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**Physical Studies on Deposited Snow. I.\***  
**Thermal Properties.**

by

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**Introduction**

The authors, Zyungo YOSIDA, Hirobumi OURA, Daisuke KUROIWA, Tosio HUZIOKA, Kenji KOJIMA, Sin-iti AOKI and Seiiti KINOSITA, have cooperated in studying experimentally the physical properties of deposited snow at the Institute of Low Temperature Science since 1945. Although their studies were of preliminary nature they were extended almost over all physical properties of deposited snow. The authors report in this booklet their studies on the thermal properties of deposited snow, intending to publish the studies on other properties such as optical, acoustical, dielectrical and mechanical properties in books which will be published in the near future.

The experiments were carried out on the ground surrounding the building of the Institute or in the cold rooms attached to it. Each of the cold rooms has an area  $4\text{ m} \times 4\text{ m}$  and is kept at the temperature of about  $-25^{\circ}\text{C}$  throughout the year by ammonium refrigerating machines. They can be cooled as low as about  $-50^{\circ}\text{C}$  if it is needed.

The Institute is in the grounds of Hokkaido University which is located in the northern part of Sapporo city. Sapporo is the largest city in Hokkaido, the most northern island of Japan; the geographical position of the city is  $141^{\circ}21'\text{E}$ ,  $43^{\circ}4'\text{N}$ . It is situated at the foot of a mountain-range about 1,000 meters high and on its northern and northeastern sides extends the Ishikari plain. The coast of the Japan Sea is not more distant than 16 km from it in northwesterly direction. Snow crystals which fall in Sapporo carry on their surface cloud droplets frozen to it almost without exceptions. Graupels fall frequently.

In the following table are listed:

- A. mean monthly maximum depth of snow in cm
- B. mean monthly maximum temperature in  $^{\circ}\text{C}$
- C. mean monthly temperature in  $^{\circ}\text{C}$
- D. mean monthly minimum temperature in  $^{\circ}\text{C}$
- E. mean monthly wind velocity in m/sec,

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mean values being based on many year observations made by the Meteorological Observatory of Sapporo. The greatest depth of snow and the lowest temperature observed in Sapporo up to the present are 169 cm. and  $-28^{\circ}\text{C}$  respectively.

	Oct.	Nov.	Dec.	Jan.	Feb.	March	April
A (cm)	1	15	46	71	87	75	21
B ( $^{\circ}\text{C}$ )	15.8	7.9	0.9	- 1.9	- 0.8	2.7	10.5
C ( $^{\circ}\text{C}$ )	10.2	3.6	- 3.0	- 6.6	- 5.1	- 1.4	5.3
D ( $^{\circ}\text{C}$ )	4.1	- 2.9	- 7.8	- 11.6	- 10.9	- 6.4	0.1
E (m/sec)	2.3	2.9	2.9	2.9	2.9	3.5	4.1

As will be seen from the temperatures shown in the above table the snow cover deposited in Sapporo remains unmelted throughout winter until it begins to thaw at the middle of March, unlike the snow in the main island of Japan where the temperature is higher and the snow is sometimes melting even while it is falling. Under these circumstances the authors' studies were made mainly on the snow which is kept at temperatures below the melting point of ice.

## I. Thermal Properties of Deposited Snow

### § 1. Heat conductivity of deposited snow

A large number of authors have made measurements on the heat conductivity of snow and it may seem needless to add one more measurement. However almost all the measurements hitherto made were directly on actual snow cover laid on the ground which was not of homogeneous structure but composed of many snow layers of different properties. Therefore, only a mean value of the conductivities of many layers was obtained. Under these circumstances it seems desirable to make measurement on homogeneous snow of definite property. The authors tried to determine the heat conductivity of a small block of snow cut out of a single snow layer which must be almost homogeneous in all its properties.

Since the direct determination of heat conductivity  $\mu$  of snow by the use of heat flow of a known intensity seemed to be experimentally difficult, the authors took their way towards determination of temperature conductivity  $\kappa$  of snow which gave heat conductivity when multiplied by specific heat  $c$  and density  $\rho$  of snow. In order to determine the temperature conductivity of a body, change of temperature with time must be observed at one or more points in the interior of the body. In the case of a small block of snow, however, insertion of a thermometer into it makes the measurement uncertain since unavoidable heat flow along the thermometer disturbs the original temperature change considerably because of the small heat capacity of snow. Moreover the fragile property of snow sometimes makes the insertion of thermometer unpracticable. Such being the case the following means was devised which allowed the temperature conductivity  $\kappa$  of snow to be obtained with no use of thermometer (1).

The block of snow is enclosed in an air-tight vessel of which the wall can at will be kept at one of two different temperatures  $\theta_1$  and  $\theta_2$  below  $0^\circ\text{C}$ . One keeps the wall of the vessel at the temperature  $\theta_1$  long enough to set up a uniform temperature distribution equal to  $\theta_1$  throughout the interior of the snow block. Then one changes suddenly the temperature of the wall from  $\theta_1$  to  $\theta_2$ . The temperature of snow begins to change from  $\theta_1$  to  $\theta_2$  on the whole surface of the block of snow and this change of temperature gradually proceeds towards its centre, finally making the block uniform in temperature equal to  $\theta_2$ . During this time the mean temperature  $\bar{\theta}$  of snow block changes from  $\theta_1$  to  $\theta_2$  and its mode of change can be given by a mathematical function

$$f(A\kappa t), \quad (1)$$

where  $A$  is a constant determined by the shape and dimensions of the vessel while  $t$  is the time passed since the temperature of the wall of the vessel was changed from  $\theta_1$  to  $\theta_2$ . It is now shown that this function comes to change in proportion to  $\exp(-A\kappa t)$  after the value of  $(A\kappa t)$  has exceeded 0.3 or 0.4. Therefore  $\kappa$  of snow can be found if any quantity which changes its value in proportion to the mean temperature  $\bar{\theta}$  is found and observed. The pressure of air included in the snow block is one of such quantities as understood by the following reasoning. Let pressure, density and absolute temperature of air at a point in the interior of the snow be denoted by  $p$ ,  $\sigma$ ,  $T$  respectively.  $\sigma$  and  $T$  are variable as the position of the point is varied but  $p$  has the same value throughout the interior of snow and the following relation

$$p = c\sigma T \quad (2)$$

holds between them. The letter  $c$  here means a constant independent on  $p$ ,  $\sigma$  and  $T$ . The invariable mass of the air enclosed in the vessel

$$m = \int \sigma dv \quad (3)$$

and the above equation (2) gives

$$p = cm/\int dv/T, \quad (4)$$

the integral being extended throughout the space  $v$  occupied by the air in cases of both (3) and (4). If  $T$  is expressed by  $T = T_0 + \theta$ , where  $T_0$  is absolute temperature of  $0^\circ\text{C}$ , the above equation (4), as long as  $\theta$  is small compared to  $T_0$  as in the present case, is transformed to

$$p = \frac{cmT_0}{v} \left(1 + \frac{\bar{\theta}}{T_0}\right), \quad (5)$$

which shows that  $p$  is changed in proportion to (rigorously speaking, in linear relation to) the change of mean temperature  $\bar{\theta}$ . Moreover,  $p$  can easily be observed by using a capillary tube to connect a manometer to the air-tight vessel containing

the snow. In reality the manometer measures the pressure of the water vapour evaporating from snow in addition to that of the air, but this circumstance does not matter because the former is also proportional to  $\bar{\theta}$  if  $\bar{\theta}$  is confined to change within a small range, that is, if the difference between  $\theta_1$  and  $\theta_2$  is not large.

The experiment was done by means of an apparatus shown schematically in Fig. 1. A circular metal cylinder Q and two metal sheets P and P—each of them being made of plate metal of thickness 1 mm—enclosed a cylindrical space into which the sample of snow was put. The cylinder Q was pressed carefully into one of the snow layers composing the snow cover in such a way that it was filled with snow with no left space. The snow was then cut by a metal plate at the end planes of the cylinder and the metal plates P and P were pressed firmly onto its ends by bolts. Lines of contact between the metal cylinder and the plates were packed air-tight by vacuum compound and wool matt C was put on the wall of the cylinder in order that it might be maintained always at the same temperature as the metal plates P and P. Two wooden pails  $A_1$  and  $A_2$  contained mixtures of water and ethyleneglycol, the mixture in each pail being kept at the temperatures  $-1^\circ\text{C}$  and  $-6^\circ\text{C}$  respectively by adding to it snow or hot water from time to time. The mixture in  $A_1$  was continually drawn out by a gear pump G and was pushed into rectangular metal boxes W and W attached to the metal plates P and P,

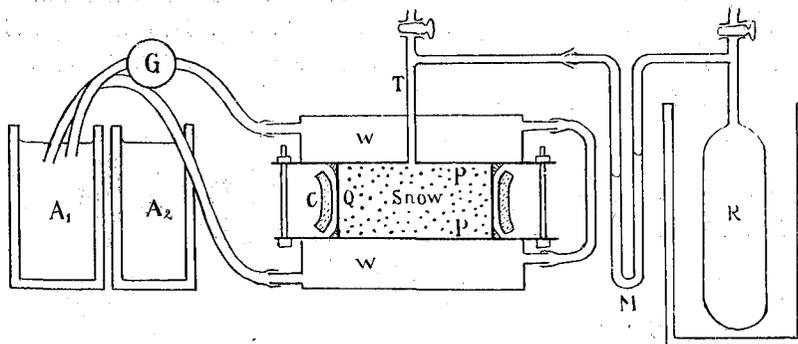


Fig. 1 Experimental equipment for measuring the heat conductivity of snow.

keeping in this way the cylinder Q and the metal plates P and P at the temperature  $-1^\circ\text{C}$  for about 1.5 hours. The circulating mixture was then replaced by that in the pail  $A_2$ . The meniscus of oil in the U-tube manometer M connected to the cylindrical space by the capillary tube T began to be depressed indicating that the pressure  $p$  of air contained in the snow began to diminish. The pressure of air in the right half of the manometer was maintained constant by means of an air reservoir R which was immersed in mixture of snow and water and was kept at the temperature  $0^\circ\text{C}$ .

