A Pincer Randomization Method
for Valuing American Options*

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For European option we can obtain their exact values by using the so-called Black-Scholes formula, whereas there is no explicit exact solution for American counterparts despite that many researchers have attempted to obtain the solution. There have been many approximation methods developed for valuing American options, but they are not inclusive because they have both drawbacks and advantages with respect to their speed and accuracy. This paper develops an interpolation or pincer approximation based on the randomization methods due to Carr (1998) and Kimura (2004), using a pair of lower and upper bounds for option values derived by the Theta property. From numerical comparisons with other approximations, we see that our approximation has sufficient accuracy and efficiency for practical applications.

JEL Classification: G12, G13
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1. Introduction

Variety has come to options market nowadays since Black and Scholes (1973) and Merton (1973) published the seminal papers. In particular, the valuation of American options written on dividend-paying assets is an important issue in the market, because it can be widely applied to many other types of problems such as real options. Since McKean (1965) and Merton (1973) formulated the American option valuation as a free boundary problem, many researchers and practitioners have attempted to solve the analytical valuation problem of American options. However, no closed-form formulas have not yet been obtained. The difficulty is due to the possibility of early exercise whose unknown boundary must be determined as a part of the solution. Some researchers have made efforts toward developments of numerical approximation methods for pricing American options.

A simple and intuitive approximation was developed by Johnson (1983),

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which is based on analytical lower and/or upper bounds for the option value. Approximations are generated either by multiplying a coefficient to a single bound or by combining a pair of lower and upper bounds with weight coefficients. These coefficients are statistically estimated from a large set of the values of options actually traded in the market. In this sense, approximations based on bounds are of experimental or implied nature. Also, Broadie and Detemple (1996) have developed more sophisticated approximations in the same spirit. They obtained a lower bound of American call value by a capped American call option, while they numerically computed an upper bound through the integral representation of the early exercise premium; see Carr et al. (1992) and Kim (1990). Using these bounds, they proposed two kinds of approximations called LBA (lower-bound approximation) and LUBA (lower-and-upper-bounds approximation). Numerical comparisons in Broadie and Detemple (1996) shows that both approximations can be quickly computed and that LUBA is more accurate than LBA.

Another fast and accurate approximation among existing methods is the randomization method proposed by Carr (1997), which is based on an American option with a random maturity. The random maturity follows the $n$-stage Erlangian distribution with mean equal to the original maturity. As $n \to \infty$, the Erlangian distribution converges to a point mass concentrated at the mean. Hence, for large $n$, it can be considered that the value of American option with random maturity approximates the true value. Although this idea is easy to understand, the Erlangian distribution is not suitable for obtaining a simple formula for the $n$-th approximation. In fact, Carr’s formula for the American put value is given by a recursion of complex triple sums. To obtain a more tractable formula, an alternative randomization method has been recently developed by Kimura (2004), which used an order statistic for the random maturity. It has been shown that Kimura’s randomization method is much simpler than Carr’s method, providing almost the same accuracy. However, numerical solutions tend to behave unstably when we use high-precision computation. To remove this instability, we consider to refine Kimura’s randomization by treating approximations with low precision as bounds of the true value. Interpolating these bounds, we propose approximations for the option value and the early exercise boundary. This hybrid scheme is called a pincer randomization in this paper.

The rest of this paper is organized as follows: In Section 2, we provide some preliminaries for the analysis. The primal focus is on the American put option because the call case can be analyzed by put-call symmetry relations. Section 3 provides an idea of the pincer randomization method. To examine the accuracy of our method, numerical comparisons with other approximations are shown in Section 4. Finally, we give a conclusion and some comments on future research in Section 5.
2. Preliminaries

2.1 Basic framework

Let \( S_t \) be the stock price in the capital market following the efficient market hypothesis. Assume that \( S_t \) is a risk-neutralized process governed by the stochastic differential equation

\[
\frac{dS_t}{S_t} = (r - \delta)dt + \sigma dW_t, \quad t \geq 0,
\]

where \( r > 0 \) is the risk-free interest rate, \( \delta \geq 0 \) is a continuous dividend rate, \( \sigma > 0 \) is a volatility of the asset returns, and \( (W_t)_{t \geq 0} \) is a standard Wiener process on a filtered probability space \( (\Omega, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P}) \), where \( (\mathcal{F}_t)_{t \geq 0} \) is the natural filtration corresponding to \( W \) and the probability measure \( \mathbb{P} \) is chosen so that the stock has mean rate of return \( r \).

