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Alternative Programming Models for  
Interregional Analysis

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Thanks to vigorous developments in modern economic theory, we did not experience such rampant inflations after World War II as we did after World War I, nor have we suffered from such bitter depressions as we did in 1930's. As economic theory proves to be successful in controlling the aggregate behavior of the economy, economists have been encouraged to tackle more difficult problems—analysis of the interaction and the behavior of finer breakdowns of the economy. Instances of such efforts are the interindustry and the interregional analyses.

This paper concerns the second type of analyses. It aims (1) to review alternative programming models so far devised for the interregional analyses, (2) to suggest a new programming model using the parametric quadratic programming method, and (3) to evaluate the usefulness and the limitations of all these models in applying them to the actual interregional analyses in the agribusiness sector.

### 1. The Transportation Model

The transportation model developed by Hitchcock [13] and Koopmans [16] to solve a problem of efficiently distributing a homogeneous commodity from a number of producing areas to various consumer markets :

to maximize

$$(1.1) \sum_{k=1}^p \sum_{g=1}^q t_{kg} Z_{kg}$$

subject to

$$(1.2) \sum_{g=1}^q Z_{kg} = b_k \quad (k=1, \dots, m),$$

$$(1.3) \sum_{k=1}^p Z_{kg} = d_g \quad (g=1, \dots, q),$$

and to

$$(1.4) Z_{kg} \geq 0,$$

where,

$b_k$  : denotes supply of the commodity to be distributed from the  $k$ th area,

$d_g$  : demand of the commodity to be filled in the  $g$ th market,

$t_{kg}$  : costs of transporting a unit of the commodity from the  $k$ th area to the  $g$ th market, and

$Z_{kj}$ : quantity of the commodity to be shipped from the  $k$ th area to the  $j$ th market, is the first programming model that is employed in the interregional analyses, See, e, g., [15] or [26].

The simplicity of problem formulation and the existence of an efficient solution procedure both make this model very attractive. However, it has some price. Rather strict assumptions underlying this model discourage its wide application in regional analyses. It fails to provide an adequate analytical tool when the production system as well as the distribution system plays an important role. It does not have enough room to consider the complementary, supplementary and substitutional relationships in the production of the commodity.

## 2. The Interregional Linear Programming Model

Beckman and Marshak [1] paid their full attention to the production system as well as to the distribution system when they attempted an activity analysis approach to location theory. This attitude was followed by Lefebvre [17] and Stevens [22] in their interregional linear programming model. Heady and Egbert [10] independently developed the similar type of programming model and have been vigorously applying it to various types of interregional problems in agriculture.

The interregional linear programming model can be visualized as a combination of the production and allocation model and the transportation model. All production activities including some processing activities, all productive resources and all produced commodities are *addressed* to the pertinent producing area or to the pertinent market. The production activities in one area are related to those in other areas through the transportation of intermediate commodities such as feed grains and manufacturing milk. They are also related to various consumer markets via transportation of final commodities such as fluid milk and butter. The model also considers the immobile resources such as land and labor in some busy growing seasons, which cannot be transported. The transportation activities are *addressed* to an ordered pair of places, i.e., their origin and their destination, to which they are assigned. The transportation activities of intermediate commodities are referred to as the procurement

activities and those of final commodities as the marketing activities.

Resource requirements, commodity production and costs per unit level of production activities, procurement activities and marketing activities are all assumed to be constant within the relevant range of their operations, just like in the production and allocation model and in the transportation model. Revenues per unit level of marketing activities are also assumed to be fixed. Consequently, net revenue per unit level of marketing activities are assumed to be constant within the relevant range of their operations.

It should be noted that the constant gross revenue per unit level of marketing activities implies that constant price of final commodities in the consumer market. Thus the demand for final commodities are infinitely elastic within the relevant range. However, the quantity of all final commodities demanded in each market are limited. The price, therefore, the demand of the intermediate commodities are not specified in the model. The demand for intermediate commodities are indirectly *derived* from the demand for the final commodities for whose production they are consumed. The interregional linear programming model actually calculates the demand for and the implicit prices of all intermediate commodities and of all productive resources, given the quantity demanded and the price of all final commodities in each market. Finally the natural endowments of all primary commodities or resources are assumed to be given in each producing area.

