Coupled Oscillators in Quenched Random Potential

Tomoaki NOGAWA,[1] Hajime YOSHINO[2] and Hiroshi MATSUKAWA[3]

[1]*Division of Physics, Graduate School of Science, Hokkaido University, Sapporo 060-0810, Japan*
[2]*Department of Earth and Space Science, School of Science, Osaka University, Toyonaka 560-0043, Japan*
[3]*Department of Physics, College of Science and Engineering, Aoyama Gakuin University, Sagamihara 229-8558 Japan*

(Received June 30, 2005)

We study a synchronization of coupled oscillators in quenched random potential by numerical simulations as a model of sliding charge density waves and flux line lattices. By changing external driving force, we find a percolation transition of a cluster with a same frequency in a finite time observation. Percolating cluster, however, becomes unstable in the long time limit while finite size systems fall into limit cycle motion.

Collective translational motion of periodic systems in condensed matter such as charge density waves (CDW)[1)] and flux line lattices (FLL)[2)] has rich physics on an aspect of nonlinear dynamics with large degrees of freedom. The spatial periodicities of these systems are destroyed by random pinning potential originated to impurities or lattice defects. They also yield spatially inhomogeneous dynamics, so-called "plastic flow", under an external driving force, such as electric field for CDW. It is argued that these systems show a solidification transition in nonequilibrium steady state as driving force is increased.[3),4)] This ordered state is called "moving solid". In this article, we numerically investigate this transition as a synchronization problem of coupled oscillators in quenched random potential.

At first, we introduce a model to describe the dynamics of periodic systems, which is called driven random-field XY model.[5)–8)] Phase field $\theta(\boldsymbol{r}, t)$ is employed as a dynamical degree of freedom, with which a deformed density wave is expressed as $\rho_0 \cos\big(\boldsymbol{q}\cdot\boldsymbol{r} + \theta(\boldsymbol{r}, t)\big)$. Here $\boldsymbol{q}$ is a reciprocal lattice vector. The translational velocity of periodic structure is proportional to phase velocity as $\dot{\theta}/q_{||}$. We suppose an existence of semi-macroscopic domains, in which phase is always uniform. The interaction energy between neighboring domains are supposed to be sinusoidal coupling, 1-cos$(\theta_i - \theta_j)$, which has 2π translational symmetry for phase difference. The phase variables are regularly put on the three dimensional ($d = 3$) simple cubic lattice in periodic boundary condition. The overdamped equations of motion for the phases of domains are as follows.

$$\dot{\theta}_i = -\frac{J}{2d}\sum_j \sin(\theta_i - \theta_j) - \sin(\theta_i - \beta_i) + f \ .$$

The first term on the right hand side indicates nearest neighbor coupling. In this article, we show the results for $J = 1.0$. The second term is a random pinning force

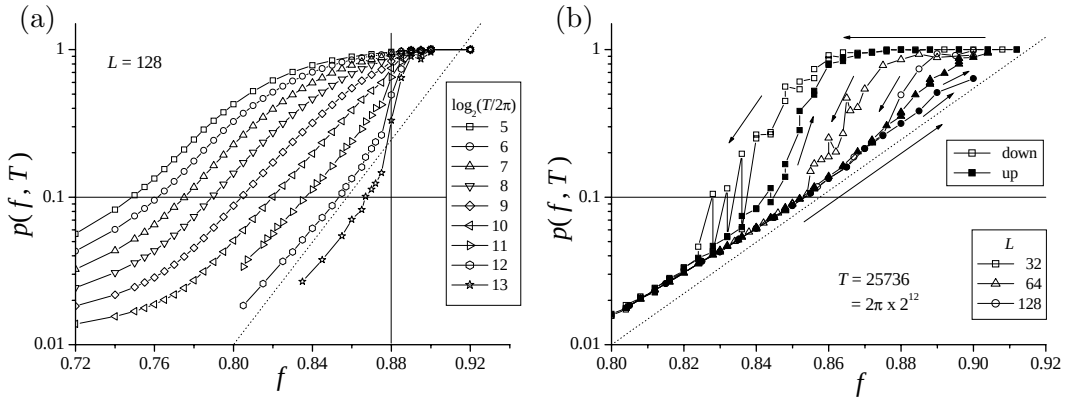

Fig. 1. (a) The fraction of connected bonds as a function of driving force. Uniform initial condition is employed. The horizontal line indicates $p(f,T) = p_c = 0.10$. Dotted line shows $p(f,T) \propto \exp(f/0.025)$. (b) The relation between the fraction of connected bonds and driving force. Both sweeping up and down cases are shown for several system sizes, L=32, 64 and 128.

which prevents from homogeneous oscillation. Here, β_i's are introduced as uniform random variables between 0 and 2π. The last term f is an external driving force. By integrating these equations numerically, we investigate f dependence of dynamics in steady state.