An example of the results of measurement is shown in Fig. 2. The lower

figure shows the course of changes in the mean temperature  $\bar{\theta}$  and the temperatures  $\theta_{w_1}$ ,  $\theta_{w_2}$  which the circulating mixture of water and ethyleneglycol showed in each of the metal boxes W and W. The mean temperature  $\bar{\theta}$  shown here was calculated by a formula

$$\frac{\bar{\theta} - \theta_2}{\theta_1 - \theta_2} = \frac{p - p_2}{p_1 - p_2}, \quad (6)$$

where  $p_1$  and  $p_2$  are the initial and final pressures respectively while  $p$  is the pressure at any instant  $t$ .  $\theta_1$  and  $\theta_2$  were taken as equal to the mean value of  $\theta_{w_1}$  and  $\theta_{w_2}$ . The linear relation between  $\log(\bar{\theta} - \theta_2)$  and the time  $t$  shown in the

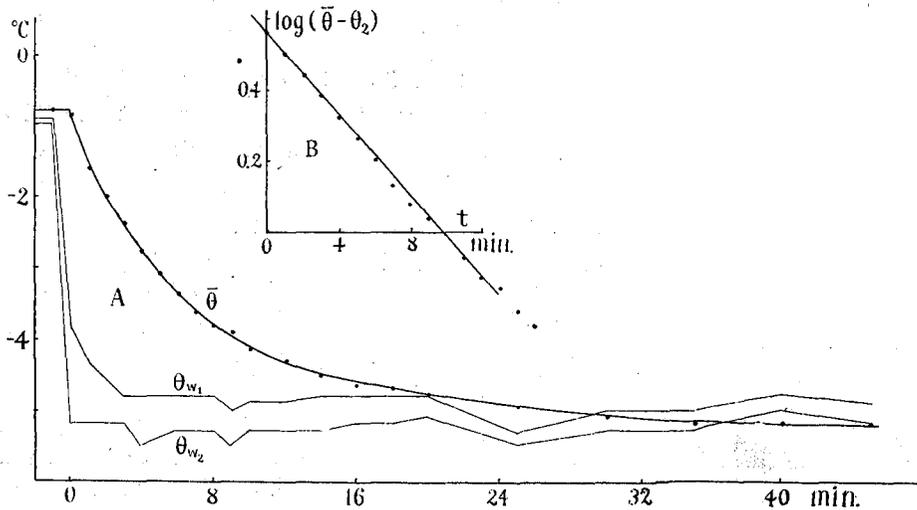


Fig. 2 Upper figure: linear relation between  $\log(\bar{\theta} - \theta_2)$  and time  $t$ .  $\bar{\theta}$ ,  $\theta_2$  are mean temperature and final temperature of the air contained in snow respectively. Lower figure: course of change in  $\bar{\theta}$ ,  $\theta_{w_1}$  and  $\theta_{w_2}$ .  $\theta_{w_1}$ ,  $\theta_{w_2}$  are temperatures of cooling mixture in the metal boxes W, W shown in Fig. 1.

upper figure shows that  $(\bar{\theta} - \theta_2)$  can be fairly well represented by a formula  $(\theta_1 - \theta_2) \cdot \exp(-A\kappa t)$  which can be deduced by the reasoning explained in the former paragraph.

The slope of the straight line in the upper figure of Fig. 2 gives the value of  $(A\kappa)$ . When the space containing snow has a cylindrical form as in the present case the constant  $A$  turns out to be

$$\frac{\pi^2}{a^2} + \frac{\lambda^2}{b^2}, \quad (7)$$

where  $a$  and  $b$  are the height and the radius of the cylindrical space respectively while  $\lambda$  is the first root of the equation  $J_0(x) = 0$  ( $J_0(x)$ : Bessel function of the first

kind). In the authors' case  $a$  was 5.0 cm and  $b$  was 4.85 cm. Since  $\pi=3.142$  and  $\lambda=2.405$ , the value of the constant  $A$  is found to be  $0.641/\text{cm}^2$ . The value of the temperature conductivity  $\kappa$  of snow is obtained by dividing the experimental value of  $(A\kappa)$  by this value of  $A$ . The heat conductivity  $\mu$  can be found by multiplying  $\kappa$  by the specific heat of ice and the density of the snow.

$\mu$  was found to lie in the range from  $1 \times 10^{-4}$  to  $7 \times 10^{-4}$  cal/deg·sec·cm for the range of snow density from 0.07 to 0.4 gr/cm<sup>3</sup>. The individual experimental values of  $\mu$  are shown against the density  $\rho$  by dots in Fig. 3 in the scale of  $\log_{10} \mu$ . The distribution of the dots is roughly represented by the straight broken line of which the mathematical expression is

$$\log_{10} \mu = -4 + 2\rho, \quad (8)$$

where  $\mu$  and  $\rho$  should be expressed by the units (cal/deg·cm·sec) and (gr/cm<sup>3</sup>) respectively. The continuous curves marked by letters  $a, j, d$  represent respectively the following mathematical relations between  $\mu$  and  $\rho$  presented by ABELS (2), JANSSON (2) and DEVAUX (3)

$$\text{ABELS : } \mu = 0.0068 \rho^2$$

$$\text{JANSSON : } \mu = 0.00005 + 0.0019 \rho + 0.006 \rho^2$$

$$\text{DEVAUX : } \mu = 7 \times (1 + 100 \rho^2) \times 10^{-5}.$$

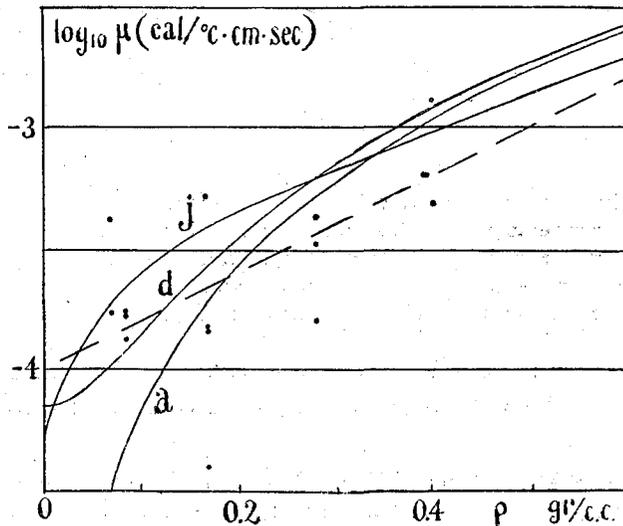


Fig. 3

Mean relation between heat conductivity  $\mu$  and density  $\rho$  of snow. Broken line shows the relation obtained by the present authors. Three full lines marked by  $a, d$  and  $j$  are those presented by ABELS, DEVAUX and JANSSON respectively.

The temperature conductivity  $\kappa$  was determined not only by changing the temperature of the circulating mixture from  $\theta_1$  to  $\theta_2$  but also by changing it backwards from  $\theta_2$  to  $\theta_1$ . These two courses of experiment did not always give the same value for  $\kappa$ , sometimes giving larger value in the backward course than in the forward one and sometimes a smaller. Even in one and the same course

changes were often found in the value of  $\kappa$  which was shown by the fact that the relation between  $\log_{10}(\bar{\theta} - \theta_2)$  and  $t$  was represented by a continuation of two or three segments of straight line with different slope as illustrated in Fig. 4. Each dot in Fig. 3 belonging to the same density represents each value of these different

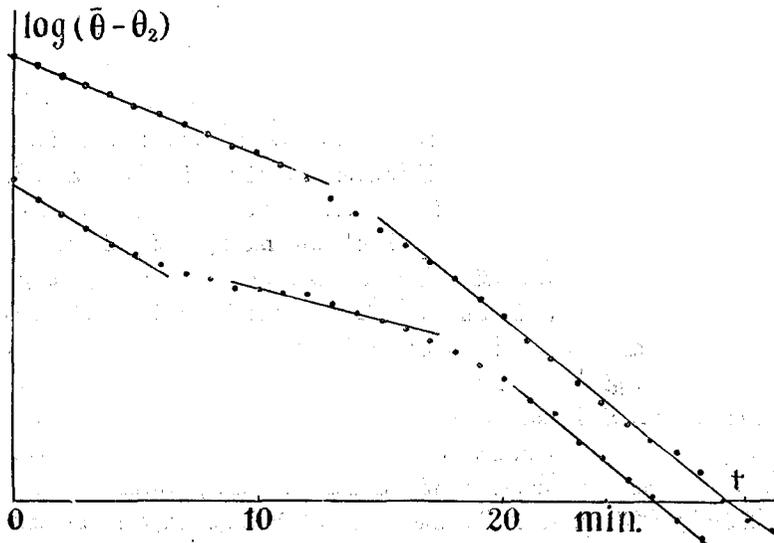


Fig. 4 Examples of  $\log(\bar{\theta} - \theta_2) - t$  curve composed of segments with different slopes.

values of  $\mu$  shown by the same sample of snow of a given density. In most cases thin clearance of one or two mm width was found between the surface of the snow mass and the wall enclosing it when the experiment was completed. This fact shows that the density of snow was slightly increased during the experiment but this increase in density is clearly too small to explain the above stated change in  $\kappa$ . It is thought that heat changed the structure of snow while it was flowing through the snow. The temperature gradient prevailing in the snow without which no heat flow could take place would have built up minute temperature differences between the individual ice grains composing snow. On account of these minute temperature differences evaporation and condensation of water vapour would have taken place on the surface of ice grains, which the authors think made the structure of snow change to such a degree that the above noted change in temperature conductivity appeared.

## § 2 The diffusion of water vapour through snow cover caused by temperature difference

Since the pressure of water vapour in equilibrium with ice becomes large as the temperature rises, the pressure of water vapour existing in the part of higher temperature of snow should be larger than that of water vapour in the part of

lower temperature. On account of such a difference of pressure a transfer of water vapour should take place from the former part of snow to the latter by diffusion phenomenon. This diffusion of water vapour, of course, occurs in the air included in the narrow space between ice particles composing snow according to the common law of diffusion expressed mathematically as

$$\frac{\partial p}{\partial t} = D_0 \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} \right), \quad (1)$$

where  $p$  and  $D_0$  are partial pressure of the water vapour and its diffusion coefficient through air respectively. However the discussion on this microscopic diffusion of water vapour taking place in the narrow spaces between the ice particles will be postponed to section 8 below. Here the macroscopic diffusion of water vapour through snow will be studied from the point of view of regarding snow as a continuous medium. Since, in this case, the ice particles present themselves as obstacles to the diffusion of water vapour, and, moreover, since it must be expected that they themselves give out or take in the diffusing water vapour as was noted at the end of the previous section, the law of macroscopic diffusion may have to be expressed by some mathematical expression different in form from equation (1). But it is assumed here that macroscopic diffusion is still governed by the common law of diffusion, namely that, if the mean value of water vapour pressure in the vicinity of the point  $(x, y, z)$  be denoted by  $p$ , it is governed by the following differential equation:

$$\frac{\partial p}{\partial t} = D \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} \right). \quad (2)$$

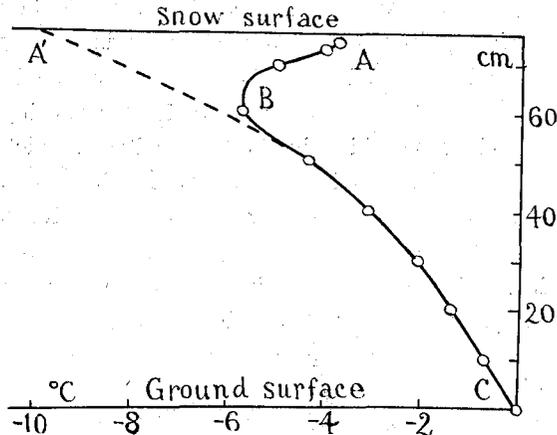


Fig. 5 Typical example of vertical temperature distribution in snow cover. In its uppermost layer the distribution curve takes the form AB in day-light hours while it takes the form A'B at night.

$D$  is then the macroscopic diffusion coefficient of water vapour through snow.

In these districts about Sapporo where the mean air temperature in winter is kept below  $0^{\circ}\text{C}$ , the temperature in snow cover lying on the ground is in general lower in its upper part than in its lower part. The full line in Fig. 5 represents the vertical temperature distribution in snow cover in day-light hours. In the surface layer of about 20 cm thickness the temperature undergoes daily change and its distribution

curve is at night transformed to such a form as shown by the dotted line in Fig. 5. Below this surface layer temperature of snow changes very slowly with the seasonal change of atmospheric temperature.