Let \( P \equiv P(t, S_t) = P_t(S_t; K, \delta) \), \( 0 \leq t \leq T \)

be the value of American put option written on \( (S_t)_{t \geq 0} \) with maturity date \( T > 0 \) and exercise price \( K > 0 \). Similarly, let \( C \equiv C(t, S_t) = C_t(S_t; K, r, \delta) \) \( (0 \leq t \leq T) \)
denote the value of the associated American call option with the same parameters as those in the put option. McDonald and Schroder (1998) proved that a parity relation holds between these two options, i.e.,

\[
C(t, S_t; K, r, \delta) = P(t, K; S_t r).
\]

Because of the parity relation between \( P \) and \( C \), we focus on the American put in this paper.

From the theory of arbitrage pricing, the fair value of the American put option at time \( t \) is given by solving an optimal stopping problem

\[
P(t, S_t) = \text{ess sup}_{T \in [t, T]} \mathbb{E}[e^{-r(T-t)}(K - S_T)\mid \mathcal{F}_t], \quad t \in [0, T],
\]

where \( T_t \) is a stopping time of the filtration \( (\mathcal{F}_t)_{t \geq 0} \) and the conditional expectation is calculated under the risk-neutral probability measure \( \mathbb{P} \). Solving the optimal stopping problem (2.3) is equivalent to find the points \( (t, S_t) \) for which early exercise is optimal. Let \( S \) and \( C \) denote the stopping region and continuation region, respectively. The stopping region \( S \) is defined by

\[
S = \{(u, x) \in [0, T] \times \mathbb{R} \mid P(u, x) = (K - x)^+\}.
\]

Of course, the continuation region \( C \) is the complement of \( S \) in \([0, T] \times \mathbb{R}_+\). The boundary that separates \( S \) from \( C \) is referred as the early exercise boundary, which is defined by

\[
B_t = \sup\{x \in \mathbb{R}_+ \mid P(t, x) = (K - x)^+\}, \quad t \in [0, T].
\]
The American put value $P$ and the early exercise boundary $(B_t)_{t \in [0, T]}$ can be obtained by jointly solving a free boundary problem, which is specified by the Black-Scholes-Merton partial differential equation

$$\frac{1}{2} \sigma^2 S^2 \frac{\partial^2 P(t, S)}{\partial S^2} + (r - \delta) S \frac{\partial P(t, S)}{\partial S} + \frac{\partial P(t, S)}{\partial t} - r P(t, S) = 0$$  \hspace{1cm} (2.6)

subject to the boundary conditions

$$\lim_{S \to \infty} P(t, S) = 0, \hspace{1cm} (2.7)$$

$$\lim_{S \to K} P(t, S) = K - B_t, \hspace{1cm} (2.8)$$

$$\lim_{S \to K} \frac{\partial P(t, S)}{\partial S} = -1, \hspace{1cm} (2.9)$$

and the terminal condition

$$P(T, S) = (K - S)^+. \hspace{1cm} (2.10)$$

Equation $(2.8)$ is usually called the value matching condition and Equation $(2.9)$ is the smooth pasting condition. These conditions guarantee that premature exercise strategy on the early exercise boundary $B_t$ will be optimal.

### 2.2 Randomization methods

#### 2.2.1 Carr’s randomization

Carr’s randomization method consists of the following three steps:

1. Randomize the maturity $T$ by an exponentially distributed random variable $\tilde{T}$ with mean $\mathbb{E}[\tilde{T}] = \lambda^{-1} = T$ in order to value the so-called Canadian option.

2. Extend the result to the case that $\tilde{T}$ is distributed as the $n$-stage Erlangian distribution with the same mean $\mathbb{E}[\tilde{T}] = \lambda^{-1} = T$.

3. Take the limit of the randomized option value by letting $n \to \infty$ to obtain the underlying American option value.

Let $h_n(t) (t \geq 0)$ denote the pdf of the $n$-stage Erlangian distribution with mean $\frac{1}{\lambda} = \tilde{T}$, i.e.,

$$h_n(t) = \frac{(nt/T)^{n-1}}{(n-1)!} \frac{n}{T} e^{-nt/T}, \hspace{1cm} t \geq 0.$$  \hspace{1cm} (2.11)

Figure 1 illustrates that a sequence of $|h_n(t)|$ converges to Dirac’s delta function concentrated at the mean $T (= 1)$ as $n$ gets large.

