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Development of magnetohydrodynamic simulation code using the constrained interpolation profile method

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Abstract. A three-dimensional magnetohydrodynamic (MHD) simulation code has been newly developed. In this code, the physical variables are defined in the staggered mesh system and each of the MHD equations is split into the advection phase and the non-advection phase. The constrained interpolation profile method, which gives a highly accurate solution with less numerical diffusion, is applied to the calculation for the advection phase. In contrast, the non-advection phase is calculated using the finite difference method. The developed code is tested for the relaxation processes of the mass density, plasma pressure and magnetic field in the Large Helical Device (LHD).

1. Introduction

For the realization of the nuclear fusion reactor, it is important to understand the magnetohydrodynamic (MHD) equilibria and the stability. In helical devices, such as the Large Helical Device (LHD) [1] at the National Institute for Fusion Science (NIFS) of the National Institute of Natural Sciences (NINS) in Japan, the systematic three-dimensional (3D) MHD studies are also needed for successful operations. The fully 3D equilibrium analysis [2–4] and the fully 3D nonlinear MHD stability simulation [5] for the LHD plasmas have been made. The results of these studies, however, do not completely agree with the results of the LHD experiments. Thus, there is a need to develop a new MHD simulation code which properly reproduces the experiment results.

The constrained interpolation profile (CIP) method [6] is one of the numerical schemes to solve the nonlinear hyperbolic equations. Highly accurate and stable solutions with less numerical diffusion can be given in hydrodynamic and space plasma simulations using the method [7, 8]. In the present study, a new 3D MHD simulation code based on the CIP method is under development to analyze the LHD experiment results.

2. Numerical model

In this study, the following normalized MHD equations are solved:

$$\frac{\partial \rho}{\partial t} + (\mathbf{V} \cdot \nabla)\rho = -\rho(\nabla \cdot \mathbf{V}), \tag{2.1}$$

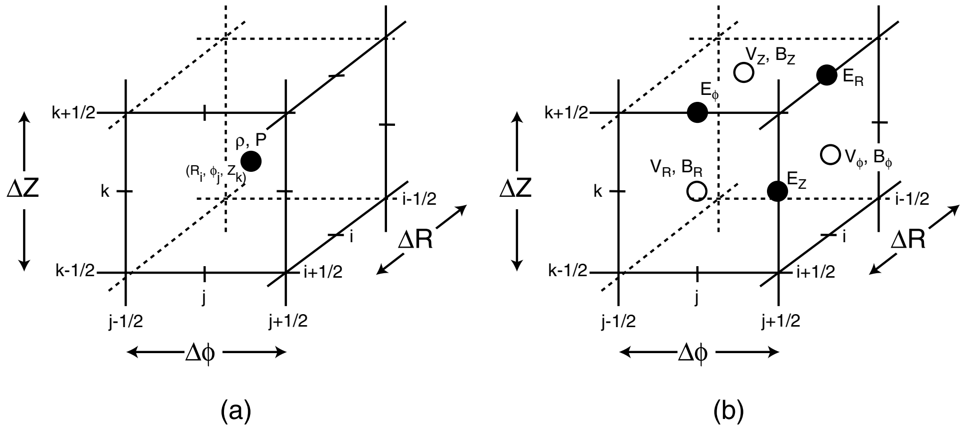

Figure 1. Location of (a) scalar and (b) vector quantities in the staggered mesh system. i,j and k represent the labels of R, ϕ and Z directions, respectively.

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla)\mathbf{V} = -\frac{1}{\rho}(\nabla P - \mathbf{J} \times \mathbf{B}), \tag{2.2}$$

$$\frac{\partial P}{\partial t} + (\mathbf{V} \cdot \nabla)P = -\gamma P(\nabla \cdot \mathbf{V}), \tag{2.3}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \tag{2.4}$$

$$\mathbf{E} = \eta \mathbf{J} - \mathbf{V} \times \mathbf{B}, \tag{2.5}$$

and

$$\mathbf{J} = \nabla \times \mathbf{B}, \tag{2.6}$$

where ρ, $\mathbf{V}$, P, $\mathbf{B}$, $\mathbf{J}$ and $\mathbf{E}$ are the mass density, velocity, pressure, magnetic field, current density and electric field, respectively. The quantities γ and η are the specific-heat ratio and the resistivity.

Equations (2.1)–(2.3) are split into two phases, i.e. the advection phase and the non-advection phase. The CIP method is applied to the calculation for the advection phase. In contrast, the non-advection phase is calculated using the finite difference method. The induction equation (2.4) is also solved with the finite difference method.

The cylindrical coordinate system (R, ϕ, Z) is adopted, and the number of grid points are set to be 56, 490 and 56 in each direction. The physical variables are defined in the staggered mesh system as shown in Fig. 1 [9]. In this system, the scalar quantities such as ρ and P are located at the center of the zone. One assumes here that the components of both velocity and magnetic field, $\mathbf{V}$ and $\mathbf{B}$, are located at the same face. $\mathbf{E}$ are located along the zone edge. Through the above definition, the constraint $\nabla \cdot \mathbf{B} = 0$ is preserved in the finite difference computation of (2.4).