Since the temperature difference in the horizontal directions ( $x, y$ ) is much smaller than the above stated temperature difference in the vertical direction  $z$ , the temperature in snow can be regarded as varying only with coordinate  $z$  measured upwards from the ground surface. Then the following simplified equation (3) can be used in place of equation (2)

$$\frac{\partial p}{\partial t} = D \frac{\partial^2 p}{\partial z^2}, \quad (3)$$

because absence of temperature variation in the horizontal direction ( $x, y$ ) makes the water vapour pressure  $p$  non-dependent on the coordinates  $x$  and  $y$ .

The water vapour pressure  $p$  at any point cannot exceed its saturation value  $p_s$  which is determined by the snow temperature  $\theta$  at that point. In these points where  $\frac{\partial p}{\partial t}$  has a positive value the pressure  $p$  must be striving to exceed  $p_s$  with the result that water vapour condenses on the surfaces of the ice grains composing the snow. On account of this density of snow at such points must be increased. On the contrary in these places where  $\frac{\partial p}{\partial t}$  is negative, both evaporation of water vapour from the ice grains and decrease in snow density take place. Although the actual pressure  $p$  of water vapour existing in snow is in general not exactly the same as the saturation pressure  $p_s$ , it can be supposed that the difference between them is very small because transfer of water vapour by diffusion phenomenon must be carried on very slowly. Then  $\frac{\partial^2 p}{\partial z^2}$  on the right side of equation (3) can be replaced by  $\frac{\partial^2 p_s}{\partial z^2}$  which in its turn can be regarded as being proportional to the second derivative of the snow temperature  $\theta$ , that is, to  $\frac{\partial^2 \theta}{\partial z^2}$ , since  $p_s$  increases roughly in proportion to the increase in  $\theta$ . On the other hand  $\frac{\partial p}{\partial t}$  can be assumed to be proportional to the time rate of change in the snow density  $\rho$ , that is, to  $\frac{\partial \rho}{\partial t}$ , by the reasonings mentioned at the beginning of this paragraph. In this way it turns out that there holds the relation

$$\frac{\partial \rho}{\partial t} \text{ is in proportion to } \frac{\partial^2 \theta}{\partial z^2}, \quad (4)$$

in which the magnitude of  $\frac{\partial^2 \theta}{\partial z^2}$  can be estimated from the curvature of the vertical distribution curve of temperature in the snow cover.

In Fig. 5 which illustrates the vertical temperature distribution in snow cover,  $\frac{\partial^2 \theta}{\partial z^2}$  is positive (or negative) where the curve of distribution is concave (or convex)

towards the right. The curve is most concave in the vicinity of point B in daylight hours and the degree of this concavity seems to be higher than enough to compensate the degree of convexity which it shows at that point at night. Therefore in the vicinity of point B an increase is expected in snow density which is caused by accumulation and condensation of water vapour, in addition to the one due to sedimentation of the snow cover. The curve is nearly straight from point B to point C. No appreciable change in density due to condensation or evaporation of water vapour will take place in this range. But, since the water vapour condensing in the vicinity of point B is certainly being supplied from the lower part of the snow cover as well as from the layer above the point, some evaporation must be expected in this range. Particularly near the ground surface it may amount to a considerable quantity.

In order to ascertain if the above stated expectation is true or not the following experiments were carried out (4). Rectangular cages ( $9\text{ cm} \times 6\text{ cm} \times 3\text{ cm}$ ) were made of fine wire gauze and snow was put in each of them. Each cage was then enclosed in another cage of wire gauze slightly larger than it with a clearance of about 5 mm between them. At the end of February when the depth of snow cover was 160 cm a vertical hole was made therein and horizontal holes were made in its wall at heights 140 cm, 90 cm and 10 cm above the ground. After the cages were placed at the inner ends of these horizontal holes, the latter were filled in with snow. The positions of the cages in the snow cover are shown by rectangles marked (a), (b) and (c) in the figure showing snow cover depth of Fig. 6. Every day the cages were taken out from the horizontal holes, carried to the laboratory room in air tight cans and the inner cages containing snow were weighed there by chemical balance. Changes in weights of cages observed during a day amounted to more than several mg, while the absolute weights themselves were about 60 or 70gr. The position of the uppermost case (a) at 140 cm height corresponds to point B in Fig. 5 where the accumulation and condensation of water vapour was expected. The middle cage (b) and the lowest cage (c) located at 90 cm and 10 cm heights respectively are in the range where some evaporation of water vapour would take place.

Since the snow within the inner cage was separated by the clearance between the inner and outer cages from the snow composing the snow cover while the cage was buried in it, any change observed in the weight of inner cage must have been due to condensation or evaporation of water vapour which had taken place in the snow actually contained in it. Thawing of snow was discernible near the end of March. It could present itself as a source of change in the weight of snow contained in the cage. But, in so far as the period up to March 20 was concerned, thawing was entirely excluded because the mean temperature of air was kept much below  $0^{\circ}\text{C}$  in this period, as shown in Fig. 6.

The observed change in weight of each cage is shown in the upper part of Fig. 6. During the period of 22 days from Feb. 25 to March 19 the weight of the

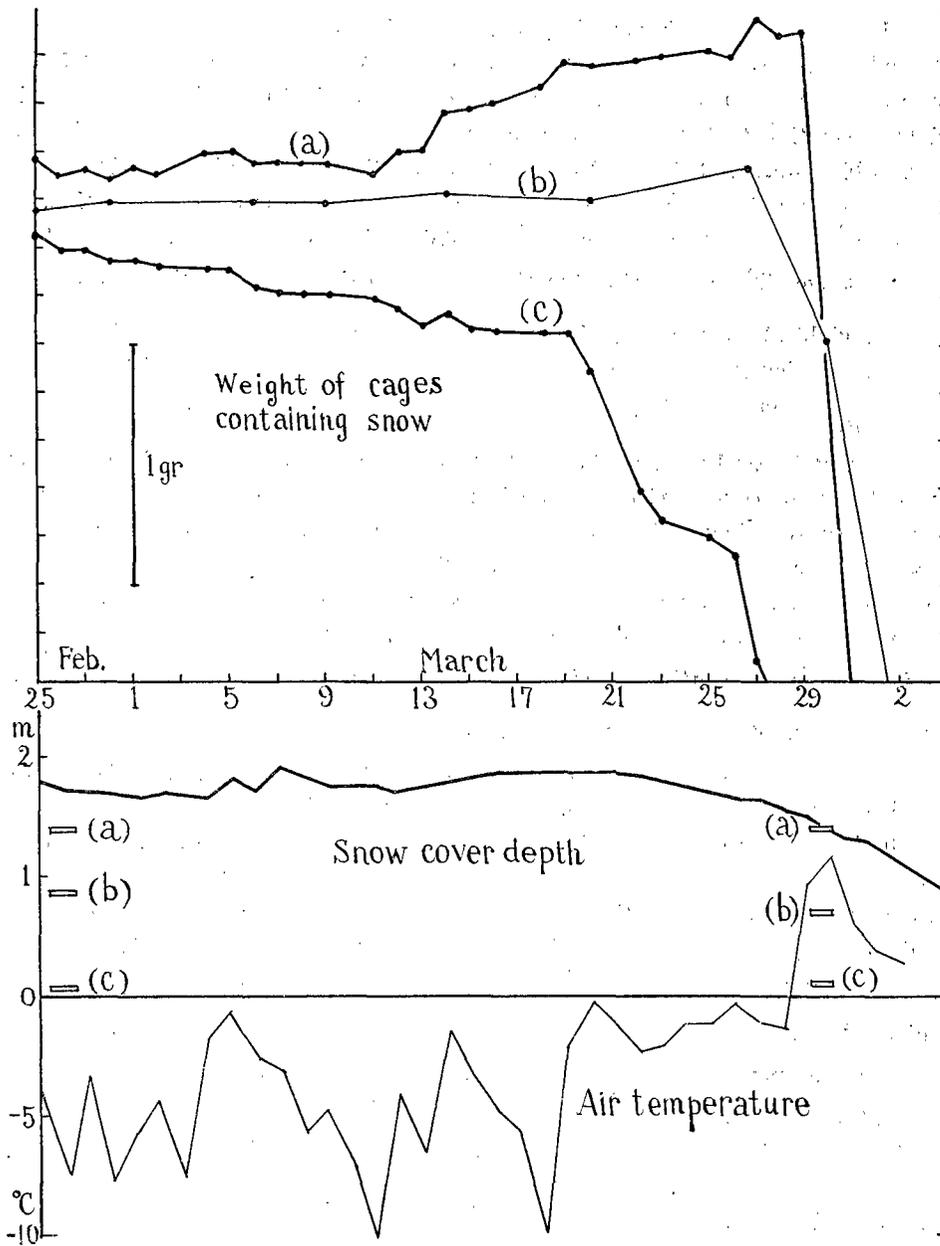


Fig. 6 Course of change in the weight of cages containing snow, depth of snow cover and air temperature during the period from Feb. 25 to April 4. Rectangles marked (a), (b), (c) show the positions of cages in the snow cover.

Initial weight of cages: (a) 70.523 gr; (b) 52.652 gr; (c) 60.723 gr.

uppermost snow cage (a) was increased by 410 mg. Its position in the snow cover was a little altered in this period. An increase in weight as small as 50 mg was observed in the middle cage (b), which had been lowered in position by about 20 cm at the end of the period. The lowest cage (c) showed in this period a decrease in weight amounting to 400 mg. These results of observation are in accord with the results which were anticipated by the above mentioned theoretical considerations, although a small increase instead of a decrease was observed in the weight of snow cage (b). The density  $\rho$  of snow contained in the cage and the mean time rate of change  $\frac{d\rho}{dt}$  of  $\rho$  during the above noted period at each height were:

Height above the ground (cm)	140	90	10
Snow density $\rho$ (gr/cm <sup>3</sup> )	0.33	0.25	0.28
$\frac{d\rho}{dt} \times 10^{10}$ (gr/cm <sup>3</sup> ·sec)	+ 10.1	+ 1.3	- 10.0

Now that it has been ascertained that water vapour evaporated in the lower part of the snow cover and condensed in its upper part, it can be concluded that water vapour was flowing incessantly upwards through it. With this flow of water vapour, heat must have been transferred since latent heat taken by the water vapour from snow for evaporation at the lower part of snow cover must have been given up to the snow by its condensation at the upper part. If it is assumed that  $\frac{d\rho}{dt}$  increases in proportion to height in the lower part of snow cover it is known from the values of  $\frac{d\rho}{dt}$  shown in the above table that  $4.6 \times 10^{-8}$  gr of water vapour evaporated for one second per horizontal one cm<sup>2</sup> from the snow lying below the level 80 cm high above the ground. Water vapour of this amount crossed upwards every horizontal one cm<sup>2</sup> area at this level in one second carrying with it a quantity of heat  $q' = (680 \text{ cal/gr}) \times (4.6 \times 10^{-8} \text{ gr/cm}^2 \cdot \text{sec}) = 3.1 \times 10^{-5} \text{ cal/cm}^2 \cdot \text{sec}$ . Here 680 cal/gr is the latent heat of evaporation of ice. The quantity of heat  $q$  which was transferred by heat conduction upwards in the snow cover can be gotten by the product (heat conductivity of snow  $\mu$ )  $\times$  (vertical temperature gradient  $\frac{d\theta}{dz}$ ). Formula (8) given by the authors in section 1 gives for  $\mu$  the value  $4 \times 10^{-4}$  cal/sec·cm·deg for a snow density of 0.3 gr/cm<sup>3</sup> and  $\frac{d\theta}{dz}$  can be estimated as 4°C/m in the situations of the authors' experiments.  $q$  then turns out to be  $1.6 \times 10^{-5}$  cal/cm<sup>2</sup>·sec. It is contradictory that  $q'$  is greater than  $q$  since theoretically  $q'$  must be only a part of  $q$ . It is thought that the origin of this contradiction lies in the roughness of the experiments and of the assumptions made for calculations. But the fact that  $q'$  was found to be of the same order of magnitude as  $q$  makes one suppose that a considerable fraction of heat quantity which is considered commonly to be transferred in snow by heat conduction is in reality transferred not by pure conduction but by water vapour diffusion through the snow. A more detailed discussion based on numerical considerations will be offered on this subject below in section 8.

