COVARIANCE ANALYSIS: REVISITED AS A METHOD OF COMBINING TIME-SERIES AND CROSS-SECTION DATA IN PRODUCTION FUNCTION ANALYSIS

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I. Introduction

In an econometrical production function analysis, it is more desirable to use a data base consisting of a time series of cross-section samples than using either time-series or cross-section samples separately. A primary reason is that data of combining both types are potentially richer in information than either one of them. However, the question of whether or not it is possible to consider time-series and cross-section factors explicitly in production function analysis has for long been a subject of disputes[1]. Unlike other inputs which are measured directly in physical terms, the two factors can not be handled in the same manner. Further, general methods of direct scaling techniques have not yet been devised.

Granted that differences in both factors exist in a time series of cross-section samples, difficulties arise in the possibility that different levels of both factors are systematically related to variations in inputs of other resources. Thus a production function estimated by means of OLS with no consideration of both factors is liable to be misinterpreted due to bias in the estimated parameter.

In this studies, an attempt has been made to describe the theoretical framework of covariance analysis as a method of handling this problem. Covariance analysis was first applied to econometrical production function analysis by HOCH (1962) and MUNDLAK (1961) almost at the same time as an effective way of combining time-series and cross-section data. Besides, other methods of combining time-series and cross-section data have been suggested by NERLOVE (1967) and WALLACE and HUSSAIN (1969). However, as the results of simulation performed by JOHNSON and LYON (1973) suggest, one is well-advised to explore analysis-of-covariance estimator when he is accessable to multiple cross-section samples.

1) MARSCHAK and ANDREWS (1944) discussed this problem for the first time by plac-

ing emphasis on the differences in managerial ability associated with each firm. Besides, HOCH (1958), GRILICHES (1957) and NERLOVE (1965) argued on this problem more extensively.

II. Identification of Production Function and Profit Maximization

For simplicity, we represent Cobb-Douglas production function explicitly with one input and this can be extended to a general model of multiple inputs.

$$X_{0it} = A_0 X_{1it}^{a_1} E_{0it}, \quad (a_1 > 0) \tag{2.1}$$

$$E_{0it} = K_{0t} M_{0i} U_{0it}, \quad (i = 1, 2, \cdots, I, \ t = 1, 2, \cdots, T)$$

In (2.1), X_{0it} is physical output of firm i in year t and X_{1it} is input. The terms K_{0t} and M_{0i} are respectively time-series and cross-section factors and U_{0it} is stochastic error term. Further, a_1 is parameter of input X_1 and if the size of this parameter is greater than, equal to or less than one, then, correspondingly, returns to scale are increasing, constant, or decreasing. If we assume that producers are engaged in maximizing profit with respect to input under given price, necessary condition for profit maximization can be represented as following.

$$a_1 \frac{X_{0it}}{X_{1it}} = P_1 E_{1it} \tag{2.2}$$

$$\left(E_{1it} = K_{1t} M_{1i} U_{1it}, \ P_1 = P_{X1}/P_{X0}, \ E(U_{rit}) = 0, \ (r = 0, 1)\right)$$

In (2.2) K_{1t} and M_{1i} are also time-series and cross-section factors which can be thought of as indicating the effects of time and firm respectively. Equation (2.2) can be interpreted as pointing behavioral pattern peculier to firm i in year t. And K and M in (2.1) and (2.2) are assumed to render non-negligible influences to the levels of inputs employed. Year effect K includes not only institutional or entrepreneurial restrictions imposed upon year t but also such effects as price deviations, weather conditions and neutral technological changes. Likewise, M is a kind of human factor associated with management including levels of knowledge and education, experiences and personal response to risk all of which can be construed as factors related to technical or price efficiency of firm i[2].

Sufficient condition for profit maximization is $a_1 < 1$ which implies the assumption of perfect competition. In behavioral equation (2.2), E_{1it} reduces to one and producers can maximize profit completely if there are no disturbances at all. However, this is too strict a restriction and it is rather plausible to take deviations from complete profit maximization into consideration. The assumption of disturbance terms K and M may depends on characteristics of the industry to which a firm belong. Further, the assumption may differ within an industry according to specifications of outputs. In agriculture, for instance, the conception of human factor M is not so explicit in the case of producing sugar beet or corn whose productions are supposed to hinge greatly on such factors as weather condition and technological innovation. However, in the case of milk production which is composed of several subactivities such as roughage production, feed reservation, maintenance and selection of milk cattle, the difference of M can hardly be overlooked.

