring parameters of a production which is basad on the theory of marginal productivity and unifies the law of production with that of distribution."

The following table shows values of coefficients ofthe production functions he estimated.

Table Ⅴ

	b_2	b_3	b_4	sum	R	sample No.	Year
Rice	.56	.19	.15	1.00	.89	69	1951
Barley	1.10	0	0	1.10	.93	14	〃
Wheat	1.23	-.36	0	.87	.95	11	〃
Sweet potato	.85	.29	0	1.14	.94	11	〃
Tea	.29	.30	.46	1.05	.93	30	〃
Tangerine	1.06	0	0	1.06	.93	19	〃

Here b_2, b_3 and b_4 are parameters of area under cultivation of the crop, labor hours (hired labor inclusive), and liquid and fixed capital respectively.

Marginal value products, estimted at the arithmetic means of resource inputs of resource services by crops are illustrated in Table Ⅵ.

Table Ⅵ

	Land	Rent	Labor	Capital
unit	1 tan	1 tan	1 hr	100 yen
Rice	8.300 yau	585 yen	11 yen	41 yen
Barley	5,900	600	0	0
Wheat	11,900	528	-14	0
Sweet potato	18,100	579	34	0
Tea	11,500	597	29	51
Tangerine	79,200	723	0	0

Using these and other figures, he tried to answer various problems of agricultural productionin in Japan such as underemployment, capital and valuation of land. Methods he used to answer these problems were more or less similar to those employed by forerunners mentioned before, so they will not be discussed here.

Further, he tried to give a formula for the measurement of marginal efficiency of capital by using marginal value product of capital.

He argues that according to Lord Keynes marginal efficiency of capital is given by the following equation with C, supply price of capital, and Q_1, $Q_2 \cdots\cdots$, Q_n, prospective annual yield of investment.

$$C = Q_1/(1+m) - Q_2/(1+m)^2 - \cdots\cdots\cdots + Q_n/(1+m)^a \qquad (I.\ 8)$$

Here he assumes expectation of Q_1 is the same for all Q′s.

$$E(Q) = Q_1 = Q_2 = \cdots\cdots\cdots = Q_n \qquad (I.\ 9)$$

By reducing the right side (1.8) into one term, he gets

$$C = Q_1/(1/n + m),$$

and $m = (Q_1 - Cn)/C$ (I. 10)

Here, Q_1 equals marginal value product of capital.

Thus, he computed the marginal efficiency of capital for rice and tea production. For rice m is .0964, and for tea m is .395. He summarizes the above results as follows. Marginal efficiency of capital in barley, wheat, sweet potato, and tangerine production is negative, and it is natural that in these production sectors there is no expectation of capital investment from outside. And internal capital rationing is seen in rice production, and is external rationing observed in tea production.

Now we come back to and examine carefully his methods of measurement of marginal efficiency of capital.

We examine his equation (I. 9). He assumed that Q′s are all equal. However, this assumption is not consistent with Keynes′ definition of marginal efficiency of capital. To quote Lord Keynes: "Finally, there is the distinction, the neglect of whict has been the main cause of confusion and misunderstanding, between the increment of value obtainable by using an additional quantity of capital in the existing situation, and the series of increments which it is expected toobtain over the whole life of the additional capital asset;………i. e. the distinction between Q_1 and the complete series Q_1, Q_2……………Q_r………This involves the whole question of the place of expectation in economic theory. Most discussions of the marginal efficiency of eapital seem to pay no attention to any member of the series except Q_1. Yet this cannot be legitimate except in a static theory, for which all the Q′s are equal. The ordinary theory of distribution, where it is assumed that capital is getting now its marginal productivity (in some sense or other), is only valid in a stationary state…………"

By the assumption of equaltity of all Q′s Mr. Tsuchiya seems to have assumed the economic situation as a stationary or static state. If so, where can the marginal efficiency of capital play its role? If expectation cannot enter the picture what is the meaning of the marginal productivity or efficiency of capital? Lord Keynes further points out the probable case of misunderstanding in page 141, but, as it is rather long, we shall be satisfied with just stating that the main problem of his was really of dynamic nature that relates to changes in economic situations in the future such as changes in labor cost, i.e. in the wage unit, or from inventions and new techniques.

Thus, Mr. Tsuchiya's formula to measure the marginal efficiency of capital is nothing but an equation, modified to a certain purpose, which gives a discount rate when future income is known with certainty and the income is the same in each year.