Then the interregional linear programming problem is formulated as to maximize the profit of all producing areas as a whole under the above production, distribution and market framework. Formally, to maximize

$$(2.1) \quad -\sum_{j=1}^n \sum_{k=1}^p c_{jk} X_{jk} - \sum_{i=1}^m \sum_{h=1}^p \sum_{k \neq h} s_{ihk} Y_{ihk} + \sum_{i=1}^m \sum_{k=1}^p \sum_{g=1}^q (e_{ig} - t_{ikg}) Z_{ikg}$$

subject to

$$(2.2) \quad -\sum_{j=1}^n a_{ijk} X_{ijk} - \sum_{h \neq k} Y_{ihk} + \sum_{h \neq k} Y_{ikh} + \sum_{g=1}^q Z_{ikg} \leq b_{ik} \begin{pmatrix} i=1, \dots, m \\ k=1, \dots, p \end{pmatrix},$$

$$(2.3) \quad \sum_{k=1}^p Z_{ikg} = d_{ig} \begin{pmatrix} i=1, \dots, m \\ g=1, \dots, q \end{pmatrix},$$

and to

$$(2.4) \quad \begin{aligned} X_{jk} &\geq 0, \\ Y_{ihk} &\geq 0 \quad (h \neq k), \\ Z_{ikg} &\geq 0, \end{aligned}$$

where,

$a_{ijk}$ : denotes quantity of the  $i$ th commodity or resource required, if positive, or produced, if negative, by a unit level of the  $j$ th production activity in the  $k$ th area,

$b_{ik}$ : natural endowments of the  $i$ th commodity or resource in the  $k$ th area,

$c_{jk}$ : cost per unit level of the  $j$ th production activity in the  $k$ th area,

$d_{ig}$ : quantity of the  $i$ th commodity demanded in the  $g$ th area,

$e_{ig}$ : price of the  $i$ th commodity in the  $g$ th market,

$s_{ihk}(h \neq k)$ : costs of procuring a unit of the  $i$ th commodity from the  $h$ th area for the  $k$ th area,

$t_{ikg}$ : costs of marketing a unit of the  $i$ th commodity of the  $k$ th area in the  $g$ th market,

$X_{jk}$ : level of the  $j$ th production activity in the  $k$ th area.

$Y_{ihk}(h \neq k)$ : quantity of the  $i$ th commodity procured from the  $h$ th area for the  $k$ th area,

$Z_{ikg}$ : quantity of the  $i$ th commodity shipped from the  $k$ th area and sold in the  $g$ th market.

The who'e expression in the above can be rewritten in a more amiable form using matrix notations :

$$(2.1a) \quad [-c, -s, e-t] \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

subject to

$$(2.2a) \quad [-A_1, -A_2, A_3] \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \leq b,$$

$$(2.3a) \quad A_4 Z = d,$$

and to

$$(2.4a) \quad \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \geq 0,$$

where

- $A_1$ : is an  $mp$  by  $np$  matrix of  $a_{ijk}$ 's in (2.2),  
 $A_2$ : an  $mp$  by  $m(p-1)$  matrix of the coefficients of  $Y_{ihk}$ 's in (2.2), each column of which consists of 1, -1 and  $mp-2$  zeros,  
 $A_3$ : an  $mp$  by  $mpq$  matrix of the coefficients of  $Z_{ikg}$ 's in (2.2), each column of which consists of 1 and  $mp-1$  zeros,  
 $A_4$ : an  $mq$  by  $mpq$  matrix of the coefficients of  $Z_{ikg}$ 's in (2.3), each column of which consists of 1 and  $mq-1$  zeros.  
 $b$ : an  $mp$ -component column vector of  $b_{ik}$ 's,  
 $c$ : an  $np$ -component row vector of  $c_{jk}$ 's,  
 $d$ : an  $mq$ -component column vector of  $b_{ig}$ 's,  
 $e-t$ : an  $mpq$ -component row vector of  $e_{ig}-t_{ikg}$ 's,  
 $s$ : an  $m(p-1)$ -component row vector of  $s_{ihk}$ 's,  
 $X$ : an  $np$ -component column vector of  $X_{jk}$ 's,  
 $Y$ : an  $m(p-1)$ -component column vector of  $Y_{ihk}$ 's,  
 and  
 $Z$ : an  $mpq$ -component column vector of  $Z_{ikg}$ 's,