2) The definition of these two terms is not unique and somewhat ambiguous. For instance, MARSCHAK and ANDREWS (1944) included the luck of a particular year into technical efficiency, however, HOCH (1962) thought it as a component of error term. In this studies, we will follow the latter regarding this problem. For the meanings of both technical and price efficiences, see FARRELL (1957). However, the definitions of these two terms suggested by FARRELL are not necessarily applicable to other cases where assumptions given there are not satisfied.

With the conception mentioned in the above in mind, we transform (2.1) and (2.2) into logarithm and rewrite in matrix notation as following.

$$AX-P+A_0=E \tag{2.3}$$

$$A=\begin{vmatrix}1 & -a_1\\ 1 & -1\end{vmatrix},\quad X=\begin{vmatrix}x_{0it}\\ x_{1it}\end{vmatrix},\quad P=\begin{vmatrix}0\\ p_1\end{vmatrix},\quad A_0=\begin{vmatrix}-a_0'\\ a_1'\end{vmatrix},$$

$$E=\begin{vmatrix}k_{0t}\\ k_{1t}\end{vmatrix}+\begin{vmatrix}m_{0i}\\ m_{1i}\end{vmatrix}+\begin{vmatrix}u_{0it}\\ u_{1it}\end{vmatrix}$$

In the above expression, $a_r'=\text{In}\, a_r$ and other small letters mean logarithm. Again, we simplify (2.3) by rewritting with variables defined as deviations from sample means.

$$Ay=v \tag{2.4}$$

$$(y=X-\bar{X}\,,\quad v=E-\bar{E})$$

Equation system (2.4) can be represented as following.

$$y_{0it}-a_1 y_{1it}=v_{0it} \tag{2.5}$$

$$y_{0it}-y_{1it}=v_{1it} \tag{2.6}$$

In order to get some insights into the implications K and M have on (2.5) and (2.6), we assume that in (2.1) E_{0it} equals U_{0it} and in (2.2) E_{1it} is one. This corresponds to conventional way of defining production function with no consideration of disturbance terms. In this case, the equation system in (2.5) and (2.6) is given in the following form.

$$y_{0it}-a_1 y_{1it}=z_{0it} \tag{2. 5-a}$$

$$y_{0it}-y_{1it}=0 \tag{2. 6-a}$$

$$(z_{0it}=\text{In } U_{0it}-\overline{\text{In } U_{0it}})$$

The reduced form of (2.5–a) and (2.6–a) is expressed as

$$y_{0it}=Dz_{0it} \tag{2. 5-b}$$

$$y_{1it}=y_{0it} \tag{2. 6-b}$$

likewise, reduced form of (2.5) and (2.6) can be written as

$$y_{0it}=D(v_{0it}-a_1 v_{1it}) \tag{2. 5-c}$$

$$y_{1it}=D(v_{0it}-v_{1it}) \tag{2. 6-c}$$

where $D=(1-a_1)^{-1}$.

From the above two reduced forms, it is clear that y_{1it} in (2.6–b) is dependent on the dependent variable $y(y_{0it})$ while in (2.6–c) it is not true since y_{1it} is expressed as a function of v_{0it}. This hypothesis is fully justified so far as v_{0it} in (2.6–c) includes not only the stochastic error term but also K and M that are assumed as systematically related to variations in inputs of conventional resources.It is trivial to show how the OLS estimator of a_1 will be turned out in both cases. For the convenience of explanation to be followed, we will show the result of the latter case with following assumptions.