Again we check the equation (I. 10). Here Q_1 is a marginal value product of both liquid and fixed capital. This quantity is not suitable to figuring out the marginal efficiency of capital, especially of fixed capital or capital equipment. Thus, because of the theoretical defects in the deduction and estimation of marginal efficiency of capital most empirical statements on capital rationing lose their foundation.

Heretofore, we discussed and evaluated studies on the Douglas production function made by Japanese researchers. We summarize this review as follows.

1. The Douglas production function has been used widely since 1940's.
2. The objectives in using the function were various, but they may fall under three categories.
    a. A search for coincidence of coefficients of production functions (b_2, b_3, b_4, $\cdots b_n$) with distributive shares of resources in the total costs.
    b. Valuation of factors.
    c. A search for efficiency in the use of the factors.

We will make a comment or two at this point and close this part.

Since the researchers' objectives were broad, they did their sampling of data in such a way that the date cover wide areas that include various climatic conditions and techniques of production in one population. In those instances where the area is restricted, the sample size was small. This restricts the use of the estimated production functions on farms.

Much discussion has been made by many researchers on the coincidence of marginal productivity of resources of production and proportion of cost of the resources in the total costs. However serious this problem may be to the theory itself, this is probably not a serious problem to people on farms.

objectives of production function studies have changed after the war; the valuation problem has come into picture. But the way of valuation is sometimes far from reality, and sometimes contradictory to the relevant theory.

There are various ways of utilizing the Douglas production function; political, managemental, and uses of an intermediate nature.

In general, the researchers were concerned with the political use of the function and only slightly interested in the improvement of farm businesses. We think this represents a defect in the use of the production function analysis in Japan.*

In this part, we review and discuss our results of the applcation of the Douglas functions to Hokkaido crop production, mainly rice production.**

The First Study.

The first study on rice production was made at a town near the city of Sapporo in 1956.

The sampling was done neither in random nor in stratified fashion. The samples were taken at a concentrated area of paddy farms. The number of farms in the sample was 25. However, most of the sample farms had some area of upland crop fields. Thus, in order to derive a single production function 8 farms were taken out as they had 50% or more of upland field in the total farming fields. We have now only 17 samples. And the objectives of this study were as follows.

1. To determine if there are significant differences in production coefficients between resources used in Hokkaido and the same resources in other prefectures.
2. To interpret the results in terms of economics.

The production function of the Cobb-Douglas type was used.

Sample means and variances are tabulated as follows.

$\overline{(\log X1)}=2.21113$	$\overline{(\log X2)}=2.76234$	$\overline{(\log X3)}=2.46181$	$\overline{(\log X4)}=4.89649$
(162.60)	(578.55)	(289.61)	(78,794.)
S11= .23883	S12= .20861	S13= .22003	S14= .16003
	S22= .32152	S23= .19369	S24= .15931
	S32=S23	S33= .22467	S34= .17414
	S42=S24	S43=S34	S44= .43613

* There are many agricultural economists who have been and are working on theoretical and empirical production functions. I sincerely regret that I could not cover all of them in the limited space of this paper.

** We have several empirical studies of production functions on upland crops in Hokkaido made by Mr. T. Kyono and Mr. Y. Maruyama of Hokkaido University. However, they are not taken up in this paper since the functions lack significance tests of the coefficients.

Here X_1, X_2, X_3 and X_4 refer to rice yields in <u>Hyo</u> (about 2 bushels), labor days, land in <u>se</u> (1/100 ha.),and fertilizer inputs in <u>yen</u> respectively.

The production function

$$X_1 = .686 X_2^{.1247} X_3^{.9019} X_4^{-.0388} \qquad \text{(II. 1)}$$

is worked out.

b1 significantly different from zero at 1% probability level.
b2 〃 〃 〃 〃 inbetween 20 to 30% 〃 〃.
b3 〃 〃 〃 〃 1% 〃 〃.
b4 〃 〃 〃 〃 more than 50% 〃 〃.

Now X4 is dropped off the production function, because this has a negligible effect on yields, and it might be possible to look upon b4 as zero. Thus X4 =1, and the original variables are reduced by one. A function with X1, X2 and X3 is worked out as follows.

$$X_1 = .527 X_2^{.12245} X_3^{.87377} \qquad \text{(II. 2)}$$

b1 significant at in between 1% probability level.
b2 〃 〃 20% to 30 〃 〃.
b3 〃 〃 1% 〃 〃.