For a continuous function $g(t) (t \geq 0)$, define

$$g_n^+(T) = \mathbb{E}[g(\tilde{T})] = \int_0^\infty g(t) h_n(t) dt.$$  \hspace{1cm} (2.11)
Then, we have

\[
\lim_{n \to \infty} g_n^*(T) = g(T)
\]

that is the mathematical essence of Carr’s randomization method.

### 2.2.2 Kimura’s randomization

Instead of the \( n \)-stage Erlangian distribution, Kimura (2004) adopted an order statistic for the random maturity. In much the same way as in Carr’s randomization, his method can be written by the following three steps:

1. Randomize the maturity \( T \) by an exponentially distributed random variable \( \tilde{T} \) with mean \( \mathbb{E}[\tilde{T}] = \lambda^{-1} = T \) in order to value the Canadian option.

2. Extend the result to the case that \( \tilde{T} \) is distributed as an order statistic with the same mean \( \mathbb{E}[^{\tilde{T}}] = \lambda^{-1} = T \).

3. Take the limit of the randomized option value by letting \( n, m \to \infty \) to obtain the underlying American option value.

The order statistic used by Kimura is defined as follows: Let \( X_1, \cdots, X_{n+m} \) be iid random variables with parameter \( \alpha > 0 \), and let \( X_{(i)} \) denote the \( i \)-th smallest of these random variables \( (i = 1, \cdots, n+m) \). Then, the pdf of \( X_{(n+1)} \) is given by

\[
f_{n,m}(t) = \frac{(n+m)!}{n!(m-1)!} (1-e^{-\alpha t})^n \alpha e^{-\alpha m^2 t}, \quad t \geq 0.
\]

The mean and variance of \( X_{(n+1)} \) are given by

\[
\mathbb{E}[X_{(n+1)}] = \frac{1}{\alpha} \sum_{i=0}^n \frac{1}{m+i},
\]
\[
\mathbb{V}[X_{(n+1)}] = \frac{1}{\alpha^2} \sum_{i=0}^n \frac{1}{(m+i)^2}.
\]
In addition, the modal value of $X_{n+1}$ is given by

$$
\mathbb{M}[X_{n+1}] = \arg \max_t f_{n,m}(t) = \frac{1}{\alpha} \log \frac{n+m}{m}.
$$

Figure 2(a) and 2(b) show the convergence of the pdf as $n(=m) \to \infty$ for the cases that (a) $\mathbb{E}[X_{n+1}] = T = 1$ and (b) $\mathbb{M}[X_{n+1}] = T \equiv 1$. From these figures we see that there is no difference between these two cases and the pdfs converge to Dirac’s delta function concentrated at the mean $\mathbb{E}[X_{n+1}]$. By setting either $\mathbb{E}[X_{n+1}] = T$ or $\mathbb{M}[X_{n+1}] = T$, $X_{n+1}$ can be another candidate for the random maturity $\bar{T}$, because $\lim_{n,m \to \infty} \mathbb{V}[X_{n+1}] = 0$. Kimura (2004) adopted the mode-matching $\mathbb{M}[X_{n+1}] = T$ in his randomization for computational convenience, because there is no significant difference between the two matchings for large numbers of $n$. For the mode-matching, $\alpha$ can be determined by

$$
\alpha = \frac{1}{T} \log \frac{n+m}{m}.
$$

For a continuous function $g(t) (t \geq 0)$, define

$$
\tilde{g}_{n,m}(T) = \mathbb{E}[g(\bar{T})] = \int_0^\infty g(t) f_{n,m}(t) dt.
$$

Kimura (2004, Proposition 3) showed that the sequence $(\tilde{g}_{n,m})_{n,m \geq 1}$ can be efficiently computed by using the recursion
\[
\begin{align*}
  g_{0,m}^* &= \int_0^\infty m a e^{-m t} g(t) \, dt \\
  g_{n,m}^* &= \frac{n+m}{n} g_{n-1,m}^* - \frac{m}{n} g_{n-1,m+1}^*, \quad n \geq 1.
\end{align*}
\]

and that
\[
\lim_{n,m \to \infty} g_{n,m}^*(T) = g(T). \tag{2.19}
\]

### 2.3 Canadian options

It can be seen that the Canadian case is a common starting point in these randomization methods. Hence, we briefly summarize the fundamental results for the Canadian options.