$$E(v_r)=0\,,\quad \text{Var}(v_r)=\sigma_{rr}\quad (r=0,1),\ s=1,2,\cdots,IT$$

With the given assumptions, OLS estimator of a_1 can be expressed as,

$$\hat{a}_1=\frac{\sum\limits_s y_{0s}y_{1s}}{\sum\limits_s y_{1s}^2}=\frac{\sum\limits_s y_{1s}(a_1 y_{1s}+v_{0s})}{\sum\limits_s y_{1s}^2}=\frac{a_1\sum\limits_s y_{1s}^2+\sum\limits_s y_{1s}v_{0s}}{\sum\limits_s y_{1s}^2}$$

$$=a_1+\frac{\sum\limits_s y_{1s}v_{0s}/s}{\sum\limits_s y_{1s}^2/s}$$

The numerative of the second term on the right side is a sample covariance so that under the general conditions[3] we obtain

$$\text{plim} \sum_s y_{1s} v_{0s}/s = \overline{E}(y_{1s} v_{0s}) = E(y_{1s} v_{0s}) = D(\sigma_{00} - \sigma_{01}), \text{ similary}$$

$$\text{plim} \sum_s y_{1s}^2/s = \overline{E}(y_{1s}^2) = E(y_{1s}^2) = D^2(\sigma_{00} - 2\sigma_{01} + \sigma_{11})$$

$$\text{plim } \hat{a}_1 = a_1 + \frac{D(\sigma_{00} - \sigma_{01})}{D^2(\sigma_{00} - 2\sigma_{01} + \sigma_{11})} \qquad (2.7)$$

In (2.7), if we impose the restriction, $E(\sigma_{01})=0$, it becomes

$$\text{plim } \hat{a}_1 = a_1 + \frac{D^{-1}\sigma_{00}}{(\sigma_{00} + \sigma_{11})}$$

and from (2.1) a_1 is positive, then, together with sufficient condition for profit maximization we know that a_1 lies between zero and one. Thus, the difference between estimated and true value of parameter a_1 is turned out to be positive and its size depends on σ_{rr}. It is verified that X_1 is no more exogenous when disturbance terms M and K enter (2.1) and (2.2). This means that we can not estimate unbiased parameter of a_1 by means of OLS.

We illustrated that y_1 is an endogenouus variable in equation system of (2.5) and (2.6) and we have to satisfy identification condition to estimate a_1. However, there exists no predetermined variable either in the form of exogenous or lagged endogenous variable. NERLOVE (1965) suggested some implications on this problem. He satisfied the conditions for identification by introducing lagged terms of independent variables corresponding to y_{1it-1} in our system. However, a little reflection will suffice to convince one that it is highly implausible to assume that v_{1it} in (2.6) is independent of lagged variable y_{1it-1}. In our model, it is assumed that firm factor M is constant within the period of our observation and also independent of time. So it is natural to imagine that m_{1i} which is a component of v_{1it} determines the lagged value of endogenuous variable y_{1it-1} to exactly the same extent as it determines the current value of y_{1it}. The results implies that method of this kind fails to provide a consistent estimator of a_1 in our simultaneous equation system. So far as our specifications of production function in (2.1) and behavioral equation in (2.2) are not irrelevant, alternative approach is needed to obtain consistent estimator of a_1.

3) See GOLDBERGER (1964), Ch. 7.

III. Constrained Profit Maximization and the Derivation of Covariance Analysis Estimator

Before we proceed, we adopt two additional assumptions suggested by

HOCH (1958) in order to derive unbiased estimator of covariance analysis. The newly adopted two assumptions are as following.

1) In behavioral equation (2.2), a producer maximizes profit regarding to anticipated output rather than actual output.
2) The error terms both in production function (2.1) and behavioral equation (2.2) are independent each other.

By rewritting production function (2.1) in logarithmic form and variables defined as deviations from sample means, we obtain

$$x_{0it}=a_1x_{1it}+e_{0it}\,,\quad (e_{0it}=k_{0t}+m_{0i}+u_{0it}) \tag{3. 1}$$

where we substituted y and v in (2.4) by x and e for distinction. And by following MUNDLAK (1963) we can define k_{0t} and m_{0i} as follows with no lost of generality.

$$\sum_t k_{0t}=\sum_i m_{0i}=0 \tag{3. 2}$$

By the normalization of disturbance terms in (3.2), we can easily verify that following intriguing relationships establish between residual term E and stochastic error term U.