At any rate it is not strange that heat conductivity of snow changes while heat is flowing through it as was pointed out in the previous section; now that both evaporation of water vapour from and its condensation on the ice particles composing snow cover have been ascertained experimentally, because evaporation and condensation cannot occur without producing changes in the internal structure of snow.

### § 3. Experimental determination of coefficient $D$ of macroscopic diffusion of water vapour through snow

Since it has been known that a considerable part of transference of heat through snow is effected by diffusion of water vapour it is desirable to study in more detail the diffusion phenomenon of water vapour in snow. For this purpose an attempt was made to determine the value of diffusion coefficient  $D$  as the first step (4), (5).

The bottom of a small circular can (diameter: 5.5 cm, height: 3.5 cm) was taken off and a fine wire gauze was stretched in its place. Four such cans  $C_1$ ,  $C_2$ ,  $C_3$  and  $C_4$ , after having been filled with snow, were piled up one upon another within a metal tube  $T$  as shown in Fig. 7. The upper and lower ends of the pile were maintained at the temperatures  $-1^\circ\text{C}$  and  $-6^\circ\text{C}$  respectively for several hours. Maintenance of temperatures at these values was effected by attaching to the ends of the pile closed metal boxes  $R_1$ ,  $R_2$  in which non-freezing mixture of water and ethyleneglycol having the above stated temperatures was kept circulating. A circular disk  $P$  of metal was placed between the bottom of the lowest can  $C_4$  and the top of the lower box  $R_2$  in order to catch water vapour escaping downwards from the pile. The outside of tube  $T$  was surrounded by heat insulating mat  $F$  in order to keep heat from escaping sideways from the pile. The air temperature of the laboratory room was a few degrees below  $0^\circ\text{C}$ .

The water vapour would evaporate from the snow near the warmer end of the pile and diffuse through it towards its colder end condensing on the snow nearby it. Therefore the can at the warmer end would decrease in weight while that at the colder end would increase in weight. The measurement of these changes

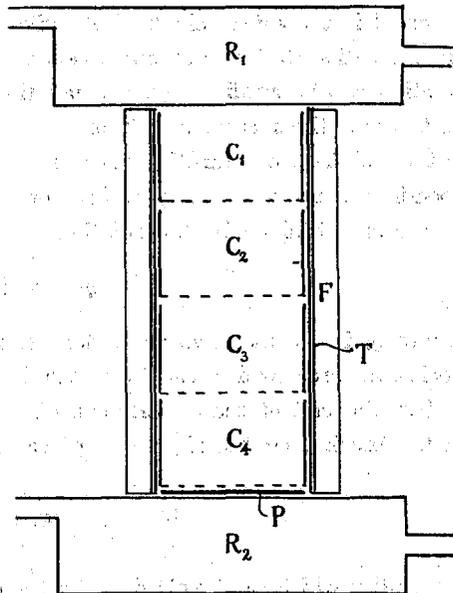


Fig. 7 Equipment for determination of the coefficient  $D$  of macroscopic diffusion of water vapour through snow.

in weight of the cans at the ends of the pile as well as of those in the middle part give the value of the diffusion coefficient  $D$  as will be explained in the following.

If the ratio of the density and the pressure of water vapour is denoted by  $\beta$ ,

$$-\beta D \frac{\partial p}{\partial z} = w \quad (1)$$

is the quantity of water vapour crossing in a unit time a unit horizontal area at the point  $z$  ( $z$ : coordinate downwards along the pile).  $w$  is a function of  $z$  as well as of time  $t$ . Let the values which any quantity  $u$  takes at the top and bottom of the can  $C_i$  be designated by  $u(i-1)$  and  $u(i)$  respectively. For instance, let  $w(1)$  and  $w(2)$  show the values of  $w$  at the top and bottom of the second can  $C_2$  respectively. Then, since the mean value of  $\frac{\partial p}{\partial z}$  in can  $C_1$  can be put equal to

$\frac{1}{2} \left\{ \frac{\partial p}{\partial z}(i-1) + \frac{\partial p}{\partial z}(i) \right\}$  approximately, the following relation can be deduced on account of equation (1):

$$\beta D \{p(0) - p(4)\} = \frac{l}{4} \left\{ \frac{w(0)+w(1)}{2} + \frac{w(1)+w(2)}{2} + \frac{w(2)+w(3)}{2} + \frac{w(3)+w(4)}{2} \right\}, \quad (2)$$

where  $l$  is the total height of the pile. Although  $w$ 's are quantities varying with time, relation (2) holds at any instant during the experiment. Therefore the same relation can be applied to the mean time values taken for the time of experiment of the quantities such as  $p$ 's or  $w$ 's. The mean values of  $p(0)$  and  $p(4)$  can be put equal to the saturation pressure of water vapour with respect to ice corresponding to the mean temperatures of the circulating non-freezing mixtures. Those of  $w$ 's are obtained by the relation

$$q_i = w(i) - w(i-1) \quad (3)$$

where  $q_i$  is the mean value of decrease in weight of can  $C_i$  in a unit time per unit horizontal area which can be determined by weighing the cans at the beginning and at the end of the experiment. Then the mean value of the quantity enclosed in the bracket on the right side of equation (2) is found to be equal to

$$Q = \frac{1}{2} \{7q_1 + 5q_2 + 3q_3 + q_4\}, \quad (4)$$

a quantity which can be obtained by the experiment. It is to be noted that  $w(0) = 0$  since no flow of water vapour occurs at the top of the pile. In this way the following relation is obtained by which the diffusion coefficient  $D$  is to be determined:

$$D = \frac{lQ}{4\beta \{p(0) - p(4)\}} \quad (5)$$

The results of the experiments are listed in the next table.

Number of experiment	1	2	3	4	5
Density of snow (gr/cm <sup>3</sup> )	0.08	0.14	0.25	0.34	0.51
Temperature of the mixture in the uppermost box (°C)	-1.2	-0.4	-0.3	-9.1	-1.9
Temperature of the mixture in the bottom box (°C)	-6.7	-7.0	-4.9	-1.4	-8.8
$p(0) - p(4)$ (mmHg)	1.58	1.90	1.43	1.96	1.75
Time of experiment (10 <sup>3</sup> sec)	16.5	16.9	20.1	18.0	16.2
$D$ (cm <sup>2</sup> /sec)	0.7	1.0	0.9	0.8	0.9

The diffusion coefficient  $D$  of water vapour through snow seems to be almost independent of the density of snow. Its value is four or five times as large as the diffusion coefficient  $D_0$  of water vapour through air which is 0.22 cm<sup>2</sup>/sec at 0°C and one atmospheric pressure. It is to be noted that gravity has no effect on the diffusion phenomenon since no particularity was found in experiment No. 4 shown in the above table, in which experiment the direction of diffusion was reversed by making the upper end of the pile cold instead of warm. Since the ice grains composing snow must be great obstacles to the water vapour diffusing through it, the above result that  $D$  is greater than  $D_0$  seems to be an unexplainable contradiction. However ice grains do not act as mere obstacles on the diffusing water vapour unlike the manner in which sand grains hinder the movement of water vapour diffusing through sand, but they can produce or absorb the diffusing water vapour themselves by evaporation or condensation process. The above mentioned contradiction will be resolved by the latter fact in the following way.

Let a train of ice grains situated along a straight line placed parallel to the direction of temperature gradient existing in the snow be imagined. Since the heat conductivity of ice is one hundred times as large as that of air the actual temperature gradient must exist in the air space between a pair of adjacent particles but not in the ice particles practically. If the mean lengths of the air spaces and of the ice particles in the direction of the above stated straight line are represented by  $a$  and  $b$  respectively the macroscopic temperature gradient  $G$  which is observed by the experiment is connected to the actual microscopic temperature gradient  $G_0$  present in the air space by the relation

$$(a+b)G = aG_0, \quad (6)$$

which shows that  $G_0$  is much larger than  $G$ . Under these circumstances water vapour evaporates from one of the surfaces facing each other of two adjacent ice particles and condenses on the other after diffusing through the narrow air space between them. For the water vapour to traverse by diffusion a length in snow it needs not to traverse its whole length but only a fraction of it. The ice particles

are far from obstacles for the diffusing water vapour; the spaces occupied by them are regions freely opened to it, although it must abandon its form of vapour and fuse into the ice particles in these regions. In other words the water vapour does not diffuse forcing a way through the snow but it is delivered hand to hand by each of the ice particles, the only thing which it must do by itself being to diffuse across the narrow spaces between the ice particles. This ease of the diffusion of water vapour through snow is the cause which makes the diffusion coefficient appear larger through snow than through the open air.

It will be shown later in § 6 by microscopic observation made on an assemblage of snow crystals subject to the influence of a temperature gradient that the above-mentioned hand to hand delivery of water vapour is actually taking place between the snow crystals.

#### § 4 The metamorphosis of snow crystals in an atmosphere of uniform temperature

Snow crystals deposited on the ground gradually change their shapes and join themselves together making a coherent structure of the deposited snow cover. Such changes take place in the snow cover even if it experiences no melting because the atmospheric temperature is kept below  $0^{\circ}\text{C}$  throughout the winter season. This is the case with the snow cover in the vicinity of Sapporo. The authors observed the process of metamorphosis of snow crystals at temperatures below  $0^{\circ}\text{C}$  under a microscope in one of the low temperature rooms of their Institute. An attempt was made to place crystals under circumstances which resembled those of the actual snow cover in as many points as possible. For example, the snow crystals were placed not on a microscopic slide glass but on a fine net woven with single silk fibres (diameter of the fibre:  $12\ \mu$ , distance between fibres:  $0.5\ \text{mm}$ ) in order to avoid the clinging of the snow crystals to the glass surface which evidently disturbed the natural mode of metamorphosis of the snow crystals.

Although more or less vertical temperature gradient is usually present in the interior of the snow cover the metamorphosis of snow crystals was studied at first in the state of absence of any temperature gradient (6) (7). A small glass box ( $15 \times 15 \times 3\ \text{mm}^3$ ) was filled with snow and a small vertical hole was made through it. The above described silk net was stretched at the middle height of the hole in the snow and several snow crystals to be observed were put on it. The authors believed that in this manner they could put the snow crystals in a state not so different from that in the actual snow cover. The glass box was enclosed in a metal box made of brass plates 1 mm thick and put in a thermostat placed in the cold room which was kept constantly at the temperature of  $-25 \pm 2^{\circ}\text{C}$ . Enclosure of the glass box in the metal box was necessary in order to secure uniformity of temperature throughout the snow. The temperature of the thermostat was regulated to several temperatures within the range from  $0^{\circ}\text{C}$  to  $-18^{\circ}\text{C}$ . The glass box was

taken out from the thermostat from time to time and the crystals on the silk net were observed and photographed through a microscope. The observations were continued for a month or two.