For \( \lambda > 0 \) and \( S = S \), let \( p^* = p^*(\lambda, S) \) denote the European-style Canadian put value at time \( t \in [0, T] \). Note that \( p^*(\lambda, S) \) does not depend on \( t \) by virtue of the memoryless property. Then, we have

**Proposition 1 (Kimura (2004))** The value of the European-style Canadian put option is given by
\[
p^*(\lambda, S) = \begin{cases} 
  \xi(S) + \frac{\lambda}{\lambda + \delta} K - \frac{\lambda}{\lambda + \delta} S, & S < K \\
  \gamma(S), & S \geq K,
\end{cases} \tag{2.20}
\]

where
\[
\xi(S) = \frac{1}{\theta_1 - \theta_2} \frac{\lambda}{\lambda + \delta} \left( 1 - \frac{r - \delta}{\lambda + r} \theta_2 \right) \left( \frac{S}{K} \right)^{\theta_1}, \quad S < K \tag{2.21}
\]
\[
\gamma(S) = \frac{1}{\theta_1 - \theta_2} \frac{\lambda}{\lambda + \delta} \left( 1 - \frac{r - \delta}{\lambda + r} \theta_1 \right) \left( \frac{S}{K} \right)^{\theta_2}, \quad S \geq K \tag{2.22}
\]

and the parameters \( \theta_1 = \theta_+ > 0 \) are \( \theta_2 = \theta_- < 0 \) are two roots of the quadratic equation
\[
\frac{1}{2} \sigma^2 \theta^2 + \left( r - \delta - \frac{1}{2} \sigma^2 \right) \theta - (\lambda + r) = 0,
\]
\[i.e.
\theta_+ = \frac{1}{\sigma^2} \left\{ - \left( r - \delta - \frac{1}{2} \sigma^2 \right) \pm \sqrt{ \left( r - \delta - \frac{1}{2} \sigma^2 \right)^2 + 2\sigma^2(\lambda + r) } \right\}. \tag{2.23}
\]

For \( \lambda > 0 \) and \( S = S \), let \( P^* = P^*(\lambda, S) \) denote the American-style Canadian put value. Then, we have

**Proposition 2 (Kimura (2004))** The value of the European-style Canadian put option is given by
\[ P^*(\lambda, S) = \begin{cases} K - S, & S \leq B^* \leq K \\ p^*(\lambda, S) + e^*(\lambda, S), & S > B^* \end{cases} \]  

where \( B^* = B^*(\lambda) \) is the Laplace-Carlson transform of the time-reversed early exercise boundary \( \bar{B}_\tau = B_{T - \tau}, (\tau \geq 0) \), i.e.,

\[ B^*(\lambda) = \int_0^\infty \lambda e^{-\lambda t} \bar{B}_t \, dt, \quad \lambda > 0 \]  

and

\[ e^*(\lambda, S) = -\frac{1}{\theta_2} \left( \theta_1 \xi(B^*) + \frac{\delta}{\lambda + \delta} B^* \right) \left( \frac{S}{B^*} \right)_{\theta_1}, \quad S > B^*. \]  

Applying the value matching condition (2.8) to the option value \( P^*(\lambda, S) \) in (2.24), we can obtain the following results that specify the early exercise boundary \( B^* \) of the American-style Canadian put option.

**Proposition 3 (Kimura (2004))**

(i) The early exercise boundary \( B^* \) of the American-style Canadian put option satisfies the equation

\[ \lambda \left( \frac{B^*}{K} \right)^{\theta_1} = r(\theta_1 - 1) - \delta \theta_1 \frac{B^*}{K}. \]  

(ii) For the limiting case \( \lambda \to 0 \), we have

\[ B^*(0) = \lim_{\tau \to \infty} \bar{B}_\tau = \frac{r(\theta_1 - 1)}{\theta_1} K = \frac{\theta^*_2}{\theta_2 - 1} K, \]  

where \( \theta^*_i = \lim_{\tau \to \infty} \theta_i (i = 1, 2) \). In particular, if \( \delta = 0 \), then

\[ B^*(0) = \lim_{\tau \to \infty} \bar{B}_\tau = \frac{K}{1 + \frac{\sigma^2}{2r}}. \]  