$$\begin{aligned}
e_{0.t}&=k_{0t}+\qquad +u_{0.t}\\
e_{0i.}&=\qquad +m_{0i}+u_{0i.}\\
e_{0..}&=\qquad\qquad u_{0..}
\end{aligned}$$

$$e_{0.t}=\frac{1}{I}\sum_i e_{0it}\,,\quad e_{0i.}=\frac{1}{T}\sum_t e_{0it}\,,\quad e_{0..}=\frac{1}{I.T}\sum_t\sum_i e_{0it}$$

$$e_{0it}-e_{0.t}-e_{0i.}+e_{0..}=u_{0it}-u_{0.t}-u_{0i.}+u_{0..} \tag{3. 3}$$

If we simplify (3.3) as

$$e_0^{IT}=u_0^{IT}$$

the simultaneous equation system (2.3) can be represented as following.

$$x_0^{IT}-a_1x_1^{IT}=u_0^{IT} \tag{3. 4}$$

$$x_0^{IT}-x_1^{IT}=u_1^{IT} \tag{3. 5}$$

$$(x_r^{IT}=x_{rit}-x_{r.t}-x_{ri.}+x_{r..})\,,\quad (r=0,\,1)$$

The reduced form of (3.4) and (3.5) can be written as following.

$$x_0^{IT}=D(u_0^{IT}-a_1u_1^{IT}) \tag{3. 6}$$

$$x_1^{IT}=D(u_0^{IT}-u_1^{IT}) \tag{3. 7}$$

By adjusting disturbance terms of both time-series and cross-section in (3.3), it was possible to eliminate the effects of both factors from the reduced form of (3.6) and (3.7). However, x_1^{IT} in (3.7) is no more independent of the error term of production function than y_{1it} in (2.6–c) is. By what we have described hitherto, it is clear that the OLS estimator in (3.6) and (3.7) is still subject to simultaneous equation bias.

In order to secure unbiased estimator of a_1, we proceed by introducing two additional assumptions mentioned in the above. Thus, we derive the necessary condition for profit maximization not from (2.1) but from following production function of anticipated output.

$$\bar{X}_{0it} = A_0 X_{1it}^{a_1} K_{0t} M_{0i} q \tag{3.8}$$

In (3.8) $\bar{X}_{0it}$ means anticipated output and q corresponds to U_{0it} in (2.1), however, it is different from U_{0it} itself, q is rather something of $f(U_0)$ or any other function of parameters of $f(U_0)$[4]. Necessary condition derived from (3.8) can be written as following.

$$\frac{a_1 \bar{X}_{0it}}{X_{1it}} = P_1 M_{1i} K_{1t} U_{1it} \tag{3.9}$$

From (2.1) and (3.8), we can easily derive the relationship between anticipated and actual outputs.

$$\bar{X}_{0it} = (X_{0it}/U_{0it})\, q \tag{3.10}$$

By substituting (3.10) in (3.9),

$$\frac{a_1 (X_{0it}/U_{0it})}{X_{1it}} = P_1 M'_{1i} K_{1t} U_{1it}\,. \tag{3.11}$$

In (3.11) $M'_{1i} (=M_{1i}/q)$ can be construed as expressing the extent to which a producer is ignorant of perfect information. By rewritting (3.11) in logarithm and define variables as deviations from sample means, we obtain the following expression.

$$x_{0it} - x_{1it} = u_{0it} + m'_{1i} + k_{1t} + u_{1it} \tag{3.12}$$

If we put $m'_{1i} + k_{1t} + u_{1it} = e'_{1it}$, from (3.3) and (3.12) our behavioral equation is represented in the following form.

$$x_0^{IT} - x_1^{IT} = u_0^{IT} + u_1^{IT} \tag{3.13}$$

Thus, from (3.4) and (3.13), we can derive reduced form that satisfies condition for the unbiased estimator of a_1.

$$x_0^{IT} = D\left[(1-a_1)\, u_0^{IT} - a_1 u_1^{IT}\right] \tag{3.14}$$

$$x_1^{IT} = -Du_1^{IT} \tag{3.15}$$

Equation (3.15) is different from (2.6-c) and (3.7) in the respect that stochastic error term of production function does not appear on the right side.