The solution to the above programming problem (2.1) through (2.4) (or equivalently to (2.1a) through (2.4a)) provides the level of all production activities in each area, the flow of intermediate commodities between each producing area and the flow of final commodities between each producing area and each consumer market that maximize the profit of all producing areas as a whole.

The dual of the above programming problem appears as follows; to minimize

$$(2.1b) \quad U_1 b + U_2 d,$$

subject to

$$(2.2b) \quad [U_1, U_2] \begin{pmatrix} -A'_1, O_1 \\ -A'_2, O_2 \\ A_3, A_4 \end{pmatrix} \geq \begin{pmatrix} -c' \\ -s' \\ e' - t' \end{pmatrix},$$

$$(2.4b) \quad U_1 \geq 0$$

where

$O_1$ : is an  $np$  by  $mq$  matrix of zeros,

$O_2$ : an  $m(p-1)$  by  $mq$  matrix of zeros,

$U_1$ : an  $mp$ -component row vector of  $U_{ik}$ 's, which in turn indicates

the implicit price of the  $i$ th commodity or resource in the  $k$ th area,  $U_2$ : an  $mq$ -component row vector of  $U_{i_0}$ 's, which in turn indicates the implicit price of the  $i$ th commodity in the  $g$ th market. It should be noted that  $U_{i_0}$ 's, are not restricted to be nonnegative, since the commodity constraints in each market (2.3) or (2.3a) are written as equality in the primal problem. See, e.g., [7] or [8].

Since the solution of the linear programming problem also solves its dual problem, the solution to the above interregional linear programming problem provides the implicit price of all commodities (or resources) both in each producing area and in each consumer market, which are relevant at equilibrium. For instance, it provides the optimum level of rent, wage rate and implicit price of the intermediate commodities in each producing area.

Comparison of this interregional linear programming model with the transportation model in the previous section reveals (1) that more details of production system are specified in this model than in the previous model. (2) The natural resource endowments rather than the supply of commodities are given as fixed in this model, which allows more flexible supply of commodities (with respect to parametric changes in prices) in this model than in the previous model. (3) Since several commodities can be simultaneously considered in this model, this model enables us to analyse the complementary, supplementary and substitutional relationships in production and resource use.

Thus it is clear that the interregional linear programming model provides a better analytical tool of the interregional problems than the transportation model. However, the representation of demand structure is not yet very adequate in this model. (1) It does not allow the flexibility of prices. (2) Nor does it consider the complementary or substitutional relationships between the demand of final commodities. One possible implication of these inadequacies may be as follows. This model will underestimate the importance of technological developments in the production or in the distribution system which influences the supply and eventually the demand of the specific commodities via market mechanism, since it does not consider possible substitution effects relevant to the



commodities.

## INTERREGIONAL INTERINDUSTRY MODELS

The interindustry model as applied to interregional analyses [2] or [14] can be regarded as a variant of the interregional linear programming model, in so far as methodology is concerned. Only, (1) the interindustry model lacks the objective function. Therefore the main effort of interindustry analyses tends to the study of feasible solutions which are possible in the general setup of industrial interdependence. (2) Usually, it lacks the transportation activities. Activities are *addressed* to the specific regions in this model. However, it does not add any spatial significance, since the regional difference can be overcome without any expense from the economy. Activities are *addressed* just for identification purpose. (3) Usually, it contains very few scarce resources. Capital is the only scarce resource considered in many interindustry models. (4) Activities are identified as major industries, which are much coarser breakdowns of the economy than the activities in the interregional linear programming models are.

