A novel method to stabilize the channeled spectropolarimeter is described. The fluctuating retardations of the high-order retarders used in the spectropolarimeter are calibrated in parallel to the measurement of the wavenumber-dependent state of polarization (SOP) of light. Both the calibration of the retarders and the measurement of the SOP can be made simultaneously using a single light to be measured, and hence the resultant wavenumber-dependent SOP is almost immune to the fluctuation of the retardations. The effectiveness of this method is experimentally demonstrated with the retardation fluctuations induced by the temperature change up to 40°C. © 2006 Optical Society of America
Channeled spectropolarimetry\cite{1,2} is a snapshot method for measuring the spectral dependence of the state of polarization (SOP) of light. This method uses no mechanical or active elements for polarization control, such as a rotating compensator or an electro-optic modulator, and all the spectrally-resolved Stokes parameters can be completely determined at once only from a single spectrum. Furthermore, the channeled spectropolarimeter has a simple optical system and hence is suited for use in many applications.\cite{3,4}

In spite of these advantages, however, the channeled spectropolarimetry has a drawback that it is quite susceptible to the environmental perturbations, such as the temperature change. That is why it uses high-order, namely fairly thick, birefringent retarders whose retardations are heavily dependent on the environmental conditions. Although this drawback can be avoided by using additional instruments such as the temperature controller or the temperature monitor, the use of these instruments sacrifices the major merits of the channeled spectropolarimeter, namely the simplicity and the compactness of the configuration.

To overcome this drawback of the channeled spectropolarimetry, a novel method to compensate the fluctuations in the high-order retarders is developed. The authors discovered that the channeled spectropolarimeter has inherently possessed the potential for the “self-calibration” feature for the retarders. We use this feature to compensate
the retardation fluctuations during the measurement. This method enables us to calibrate the retarders in parallel to the SOP measurement only using a single light to be measured its SOP.

We first briefly recall the principle of the channeled spectropolarimetry. The basic configuration of the channeled spectropolarimeter is illustrated in Fig. 1. The polychromatic light under measurement of its SOP successively passes through two high-order birefringent retarders $R_1$ and $R_2$, and an analyzer $A$, and then is launched into a spectrometer. The fast axes of $R_1$ and $R_2$ intersect at 45°, and the transmission axis of $A$ is aligned to the fast axis of $R_1$. Using $\sigma$ to be the wavenumber, the reciprocal of the wavelength, and $S_k(\sigma), (k = 0...3)$ to be the spectrally-resolved Stokes parameters of the light under measurement, the channeled spectrum obtained from the spectrometer varies with $\sigma$ as

$$P(\sigma) = (1/2)S_0(\sigma)$$

$$+ (1/4)|S_{23}(\sigma)|\cos[\phi_2(\sigma) - \phi_1(\sigma) + \text{arg}\{S_{23}(\sigma)\}]$$

$$- (1/4)|S_{23}(\sigma)|\cos[\phi_2(\sigma) + \phi_1(\sigma) - \text{arg}\{S_{23}(\sigma)\}]$$

$$+ (1/2)|S_1(\sigma)|\cos[\phi_2(\sigma) + \text{arg}\{S_1(\sigma)\}], \quad (1)$$

where $S_{23}(\sigma) = S_2(\sigma) + iS_3(\sigma)$, and “arg” stands for the operator to take the argument. Note that $\text{arg}\{S_1(\sigma)\}$ takes either 0 or $\pi$ depending on the sign of $S_1(\sigma)$. Since the phase retardation $\phi_j(\sigma), (j = 1, 2)$ of the high-order birefringent retarder $R_j$ greatly and almost linearly increases with $\sigma$, $P(\sigma)$ consists of one slowly-varying
and three quasi-cosinusoidally varying components. The four components can be separated from one another by use of the frequency filtering technique. All the Stokes parameters \( S_k(\sigma), (k = 0\ldots3) \) can be independently determined by demodulating the amplitudes and phases of the extracted respective components. Note that the light under measurement should have the sufficiently broad spectrum, so that the SOP information can be modulated on the resultant channeled spectrum.

