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# Josephson spin current in triplet superconductor junctions

Yasuhiro Asano
*Department of Applied Physics, Hokkaido University, Sapporo 060-8628, Japan*
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This paper theoretically discusses the spin current in spin-triplet superconductor/insulator/spin-triplet superconductor junctions. At low temperatures, a midgap Andreev resonant state anomalously enhances not only the charge current but also the spin current. The coupling between the Cooper pairs and the electromagnetic fields leads to a Fraunhofer pattern in the direct current spin flow in magnetic fields and an alternative spin current under applied bias voltages.

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Although the supercurrent usually refers to the dissipationless charge flow in superconductors or the mass flow in superfluid He, the spin flow carried by spin-triplet Cooper pairs is also undoubtedly a supercurrent.[1] These supercurrents have a common feature: the spatial gradient of the order parameter drives the supercurrent. It is well known that charge and mass supercurrents are possible under a spatial gradient in the macroscopic phase $\varphi$. On the other hand, the spatial gradient of the $\boldsymbol{d}$ vector causes a spin supercurrent. In the mean-field theory of superconductivity, $\boldsymbol{d}$ characterizes the order parameter of spin-triplet pairs as $\hat{\Delta}(\vec{r}) = i\boldsymbol{d}(\vec{r})\cdot\hat{\boldsymbol{\sigma}}\hat{\sigma}_2 e^{i\varphi}$ where $\hat{\sigma}_j$ for $j=1-3$ are the Pauli matrices. In $^3$He, the angular momentum vector of a Cooper pair points in the direction normal to the surrounding wall. A spin current is expected near a curved wall because $\boldsymbol{d}$ and the momentum vector align with each other due to the dipole-dipole interaction. Through the spin dynamics in superfluids, the spin current can be detected by nuclear magnetic resonance experiments.[2–4]

In bulk superconductors, however, a spin current is usually not expected because the weak spin anisotropy fixes $\boldsymbol{d}$ homogeneously in a certain direction of crystal lattice. So far the generation of spin flow has been discussed in curved structures of superconductors[5] and superconducting weak links.[5,6] To realize spin current in experiments, we should consider simpler structures such as superconductor/insulator/superconductor (SIS) junctions of triplet superconductors. Generally speaking, spin-triplet pairs are characterized by odd-parity $p$- or $f$-wave symmetry. In SIS junctions of such unconventional superconductors, a midgap Andreev resonant state (MARS) governs the low-energy transport.[7] So far it has been pointed out that the MARS drastically enhances the electric charge transport but it suppresses the thermal transport.[8] The effects of the MARS on spin transport are still an open question. In contrast to spin-triplet pairs of $^3$He, Cooper pairs in superconductors have a charge degree of freedom that couples with electromagnetic fields. This feature may also enable us to switch the spin current by using electric fields and/or magnetic fields.

In this paper, I calculate analytically the spin current in SIS junctions based on the mean-field theory of superconductivity. When the MARS is formed at the junction interface, a low-temperature anomaly appears in the spin current as well as in the charge current.[9,10] The spin current shows a Fraunhofer pattern in applied magnetic fields. I also show that an applied bias voltage across the junctions causes an alternating current of spin.

Throughout this paper, we take units of $\hbar=k_B=c=1$, where $k_B$ is the Boltzmann constant and $c$ denotes the speed of light. Vectors in real and momentum space are indicated by an overarrow while those in spin space are described by bold italic characters.

Electronic states in SIS junctions are described by the Bogoliubov–de Gennes (BdG) equation

$$\int d\vec{r}' \begin{bmatrix} \hat{h}(\vec{r},\vec{r}') & \hat{\Delta}(\vec{r},\vec{r}') \\ -\hat{\Delta}^*(\vec{r},\vec{r}') & -\hat{h}^*(\vec{r},\vec{r}') \end{bmatrix} \begin{bmatrix} \hat{u}(\vec{r}') \\ \hat{v}(\vec{r}') \end{bmatrix} = E \begin{bmatrix} \hat{u}(\vec{r}) \\ \hat{v}(\vec{r}) \end{bmatrix}, \tag{1}$$

$$\hat{h}(\vec{r},\vec{r}') = \delta(\vec{r}-\vec{r}')\left\{ -\frac{\vec{D}_{\vec{r}}^2}{2m} + V(\vec{r}) - \mu \right\}\hat{\sigma}_0, \tag{2}$$