In conclusion, the aim of interindustry model is quite different from that of interregional linear programming model, though their methodological frameworks are very similar. The interindustry model can be said to be more inclined to the analysis of the aggregate behavior of the national economy than to the analysis of more detailed behavior of the regional economy.

### 3. The Interregional Quadratic Programming Model

One possible way of describing demand structure is to express the price of each commodity as a function of the quantity of all commodities sold in the market. The simplest function of all is a linear function. This is exactly the way where both Maruyama and Fuller [18] and Takayama and Judge [23] independently incorporated the demand structure into their interregional quadratic programming models.

Thus the market prices,  $e_{ig}$ , in (2.1) are now rewritten as follows:

$$(3. j) \quad P_{ig} = e_{ig} + \sum_{r=1}^m d_{ig,r} \sum_{k=1}^p Z_{rkg} \begin{pmatrix} i = 1, \dots, m \\ g = 1, \dots, q \end{pmatrix}.$$

where,

$d_{ig,r}$  denotes amount of change in the price of  $i$ th commodity caused by a unit increase in the quantity of  $r$ th commodity sold in the  $g$ th market,

$e_{ig}$ : sum of the effects on the price of  $i$ th commodity caused by all exogenous factors, e.g., per capita income, size of population and others in the  $g$ th market,

$P_{ig}$ : price of the  $i$ th commodity in the  $g$ th market.

It should be noted that  $\sum_{k=1}^n Z_{rkq}$  in the second term on the right hand side of (3. j) summarizes the quantity of the  $r$ th commodity shipped from all producing areas and sold in the  $g$ th market. Therefore this term as a whole represents the effect on the price of  $i$ th commodity of the quantity of all commodities (including the  $i$ th) sold in the  $g$ th market.

(1) For normal commodity, the sign of  $b_{ig,i}$  is expected to be negative. In other words, a greater volume of the  $i$ th commodity is demanded only when its price is reduced, other things being equal. (2) If the sign of  $d_{ig,r}$  ( $i \neq r$ ) is positive, then the  $i$ th and the  $r$ th commodities are complements. (3) If the sign of  $d_{ig,r}$  ( $i \neq r$ ) is negative, then they are substitutes. (4) If  $d_{ig,r}$  is equal to zero, they are mutually independent. In conclusion, the second term on the right hand side of (3. j) summarizes the flexibility of prices and the complementary and substitutional relationships of demand.

The prices in the producing area,  $e_{ig} - t_{ikg}$  in (2. 1) are now rewritten as follows:

$$(3. k) \quad R_{ikg} = e_{ig} - t_{ikg} + \sum_{r=1}^m b_{ig,r} \sum_{k=1}^p Z_{rkq} \begin{pmatrix} i=1, \dots, m \\ k=1, \dots, p \\ g=1, \dots, q \end{pmatrix},$$

where  $R_{ikg}$  denotes the production revenue per unit (or the producer price) of the  $i$ th commodity shipped from the  $k$ th area and sold in the  $g$ th market.

The whole expression (2. 4) through (2. 1) is now rewritten as follows: to maximize

$$(3. 1) \quad - \sum_{j=1}^n \sum_{k=1}^p c_{jk} X_{jk} - \sum_{i=1}^m \sum_{k=1}^p \sum_{g=1}^q S_{ihk} Y_{ihk}$$

$$+ \sum_{i=1}^m \sum_{k=1}^p \sum_{g=1}^q \left( e_{ig} - t_{ikg} + \sum_{r=1}^p b_{ig,r} \sum_{k=1}^m Z_{ikg} \right) Z_{ikg}$$