An example of sequence of shapes which an isolated snow crystal of dendritic type showed in the course of its metamorphosis is shown in Fig. 8 of Pl. I. At first the edges of the crystal became round and its branches and twigs became slender at their roots. Observation from the side of the crystal showed that the branches changed their forms from their original tape-like ones to rod-like ones as shown in Fig. 9 of Pl. VIII. In Fig. 9 the horizontal long figure looking like a bamboo-stalk is the side view of one of the branches of a snow crystal. The crystal shown in Fig. 8 is not the same one as shown in Fig. 9; but the states (a) and (c) of Fig. 9 seem to correspond to the states (a) and (c) of Fig. 8 respectively. The branches and twigs of the crystal were then separated at their roots and the whole of the crystal became an assemblage of long ice rods. After that these ice rods increased their thickness at the same time decreasing their length. In this stage there were seen many short rods and small ice lumps scattered on the silk fibre net.

The snow crystals of the dendritic type which the authors observed in Sapporo, in most cases carried many ice droplets on their surfaces. Two series (A) and (B) of photographs showing the mode of metamorphosis of such crystals are shown in Fig. 10 and Fig. 11 of Pl. II and Pl. III. (A) series is the case when the metamorphosis took place at low temperature of  $-15^{\circ}\text{C}$  while in the case of series (B) the temperature was higher, that is, it was  $-6^{\circ}\text{C}$ ,  $-0.1^{\circ}\text{C}$  and  $-6^{\circ}\text{C}$  in the first 7 days, from the 7th day to 28th day and after the 28th day respectively.

The snow crystal with ice droplets changed its shape nearly in the same manner as the one without droplets did. The ice droplets united themselves into a small number of larger droplets which in their turn dissolved in the body of the crystal. In this case the crystal branches and twigs which had been separated at their roots were much thicker than in the case of snow crystal without droplets because the ice droplets had fed the branches and twigs with their moisture. Comparison of the two series of photographs with each other shows clearly that the metamorphosis is accelerated as the temperature is raised. One will see that state (d) of series (A) corresponds to a state between (b) and (c) of series (B). (d) state of series (A) was reached at  $-15^{\circ}\text{C}$  in 10 days after the beginning of metamorphosis while the same state was reached in less than 5 days at  $-6^{\circ}\text{C}$  in the case of series (B).

One of the branches of the crystal of series (B) going out to the upper left from the crystal centre was cut at its root in stage (c) and was moved as a whole upwards some distance to join there with the upright branch in stage (d). It cannot be imagined altogether that the branch moved by itself. Although not shown in the photographs the tip of this branch was in contact with one of the branches of another snow crystal lying near the present crystal on the silk net. These two

branches froze to each other at their contact point and, after they had been cut at their roots from their mother crystals, they formed an isolated individual body. The position of the center of gravity of this body was then displaced as the metamorphosis went on and it tilted from its original position since it was supported only at a few points by the silk net. The above noted movement of the branch was caused by this tilting due to the displacement of the centre of gravity.

The ice droplets stuck to the snow crystal are supposed to have their origin in the super-cooled water cloud located in the lower atmosphere in winter (8). The super-cooled water droplets of the cloud freeze to the snow crystal while it is falling through it. The authors placed in the cold room a wooden box on whose bottom a shallow vessel containing hot water was put. Water vapour evaporating from the surface of the hot water condensed on the inner side of the lid of the box as hoar crystals with many ice droplets on their surface. These hoar crystals were then put on the silk fibre net and observed through microscope (6). Side view of a dendritic hoar crystal on which the ice droplets were produced in such a way is shown in Fig. 12 of Pl. VIII in a series of photographs illustrating sequent stages of its metamorphosis at  $-20^{\circ}\text{C}$ . Most of the ice droplets showed the form of short hexagonal prism immediately after it had been taken out from the wooden box as shown in photograph (a). The hexagonal form of the prisms rapidly disappeared and after 5 minutes they looked like small circles when seen in the direction perpendicular to the plane of the crystal. But it took about 2 hours for them to take round forms also in their side views. Photograph (b) of Fig. 12 is the state of the hoar crystal 5 hours after photograph (a). After that the branch underwent change in form as shown in the subsequent photographs (c), (d) and (e). In photograph (e) no difference in form can be distinguished from the branch of snow crystal in state (c) of Fig. 9.

The ice droplets attached to the natural snow crystal are round in their forms, but, in the most rare case, hexagonal droplets are found among them as pointed out in the book of BENTLEY and HUMPHREYS (9). Therefore the hexagonal ice droplets attached to the hoar crystal made by the present authors were not common ones as in the case of snow crystal of natural origin. NAKAYA suggests that the hexagonal droplet found on snow crystal may have appeared as the result of subsequent condensation of water vapour on an ice droplet which had had initially a round form (10).

The present authors then made hoar crystals having no ice droplets on their surface by reducing the temperature of the hot water put in the above described wooden box. Then on those hoar crystals in the cold room fine droplets of water were sprayed. The water droplets stuck frozen to the hoar crystals in the form of round ice droplets as shown in photograph (a) of Fig. 13, Pl. IX. Such hoar crystals were then put in a box in which water vapour was slowly evaporating from a mass of snow heated by electric power. After a while the round droplets on the surface of the hoar crystals changed to hexagonal prisms as shown in photograph (b) of

Fig. 13. There is almost no doubt that the water vapour evaporating from the mass of snow condensed on the round ice droplets, forming over them mantles of ice with polyhedral outline. Most of the prisms had their hexagonal axis perpendicular to the surface of the crystal. This fact shows that the round ice droplets which had changed into such prisms had not been separated individuals merely attached to the hoar crystal but they had been united to the crystal being continuous in their crystalline structure to that of the hoar crystal.

Three of the hexagonal prisms indicated by arrows in photograph (b) of Fig. 13 seem to lie on the crystal surface with their axes parallel to it. Such prisms cannot be complete continuations of the hoar crystal since the hexagonal axis of the latter is perpendicular to its surface. On the other hand they must not be entirely independent of the hoar crystal because the direction of their axes show a definite relationship to those of the edges of the crystal. This may be a kind of the phenomenon "Orientierte Substanzabscheidung (Epitaxie)", a review of which was given recently by NEUHAUS (11). It should be noticed that the round ice droplets which produced them had been located on the edge of the hoar crystal or on the margin of reliefs on its surface. Such situations seem to have relationship to the peculiar direction of the prism axes.

It is supposed that the hexagonal prisms which appeared on the crystal surface in the first experiment were produced in the following way. Since the temperature of the hot water in the box was high in this case the water vapour evaporating from it was plentiful enough to produce water droplets in the air. The hoar crystals which were being made by condensation of water vapour chanced to catch the water droplets when they came close to their surfaces being conveyed by the air current caused by heat convection. Water vapour condensed subsequently on these droplets frozen to the hoar crystal giving them crystalline appearance. The same thing is supposed to be taking place in the cloud while snow crystals are falling through it. Therefore the ice droplets attached to snow crystals would be showing crystalline form in the cloud or immediately after they have crossed through it.

Graupel, being an assemblage of a very large number of ice droplets deposited on a dendritic snow crystal, changed its shape in a manner different from the former cases of dendritic crystals. The ice droplets united themselves into larger ones which were linked together in a form resembling an entangled string of beads. The string of linked droplets then became thicker and thicker until in the final stage the appearance of the whole could not be distinguished from that of the assemblage of gravel-like particles found in old snow cover. Series (A) and (B) of photographs shown in Fig. 14 and Fig. 15 of Pl. IV and V show two examples of metamorphosis of graupel. Series (A) is the case of temperature  $-15^{\circ}\text{C}$ . Initial state (a) changed to state (d) in 10 days. In series (B) the graupel was kept at  $-6^{\circ}\text{C}$  for the first 7 days, at  $-0.1^{\circ}\text{C}$  until the 28th day and at  $-6^{\circ}\text{C}$  thereafter.

In 2 days it changed from state (a) to state (b) which had obviously advanced in metamorphosis beyond a state corresponding to state (d) of series (A). Here the promotion of metamorphosis by the rise of temperature seems much more distinct than in the case of above described dendritic snow crystal with ice droplets.

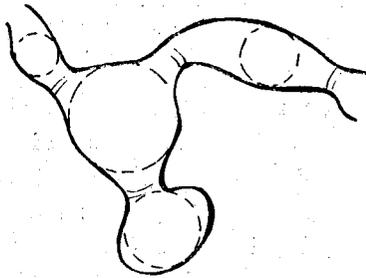


Fig. 16

Circles touching internally the ice texture of snow.

It is very desirable to express definitely the rate of change in forms in some quantitative way but this is a very difficult matter. Therefore it seemed necessary to take the following provisional way of expressing the dependence of metamorphosis on the temperature. As noted above, graupel consists of linkages of ice rods and spheroids in every stage of metamorphosis. The diameters of ice rods and spheroids of a graupel were measured as its metamorphosis proceeded and the mean value  $d$  of them was taken for a mark of the degree of the metamorphosis.

Fig. 16 shows a part of the linkage; the diameters of the circles drawn with broken lines touching internally with the contour of the figure were used for the diameters of ice rods and spheroids.  $d$  increases as the metamorphosis proceeds as shown in Fig. 17, in which the time  $t$  elapsed since the beginning of metamorphosis is taken along the abscissa in units of days. The relation between  $d$  (mm) and the time (days) is given by the following formula:

$$d = d_0 + At^c, \quad (1)$$

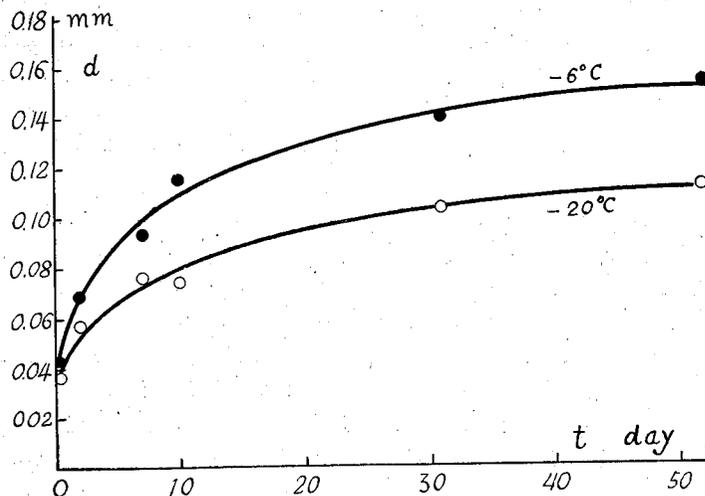


Fig. 17 The mean value  $d$  of the diameters of circles touching internally the ice texture of snow increases with time  $t$ .

where  $d_0$  is the mean value of diameters of ice droplets composing the initial graupel which is generally of a magnitude between 0.02 mm and 0.03 mm.  $A$  and  $c$  are constants which depend on the temperature.  $A$  is about 0.055 mm at  $0^\circ\text{C}$  and decreases with the decreases of temperature attaining 0.035 at  $-20^\circ\text{C}$ . The dependence of  $c$  on the temperature  $\theta$  ( $^\circ\text{C}$ ) can be roughly expressed by the relation

$$c = 0.65 + 0.013 \theta. \quad (2)$$

The photographs (a) and (b) shown in Fig. 18, Pl. IX are side views of a snow crystal belonging to the type 'capped column'. Photograph (b) was taken by placing the crystal between the crossed nicols of a polarisation microscope after it had already undergone a considerable metamorphosis. Since the body of the crystal looks bright on the dark background the optic axis of the crystal must lie in the plane of the paper, which result is expected from the structure of the crystal. Photograph (a) shows the initial state of the crystal through an ordinary microscope but it too looked bright when placed between the nicols. When seen parallel to the column between nicols it looked dark in cases of both (a) and (b). Therefore it can be stated that metamorphosis does not alter the direction of inner crystalline structure of a snow crystal although it brings about a large change in its outer shape.