$\mathbf{V}$ and $\mathbf{B}$ at the undefined grid points are required to calculate $\mathbf{E}$ in (2.5). The Lorentz force $\mathbf{J} \times \mathbf{B}$ and ρ are also required to deal with the momentum equation (2.2). These quantities are interpolated from the corresponding values and their first spatial derivatives at the neighboring grid points in the sense of the cubic Hermite interpolation.

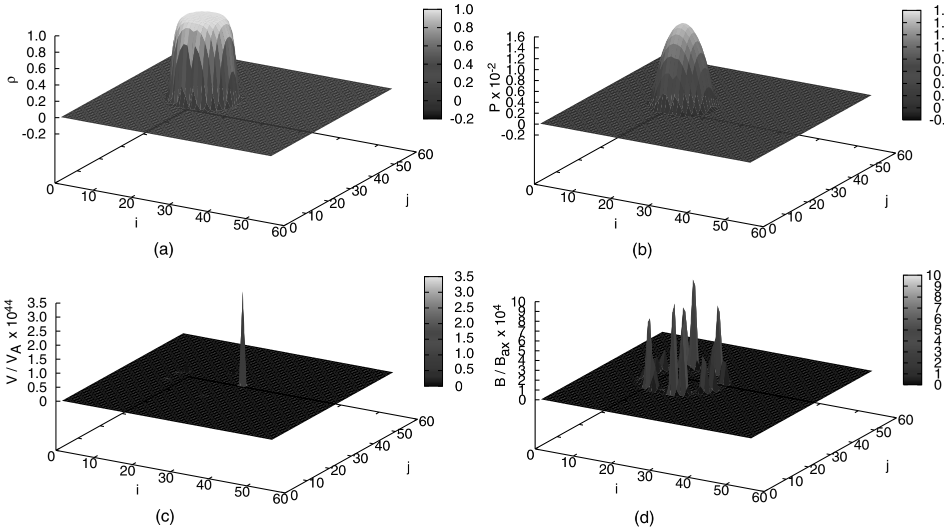

Figure 2. Bird's-eye view of (a) mass density, (b) pressure, (c) speed, and (d) magnetic field strength on the horizontally elongated plan at $t = 0.2$. i and j are the labels of R and Z directions. V_{A} is the Alfvén speed and B_{ax} the magnetic field strength on the vacuum magnetic axis.

Initial values of ρ and P are set as

$$\rho = \rho_0(1 - r^8), \tag{2.7}$$

and

$$P = P_0(1 - r^8)(1 - r^2), \tag{2.8}$$

where r is the normalized minor radius, while ρ_0 and P_0 denote the mass density and the pressure on the magnetic axis. The vacuum magnetic field calculated using the KMAG code [10] in the $R_{\mathrm{ax}} = 3.6$ m case is used as the initial condition. One also assumes $\mathbf{V} = 0$ at the initial state.

3. Test calculation

A numerical test has been made for the relaxation processes of ρ, P and $\mathbf{B}$ which started with the above-described initial conditions ($\rho_0 = 1$, $P_0 = 0.016$). It is noted that $P_0 = 0.016$ represents $\beta_{\mathrm{ax}} = 3.2\%$. Bird's eye views of ρ, P, $|\mathbf{B}|$ and $|\mathbf{V}|$ on the horizontally elongated plane at $t = 0.2$ are shown in Fig. 2. Unfortunately, no reasonable quantities can be obtained by the present code. Even after a short time, $|\mathbf{V}|$ and $|\mathbf{B}|$ have diverged. This shows that there are some problems in the computation of (2.2) and (2.4). So, the present code requires some conditions to prevent $\mathbf{V}$ and $\mathbf{B}$ from diverging. In addition, an improved method to interpolate $\mathbf{V}$ and $\mathbf{B}$ at the undefined grid points should be developed.

4. Summary and future plans

A MHD simulation code has been newly developed. In this code, the MHD equations are split into the advection and non-advection phases, which are respectively solved by the CIP and finite difference method. The developed code has been applied to the LHD plasma, however, reasonable quantities have not been obtained for the time being.

Our future plans are as follows

(a) The viscosity term will be added to (2.2) and (2.3) to prevent $\mathbf{V}$ from diverging.

(b) The method of characteristic constrained transport (MOCCT) scheme [8, 11] will be adopted to estimate the quantities at the undefined grid points.

(c) The precision of the finite difference computation will be raised.

(d) After completion of this simulation code, the equilibrium pressure in the peripheral region of the LHD will be studied. The peripheral pressure profile obtained by the LHD experiments has not been reconstructed by any equilibrium codes. We will also study the nonlinear stability/instability in the LHD.

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