Confidence limits of b2 and b3 are

$$-.08939 < b2 < .33384$$
$$.62087 < b3 < 1.12667.$$

From these figures and other figures in the previous parts of this paper, and taking smallness of number of samples into account, we might be safe not to say that the differences in production coefficients between Hokkaido rice farms and those of southern prefectures are significant.

Using Tintner's method of linearity test and the Crount method, linearity of the above function was tested. The hypothesis of linearity is not rejected at 1% level.

Interpretation of this function is rather difficult because of the smallness of sample size. However, one point that is clear from these data and results is that there is only a slight difference in land productivity among farmers in the sample. This finding reflects the fact that these farms are located on one soil class, alluvial soil, and most of them are old settlers with an almost uniform level of production.

Family labor is the main source of family farms. The most important feature of family farms in this area is that they are not concerned much about their own labor inputs. Hence, the snbjective wages of their labor is low. They work from April through November, but some report only 5 months of field work a year. Small sized farms are rather ambiguous on this point. On the other hand, large sized farms usually keep books and know how manydays they worked for each phase of the rice production activities. Consequently their labor days are rather small. This may be a causes of a low labor coefficient in our production function.

Fertilizer inputs depend on operator's judgment on the year's weather, prospects of yields and varieties they plant. Usually small sized farms use more fertilizers than large sized farms. As a matter of fact, fertilizer inputs distributed widely with a very small correlation with yields. ($S14/S44=.366$) In this sense, it is fairly difficult to derive a well-behaved fertilizer response function from farms such as we surveyed.

The linearity test indicates that there is no evidence to reject the hypothesis of constant returns to scale in this area.

If it is desired to determine the kind of factor combination between land and labor which gives the least cost or maximum profits, however, we are not sure if we can give a definite answer because the price of land and of labor is arbitrary and indeterminate.

This function was not successful in deriving useful results for farmers. Still it gives us information on the general structure of resource use in the rice producing area and of the efficiency of use of each resource service.

A little more operational meaning will be given to this function by figuring out marginal products of resource services. Marginal products of land and labor are as follows.

$MP_2=.55$ ($MVP_2=220$ yen) $MP_3=.767$ ($MVP_3=3,069$ yen)

The wage rate of hired farm labor was about ¥500 per day during rice seedling transplanting season and harvesting season in 1956. Two hundred and twenty yen may be a reasonable estimate of family labor wages per day, since there are many labor days that are spent loosely on farms during slack seasons of the rice production, and thus overestimated in the sense above mentioned. However, if confidence limits of production coefficients are taken into account, the coefficients will not give us information useful enough to provide recommendations to farmers.

This first study on the Douglas production function was not satisfactory. This was partly because of the smallness of sample number. This was inevitable because 1) the objectives of the field survey were not to derive production funcitons per se and 2) a systematic sampling method was not used.

The Second Study.

We still wanted to derive production functions of the Cobb-Douglas type out of farm data, and to grasp an over-all picture of Hokkaido rice production as a first approximation to a deeper understanding of rice prodnction economy of some specific areas of Hokkaido.

To this purpose we used as a source "Rice Production Cost Statistics, 1955" by the Statistics Bureau of the Ministry of Agriculture. The samples cover most of rice producing areas of Hokkaido, and the sample size was sixty.*

Objectives of this study are as follows.

1. To determine if a difference exists in production structure between large sized farms and small sized ones.
2. To estimate the marginal products of resource services, and to find if resources are being applied in optimum amounts.

The farms in the sample are book-keeping farms which have higher than average incomes and managerial capacities.

Sample means and variances for the large sized farms and the small sized farms *are tabulated as follows.*

Large sized farms:

$\overline{(\log X1)}=3.8656$	$\overline{(\log X2)}=3.4495$	$\overline{(\log X3)}=3.6129$
(7, 338. 4)	(2, 815. 1)	(4, 101. 0)
$S11=.8812$	$S12=.6957$	$S13=.5850$
	$S22=.7256$	$S23=.5634$
		$S33=.6378$
$\overline{(\log X4)}=3.2976$	$\overline{(\log X5)}=3.3837$	
(1, 984. 3)	(2, 419. 4)	
$S14=.5645$	$S15=.8983$	
$S24=.5806$	$S25=.7268$	
$S34=.5293$	$S35=.4918$	
$S44=.6564$	$S45=.6464$	
	$S55=1.9317$	

$\overline{(\log X1)}=3.5274$	$\overline{(\log X2)}=3.1642$	$\overline{(\log X3)}=3.3556$
(3, 368. 2)	(1, 459. 5)	(2, 267. 8)
$S11=2.1313$	$S12=1.9762$	$S13=1.7963$
	$S22=2.0199$	$S23=1.7967$
		$S33=1.8991$

* We owe greatly to Mr. Y. Maruyama, graduate student of Hokkaido University, for the selection of data and computation of the functions.

