Statistical significance of the fine structure in the frequency spectrum of Aharonov-Bohm conductance oscillations

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We discuss a statistical analysis of Aharonov-Bohm conductance oscillations measured in a two-dimensional ring, in the presence of Rashba spin-orbit interaction. Measurements performed at different values of gate voltage are used to calculate the ensemble-averaged modulus of the Fourier spectrum and, at each frequency, the standard deviation associated to the average. This allows us to prove the statistical significance of a splitting that we observe in the h/e peak of the averaged spectrum. Our work illustrates in detail the role of sample specific effects on the frequency spectrum of Aharonov-Bohm conductance oscillations and it demonstrates how fine structures of a different physical origin can be discriminated from sample specific features.

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The investigation of Aharonov-Bohm (AB) conductance oscillations in mesoscopic devices permits to study different aspects of phase-coherent transport of electrons. One of the aspects that has recently attracted considerable attention is the effect of the electron spin.[1] It has been theoretically predicted that in the presence of spin-orbit interaction (SOI), the electron spin modifies the properties of AB conductance oscillations in an observable way.[2,3]

Experimental attempts have been reported in which features observed in either the envelope function of the AB oscillations or their Fourier spectrum were attributed to the presence of Rashba SOI.[4,5] In a few cases[5] these claims were based on the interpretation of *single* magnetoconductance measurements. The interpretation of such experiments is difficult, however, due to the sample specific nature of the h/e oscillations. In particular, a certain scatterer configuration in the ring might cause features that are similar to those due to SOI. In the analysis of past experiments this possibility has not been considered thoroughly.

In this paper we show experimentally how sample specific effects in the Fourier spectrum (FS) of the AB oscillations can be *quantifiably* suppressed in a controlled way. In particular, we perform a statistical analysis of the ensemble averaged FS. At each frequency, the mean Fourier amplitude and standard deviation are calculated. We find features in the averaged FS that are significantly larger than the standard deviation. These features can therefore be discriminated from remnant sample specific effects and their origin attributed to a different physical phenomenon.

The AB oscillations used in our analysis have been measured in a two-dimensional ring fabricated using an InGaAs-based heterostructure (Fig. 2, bottom inset), in which Rashba SOI is particularly strong[6] (Fig. 2, top inset); $\alpha \approx 0.8 \times 10^{-11}$ eVm. The mean radius and width, respectively, 350 nm and 180 nm, are smaller than the mean-free path ($\simeq 1$ μm) and transport is quasiballistic.[7] A gate electrode covering the ring permits to change the Fermi energy as well as the strength of the SOI (the maximum expected gate induced change is 20%–30%).[6]

The magnetoconductance of the ring ($G(B)$) was measured at different values of the gate voltage V_g ranging from -55 mV to 195 mV at a temperature of 300 mK. Three $G(B)$ curves, measured at different V_g, are shown in the left column of Fig. 1. Clearly visible in each $G(B)$ curve are a background increasing with magnetic field (due to the classical dynamics of the electron in a laterally confined geometry[8]), aperiodic conductance fluctuations, and periodic AB oscillations.[9] The middle column shows the AB oscillations obtained by subtracting the background from the $G(B)$ curve. Due to the small period of the oscillations, only the envelope function is visible. The right column shows the h/e peak in the FS.

Both the envelope function of the AB oscillations and its FS depend strongly on V_g. This strong V_g dependence is expected and well known for random interference of electronic waves and shows that in a single $G(B)$ trace sample specific effects dominate the behavior of the measured AB oscillations. This precludes the attribution of a special meaning to any feature observed in a single measurement, since features appear and disappear randomly with V_g. Therefore, in practice, it is not possible to draw firm conclusions about the effect of SOI on the AB oscillations from a single $G(B)$ measurement.[5]

In order to put in evidence subtle effects possibly present in the AB oscillations, it is necessary to suppress sample specific features in a controlled way. This can be achieved by studying an *ensemble of measurements*,[4] i.e., by averaging the modulus of the Fourier spectrum, $|G(\nu)|$, over different scatterer configuration. The ensemble averaged FS, $\langle|G(\nu)|\rangle$, is expected to be a smooth function, peaked at the frequency that corresponds to the mean radius of the ring. Superimposed on top of this smooth function, spin (or other) effects may show up as a splitting or another well-defined structure.