subject to

$$(3.2) \quad - \sum_{j=1}^n a_{ljk} X_{ljk} - \sum_{h \neq k} Y_{ihk} + \sum_{h \neq k} Y_{ikh} + \sum_{g=1}^q Z_{ikg} \leq b_{ik} \quad \left( \begin{matrix} i=1, \dots, m \\ k=1, \dots, p \end{matrix} \right)$$

and to

$$X_{jk} \geq 0,$$

$$(3.4) \quad Y_{ihk} \geq 0 \quad (h \neq k)$$

$$Z_{ikg} \geq 0,$$

The constraints (2.3) on the quantity demanded of the interregional linear programming model are now replaced by the demand relationships (3.2) that is introduced into the objective function (3.1). It should be noted that  $Z_{ikg}$ 's appear twice in the objective function (3.1). In other words it is of the second degree in  $Z_{ikg}$ . Therefore, the programming problem (3.1) through (3.4) are not a linear programming problem but a quadratic programming problem for which several solution procedures, e. g., [11] and [25], have been developed.

The whole expression (3.1) through (3.4) can be rewritten in a more amiable form using matrix notations:

to maximize

$$(3.1a) \quad [-c, -s, e-t] \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + Z'DZ,$$

subject to

$$(3.2a) \quad [-A_1, -A_2, A_3] \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \leq b,$$

and to

$$(3.4a) \quad \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \geq 0,$$

where  $D$  is an  $mpq$  by  $mpq$  square matrix of  $d_{ig,r}$ 's.

The solution to the interregional quadratic programming problem (3.1) through (3.4) (or equivalently to (3.1a) through (3.4a)), provides the level of all production activities in each area, the flow of intermediate commodities between each producing area and the flow of final commod-

ities between each producing area and each consumer market that maximize the profit of all producing areas as a whole. It also provides the price and the quantity of all final commodities demanded at equilibrium in each consumer market, while they are assumed to be given in advance in the interregional linear programming model. Furthermore, it provides the price of all final commodities received at equilibrium in each producing area, which are also assumed to be given in advance in the interregional linear programming model. Finally, the solution to the interregional quadratic programming problem also provides the implicit price of all commodities (or resources) at equilibrium in each producing area. It should be noted that the optimal values in the above are based on the perfect collusion among of all producing areas.

#### AN INTERREGIONAL PARAMETRIC QUADRATIC PROGRAMMING MODEL.

Takayama and Judge [23] observed this interregional problem from a slightly different angle. They followed Enke [5] and Samuelson [21], and assumed the perfect competition between each producing area. In order to attain the equilibrium under this assumption, they maximized the net social payoff as defined by Samuelson subject to the same constraints on commodity balances as (3.2). Change of the maximand from profit to net social payoff required only a slight modification of the objective function (3.1), since they also employed linear price functions to represent the demand relationships. Their formulation appears as follows :

to maximize

$$(3.1c) \quad - \sum_{j=1}^n \sum_{k=1}^p c_{jk} X_{jk} - \sum_{i=1}^m \sum_{h=1}^p \sum_{k=1}^p s_{ihk} Y_{ihk} \\ + \sum_{i=1}^m \sum_{k=1}^p \sum_{g=1}^b \left( e_{ig} - t_{ikg} + \frac{1}{2} \sum_{r=1}^m b_{ig,r} \sum_{k=1}^p Z_{rkg} \right) Z_{ikg},$$

subject to (3.2) and to (3.4).

The necessary modification may be highlighted by rewriting (3.1c) in matrix notations :

to maximize

$$(3.1d) \quad [-c, -s, e-t] \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \frac{1}{2} Z'DZ,$$

subject to (3.2) and (3.4).

In actual interregional problems it is very difficult to determine whether perfect collusion or perfect competition better represents the actual pattern of competitive relationships in the agribusiness sector. Actual pattern of collusion or competition rather lies somewhere in between perfect collusion and perfect competition. Then why do we not specify every pattern of collusion or competition that lies between the two extremes and check them all including the two simple extremes? This is exactly the way where Maruyama and Fuller (19) approached the problem of finding the degree of competition or collusion that prevails in the agribusiness sector.