$\overline{(\log X4)} = 3.0148$ $\overline{(\log X5)} = 3.1523$

(1, 034. 7)	(1, 420. 0)
S14=2. 0287	S15=1. 6291
S24=2. 0390	S25=1. 6331
S34=1. 9366	S35=1. 4672
S44=2. 3582	S45=1. 6289
	S55=3. 2298

Here X1, X2, X3, X4, and X5, refer to gross income from rice production in ¥1000, land in 1/10 tan, hired and family labor hours, variable costs such as fertilizers, farm drugs etc. in 100 yen and fixed capital in 100 yen respectively.

The estimated production functions are as follows:

Large sized farms:

$$X1 = 2.1168\ X2^{.6172}\ X3^{.3647}\ X4^{-.1758}\ X5^{.1989} \qquad \text{(II. 3)}$$

Here b0 significantly different from zero at 1% probability level.
b2 〃 〃 〃 〃 〃 〃 〃
b3 〃 〃 〃 〃 in between 2 to 5% 〃 〃.
b4 〃 〃 〃 〃 〃 30 to 40% 〃 〃.
b5 〃 〃 〃 〃 1% 〃 〃 .

Small sized farms:

$$X1 = .8257\ X2^{.8043}\ X3^{.0972}\ X4^{.0737}\ X5^{.0164} \qquad \text{(II. 4)}$$

Here bo significantly different from zero at 1% probability level.
b2 〃 〃 〃 〃 〃 〃
b3 〃 〃 〃 in between 50 to 60% 〃 〃.
b4 〃 〃 〃 〃 60 to 70% 〃 〃.
b5 〃 〃 〃 〃 70 to 80% 〃 〃.

From (II. 3) and (II. 4) and the siginificance tests we may conclude that the production structures of these two stratified farms are different to the extent that will be discussed below.

In the producion function for small sized farms production coefficients of X3, X4, and X5 are considered to be zero. This means that in small sized farms 1) there are more family labor hours available and used than are required for farming small paddy fields efficiently, 2) fertilizer inputs are in excess of the optimum because farmers in this size group are trying to raise yields as much as possible by using more and more of nitrogenous fertilizers, and 3) they have almost the same sets of farm machinery irrespective of their size differences. Thus, it is natural that the only factor which plays an important role in production is land.

On the other hand, large sized farms have a different production behavior. Land has a relatively high producton coefficient. However, labor and fixed capital play an equally important role in producton. Fertilizer inputs are in the same situation as for small sized farms. In general, we can say that the average level of fertilizer inputs is greater than its optimum level of application.

Marginal value products of resource services of land, labor, and fixed capital are figured out at the arithmetic means of resource inputs. They are as follows:

MVP2 = ¥16,080/tan MVP3 = ¥65.26/hour MVP5 = ¥.6033/yen

We worked out a different production function omitting X4 based on the same logic as was used in deriving (II. 2) from (II. 1). The function is as follows.

$$X1 = 1.996\ X2^{.5480}\ X3^{.2906}\ X5^{.1848} \qquad (II.\ 5)$$

Here b0 significantly different from zero at 1% probabiltty leval.

b2	〃	〃	〃	〃	〃	〃	
b3	〃	〃	〃	in between 5 to 10%	〃	〃.	
b5	〃	〃	〃	1%	〃	〃.	

The marginal value products are:

MVP2 = ¥;4,284/tan MVP3 = ¥51.999/hour MVP5 = ¥.5605/yen.

These figures give us understimation for all MVPi in comparison with those derived from (II. 3). This is because for both functions (II. 3) and (II. 5) the hypothesis of linearity holds and a variable with a negative coefficient was taken out of the original function (II. 3). Instead of discussing negative marginal value products of some resource services and drawing economic meanings of non-negative marginal value products of the rest of resource services keeping the variable with negative coefficients in the equation, it might be better to discuss positive marginal value products only from the view point of achieving an efficient economic alloction or combination of resources with positive market prices, after rearranging relevant variables so that they may have positive or non-negative production coeffcients. In this sense, we use the marginal value products derived from the equation (II. 5) in the following discussion.