where $\vec{D}_{\vec{r}}=\vec{\nabla}_{\vec{r}}-ie\vec{A}_{\vec{r}}$ with $\vec{A}_{\vec{r}}$ being the vector potential, the caret indicates a $2\times 2$ matrix describing spin space, $\hat{\sigma}_0$ is the unit matrix, and $\mu$ is the Fermi energy. The insulating barrier at $x=0$ is described by $V_B\delta(x)$. In uniform superconductors, the pair potential is given by

$$\hat{\Delta}(\vec{r},\vec{r}') = \frac{1}{V_{vol}}\sum_{\vec{k}} \hat{\Delta}_{\vec{k}} e^{i\vec{k}\cdot(\vec{r}-\vec{r}')}, \tag{3}$$

$$\hat{\Delta}_{\vec{k}} = i\Delta_{\vec{k}}\boldsymbol{d}_j\cdot\hat{\boldsymbol{\sigma}}\hat{\sigma}_2 e^{i\varphi_j}. \tag{4}$$

The vector $\vec{k}=(k,\vec{p})$ represents the wave number on the Fermi surface (i.e., $k^2+\vec{p}^2=k_F^2$), where $k$ and $\vec{p}$ are the wave number in the direction of the current and that in the direction transverse to the current, respectively. I consider a similar model to that in Ref. 11. We assume that the two superconductors are identical to each other except for the directions of the unit vectors $\boldsymbol{d}_j$ ($j=L$ or $R$) as shown in Fig. 1(a), where $\alpha$ is the orientation angle between $\boldsymbol{d}_L$ and $\boldsymbol{d}_R$. In unconventional superconductor junctions, the characteristic behaviors of the Josephson charge current are very sensitive to the dependence of the pair potential on the wave number. When $\Delta_{k,\vec{p}}$ and $\Delta_{-k,\vec{p}}$ have opposite signs to each other, the MARS formed at junction interfaces causes anomalous charge transport at low temperatures.[7,12] In this paper, I consider the two typical situations $\Delta_{-k,\vec{p}}=\nu\Delta_{k,\vec{p}}$ with $\nu=\pm 1$. The

 

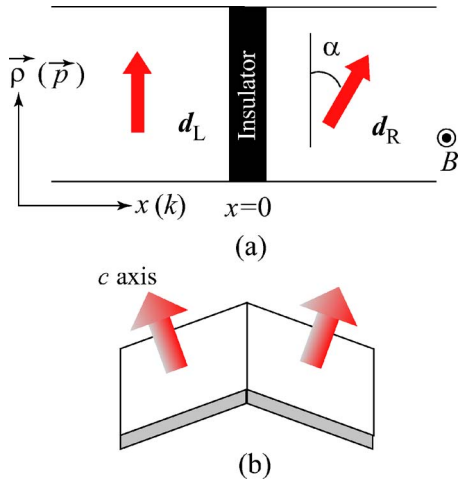


FIG. 1. (Color online) (a) A schematic figure of a SIS junction. The large arrows in the superconductors represent $\boldsymbol{d}$ which is a unit vector in spin space. In (b), we propose a SIS junction using $Sr_2RuO_4$, where $\boldsymbol{d}$ is parallel to the $c$ axis of the crystal.

MARS forms (does not form) for $\nu=-1$ ($\nu=1$). Within $p$-wave symmetries, $\nu=1$ ($\nu=-1$) corresponds to $p_y$ ($p_x$) wave symmetry. First I solve the BdG equation in the two superconductors independently and obtain the wave functions

$$\boldsymbol{\Psi}_L(x,\vec{\rho}) = \check{\chi}_L\left[\begin{pmatrix}\hat{u}\\ \hat{v}\end{pmatrix}e^{ikx}\hat{a} + \begin{pmatrix}\hat{v}\nu\\ \hat{u}\end{pmatrix}e^{-ikx}\hat{b} + \begin{pmatrix}\hat{u}\\ \hat{v}\nu\end{pmatrix}e^{-ikx}\hat{A} + \begin{pmatrix}\hat{v}\\ \hat{u}\end{pmatrix}e^{ikx}\hat{B}\right]e^{i\vec{p}\cdot\vec{\rho}},$$

$$\boldsymbol{\Psi}_R(x,\vec{\rho}) = \check{\chi}_R\left[\begin{pmatrix}\hat{u}\\ \hat{v}\end{pmatrix}e^{ikx}\hat{C} + \begin{pmatrix}\hat{v}\nu\\ \hat{u}\end{pmatrix}e^{-ikx}\hat{D}\right]\times e^{i\vec{p}\cdot\vec{\rho}},$$

$$\tilde{\chi}_j = \begin{pmatrix} e^{i\varphi_j/2}\hat{\sigma}_0 & \hat{0}\\ \hat{0} & e^{-i\varphi_j/2}\hat{\sigma}_0\end{pmatrix},$$