Above depinning threshold force $f_T = 0.10$, some of sites are depinned.[7] For f slightly larger than f_T, moving and stopping sites coexist in a same system. Time averaged local DC velocity, $\omega^i_{\rm DC} = \langle \dot{\theta}_i \rangle_{\rm time}$, is spatially non-uniform and the motion of each site is intermittent. As the driving force becomes larger and the velocity increases, the motion becomes spatially uniform and temporally periodic. The effect of pinning force becomes less important because random potential, $\cos(\theta_i(t) - \beta_i) \approx \cos(\omega^i_{\rm DC} t - \beta_i)$ fluctuates so rapidly that the system cannot follow the change.

To argue the spatial correlation of dynamics quantitatively, we remark on the phase slip process as mentioned below. When the velocities are different between neighboring domains, the phase difference between them advances by $\pm 2\pi$ at certain instance. This is called phase slip, which does not leave a change in coupling energy on the bond and is related to plastic deformation of the lattice.

We regard a bond between nearest neighbor sites (i,j), as *connected* one if phase slip does not occur between i and j, i.e., $\Delta\omega^{i,j}_{\rm DC} = |\omega^i_{\rm DC} - \omega^j_{\rm DC}| < 2\pi/T$. Here T is an observation time. We analyze percolation of clusters connected with such bonds, in other word, percolation of oscillators with same frequency. Percolating phase, which has macroscopic comoving cluster, is regarded as the moving solid phase.

In Fig. 1(a), the fraction of connected bond $p(f,T)$ for various T is plotted as a function of f. $p(f,T)$ exponentially increases with f and decreases as a power function of T as far as $p(f,T)$ is not too small and not too large. As mentioned later, percolation transition occurs when $p(f,T)$ equals $p_c(= 0.10)$ independently of T therefore a critical driving force f_c depends on observation time.

In the vicinity of $f = f_{\rm lc} \sim 0.88$, $p(f,T)$ rapidly saturates to unity even for very

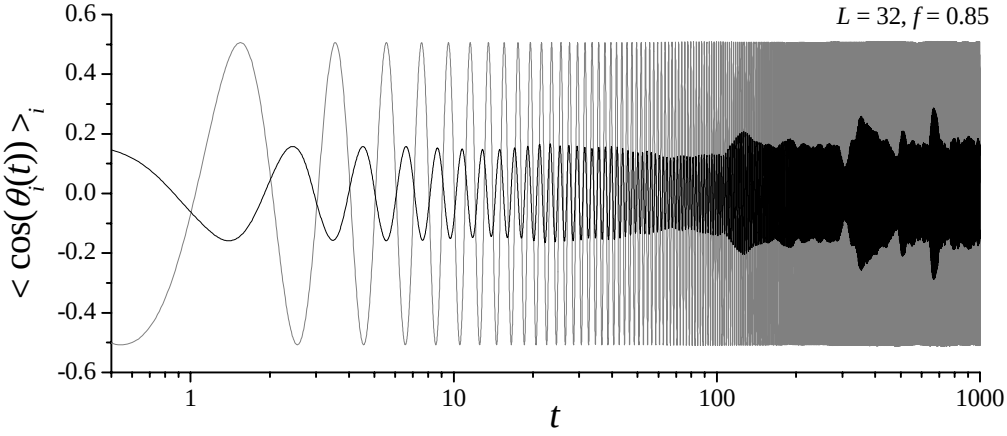

Fig. 2. Time evolution of $\langle \cos(\theta_i(t)) \rangle_i$ in steady state. Black and gray line indicates random and uniform initial conditions, respectively. The mean oscillation periods, $2\pi/\langle \omega^i_{\rm DC} \rangle_i$, are 2.14 and 2.00.

large T. Then f_c does not depend on T. For $f > f_{\rm lc}$, the whole system shows periodic motion then intermittent phase slip is suppressed. Of course, $\omega^i_{\rm DC}$ is homogeneous in this regime. $f_{\rm lc}$ is dependent on history and system size as shown in Fig. 1(b). Each sample, i.e., set of random variables $\{\beta_i\}$, shows a discontinuous hysteresis loop like a first order transition although it is smoothed by sample averaging due to the distribution of $f_{\rm lc}$. Upper branch is obtained when starting from an uniform initial condition [$\theta_i(0) = 0$ for all i] or driving force is gradually decreased by a constant step while lower branch appears with random initial conditions or f increasing simulations.

In Fig. 2, time evolution of $\langle \cos[\theta_i(t)] \rangle_i$ at a driving force in the bistable region is shown. The amplitude of oscillation indicates a spatial coherence of θ_i itself. With an uniform initial condition (gray line), limit cycle oscillation appears while a random initial condition yields chaotic motion,[6] which has similar period with that of limit cycle but no long time correlation.

$f_{\rm lc}$ becomes larger with the system linear dimension L for both type of initial conditions. Therefore the upper branch, limit cycle motion, is realized due to the finite size effect and is expected to disappear in the large size limit.