Although the last stage of metamorphosis of an isolated snow crystal of dendritic type did not present an intimate resemblance with the structure of actual snow cover, an assemblage of many dendritic snow crystals changed as a whole into a state like to that of the actual snow cover. Fig. 19, Pl. VI and Fig. 20, Pl. VII show two series of metamorphosis stages of dendritic snow crystals placed on the silk net in such a way that they partially overlapped each other. In the case of Fig. 19 it took 51 days for the crystals to attain, at temperature  $-15^\circ\text{C} \sim -20^\circ\text{C}$ , the state shown by figure (f) in which can still be seen signs of the original shape of the snow crystals. But, as shown in figures (c) and (d) of Fig. 20, the snow crystals lost almost entirely their original forms in  $10 \sim 15$  days at  $-6^\circ\text{C} \sim -7^\circ\text{C}$ . After having been kept at the same temperature for 51 days, they finally presented such an appearance as shown in figure (f), Fig. 20 which seems like that of old snow cover except for the distribution of ice particles being much more coarse in this case than in the case of old snow cover. In this case of assemblage of many snow crystals the crystal branches which originated from different crystals and had overlapped each other united themselves into thick ice rods combining in such a complicated way as seen in the structure of the actual snow cover.

The dendritic snow crystal and the graupel are widely different in their forms and it is thought that they stand in a sense at the two extremities of a series of forms which snow crystal can take. Nevertheless they approach the same final forms after metamorphosis as seen from the resemblance of the above described final form of an assemblage of dendritic crystals to the final form of graupel shown in photo-

graph (g) in Fig. 15 of Pl. V. It is then imagined that an assemblage of snow crystals, no matter what form they may have initially, takes the same state in its final state of metamorphosis. As a matter of fact young compact snow in the upper part of the actual snow cover to whose state the above final state of the crystal assemblage corresponds shows the same microscopic appearance although it must have come from varieties of snow crystals. The compact snow, however, is not unchangeable but is growing denser, ice grains composing it are becoming thick. Therefore the words "final state" used above mean only the final state reached in the course of the experiments carried out in the laboratory cold room.

### §5 Some considerations on the metamorphosis of snow crystals

(a) *Sublimation of snow crystal as the cause of metamorphosis.* All the metamorphoses of snow crystals described in the previous section took place at temperatures below  $0^{\circ}\text{C}$  with no melting phenomenon. Metamorphosis of snow crystal below  $0^{\circ}\text{C}$  can be considered to take place in two ways: by sublimation of snow crystal and by movement of crystal molecules on the crystal surface. If the saturation pressure of water vapour is variable on different parts of the crystal surface, water vapour evaporates at those parts of higher saturation pressure and condenses on the parts of lower pressure. This explains the metamorphosis due to sublimation.

The molecules on the surface of ordinary crystals have been known to move on it under some conditions especially when the crystals are growing. NAKAYA and MATSUMOTO reported recently an interesting phenomenon which seemed to indicate the existence of a liquid water film over the surface of an ice sphere even at a temperature as low as  $-7^{\circ}\text{C}$  (12). Therefore it is not impossible that a snow crystal changes its shape as a result of such a movement of its surface molecules.

In order to learn to what degree each of the above stated causes may contribute to the metamorphosis of a snow crystal, a single crystal was placed in a film of watch oil and observed under microscope for a long time. It was considered that by this means a stop could be brought to the metamorphosis due to sublimation by the suppression of evaporation or condensation of water vapour while the movement of surface molecules of the crystal would be left unhindered.

Two snow crystals, the one being dipped in oil and the other being exposed to air, were placed side by side in the cold room and their metamorphoses were observed through microscope. The one in oil showed no sign of change in its shape even for such a long time as a month while the one surrounded by air lost its original shape entirely within that time. The two photographs in Fig. 21, Pl. X show the change in shape which appeared during 3.5 years in the case of a snow crystal kept in oil at  $-25^{\circ}\text{C}$ . This crystal was preserved in order to be shown to the inspectors of the authors' Institute and was illuminated by a strong beam of light from time to time. Moreover it was two or three times in danger of melting when the cold room in which it was being preserved was being cleaned. But the

change in its shape was slight as shown in the two photographs of Fig. 21. The crystal of which the original shape is shown by (a) holds its outline unchanged in figure (b) which shows its state after 3.5 years, although minute patterns carved on its surface have disappeared almost entirely. The state shown by figure (b) corresponds to that which the crystal would take after 20 hours at  $-25^{\circ}\text{C}$  if it were kept exposed to air. In this way it may be concluded that the metamorphosis of snow crystals at temperatures below  $0^{\circ}\text{C}$  takes place by the way of sublimation but not by the movement of surface molecules of the crystals since the suppression of sublimation by the oil reduces remarkably the speed of metamorphosis.

(b) *Collection of foreign matters contained in snow crystal caused by its sublimation.* KUMAI (13), in his experiments on finding the sublimation nucleus of snow crystal, placed the snow crystal on a cellulose film for electron-microscopic use and let it sublime there. In addition to the sublimation nucleus of the dimension of about one micron at the centre of the crystal he found many minute particles of the dimension of about one one-hundredth micron on those parts of the cellulose film where the body of the crystal had been before it began to sublime. He supposed these minute particles to be the condensation nuclei of small water droplets suspended in the atmosphere which had been collected by the snow crystal while it was growing. Setting aside the question concerning the origin of the minute particles the above-stated fact makes one suppose that the subliming snow crystal becomes small leaving the foreign matters contained in it behind without drawing them in with its retiring boundary. But the following two cases with which the present authors met seem to show that retiring surface of the subliming snow crystal draws in and collects the foreign matters contained in it whether they are solid matters or whether they are substances dissolved in the crystal body.

In one case three very small dark particles, supposed to be soot particles, were found on the tip of a branch of a snow crystal subliming on the silk net as shown on the curve marked 1 in Fig. 22. The boundary of the branch represented by curve 1 retired to curve 2 and then to curve 3 carrying the dark particles with it. The broken lines drawn in the figure represent the paths of the dark particles. All the paths cut the outlines of the branch almost perpendicularly.

This fact shows that the surface of crystal retires by sublimation always in the

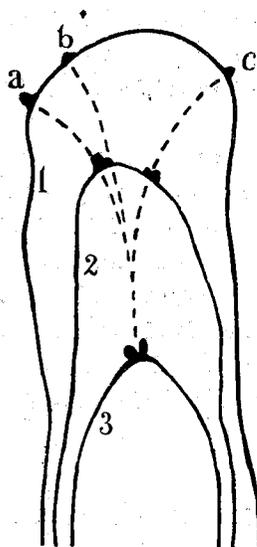


Fig. 22

Dust particles a, b, c attached by chance to the margin of a branch of a snow crystal moved following the dotted lines as the margin shrank by sublimation.

direction perpendicular to itself, at the same time keeping the dark particles attached to it. The dark particles separated from each other at first, gradually approached and united themselves at last (6).

Sometimes it was found that snow crystals became stained as they sublimed although they had been clear at first. One of such snow crystals made metamorphosis as shown in three photographs (a), (b), (c), of Fig. 23, Pl. X. The crystal shown by photograph (a) became sensibly stained with the advance of sublimation as shown in photograph (b). Finally the central part of the crystal and the tips of its branches were left attached to the silk fibres at the stage of photograph (c) and fine strings were stretched between them. These strings were likely under tension since they became wavy when their ends attached to the tips of the branches were released by the disappearance of the latter. They must have been made of some substances dissolved in the original crystal which became concentrated as the crystal sublimed (14).

In the above described case reported by KUMAI the snow crystal must have stuck to the cellulose film and the foreign matters must have been caught by the film when they became in contact with it as the crystal sublimed, otherwise they would have united themselves to clusters. However it cannot be disregarded that some of the particles found on the cellulose film might be congregations of foreign matters originally occluded in the crystal separated from each other.

(c) *Curvature of crystal surface as the cause of metamorphosis by sublimation.*

The cause of metamorphosis by sublimation of snow crystal at a temperature below the melting point of ice is commonly believed to be the non-uniformity of curvature  $K$  of the crystal surface. At those parts of the surface, such as points or edges of the crystal which have positive curvatures, the vapour pressure of the crystal surface is higher than at ditches or pits engraved on the snow crystal surface which have negative curvatures. The surface tension  $\alpha$  of ice in conjunction with positive curvature brings about a local mechanical pressure

$$\Delta P = \alpha K \quad (1)$$

which acts on the ice lying under the surface in addition to the atmospheric pressure acting uniformly on the whole surface of the crystal. This local additional mechanical pressure  $\Delta P$  causes an increase

$$\Delta \mu = V \cdot \Delta P \quad (2)$$

in the chemical potential  $\mu$  (free energy per gramme molecule) of the ice lying under the surface which increases in turn the vapour pressure  $p$  of that part of the surface by an amount

$$\Delta p = p_0 \cdot \Delta \mu / RT. \quad (3)$$

Here  $V$ ,  $R$ ,  $T$  and  $p_0$  represent molar volume of ice, universal gas constant, absolute temperature and vapour pressure of ice at the temperature  $T$  respectively. In the same way negative curvature of the surface brings about a decrease in the vapour pressure in which case all the equations (1), (2) and (3) hold with an negative value for the curvature  $K$ . Water vapour then evaporates from those parts of the snow crystal surface where the curvature is positive, diffuses through the air surrounding the crystal and condenses on those parts where the curvature is negative. Such a transfer of water vapour evidently makes the surface of the crystal smooth. This concept seems to accord with the mode of metamorphosis observed on an actual snow crystal.

Although this explanation of metamorphosis of snow crystal seems very plausible, it is nothing but a qualitative explanation. Cases are not seldom met with where a qualitative explanation was proved wrong by the quantitative consideration which was carried out later. For example, the small resistance experienced while skating has been for a long time believed due to appearance of liquid water between the skates and ice which was produced by large pressure delivered by the skates, but this qualitative explanation is now suffering criticism from the quantitative point of view. Under these circumstances an attempt was made to make a quantitative study on metamorphosis, although, on account of mathematical difficulties, not all kinds of metamorphosis of snow crystal could be taken up but only one of the simple cases (15).

The smoothing of the surface of crystal is not the only process observed to occur during metamorphosis, but also the branches and twigs of the dendritic type crystal become thin at their roots and finally cut off there. Usually the branches and twigs begin to thin at their roots when they have changed to rod-like pieces with smooth surface as shown in the previous section. The authors made a mathematical study on this thinning process of the roots under the assumption that the phenomenon is due to the curvature of those thinning parts of the crystal.