(iii) For the limiting case \( \lambda \to \infty \), we have

\[ \lim_{\lambda \to \infty} B^*(\lambda) = \bar{B}_0 = B_T = \min \left( \frac{T}{\delta}, 1 \right) K. \]  

3. A Pincer Randomization Method

By virtue of the recursion (2.18), Kimura’s randomization method is much simpler than Carr’s method, providing almost the same accuracy. However, Kimura’s method sometimes behaves unstably near the expiry in high-precision computation. The reasons for the instability are considered as

(i) the algorithm is fairly sensitive to the precision of the root \( B^* \) of the equation (2.27).
In this section, we propose a refinement to overcome these difficulties, which is based on a pair of lower and upper bounds for a true value (say TRUE), and then TRUE is sandwiched between the bounds. This method reflects some fundamental properties of the option Greek \( \Theta \) and the order statistic. It is generally known that \( \Theta \) indicates the ratio of the change in an American put value to decrease in time to expiration. Hence, the shorter the remaining time to expiration, the option value is cheaper.

3.1 Lower and upper bounds for the option value

First consider the mean-matching case. From Figure 2(a) and the Theta property of American put options that the mean-matching approximation for the option value always underestimates the true value when \( n \) and \( m \) are not large enough, giving a lower bound. Note that mean-matching approximation for the early exercise boundary provides an upper bound. Figure 3(a) shows that the lower bound is a tight one over the true value derived by the CRR bi-

![Figure 3. Lower and upper bounds](image)
nominal method (Cox et al. (1979)) with \( n = 1000 \).

In the same manner as the mean-matching case, Figure 2(b) and the Theta property shows that the mode-matching approximation always overestimates the true value when \( n \) and \( m \) are small, i.e., it gives an upper bound. For the early exercise boundary, the mode-matching approximation provides a lower bound on the contrary. Figure 3(b) shows that the upper bound is less tight than the lower bound, where TRUE values are also computed by the CRR binomial method with \( n = 1000 \).

### 3.2 Interpolating lower and upper bounds

Figure 4 illustrates a relationship between the lower and upper bounds for the early exercise boundary. This figure shows that the TRUE value is appropriately sandwiched between the bounds, and that the upper bound derived by the mean matching is a good approximation for the TRUE value. For the option value, the TRUE value is also in the bounds, and the lower bound is a good approximation for the TRUE one. From Figure 3, the mean matching provides more accurate approximations for the option value. From these observations, we employ the two interpolation methods below for valuing American put options, each of which does not have experimental nature as in LUBA (See Broadie and Detemple (1996)): Let \( L(t, S_t) \) and \( U(t, S_t) \) denote the lower and upper bound for the option value, respectively. Then, we define

- **Arithmetic Average:**

  \[
P_A(t, S_t) = \frac{L(t, S_t) + U(t, S_t)}{2},
\]

- **Geometric Average:**

  \[
P_G(t, S_t) = \sqrt{L(t, S_t) \times U(t, S_t)}
\]

for approximations of the American put value. For the early exercise boundary, we define the arithmetic and geometric averages in a similar way. As de-
scribed above, the upper bound of the early exercise boundary and the lower bound of the option value are good approximations for the TRUE values. Hence, we also add the upper-bound approximation for the early exercise boundary and the lower-bound approximation for the option value in comparisons.

To determine an optimal level of $n,m$ in the approximation $g_{n,m}$, we made a preliminary comparison between the European put value and its PR approximation. Let $p\left(t, S\right)$ denote the value of a vanilla European put option at time $t \in [0,T]$. Obviously, $p\left(t, S\right)$ can be computed by the Black-Scholes formula

$$p\left(t, S\right) = Ke^{-r(T-t)}\Phi\left(-d + \sigma \sqrt{T-t}\right) - S e^{-\delta(T-t)}\Phi\left(-d\right),$$

(3.3)

where

$$d = \frac{\log\left(S/K\right) + \left(r - \delta + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma \sqrt{T-t}}.$$  

(3.4)

Figure 5 illustrates the relative percentage errors of approximations $g_{N,N}(N=3,...,8)$ for $p(0,S)$ as functions of $S$. We see from Figure 5 that the approximations become better as $N$ increases, being sufficiently accurate for
Figure 6. Early exercise boundaries of put options \( (K=100, T=1.0, r=0.05) \)

\[ \delta=0.05, \sigma=0.2, 0.25, 0.3 \]

\[ \delta=0.02, 0.05, 0.08, \sigma=0.2 \]