where $u(v)=\sqrt{(\omega_n+(-)\Omega)/(2\omega_n)}$, $\Omega=\sqrt{\omega_n^2+|\Delta_{\vec{k}}|^2}$, $\hat{u}=u\hat{\sigma}_0$, $\hat{v}=v\hat{\Delta}_{\vec{k}}/|\Delta_{\vec{k}}|$, and $\omega_n=(2n+1)T$ is the Matsubara frequency. The incident amplitudes of a quasiparticle in the electron (hole) branch are denoted by the diagonal $2\times 2$ matrix $\hat{a}$ ($\hat{b}$). In $\boldsymbol{\Psi}_L$ ($\boldsymbol{\Psi}_R$), $\hat{A}$ and $\hat{B}$ ($\hat{C}$ and $\hat{D}$) represent the amplitudes of outgoing waves in the electron and hole branches, respectively. Second, I calculate the Andreev reflection coefficients from the boundary conditions,

$$\boldsymbol{\Psi}_L(0,\vec{\rho}) = \boldsymbol{\Psi}_R(0,\vec{\rho}),$$

$$\frac{d}{dx}\boldsymbol{\Psi}_L(x,\vec{\rho})\bigg|_{x\to 0} = \frac{d}{dx}\boldsymbol{\Psi}_R(x,\vec{\rho})\bigg|_{x\to 0} - 2z_0k_F\boldsymbol{\Psi}_R(0,\vec{\rho}),$$

with $z_0=V_B/(k_F/m)$. The Andreev reflection coefficients are defied by the off-diagonal elements of the matrix relation below:

$$\begin{pmatrix}\hat{A}\\ \hat{B}\end{pmatrix} = \begin{pmatrix}\hat{r}_{ee} & \hat{r}_{eh}\\ \hat{r}_{he} & \hat{r}_{hh}\end{pmatrix}\begin{pmatrix}\hat{a}\\ \hat{b}\end{pmatrix}. \tag{5}$$

The calculated results of the Andreev reflection coefficients are summarized as

$$\hat{r}_{he} = \frac{uv\nu\hat{\Delta}_{\vec{k}}^{\dagger}\hat{K}}{\Xi|\Delta_{\vec{k}}|}, \quad \hat{r}_{eh} = \frac{uv\hat{K}^*\hat{\Delta}_{\vec{k}}}{\Xi|\Delta_{\vec{k}}|},$$

$$\hat{K} = [(a_0c_0 - a_1c_1\sin^2\alpha)\hat{\sigma}_0 + (a_0c_1 - c_0a_1)\boldsymbol{n}\cdot\hat{\boldsymbol{\sigma}}]/h_1h_2,$$

$$a_0 = |r|^2f_-^2(\nu g_+\cos\alpha - f_+) + |t|^2(f_+\nu\cos\alpha - g_+^*)h_1,$$

$$a_1 = i\nu f_-[|r|^2g_-f_- + |t|^2h_1],$$

$$c_0 = h_1\{\gamma - (1+\nu)\cos\alpha\} - |r|^2h_2\{g_+ - u^2v^2\gamma - u^2v^2(1+\nu)\cos\alpha\},$$

$$c_1 = -i(1-\nu)[h_1 + |r|^2h_2u^2v^2],$$

$$h_1 = u^4e^{i\varphi} + v^4e^{-i\varphi} - 2u^2v^2\cos\alpha,$$

$$h_2 = 2(\cos\varphi - \nu\cos\alpha),$$

$$\Xi_\pm = |r|^2f_-^2 + |t|^2\left\{1 - 4u^2v^2\cos^2\left(\frac{\varphi\pm\alpha}{2}\right)\right\}, \tag{6}$$

with $f_\pm=u^2\pm\nu v^2$, $g_\pm=u^2e^{i\varphi}\pm v^2e^{-i\varphi}\nu$, $\gamma=e^{i\varphi}+e^{-i\varphi}\nu$, $\varphi=\varphi_L-\varphi_R$, $\boldsymbol{n}=\boldsymbol{d}_R\times\boldsymbol{d}_L$, and $\Xi=\Xi_+\Xi_-$. The normal transmission and reflection probabilities for a channel characterized by $\vec{p}$ are given by $|t|^2=k^2/(z_0^2k_F^2+k^2)$ and $|r|^2=z_0^2k_F^2/(z_0^2k_F^2+k^2)$, respectively. Finally, I obtain the spin current in SIS junctions based on the formula[5]