This means that it is possible to estimate unbiased parameter of a_1 by dint of OLS in (3.14) and (3.15).

$$\hat{a}_1 = \frac{\sum_i \sum_t x_0^{IT} x_1^{IT}}{\sum_i \sum_t (x_1^{IT})^2} = \frac{\sum_i \sum_t (-D^2)\left[(1-a_1)\, u_0^{IT} u_1^{IT} - a_1 (u_1^{IT})^2\right]}{\sum_i \sum_t D^2 (u_1^{IT})^2}$$

$$\text{plim } \hat{a}_1 = a_1 + \frac{\text{plim } \sum_i \sum_t (-D^2)(1-a_1)\, u_0^{IT} u_1^{IT}}{\text{plim } \sum_i \sum_t D^2 (u_1^{IT})^2}$$

$$= a_1 + \frac{-D^2 \text{ plim } \sum_i \sum_t (1-a_1)\, u_0^{IT} u_1^{IT}/s}{D^2 \text{ plim } \sum_i \sum_t (u_1^{IT})^2/s}, \quad (s=1, 2, \cdots, IT) \tag{3.16}$$

The numerative of second term on the right side of (3.16) is a sample covariance of u_0 and u_1 adjusted in (3.3) by two disturbance terms of time-series and cross-section. Thus, under general conditions,

$$-D^2 \text{ plim } \sum_i \sum_t (1-a_1)\, u_0^{IT} u_1^{IT}/s = -D^2(1-a_1)\, \overline{E}(u_0^{IT} u_1^{IT})$$

$$= -D^2(1-a_1)\, E(u_0^{IT} u_1^{IT}) = -D^2(1-a_1)(0) = 0$$

and from (3.16) we obtain unbiased estimator of a_1.

$$\text{plim } \hat{a}_1 = a_1 \tag{3.17}$$

By the exhaustive descriptions up to now, it has become clear that the normalization assumption in (3.2) and covariance analysis technique applied in (3.3) made it possible to get rid of two disturbance terms in (3.7). Again, the introduction of HOCH's (1958) two additional assumptions enabled us to derive unbiased estimator a_1 in (3.17).

Now we reconsider the rationale of two additional assumptions adopted in the above whether it is plausible or not. The first one that producers in general maximize their profits respect to anticipated output rather than actual output has following implications. The nature of production process is such that inputs precede output in time and, therefore, at the date when inputs are determined the actual output that will be realized is not known with full certainty, the reason being the dependence of output on the stochastic error term. Thus the firm can base its decision on anticipated output or chooses another criterion such as minimizing the expected loss resulting from deviation between anticipated and actual outputs or some other criterions peculier to each individual firm.

The second assumption that stochastic error terms U_0 and U_1 are independent each other seems to be not implausible in the sense that we have already eliminated the effects of both disturbance terms in (3.6) and (3.7), thus there remains only the stochastic error term. From what have been observed hitherto, it should be made clear that the unbiased estimator in (3.17) is justified so far as the model specifications in (2.1), (2.2) and two additional assumptions are accepted[5].

4) In this case, the anticipated output $\bar{X}_{0it}$ does not equal $E(X_{0it})$, for details on this point, see HOCH (1962).

5) For empirical analysis of this method, see HOCH (1962) and MUNDLAK (1961). And for examples applied to milk production function analysis, see MATSUBARA (1972) and CHO (1976). For details of estimation method, see SCHEFFE (1959) and MORISHIMA (1960).

IV. Concluding remarks

We have seen how a time series of cross-section samples is combined in estimating Cobb-Douglas production function. And it has been illustrated that under the conventional assumption of production function we are subject to simultaneous equation bias so far as firm and year factors are not independent of input levels of resources. However, covariance analysis method renders unbiased estimator. And this method is fully justified within the realm of our approval of the specifications of production function in (2.1) and behavioral equation in (2.2) as such.

And some points of importance in the context of applying this method can be summed up as following. First, it is necessary to secure homogeneity of sample data. Especially, is this true so far as the effects of geographical or environmental conditions of sample data are concerned. Second, covariance analysis method necessitates multiple cross-section sample. This means that covariance analysis is rather limited in its range of application since it is not always easy to find such a data base. For all the difficulties, covariance analysis method provides us with a strong tool of combining time-series and cross-section data.

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