For large sized farms the marginal value product of a labor hour is about ¥52, and this estimate is considered to be a good estimate of family labor productivity in terms of opportunity cost since a hired farm labor cost was at about the same level (¥500 per day) as this estimate assuming that labor hours of a memder of farm families are 10. This finding is affirmed by the fact that in these days farm opertors are rather inclined to hire day or resident laborers on seasonal or yearly basis to do the operator's job in the central paddy farming areas of Hokkaido.

Fixed capital has a high return of about 56 yen per 100 yen. As fixed capital is represented by total amount of capital invested in farm machinery and buildings which were used for year's prodnction of the specific crop, this fact is equivalent to saying that the farmers can replace this part of capital in about two years. This figure gives a very optimistic impression as to the profitability of investment in farm machineries which are the largest part of the capital item. In mathematical form, a rate of annual net retnrns from investment in machinery per yen, at its arithmetic mean, is

$$\frac{(\delta\overline{X}1/\delta\overline{X}5) - (1/n)}{1} = .5605-(1/n) \qquad \text{(II. 6)}$$

Thus, if we assume that the machinery life time (n) is five years on an average, we can still expect 36.05% of net returns on each unit of capital investment in machineries on an average.

Test for linearity is carried out and the function (II. 5) is found to be safely called a linear homogeneous production function. This finding is not new, and we can say that constant returns to scale are prevailing in rice prodnction in Hokkaido.

This study suggests the possibility of deriving statistically significant and economically meaningful production functions in a rice producing area homogeneous in its climatic and soil conditions.

The Third Study.

We selected a town where we made the first study on production function. Thirty six sample farms were selected according to the stratified sampling method. Our objectives, in this study, were as follows.

1. To study if statistically significant and economically meaningful production functions may be derived from farm data from a population consisting of a small area, a town.
2. To estimate marginal products of resource services and interpret the results.
3. If possible, apply these results for the improvement of farm management of this area.

In this study machinery capital is not taken as stock capital or total amount of capital after each asset value of capital is summed up, but as flow capital.

Each machinery is assessed at the present value, and the life time is taken on the specific farm. Thus, on each machinery a linear depreciation is calculated. The summation of the depreciation values gives the value of fixed capital in terms of annual flow inputs.

Sample means and variances are as follows.

$\overline{(\log X1)}=2.9780$	$\overline{(\log X2)}=2.5354$	$\overline{(\log X3)}=2.8205$	$\overline{(\log X4)}=2.8893$
(950.52)	(343.12)	(832.63)	(775.00)
S11=.3553	S12=.3218	S13=.2574	S14=.2512
	S22=.3060	S23=.2284	S24=.2486
		S33=.6149	S34=.2904
			S44=.7313

$\overline{(\log X5)}=2.7740$
(594.25)
S15=.2402
S25=.2436
S35=.3536
S45=.3369
S55=.5756

Thus a production function of the Cobb-Douglas type is computed.

$$X1=2.050\ X2^{1.0661}\ X3^{.0684}\ X4^{-.0153}\ X5^{-.0669} \qquad \text{(II. 7)}$$

Here X1, X2, X3, X4 and X5 refer respectively to gross income from rice production in 1,000 yen. land in 1/10 tan, labor (hired labor inclusive) in hour, machinery capital in 100 yen, and fertilizer and farm drugs capital in 100 yen.

b0	Siginificantly	different	from	zero at 1%	probability	level.	
b2	〃	〃	〃	〃	〃	〃	
b3	〃	〃	〃	more than 50%	〃	〃	
b4	〃	〃	〃	〃	〃 〃	〃	
b5	〃	〃	〃	〃	〃 〃	〃	

Average farm size of the sample farms is 3.43 ha and is slightly larger than that of sample farms in our second study. Still this function shows poor results of significance tests.

Our objectives were not fulfilled to the full extent. However it is still possible to derive economic inferences out of these results. Before discussing the function and the results on a purely theoretical basis, it is well to listen to the opinions of agricultural leaders and exteusion agents of the town. They all say that farmers use too much fertilizer without paying consideration to the scientific facts on the nature of yield response to fertilizer inputs. Further, peat land improvement is urgently needed to increase yields.

The first of these statements reflects our findings in the above production function analysis. The coefficient of fertilizer inputs is considered to be zero, which means that, on the average, fertilizer inputs have almost no or negligible effect on yields. On the other hand, land has a strong influence on yields. In this study we could not consider the role of peat land improvement and land improvement in general and their effect on yields. The importance of land in rice production cannot be overstimated.