$$\boldsymbol{J}_s = -\sum_{\vec{p}}\frac{T}{4}\sum_{\omega_n}\mathrm{Tr}\left[\frac{1}{\Omega}\left\{\hat{\Delta}_{\vec{k}}\hat{r}_{he}\frac{\hat{\boldsymbol{\sigma}}}{2} + \hat{r}_{he}\hat{\Delta}_{\vec{k}}\frac{\hat{\boldsymbol{\sigma}}^*}{2}\right\} - \frac{\nu}{\Omega}\left\{\hat{r}_{eh}\hat{\Delta}_{\vec{k}}^{\dagger}\frac{\hat{\boldsymbol{\sigma}}}{2} + \hat{\Delta}_{\vec{k}}^{\dagger}\hat{r}_{eh}\frac{\hat{\boldsymbol{\sigma}}^*}{2}\right\}\right]. \tag{7}$$

The electric Josephson current is also given by $\hat{\boldsymbol{\sigma}}/2\to -e\hat{\sigma}_0$ in Eq. (7).[13,14] The spin polarized in the direction of $\boldsymbol{n}\parallel\vec{z}$ flows through the SIS junction as shown in Fig. 1(a). The electric current $J_e$ and the $z$ component of the spin current $J_s^z$ in the $x$ direction result in

$$\tilde{J}_e(\varphi,\alpha) = \frac{J_e}{e\Delta_0} = \sum_{\vec{p}}\frac{|t|^2}{4}\frac{\Delta_{\vec{k}}}{\Delta_0}[F_\nu(+) + F_\nu(-)], \tag{8}$$

$$\tilde{J}_s^z(\varphi,\alpha) = \frac{J_s^z}{\Delta_0/2} = \sum_{\vec{p}}\frac{|t|^2}{4}\frac{\Delta_{\vec{k}}}{\Delta_0}[F_\nu(+) - F_\nu(-)], \tag{9}$$

$$F_\nu(\pm) = \frac{\sin(\varphi\pm\alpha)}{A_\nu(\pm)}\tanh\left(\frac{\Delta_{\vec{k}}A_\nu(\pm)}{2T}\right), \tag{10}$$

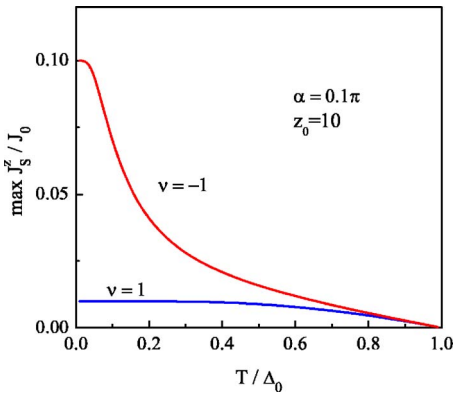


FIG. 2. (Color online) The temperature dependence of the spin current for $\alpha=0.1\pi$ and $z_0=10$. The results are normalized by the maximum value of the spin current at $z_0=0$ and $T=0$.

$$A_\nu(\pm)=\begin{cases}\sqrt{1-|t|^2\sin^2\left(\dfrac{\varphi\pm\alpha}{2}\right)}, & \nu=1,\\ |t|\cos\left(\dfrac{\varphi\pm\alpha}{2}\right), & \nu=-1,\end{cases}\tag{11}$$

where $\Delta_0$ is the amplitude of the pair potential at zero temperature. The summation in terms of $\vec{p}$ means the summation of the current over propagation channels. It is easy to confirm that $J_s$ ($J_e$) is an odd function of $\alpha$ ($\varphi$). At $z_0=0$, $J_e$ and $J_s^z$ are independent of $\nu$ because the transmission probability is unity (i.e., $|t|^2=1$). In what follows, we consider the low-transmission limit (i.e., $|t|^2\ll 1$). We show the maximum amplitude of $\tilde{J}_s^z$ as a function of temperature in Fig. 2 for $z_0=10$ and $\alpha=0.1\pi$. The maximum values are calculated from the current-phase ($\tilde{J}_s^z$-$\varphi$) relation and are normalized by $J_0=\Sigma_{\vec{p}}|\Delta_{\vec{k}}|/\Delta_0|\sin(\alpha/2)|$, which corresponds to the critical current at $z_0=0$ and $T=0$. For $\nu=1$, the temperature dependence of the spin current is described by the Ambegaokar–Baratoff formula[15] and the spin current saturates at low temperatures as shown in Fig. 2. On the other hand for $\nu=-1$, the spin current increases rapidly with decreasing temperature because the MARS is formed at the interface. The resonant transmission through the MARS causes a low-temperature anomaly not only in the charge transport[9,10] but also in the spin transport. This is because the Andreev reflection carries both electric charges and spins as shown in Eq. (7). In fact, we find the relation $\tilde{J}_e(\alpha,\varphi)=\tilde{J}_s^z(\varphi,\alpha)$. Interchange of the angle in the gauge space $\varphi$ and the angle in the spin space $\alpha$ connects the two supercurrents, which implies the duality of charge and spin.