In Fig. 3(a) percolation probability $P(f, L_{\rm sub}, T)$ is plotted as a function of $p(f, T)$. Here $L_{\rm sub}(< L)$ is a linear dimension of subsystems in real sample that is introduced to perform finite size scaling.[8] Percolation probability is given by a fraction of the subsystems which has a large connected cluster which reaches the both sides of subsystems. There are universal curves which only depend on $L_{\rm sub}$ then $P(f, L_{\rm sub}, T) = P(p(f, T), L_{\rm sub})$. The fixed crossing point, $p = p_c = 0.10$, which does not depend on $L_{\rm sub}$ is a critical point of percolation transition. This critical fraction is smaller than that of the ordinary stochastic bond percolation transition in three dimensions, $p_c = 0.2488$.[9] This is due to the attractive spatial correlation of connected bonds. The critical force, however, increases logarithmically with T,

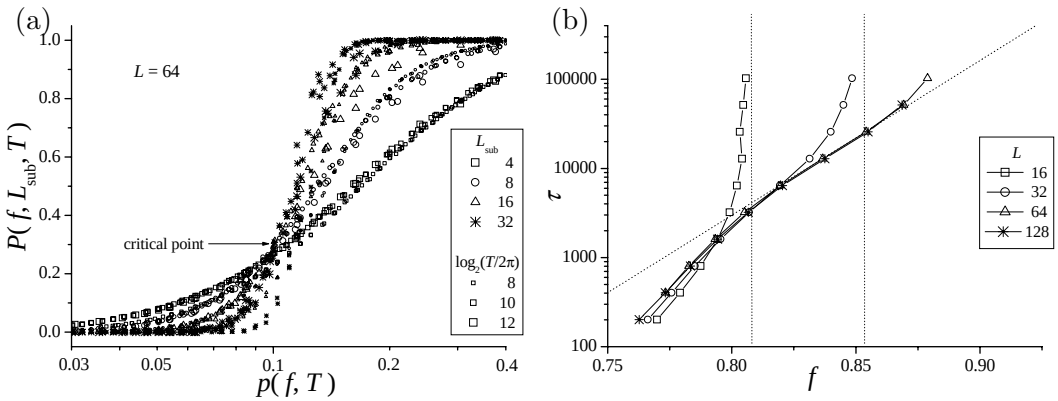

Fig. 3. (a) Percolation probabilities for various system sizes and observation times are plotted as a function of fraction of connected bonds. The true sample size is 64 here. (b) The relation between crossover time and driving force. Dotted line shows $\tau = \exp[(f - 0.60)/0.025]$. Uniform initial condition is employed for both (a) and (b).

$f_c(T) = f_0 \log(T/t_0)$. Here f_0 is a constant value and equals 0.025.

We can clearly define a crossover time $\tau(f) = t_0 \exp(f/f_0)$ from $f_c(\tau(f)) = f$. It is plotted in Fig. 3(b). In short observation time than this time scale the system behaves like a moving solid and for longer time scale it shows fluid like behavior. This exponential dependence of the relaxation time can be interpreted as thermal activation, $\tau \propto \exp(V_{\rm barrier}/k_B T_{\rm sk})$, with an effective "shaking temperature" $T_{\rm sk} \propto f^{-1} \approx \omega_{\rm DC}$ which is suggested by Koshelev and Vinokur.[4)]

In finite size systems, $\tau(f)$ diverges at $f = f_{\rm lc}(L)$, above which the system takes spatially uniform and temporally periodic motion. $f_{\rm lc}(L)$ becomes larger with L and e^{f/f_0} behavior of $\tau(f)$ holds for larger f. In a infinite size system the relaxation time is expected to be finite for finite driving force and diverges at $f = \infty$.

References

1) G. Grüner, Rev. Mod. Phys. **60** (1988), 1129.
2) G. Blatter, M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin and V. M. Vinokur, Rev. Mod. Phys. **66** (1994), 1125.
3) L. Balents and M. P. A. Fisher, Phys. Rev. Lett. **75** (1995), 4270.
4) A. E. Koshelev and V. M. Vinokur, Phys. Rev. Lett. **73** (1994), 3580.
5) S. H. Strogatz, C. M. Marcus, R. M. Westervelt and R. E. Mirollo, Phys. Rev. Lett. **61** (1988), 2380.
6) D. A. Huse, in *Computer Simulation Studies in Condensed Matter Physics* IX, ed. D. P. Landau et al. (Springer, Berlin, 1997), p. 1964.
7) T. Kawaguchi, Phys. Lett. A **251** (1999), 73.
8) T. Nogawa, H. Matsukawa and H, Yoshino, J. Phys. Soc. Jpn. **74** (2005), 1907.
9) P. Grassberger, J. of Phys. A **25** (1992), 5867.