Capital investments in farm machinery are too high. This reflects the fact that all farms keep a set of threshing, dehulling and polishing machines irrespective of their farm size. The use of these machines is limited to a short period of time after harvest. Physical wear-out is very small, however obsolescence plays an important role in this area. Other machines such as plows, weeding machines, hand-sprayers etc. do not cost much. Thus the farmers have almost the same amounts of machinery (in flow) capital. In this size category it is usually true that indivisibility of factors of production, in this case, machinery capital is found. This fact might be explained in this way. The price of rice is fixed by governmental regulation, and this stimulates farmers to produce more rice. Befors the World War II tenant farmers suffered from a heavy burden of farm rent which amounted to 40 to 60% of total annual producs. After the War, however, the burden was taken off as a consequence of Land Reform, and it turned farmers' income. A part of this income has been used for more household expenditures and for new houses, and the rest has been invested in farm machinery, and/or fertilizers. In addition to this there is an incentive (premium) payment to early delivered rice under government regulation, the amount of payment by delivery date is as follows.

Date of delivery	Incentive payment to 5 bushels of brown rice
dy Oct 1st	¥ 800
〃 10 th	¥ 600
〃 20 th	¥ 400
〃 31 th	¥ 200

Even though this incentive payment is small compared with ¥ 10,000 of 5 bushels' brown rice price, most of farmers are eager to deliver rice as early as possible. On an average they receive ¥ 10,000 more per ha. assuming that they deliver rice evenly during October and yields per ha. are a hundred bushels. Thus an average farm with 3.4 ha. of paddy fields can gain ¥34,000 more than usual. If they do not have a dehulling machine they cannot enjoy this opportunity. It is natural that the farmers in this area keep a well mechanized motor or engine driven thresher, dehulling machine and even a polishing machine. This capital situation will continue as long as this kind of price system continues.

It is believed that the economic inferences from this porduction function analysis relate closely to reality, even though the production coefficients of the resource services except land are ecnsidered to be zero.

Heretofore we used the Cobb-Douglas production function as a tool of diagnosis of farms. We could derive a statistically significant production function. However we couldn't use this function for remedial purposes since the representativeness of data is suspected. Again we derived a production function at a specific area, which was not statistically satisfactory. And, again, the use is confined to a diagnosis of general resource productivity on the farms.

How to go about improving this situation of both production function analysis and reality is the subject of the next part of this paper.

The Codd-Douglas production function studies on Hokkaid crop production, especially rice production, have been discussed heretofore, and many interesting problems have arisen in the course of the analyses.

One of the most important problems is the fertilizer use problem. This problem has complicated roots deep in the whole body of rice production as was mentioned in the previous part of this paper. Derivation of production function from farm data may not solve this problem since the data show only the present fertilizer inputs status and do not point to the direction in which the fertilization techniques are rationalized in an economic sense. The only way we shall be able to cope with this situation is, we believe, to use experimental data available at several Experiment Stations in Hokkaido. We are trying to derive fertilizer response functions from the experimental data with respect to rice this year. If farmers fail to show interest in our recommendations based on the experimental results, we are, then, going to make the same experiments on a representative farm in the area. These experiments will be of a demonstration nature, and stronger and more persuasive recommendations may be given to farmers than when only experimenaal results are available.

The machinery situation will not change or be improved in a short period of time. Of farm machines, small sized tillers and tractors are being used in paddy field of Hokkaido. Substitution of tractors for horses has to be studied on the basis of comparative costs. In this case, soil fertility must be carefully investigated and between two situations, with barn yard manure and without it, since barnyard manure keeps and improves soil structure and its fertility. Taking these factors into account, we are making a study on farm mechanization, especially on the tiller and tractor.

Labor productivities in our first and third study are estimated to be almost zero. This finding is partly due to our poor questioning of labor days on each farm, and partly due to a part of farmers' negligence of labor efficiency in that they are not always careful of the optimum labor intensity on each production activity. This may be improved by changing our questionnairs so that they may answer our questions more exactly than before.

There are many farm management problems to be solved such as crop combination, rotation, dairy management, and the subsequent problems. Thus, there is a wide horizon to production economists in Japan to make large contribution in these problem areas. Linear programming will be effectively used in upland craps producing areas in Hokkaido where farm size is larger than other prefectures of Japan.