In contrast to spin-triplet pairs of $^3$He, Cooper pairs in superconductors couple with the electromagnetic field through the charge degree of freedom of an electron. As a result, the spin current is expected to be sensitive to electromagnetic fields. Here I first consider the spin current in SIS junctions under magnetic fields. It is well known that the electric Josephson critical current exhibits the Fraunhofer pattern under magnetic fields. We apply the widely accepted method[16] to Eq. (9) and calculate the critical spin current near the critical temperature as

$$|J_s^z(\Phi)|=|J_s^z(0)|\left|\sin\left(\frac{\pi\Phi}{\Phi_0}\right)\Big/\left(\frac{\pi\Phi}{\Phi_0}\right)\right|,\tag{12}$$

where the magnetic field $B$ is parallel to the $z$ direction, $\Phi=d_0WB$ with $d_0/2$ being the penetration depth of magnetic field, and $\Phi_0$ is the flux quantum. The critical current of spin shows exactly the same Fraunhofer pattern as that of the charge current. For $\nu=-1$, the period of oscillations becomes $2\Phi_0$ at low temperatures due to the resonant transmission through the MARS.

Next I discuss the spin current under applied bias voltage. The ac Josephson effect is also well known in SIS junctions of spin-singlet superconductors. The macroscopic phase obeys the Josephson equation of motion $\partial\varphi/\partial t=2eV_{\rm bias}$ under bias voltage $V_{\rm bias}$ across the junction. The spin current under a bias voltage near the critical temperature becomes

$$\tilde{J}_s^z\propto\cos(2eV_{\rm bias}t+\varphi_0)\sin\alpha.\tag{13}$$

Bias voltages generate an alternating spin current in SIS junctions. The gauge coupling of the Cooper pair results in Eqs. (12) and (13), which represent the specific properties of the spin current in superconductor junctions. In addition, it may be possible to control the spin current by using these properties.

To observe the spin current in superconducting materials or junctions, we have to overcome two difficulties in experiments: the generation and detection of the spin current. To generate the spin current, we propose a SIS junction of the spin-triplet superconductor[17] $Sr_2RuO_4$ as shown in Fig. 1(b). A bending junction is required to have a spatial gradient of $\boldsymbol{d}$. In $Sr_2RuO_4$, $\boldsymbol{d}$ homogeneously aligns in the direction parallel to the $c$ axis of the crystal lattice. The fabrication of such a bending SIS junction may be possible because the crystal growth rate within the $ab$ plane is much faster than that along the $c$ axis. A grain boundary separates the two thin films of single crystal. At present, the chiral $p_x\pm ip_y$ wave symmetry is a promising candidate for pairing symmetry in $Sr_2RuO_4$. The formation of a MARS depends on the incident angle of a quasiparticle onto the junction interface. As a result, the MARS is weakly formed at the interface. The spin current has an intermediate quality between the spin current with $\nu=1$ and that with $\nu=-1$. Although Eqs. (9), (12), and (13) are derived in ballistic junctions, these behaviors persist even in realistic disordered junctions. This is because the MARS causes an anomalous Josephson effect[18] in the $p$-wave symmetry. The electromagnetic fields modulate the spin current as discussed in Eqs. (12) and (13). At present, unfortunately, there is no good way of directly observing the spin flow. Actually, the spin Hall effect[19] has been confirmed by spin accumulation, which is a result of the spin flow. The application of a spin-orbit device[20] may be possible to detect the spin current in superconducting junctions.

In summary, I have studied the spin supercurrent in superconductor/insulator/superconductor junctions, where superconductors are in spin-triplet unitary states. On the basis of the current formula, I calculate the spin and charge

currents analytically from the Andreev reflection coefficients of the junctions. The resonant transmission through the midgap Andreev resonant state causes a low-temperature anomaly of the spin current. The gauge coupling of the Cooper pairs with the electromagnetic field results in a Fraunhofer pattern of the spin current under magnetic fields and an alternating spin flow in the presence of bias voltages. The application of the obtained results to realistic junctions consisting of $Sr_2RuO_4$ is briefly discussed.

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