Care should be taken of the fact that the phase retardations \( \phi_1(\sigma) \) and \( \phi_2(\sigma) \) must be precisely calibrated before the Stokes parameters are determined. Although the retardations can be calibrated by use of a reference light with a known \( \arg\{S_1(\sigma)\} \) or \( \arg\{S_{23}(\sigma)\} \), the introduction of the reference light brings about the error associated with the retardation changes during the time interval between the calibration and the measurement. The high-order retarders used in the channeled spectropolarimeter are generally several tens- or hundreds-times thicker than the compensators used in the other conventional spectropolarimeters, and accordingly their retardations are much more susceptible to the environmental perturbations, such as the temperature change.

To overcome this problem, a novel method for calibrating the retarders using only the light under the SOP measurement is developed. From now on, we call this as “self-calibration method” for the channeled spectropolarimeter. The presented method allows us to carry out the calibration of the high-order retarders and the determination of the SOP simultaneously by using the measured light itself.

In the first step of the self-calibration method, we compute the complex represen-
tations of the four components in \( P(\sigma) \)

\begin{align}
F_0(\sigma) &= (1/2) S_0(\sigma), \\
F_2(\sigma) &= (1/4) |S_1(\sigma)| e^{i[\phi_2(\sigma) + \text{arg}\{S_1(\sigma)\}]}, \\
F_-(\sigma) &= (1/8) |S_{23}(\sigma)| e^{i[\phi_2(\sigma) - \phi_1(\sigma) + \text{arg}\{S_{23}(\sigma)\}]}, \\
F_+(\sigma) &= -(1/8) |S_{23}(\sigma)| e^{i[\phi_2(\sigma) + \phi_1(\sigma) - \text{arg}\{S_{23}(\sigma)\}]}.
\end{align}

by use of the Fourier transform technique modified for use in the spectral domain.\(^1\)

We next calculate the following products from the obtained components to eliminate \( \text{arg}\{S_1(\sigma)\} \) and \( \text{arg}\{S_{23}(\sigma)\} \) from them.

\begin{align}
F_2^2(\sigma) &= (1/16) |S_1(\sigma)|^2 e^{i2\phi_2(\sigma)}, \\
-F_-(-\sigma)F_+(\sigma) &= (1/64) |S_{23}(\sigma)|^2 e^{i2\phi_2(\sigma)}.
\end{align}

In both equations, the arguments are only the function of \( \phi_2(\sigma) \), and thus independent from the SOP of the incident light. This implies that \( \phi_2(\sigma) \) is determined when we compute either of their arguments. However, the absolute values of Eq.\((3a)\) and \((3b)\) may be zero, depending on the SOP of the light under measurement. To avoid this inconvenience, we take their weighted sum as

\[ 16F_2^2(\sigma) - 64F_-(-\sigma)F_+(\sigma) = \{S_1^2(\sigma) + S_{23}^2(\sigma) + S_{33}^2(\sigma)\} e^{i2\phi_2(\sigma)}. \]

The absolute value of Eq. \((4)\) is equal to the squared intensity of the completely polarized components of the incident light, and thus is always positive except for
the completely unpolarized light. Therefore, $\phi_2(\sigma)$ can be obtained by taking the argument of $16F_2^2(\sigma) - 64F_-(\sigma)F_+(\sigma)$, regardless of the SOP of the incident light.

The other phase retardation function, $\phi_1(\sigma)$, can be computed from the obtained $\phi_2(\sigma)$ by simply multiplying the thickness ratio of $R_1$ and $R_2$, provided that both retarders undergo the same environmental perturbations. Once $\phi_1(\sigma)$ and $\phi_2(\sigma)$ are determined, all the Stokes parameters can be calculated from $F_0(\sigma)$, $F_2(\sigma)$, and $F_-(\sigma)$ by use of Eqs. (2a)-(2c).