By contrasting (3.1a) with (3.1d), it is clear that the degree of competition or collusion is represented by a single scalar that multiplies the quadratic form  $Z'DZ$ . The degree of competition or collusion hinges on the value of this scalar. When it is equal to one, the above quadratic system represents the case of perfect collusion. When it is equal to one half, the same system represents the case of perfect competition. This observation led Maruyama and Fuller to introduce a "competition parameter"  $\lambda$  into the above quadratic programming model and to develop a more general interregional parametric quadratic programming model, which includes both their old model and the Takayama and Judge's titition as two special cases corresponding to the particular values of "competition parameter"  $\lambda$ . The generalized model looks as follows:

to maximize

$$(3.1p) \quad [-c, -s, e-t] \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \frac{1}{2} \lambda Z'DZ$$

subject to

$$(3.1a) \quad [-A_1, -A_2, A_3] \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \leq b$$

and to

$$(3.4a) \quad \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \geq 0$$

Solution to this generalized programming model depends on the

value of  $\lambda$  as well as on the value of other parameters in the model. The parametric quadratic programming method, e. g., [25], like the parametric linear programming method detects all values of  $\lambda$  that cause the change of basic solution to the programming problem (3. 1p) through (3. 4a). Thus by changing the value of  $\lambda$  continuously from one to two, we can sweep all critical degrees of competition and the corresponding solutions. Therefore, with this interregional parametric quadratic programming model we can be surer of the effects of the degree of collusion or competition in the agribusiness sector on the production, distribution and consumption pattern of specific commodities. In some cases the constraints on the commodities or resource balances (3. 1a) are so restrictive that the degree of competition may have very little influence on the production, distribution and consumption pattern of the specific commodities. In other cases the constraints are so loose that the degree of competition may play a dominant role in determining the production, distribution and consumption pattern of the same commodities, and we may be able to estimate the degree of competition in the agribusiness sector with a sufficiently small margin of error for practical purposes.

In concluding the discussion on interregional quadratic programming models, several comments on the general programming models are in order.

(1) Aggregation biases: In all of the above interregional programming models, all firms in a producing area are represented by a "representative firm". The use of representative firm tends to overestimate the capacity of production in each area, since only  $m$  constraints per area are considered in the model, while the actual number of constraints is at least equal to  $m$  times the number of firms in the area. In actual problems there still may be some more constraints that are omitted for the sake of simplicity. This bias is called the "aggregation bias". See, e. g., [3] and [9]. The best way to minimize this bias is to introduce as many different types of firms as the research budget allows. Furthermore, the parametric change of resource endowments  $b_{ik}$  provides useful information to reduce the rest of aggregation bias.

(2) Centralized decision making: In the above interregional parametric

quadratic programming model, only one objective function is incorporated. The singleness of objective function implies the centralized decision making, except in the case of perfect competition where the decentralized decision making is assured via perfect price mechanism. However, neither perfect competition nor centralized decision making prevails in the agribusiness sector. Each firm in the producing area seems to be making its own choices so as to maximize its very selfish interest. Plural objective functions are not amenable to the programming method. They can be intelligently managed only by the simulation method. See, e. g., [4] or [20],

(3) Time lags: All of the above interregional programming models are short run equilibrium models in nature. They are not particularly good at predicting the dynamic behavior of production, distribution and market system which is apt to leave the equilibrium. The information of market prices takes some time to reach the individual firms in producing areas. The production responses of individual firms to this new information take some more time to come into the market. Therefore they can hardly be consistent with the current market situation.

In the study of dynamic behavior of the production, distribution and market system of this type, the information feedback theory (See, [6] and [24].) seems to be superior to any other conventional economic theory. It is interesting to note that the analysis based on the information feedback theory may be most adequately handled through simulation method at the present stage of mathematical theory.

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