Let a long circular cylinder of ice with radius  $a$  be in equilibrium with water vapour having a pressure corresponding to the curvature of the cylindrical surface and let the radius be slightly changed to

$$a - \xi(t, x), \quad \xi(t, x) \ll a,$$

where  $x$  is length measured along the axis of the cylinder. Then the curvature  $K$  of the surface is given by

$$K = \frac{1}{a - \xi} + \frac{\partial^2 \xi}{\partial x^2},$$

and according to equations (1), (2) and (3), the rise  $\Delta p$  of vapour pressure of the surface above its initial value is given by

$$\Delta p = \frac{aV}{RT} p_0 \left( \frac{\partial^2 \xi}{\partial x^2} + \frac{\xi}{a^2} \right). \quad (4)$$

Water vapour evaporates from those parts of the surface where  $\Delta p$  is positive and condenses on these parts where  $\Delta p$  is negative. Then it will be a natural assumption that the rate of evaporation or condensation is proportional to  $\Delta p$ , and, since this rate must be proportional to  $\frac{\partial \xi}{\partial t}$ , the following equation holds:

$$\frac{\partial \xi}{\partial t} = \lambda \left( \frac{\partial^2 \xi}{\partial x^2} + \frac{\xi}{a^2} \right), \quad (5)$$

where  $\lambda$  is a constant having a positive value. The transfer of water vapour through the air surrounding the ice cylinder can be considered as being governed by the following differential equation of stationary diffusion:

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial x^2} = 0, \quad (6)$$

where  $r$  means the distance from the axis of the cylinder to a point situated outside it.  $u$  is the excess of pressure of water vapour in the air over that value which it would take if it were in equilibrium with initial uniform surface of the ice cylinder. On the surface of the cylinder  $u$  and  $\Delta p$  must be equal to each other and the quantity of water vapour  $-\beta D_0 \frac{\partial u}{\partial r}$  escaping from unit area of the surface within unit time must be equal to  $\sigma \frac{\partial \xi}{\partial t}$ . Here  $D_0$  is the diffusion constant of water vapour through air,  $\beta$  the ratio of density and pressure of water vapour and  $\sigma$  the density of ice. Although these conditions must be satisfied on the deformed surface of the ice cylinder, it will be allowable to consider that they are satisfied on a cylindrical surface of radius  $a$  since the deformation  $\xi$  is assumed to be small compared with  $a$ . Then the mathematical expressions for these conditions are given by

$$\left. \begin{aligned} u &= \Delta p \\ -\beta D_0 \frac{\partial u}{\partial r} &= \sigma \frac{\partial \xi}{\partial t} \end{aligned} \right\} \text{at } r = a. \quad (7)$$

Now it can be easily shown that

$$u = \frac{aV}{RT} p_0 \xi_0 e^{\lambda k^2 t} \frac{I k^2 H_{1,0}(i l r)}{H_{1,0}(i l a)} \cos(lx), \quad \xi = \xi_0 e^{\lambda k^2 t} \cos(lx), \quad \text{when } al < 1 \quad (8)$$

$$u = \frac{aV}{RT} p_0 \xi_0 e^{-\lambda k^2 t} \frac{I k^2 H_{1,0}(i l r)}{H_{1,0}(i l a)} \cos(lx), \quad \xi = \xi_0 e^{-\lambda k^2 t} \cos(lx), \quad \text{when } al > 1 \quad (9)$$

with

$$\lambda k^2 = \frac{\alpha \beta D_0 V}{\sigma RT} \cdot \frac{p_0}{a^3} f(al) \quad (10)$$

$$f(z) = (1-z^2) \{-zH_{1,1}(iz)\} / iH_{1,0}(iz) \quad (11)$$

satisfy all the equations from (4) to (7). Symbols  $H_{1,0}(\ )$ ,  $H_{1,1}(\ )$  appearing in formula (11) represent HANKEL'S cylindrical functions and  $i$  is imaginary unit  $\sqrt{-1}$ . The curve of function  $f(z)$  resembles that of a parabola with vertical axis, being 0 at  $z=0$ ,  $z=1$  and attaining the maximum value of about 0.68 at  $z=0.5$ .

The ice cylinder with uniform cylindrical surface of radius  $a$  can keep its surface unchanged in contact with water vapour having a pressure corresponding to the uniform curvature of its surface. But if the surface is deformed by any cause, non-uniformity of its curvature brought about by the deformation will cause local evaporation and condensation of water vapour at different parts of the surface, and, on account of this, the deformation cannot be kept as it is but must be changed. In one case it will be promoted and in the other it will be depressed. The initial form of the ice cylinder with uniform radius  $a$  is unstable in the former case and stable in the latter.

$\xi$  expressed by formulae (8) and (9) represent the mode of change of deformation given initially in the form  $\xi_0 \cos(lx)$  by any cause. This deformation is periodic in the direction of the cylinder axis and its period  $L$  is equal to  $2\pi/l$ . Formulae (8) and (9) show that  $\xi$  increases with time  $t$ , that is, the initial form of the cylinder is unstable, when  $la < 1$  that is  $L > 2\pi a$ ;  $\xi$  decreases with time  $t$ , that is, the initial form of the cylinder is stable, when  $la > 1$ , that is,  $L < 2\pi a$ . The mode of change in  $\xi$  is such that  $\xi$  is multiplied or divided by the same amount for each definite amount in increase of time  $t$ ; for example,  $\xi$  doubles or is halved in a time  $\tau = \log_e 2 / \lambda k^2$ . Therefore the rate of change in  $\xi$  is large when  $\tau$  is small and is small when  $\tau$  is large.

Now let the thinning part at the roots of branches or twigs of snow crystal be assumed to be represented by one periodical length  $L$  of the ice cylinder between  $x = -(L/2)$  and  $x = +(L/2)$ . Of course  $L$  should be taken as larger than  $2\pi a$  in this case, otherwise this periodical length would not become thin and could not represent the thinning parts of snow crystal at all. Numerical calculation of the value of  $\lambda k^2$  by formula (10) for  $a = 0.05$  mm, that is, for a rod-like branch or twig of snow crystal of diameter 0.1 mm, gives

$$\lambda k^2 = 4.8 \times 10^{-4} f(la) / \text{day}$$

which yields, for the maximum value of  $f(la)$ ,

$$\tau = 6 \text{ years.}$$

Since this value of  $\tau$  corresponds to the maximum value of  $f(la)$ , that is, to the case when the thinning rate is largest, it represents the possible shortest time within which the decrease  $\xi$  in radius of the roots of branches or twigs of snow crystal can double. But, as one can clearly see by the photographs shown in Pl's. I, II and

III, actual snow crystal needs not more than several days to double the reduction of its radius. (The above values of  $lk^2$  and  $\tau$  were calculated for the temperature  $0^\circ\text{C}$ . Surface tension  $a$  of ice was taken as 87 dyne/cm which value was determined in the course of theoretical study of sublimation nucleus (16). The values of other physical constants appearing in formula (10) were taken from current physical tables).

Discrepancy between theory and experiment is found not only in the rate of change of  $\xi$  but also in the length  $L$  of thinning part. As described above,  $\xi$  can increase only when  $L > \pi(2a)$ , whereas it decreases when  $L < \pi(2a)$  and stops changing when  $L$  becomes equal to  $\pi(2a)$ . But, as shown in the above indicated photographs, the length of the thinning part is, in actual snow crystal, less than twice its diameter  $2a$ .

The authors consider that these discrepancies between theory and experiment are sufficient to prove wrong the assumption that the thinning of the roots of branches is caused by the curvature of the crystal surface. Furthermore they are doubtful of the opinion that the smoothing of snow crystal surface is due to its curvature.

(d) *Elastic stress as the cause of metamorphosis of snow crystal.* If elastic stress  $S$ , such as tensile stress or shear stress, is produced in ice, its chemical potential  $\mu$  is raised by

$$\Delta\mu = VS^2/2E \quad (E: \text{coefficient of elasticity}) \quad (1)$$

and its vapour pressure is, for the same reason as described in the previous article, increased by the amount

$$\Delta p = p_0 \frac{V}{RT} \frac{S^2}{2E} \quad (2)$$

The branches and twigs of snow crystal are striving to bend themselves by their own weight and the bending moment  $G$  is greatest at their roots. If, then, branches are reduced in their radius at their roots the stress there is raised since they must meet the same bending moment  $G$  with their reduced cross section. Therefore the roots will become thinner and thinner on account of the rise of vapour pressure of their surface and finally will be cut off.

In order to study this problem in the same way as followed in the previous article (c), let a long ice cylinder of radius  $a$  bent uniformly by a constant bending moment  $G$  be supposed, and let the coordinate-axes  $x$  and  $y$  be set up in the plane in which the bending moment is acting with  $x$ -axis along the axis of the cylinder. The bending moment  $G$  produces in the interior of the cylinder stress  $X_x$  expressed by

$$S = X_x = \frac{4G}{\pi a^3} \frac{y}{a} \quad (3)$$

The rise of vapour pressure of the surface is then given by

$$(\Delta p) = p_0 \frac{8VG^2}{\pi^2 RTE} \frac{y^2}{a^8}, \quad (4)$$

which is not uniform along the periphery of a cross section of the cylinder. Since this non-uniformity makes the problem very difficult to be solved, let it be assumed that the whole periphery has a constant vapour pressure which is equal to the mean value of  $(\Delta p)$  expressed by formula (4), that is,

$$(\Delta p) = p_0 \frac{4VG^2}{\pi^2 RTE a^6}. \quad (5)$$

A decrease  $\xi$  in radius makes the vapour pressure rise over  $(\Delta p)$  by an amount  $\Delta p$ , and, if this additional rise is represented by a mean value in the sense described above, it is given by

$$\Delta p = \frac{24VG^2 p_0}{\pi^2 RTE} \frac{\xi}{a^7}. \quad (6)$$

Then the equation corresponding to equation (5) of the previous article (c) is

$$\frac{\partial \xi}{\partial t} = \lambda \xi \quad (7)$$

and, for the initial deformation  $\xi_0 \cos(lx)$  from whatever cause, the following results are obtained by the same procedure as carried out in the previous article:

$$\xi = \xi_0 e^{\lambda t} \cos(lx) \quad (8)$$

$$\lambda = \frac{24\beta D_0 VG^2 p_0}{\pi^2 \sigma RTE a^8} g(la) \quad (9)$$

$$g(z) = \frac{-z H_{1,1}(iz)}{i H_{1,0}(iz)}. \quad (10)$$

The function  $g(z)$  is positive for positive values of  $z$ , increases with increasing  $z$  and can be approximately represented by  $0.5+z$  except for small value of  $z$ . Therefore  $\xi$  always increases with increasing time  $t$  and the time  $\tau = \log_e 2/\lambda$ , within which  $\xi$  doubles, decreases as the period  $L = 2\pi/l$  decreases, contrary to the former case of curvature.

The formulae from (8) to (10) were applied to the case of root of a branch of snow crystal with diameter 0.1 mm and length 1 mm in the same way as before. The weight of the branch produces at the root a bending moment  $G$  of the value  $3.5 \times 10^{-4}$  dyne·cm and  $\tau$  turns out to be one half million years for the case  $L = 2a$ , which result shows elastic stress has entirely no effect upon the thinning of branches of snow crystal at their roots.

The present authors think that the problem of metamorphosis by sublimation

of snow crystal would not be solved by phenomenological methods such as described above but adequate explanation of the problem should be based on the internal structure of ice composing the snow crystal.