With the self-calibration method presented here, no separate calibration light source is required since the calibration is carried out with the light under measurement of its SOP. In addition, the self-calibration method uses no additional instrument such as the external thermometer, and can be incorporated without the significant increase in computational requirements, because the major part of its procedure, such as the computations of $F_0(\sigma)$, $F_2(\sigma)$, and $F_-(\sigma)$, can be shared with the demodulation of the SOP.

The feasibility of the self-calibration method is demonstrated by use of the experimental set-up in Fig. 2. A white light from a Halogen lamp passes through a polarizer P and a thin quartz plate (TQP), whose effective thickness is 76 $\mu$m, to generate a light to be measured its SOP. The channeled spectropolarimeter consists of three bonded calcite prisms and a multi-channel spectrometer (USB2000, Ocean Optics). Each part of the calcite prisms works as high-order retarders $R_1$ and $R_2$, and Glan-Thompson analyzer A. The thicknesses of $R_1$ and $R_2$ are 125 $\mu$m and 250 $\mu$m, respectively. Note
that R_1 and R_2 are contacted with each other so that they experience the same environmental condition.

Two sets of experiments were made to demonstrate the self-calibration method. In the first set, the phase retardations $\phi_1(\sigma)$ and $\phi_2(\sigma)$ of the retarders R_1 and R_2 were calibrated by the self-calibration method using the measurement light itself, and by another calibration procedure using an additional reference light; for the reference light, linearly polarized light oriented at 22.5° was employed. The calibrated retardations are shown in Fig. 3(a). The results calibrated by two different methods, shown in solid and dashed lines respectively, almost overlap with each other; the differences of the retardations between two calibration methods are less than 0.04 radian for both $\phi_1(\sigma)$ and $\phi_2(\sigma)$. The normalized Stokes parameters demodulated with the calibrated retardations by both methods are shown in Fig. 3(b). The derived Stokes parameters by both methods agree within 0.01. This result demonstrates that the self-calibration technique allows us to calibrate the retardations only from the measurement light independent from its SOP.

In the second set of the experiment, the SOP measurements were repeated while the temperature of the entire experimental system was changed from 5°C to 45°C by use of a temperature controlled bath. In this experiment, TQP, which is susceptible to the temperature change, was removed from the optical set-up so that the SOP of the light to be measured does not change with the temperature. The demodulated normalized Stokes parameters are shown in Fig. 4. In (a), R_1 and R_2 are calibrated
only before starting the temperature change by use of a reference light at $T = 25^\circ C$.

On the other hand, in (b), the retarders are calibrated at every temperature using the self-calibration method simultaneously with the Stokes parameter measurement. It is apparent that the fluctuations due to the temperature change are effectively reduced by the self-calibration method. The residual fluctuations of the normalized Stokes parameters in (b) are less than 0.04. The results show that the fluctuations in the retarders can be compensated without the temperature determination.

The self-calibration method presented here has a feature that it is independent from the SOP of the light incident on the channeled spectropolarimeter, provided that the light is not almost completely unpolarized. In addition, the calibration of the high-order retarders and the measurement of the wavenumber-dependent SOP can be made simultaneously only from a single measurement light. Accordingly, the measured SOP is almost free from the fluctuations in the high-order retarders. It should be emphasized that the channeled spectropolarimeter inherently possesses the potential for self-calibration, but this fact had not previously been recognized and exploited.

Note that the present method can be used only for the calibration of the high-order retarders. Fortunately, however, the other system parameters, such as the modulation transfer function of the spectrometer, do not change appreciably during the measurement, and hence it is sufficient to calibrate these parameters before the measurement. Consequently, the present method is promising to reduce the major instabilities of
the channeled spectropolarimeter induced by the phase fluctuations of the high-order retarders.

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