Theoretically it has been proven that a (sufficiently large) change in Fermi energy is equivalent to a complete

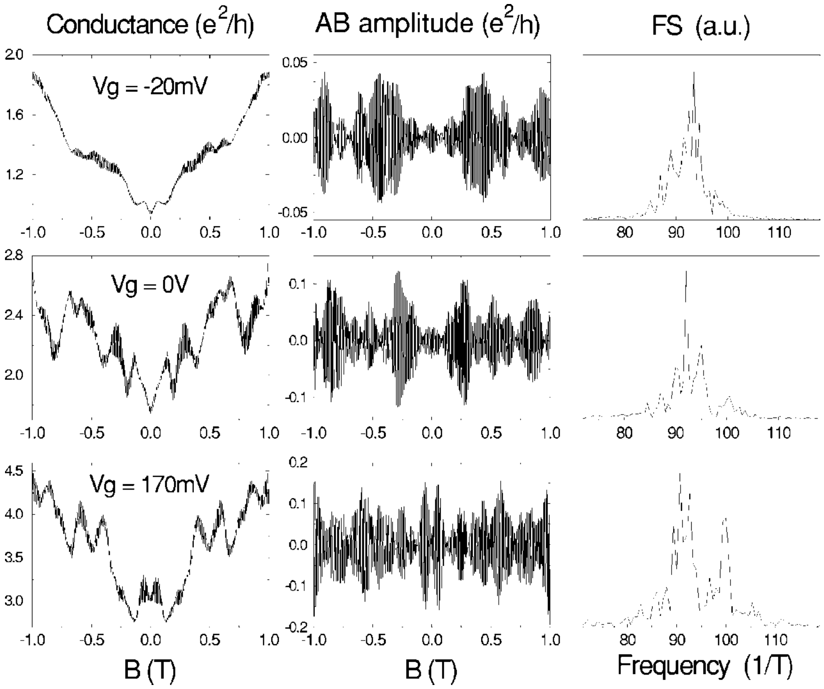

FIG. 1. The magnetoconductance of a ring measured at different gate voltage V_g at 300 mK is shown in the left column. The envelope function of the AB conductance oscillations, obtained by removing the positive magnetoconductance background, and the Fourier spectrum are shown in the middle and right column, respectively. It is apparent that both these quantities depend strongly on V_g, as it is typical for sample specific effects.

change in impurity configuration, in so far as the conductance oscillations are concerned.[10] For this reason we study the statistical properties of quantities averaged over an ensemble that consists of $G(B)$ curves measured at different gate voltage.

To show experimentally the statistical independence of $G(B)$ curves measured at different values of V_g we calculate the mean amplitude of the h/e oscillations upon averaging an increasing number N of $G(B)$ traces. The h/e oscillation amplitude in the averaged $G(B)$ is expected to be suppressed as $1/\sqrt{N}$, provided that the curves are statistically independent.[11] Figure 2 shows that this is indeed the behavior observed experimentally.[12] We have checked that also the aperiodic conductance fluctuations exhibit the same behavior, i.e., their amplitude decreases as $1/\sqrt{N}$ as well.

We note that in order to acquire the largest possible number of independent $G(B)$ curves, and therefore to obtain the largest suppression of sample specific effects, the entire range of V_g studied is rather large. This results in a sizeable (up to roughly a factor of 2) change of electron density and possibly also in a change of SOI strength, which may affect the shape of the averaged modulus of the FS, $\langle|G(\nu)|\rangle$. For this reason, we will first discuss the average over the whole V_g range and then compare it to the average over a smaller V_g range. As we will show, the final results are similar for the two different V_g ranges, which suggests that the precise extension of the V_g range used in the experiments is not very critical.