4. Computational Results

Figure 6(a) (6(b)) shows some relations between the early exercise boundary and the volatility (dividend rate). Also, Figure 7(a) (7(b)) shows some relations between the option values and the volatility (maturity). In order to check the performance of the PR method in detail, we compare them with other approximations for particular cases quoted from numerical experiments in AitSahlia and Carr (1997). Tables 1 and 2 summarize these results, in which we compute three approximations by the PR method with both the arithmetic and geometric averages and the lower-bound approximation named LB-Random. We employ the arithmetic average of the 1000- and 1001-steps binomial value as a bench mark of the TRUE value. For the methods of Kimura (2004), Carr (1997), and Geske and Johnson (1984), “N-pts” in these tables denote the number of steps of the \( N \)-point Richardson extrapolation. For the finite-difference results (Brennan and Schwartz (1977)), the parameters \( N \) and \( M \) denote the numbers of time and state steps, respectively. Also, the quadratic approximation (Barone-Adesi and Whaley (1987) and MacMillan (1886)), LBA and LUBA (Broadie and Detemple (1996)), approximations by Bunch and Johnson (1992) and Huang et al. (1996) are added for compari-
sons in these tables. See AitSahlia and Carr (1997) for details of their experiments.

The PR method performs very well and competes with the randomizations of Kimura and Carr. In addition, the PR method succeeds in the way that modified Kimura’s randomization that always underestimates the TRUE value, because the PR method provides not only much more accurate approximations for valuing put options but also better approximation than Kimura. In addition, we see from these figures that the PR method is more accurate than LBA and LUBA, which are also the lower-bound and the lower-and-upper-bounds approximations, respectively.

Table 1 shows the impacts of the initial stock price $S$. The PR method with both of arithmetic and geometric average becomes accurate as $S$ increases, because the early exercise premium relatively constitutes a smaller portion of the value for such cases. The fact is very well deserved from the viewpoints that the PR method can value European option values as accurate as the Black-Scholes formula and that we can decompose American option value into the early exercise premium plus European option value.

Table 2 demonstrates the impacts of the remaining time to maturity on the option value. For both cases of arithmetic and geometric averages, the PR method becomes accurate as the remaining time becomes long. For this tendency, we can give the same prospect from Table 1. In addition, from Tables 1
and 2, we can see that the PR method with arithmetic average is accurate enough and is greater than the one with geometric average. Clearly, this reflects the fact that $P_{\text{AR}}(t, S) \geq P_{\text{GA}}(t, S)$ for all $(t, S)$.

From the observations in Figures 3 and 4, it was considered that the lower-bound approximations for the option values would perform well. However, we see from Tables 1 and 2 that the lower-bound approximations are less accurate than other approximations. We also see from other numerical experiments that the randomization method with mean matching performs well if and only if dividend is zero for which the root $B^*$ can be computed via

$$B^* = K \left( \frac{r(t_1 - 1)}{\lambda} \right)^{1/\tau}$$  \hspace{1cm} (4.1)

without using Newton’s method. These observations would imply that the ac-
accuracy of the lower-bound (or mean-matching) approximation is highly sensitive to the computational accuracy of the root $B^∗$.

5. Conclusion

The previously established randomization methods have crucial problems such as (i) difficulty of implementation in Carr’s randomization and (ii) unstable behavior near expiry in Kimura’s randomization. To rectify these defects at the same time, we have adopted an interpolation approximation using a pair of lower and upper bounds obtained by Kimura’s randomization. The idea is due to the Theta property of American put options.

The PR method generates accurate approximations when the initial stock price is in the out-of-money or the remaining time to maturity is long. It is straightforward to interpret these properties from the fact that American option value can decomposed into the early exercise premium and the associated European option value, the latter of which constitutes a greater portion of the whole value. However, the PR method still have a tendency of underestimation from the true value, which needs a further revision of the randomization.

Mathematical essence of randomization can be interpreted as an inversion of Laplace or Fourier transforms. This interpretation enables us to apply the randomization methods including the PR method to valuing other options, e.g., exotic or path-dependent options such as Asian, lookback, barrier options and so on. This is a future theme of extensive research. Another extension of the randomization method is the case that the stock return jumps accidentally, that is, the stock price process follows a jump-diffusion process or more generally a Lévy process. This remains as a future theme, too.

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References