Figure 3 shows a typical FS of a single $G(B)$ trace (upper graph), and the result of two different kinds of ensemble averaging, using the total set of 49 $G(B)$ curves measured from $V_g = -55$ mV to 195 mV. Specifically, the middle graph is a plot of the modulus of the FS of the average magnetoconductance, obtained by first averaging the magnetoconductance curves and then calculating the modulus of the FS ($\equiv |\langle G(\nu)\rangle|$). This quantity has been studied extensively in the past, and was shown to result in a suppression the h/e oscillation amplitude.[11] For this reason, the *relative* size of the sample specific structure in the FS does not decrease upon averaging, as it is apparent from the comparison of the upper and middle graphs in Fig. 3. Therefore, this way of ensemble averaging does not allow the observation of subtle features possibly present in the h/e peak in the FS.

The bottom graph in Fig. 3 shows the ensemble average of the *absolute* value of the FS, $\langle|G(\nu)|\rangle$. This way of averaging does not suppress the h/e oscillations, since the phase information is discarded by taking the modulus of the FS of individual $G(B)$ traces before performing the average. This procedure results in a suppression of random features in the averaged FS, over the whole frequency range, as it is obvious from Fig. 3.

It was argued in Ref. 4 that this way of ensemble averaging provides information that is not easily accessible otherwise. The enhanced visibility of the third harmonic of the AB oscillations gives a direct experimental demonstration of this statement. This $h/3e$ peak is neither observable in the FS of any single $G(B)$ curve nor in the FS of the averaged magnetoconductance, but it is clearly visible in the averaged modulus of the FS, $\langle|G(\nu)|\rangle$.

The insets in Fig. 3 zoom in on the h/e peak. It is apparent again that, in comparison to the upper two graphs, the sample specific "noise" is largely suppressed in the averaged modulus of the FS (bottom graph). In this quantity, a small splitting remains visible in the averaged h/e peak. Note that also the $h/3e$ peak in the bottom main figure shows a similar structure.

Without any further analysis, it is difficult to conclude what is the origin of these features. Specifically, this is be-

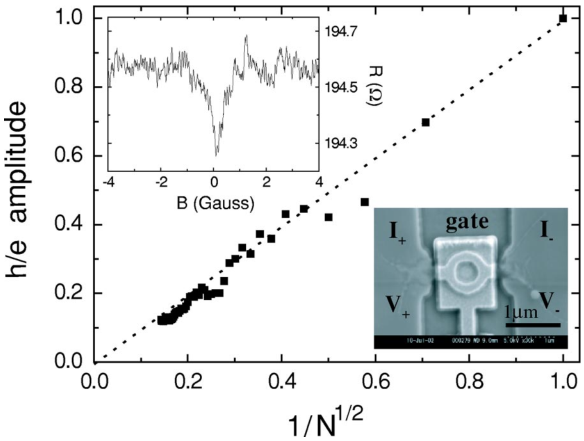

FIG. 2. Mean amplitude of the h/e oscillations in the averaged magnetoconductance, as a function of the square root of the number N of curves used in calculating the average. The mean amplitude of the h/e oscillations is calculated by integrating the modulus of the FS over the width of the h/e peak. The insets show the sample used in our investigations (bottom right) and the low-field magnetoresistance measured in a Hall bar made out of the same heterostructure used for our ring (top left). The resistance dip around zero field is due to weak antilocalization, which indicates the presence of strong Rashba spin-orbit interaction.

cause experimentally we only average over a finite number of scatter configurations so that the splitting may simply be some remnant sample specific structure. Only by quantifying the magnitude of these remnant features we can conclude if the splitting has physical significance.

We quantify the size of the remnant sample specific structure in terms of a frequency dependent standard deviation. This is obtained from the same set of N curves that we use to calculate the average modulus of the FS, $\langle|G(\nu)|\rangle$. At each fixed frequency ν, this set of curves corresponds to a set of N values of which we calculate the standard deviation $\sigma_{dis}(\nu)$. The statistical error associated with the average modulus of the FS at frequency ν is then $\sigma_{mean}(\nu)=\sigma_{dis}(\nu)/\sqrt{N}$ (central limit theorem). For an ideal ensemble average $N=\infty$ and $\sigma_{mean}(\nu)=0$, i.e., sample specific effects are completely suppressed. However, if N is finite, $\sigma_{mean}(\nu)$ is also finite.

The upper panel of Fig. 4 shows the h/e peak of the averaged modulus of the FS and, for each frequency, $\sigma_{mean}(\nu)$, plotted as an error bar. In the main panel the average has been performed on $N=16$ curves (with V_g ranging from -55 mV to 95 mV) and in the inset on $N=49$ (V_g ranging from -55 mV to 195 mV). In both cases the size of the splitting is 3–4 times larger than σ_{mean}. The splitting is therefore statistically significant, and it is *not* due to remnant sample specific structure.[13] The size of any other structure observed in these averaged quantities is too small compared to $\sigma_{mean}(\nu)$ to exclude remnant sample specific effects as their origin. This is also true for the splitting in the $h/3e$ peak visible in the bottom panel Fig. 3.

We have also performed the same statistical analysis for the Fourier *power* spectrum.[14] The ensemble averaged Fourier power $\langle|G(\nu)|^2\rangle$ and the associated statistical error, $\sigma_{power}(\nu)$ are shown in the bottom panel of Fig. 4 for $N=16$ (main figure) and $N=49$ (inset). Also here a statistically significant splitting is present in both cases.

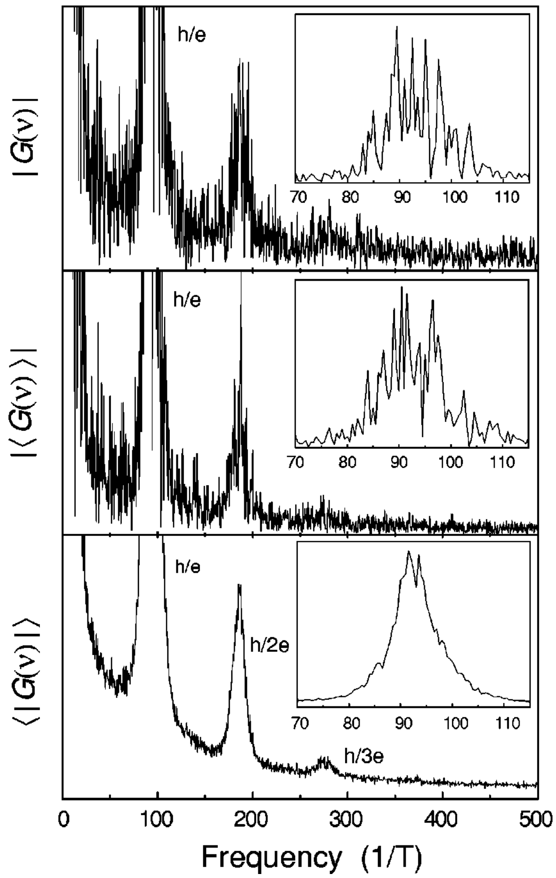

FIG. 3. The random features present in the modulus of the FS of a single mangetoresistance curve (upper graph) are *not* suppressed in the modulus of the FS of the averaged $G(B)$, $|\langle G(\nu)\rangle|$ (middle graph). However, random features are clearly suppressed in the averaged modulus of the FS, $\langle|G(\nu)|\rangle$ shown in the bottom graphs (in both the middle and in the bottom graphs, the average is taken over $N=49$ curves measured at different gate voltages). Note the much enhanced visibility of the $h/3e$ peak in $\langle|G(\nu)|\rangle$. The insets show the effect of samples specific features in the frequency range corresponding to the h/e peak. Also here the difference between the middle and bottom graphs is obvious and a splitting is clearly visible in the h/e peak of in $\langle|G(\nu)|\rangle$ (note, in the main panel, that a small splitting is also present on top of the $h/3e$ peak).

The results shown in Fig. 4 indicate that the presence of a statistically significant splitting in the h/e peak is a robust feature. It does not depend on the precise gate voltage range used in our analysis, nor on the specific quantity analyzed, i.e. the modulus of the FS or the power spectrum. The result is also robust against different procedures used to calculate the Fourier spectra from the experimental data. We have analyzed our data with and without removing the background in the magnetoconductance in different ways and using different kinds of windowing procedures. In all cases the final results show a similar, statistically significant splitting.[15]

Our results confirm the conclusion of Ref. 4, namely, that the h/e peak in the averaged FS is split in the presence of

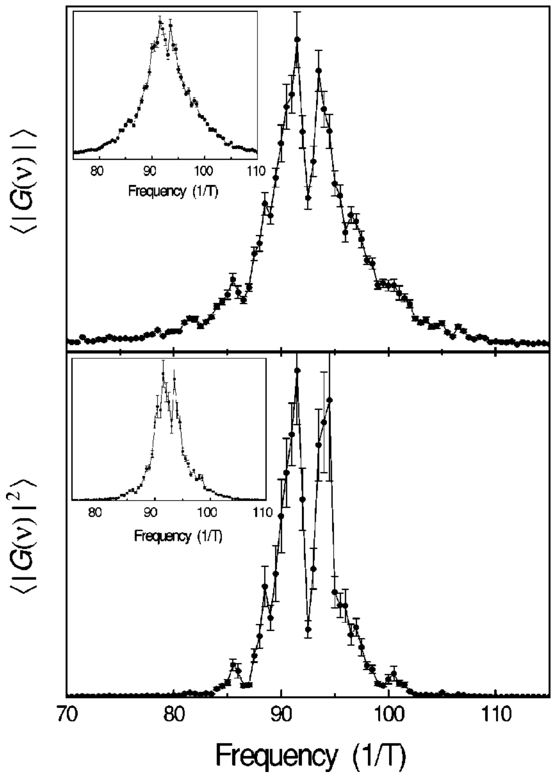

FIG. 4. The upper graph shows the h/e peak in the averaged modulus of the Fourier spectrum and the associated statistical error σ_{mean}. The average is performed over $N=16$ (main figure) and $N=49$ (inset) curves, measured at different V_g. The bottom graph shows the same statistical analysis for the power spectrum. The observed splittings are significantly larger than the corresponding σ and can therefore be discriminated from remnant sample specific features and attributed to a different physical phenomenon.

SOI, and put it on firmer grounds for two reasons. First, because the statistical analysis of the significance of the splitting had not been previously performed. Second, because the same effect is observed in a different material system.

The size of the splitting measured here is roughly 3/T, smaller than the one found in Ref. 4 ($\approx 12/T$). A direct comparison of the two results is however difficult, because the heterostructures used in the two experiments are different, as well as the radius of the rings ($0.35\ \mu$m versus $1.05\ \mu$m). It was argued in Ref. 4 that the splitting may be a manifestation of the geometrical Berry phase. Subsequent theoretical works seem to consistently conclude that the effect of Berry phase alone is too small to account for the magnitude of the previously observed splitting.[16] This is also true for the magnitude of the splitting observed here, whose precise origin remains to be identified. More experimental work is needed to discriminate between different possible mechanisms capable of accounting for the experimental results [such as SOI (Refs. 16 and 3) or Zeeman[3]]. The work presented in this paper demonstrates a well-defined experimental procedure on which future experiments can be based.

In conclusion we have discussed in detail the statistical properties of experimentally measured Aharonov-Bohm conductance oscillations and shown how random sample specific effects can be suppressed in a *quantifiable* way, up to a level that permits to reveal features of a different physical origin.

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[13] Note that although more traces are used in the average over the larger V_g range the size of the splitting, measured in terms of σ, does not increase. This is possibly due to some averaging-out of SOI effects with large V_g ranges.

[14] The (ensemble averaged) Fourier power spectrum is equal to the Fourier transform of the (ensemble averaged) magnetic correlation function $\langle C(\Delta B)\rangle$, where $C(\Delta B)=\int G(B)G(B+\Delta B)dB$.

[15] The classical background and "edge effects" were found to result predominantly in low-frequency components in the FS, far below the h/e peak. As a consequence, also the shape of the background (roughly above 25/T) is not/hardly affected.

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