(e) *Effect of elastic stress upon the metamorphosis of ice grains composing snow cover.* Although elastic stress has no effect on the metamorphosis of a single independent snow crystal, large stresses appearing in such a deposited snow layer as one hanging over the edge of a terrace or roof are expected to be sufficiently effective to promote the metamorphosis of ice grains composing the snow layer. Suppose a sheet of deposited snow of density  $\rho$  and of thickness  $2H$  projecting a length  $2L$  horizontally over an edge, and set up coordinate axes  $x$  and  $y$  in horizontal and vertical directions respectively with their origin at the height  $H$  above the edge. Then the stresses at  $x=0$  caused by the weight of projected part are found to be

$$X_x = - \frac{3LMg}{2H^3} y \quad (1)$$

$$X_y = \frac{3Mg}{4H^3} (H^2 - y^2), \quad (2)$$

where  $M = 2L \cdot 2H \cdot \rho$  is the mass of the projected part and  $g$  is the acceleration of gravity.

The actual deposited snow layer consists of ice grains of various forms and dimensions joined together in an extremely complicated way. But, for the sake of simplicity, assume that the ice grains composing snow layer are rod-like ones of various radii  $a$  but of the same length  $l$  which stand vertically and horizontally joining together at their ends in such a way to make a rectangular lattice. At the origin of the coordinates  $x$  and  $y$ , where  $X_x=0$  and  $X_y$  takes its maximum value  $(X_y)_{\max} = 3Mg/4H$ , each of the rodlike grains lying parallel to  $x$ - and  $y$ -axis are acted on by a shear force  $f = (X_y)_{\max}/N$ , where  $N = 1/l^2$  is the number of parallel rod-like grains penetrating a unit area perpendicular to their axes. This force  $f$  strives to turn the rod-like grain with a moment  $fl$  as a result of which there appears a bending moment  $G = fl/2$  at the ends of the grain. Therefore the rod-like grains are in the same situations as those of the branches of snow crystal described in the previous article but with a difference that the value of  $G$  in this case is much greater than in the case of snow crystal. Application of this conception to the case of a sheet of deposited snow of density  $\rho = 0.3\text{-gr/cm}^3$  and of thickness  $2H = 40\text{ cm}$  projecting out  $2L = 50\text{ cm}$  over an edge gives for a rod-like grain of diameter  $0.1\text{ mm}$

$$\tau = 23\text{ min}, \quad (3)$$

the time needed for the grain to make its ends double the reduction  $\xi$  in their radius.

At the top and base of the snow sheet  $X_y$  is 0 and  $|X_x|$  takes its maximum

value  $|X_x|_{\max} = 3LMg/2H^2$ . Rod-like grains lying horizontally parallel to the  $x$ -axis are acted on by tensile or compressive force  $f = |X_x|_{\max}/N$ . When a long ice cylinder of radius  $a$  which is being pulled or pushed by a force  $f$  acting in the direction of the cylinder axis is deformed for any cause in such a way that its radius is reduced by  $\xi_0 \cos(lx)$ , this deformation is enlarged according to the following formula:

$$\xi = \xi_0 e^{\lambda t} \cos(lx) \quad (4)$$

$$\lambda = \frac{2\beta D_0 V f^2 p_0}{\pi^2 \sigma R T E a^6} g(la), \quad (5)$$

where  $g(\ )$  is the same as the function expressed by formula (10) of the previous article. The ice cylinder becomes thin periodically along its axis in the same manner as in the cases described before. Application of formula (4) to the horizontal rod-like grains of radius 0.1 mm situated at the top or base of the snow sheet gives

$$\tau = 17 \text{ min.} \quad (6)$$

The small values of  $\tau$  given by expressions (3) and (6) show that elastic stresses can act very effectively on the metamorphosis of ice grains composing deposited snow layer. It should be noticed, however, that such small values of  $\tau$  appear only on thin rod-like ice grains having diameter as small as 0.1 mm.  $\tau$  increases very rapidly with increasing diameter of ice grains, in proportion to the 8th power of diameter in the case of (3) and to the 6th power in the case of (6) as seen from formula (9) of the previous article and formula (5) of the present article. Therefore thick ice grains will remain almost unaltered while thin ones are becoming thinner finally to be cut off.

(f) *Observation made on the root of branch of snow crystal when it is being cut off by sublimation.* The root of branch of snow crystal which has become very thin is cut off in the manner shown in Fig. 24 of Pl. XI. The thinning part takes the appearance of a thin string, though not reproduced in the photographs of Fig. 24, stretched between the ends of the branch and of the mother crystal which are tapered to a point directed forward to each other. The string is then suddenly cut and the pointed ends draw back with their tips rounded. The authors think that such a thin string cannot be preserved even for a little while if it is a string made of ice. It is supposed that this string is made of foreign matters which have been dissolved in the crystal in its original state and is concentrated into a string form, that is, it is supposed that this string is of the same kind as the one described in article (b) of this section which chanced to be considerably thick on account of abundance of dissolved matters.

Photographs in Fig. 25, Pl. XI give another example showing more clearly that the root of branch turns into a thin string made of substance other than ice. One of the branches of snow crystal which looked like broad leaves became very thin

at its root as shown in photograph (a). The branch seemed to be supported at its root by the mother crystal through a very short solid string. (In this case no silk fibre net was used unlike in the former case. The only thing which supported the branch in question was the mother crystal). Such a state lasted about two days and then the solid string yielded with the result that the branch became to be hung at its root as shown in photograph (c). If the string had been composed of ice alone neither could it endure for such a long time as two days nor could it bend to uphold the branch.

Since the snow crystals which the authors dealt with were all such as had fallen through dusty atmosphere over a city they would have in general dissolved an abundance of foreign matters. But it is thought that even the snow crystals falling through country clear atmosphere contain certain impurities though in small quantity since a large number of condensation nuclei is always found even in the clearest air. The snow crystals containing the very small particles which were supposed by KUMAI to be condensation nuclei (article (b)) were in fact snow crystals which fell in a deep forest (13). Then it may not be unnatural to suppose that the foreign matters dissolved in the snow crystal will exert some effect on its general metamorphosis by sublimation, now that their great influence on the thinning process of the root of branch of snow crystal has been shown.

As a case opposite to the one of separation of snow crystal into parts the authors made a study on adhesion of two ice particles when they were made to come into contact (17). Two series of photographs shown in Figs. 26, 27 of Pl. XII show two cases which advanced in opposite directions. Ice particles were put into light contact under as nearly as possible identical condition in both cases and no distinction was discernible in their contacting states so far as microscopic examination permitted. Nevertheless, in one case (Fig. 26) a short column of ice appeared as a bridge between the two ice particles at their contacting point after one hour, while in the other (Fig. 27) the contacting surface of each particles drew back and a clearance became discernible between them in the same time interval. After this the bridge became thicker and thicker making strong adhesion between those two ice particles while in the latter case no adhesion took place at all. It seems necessary for adhesion to be accomplished that a substantial union be made between the ice particles at the moment of contact, and mere geometrical contact is by itself insufficient to convert the contact into an adhesive one.

The above described adhesion of ice particles resembles the sintering of metal or glass particles at high temperatures. They may belong to one and the same phenomenon. But the form of union of two ice particles shown in Fig. 26 seems to be different from that of sintered copper particles shown by DEDRICK and GERDS (18) and also from that of sintered glass particles shown by KUCZYNSKI (19). Sintering of metal or glass particles is considered to be developed by viscous flow or volume diffusion which takes place in the material composing the particles. The

results of theories based on these conceptions were experimentally verified by the above authors. Photographs of Fig. 26 seem to show that the adhesion of ice particles is developed by condensation at the contact point of water vapour which is transferred through air to this point.

Substantial union of ice particles takes place even far below the melting point of ice at least when force is applied for a while. Fig. 28 shows the results of experiments carried out on this subject (20). Two ice spheres made by freezing tap water in a glass mould were pressed against each other by a force  $f$  gr-wt for a time  $t$  and the least force  $F$  gr-wt necessary to separate them was measured. Fig. 28 shows the relation between the ratio ( $F/f$ ) and the time  $t$ . Experimental values obtained by the use of different values of  $f$  are distinguished by different symbols. Temperature of experiment is shown in the figure. Sometimes no force was required to separate the contact, that is,  $F$  was zero even after one or two minutes of contact; this seems to correspond to the case illustrated in Fig. 27. Experiments were

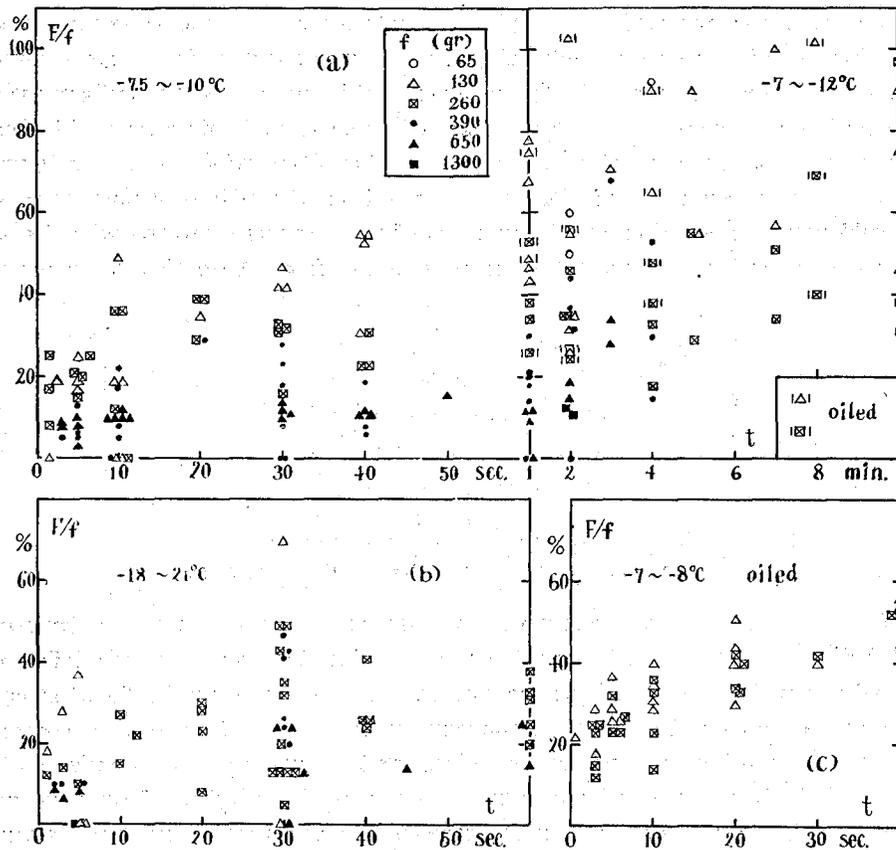


Fig. 28 Force  $F$  necessary to detach two ice spheres (diameter: 1.5 cm) which have been kept in contact by a force  $f$  during time  $t$ .

done with clean contact surface as well as with contact surface which had been coated with a film of watch lubrication oil. Figure (c) of Fig. 28 shows such oiled cases. That part of the figure corresponds to a portion of the left half of figure (a) which shows the unoiled case for the time  $t$  less than one minute. In the right half of figure (a) oiled and unoiled cases are shown together, the oiled cases being distinguished by drawing two short vertical lines on both sides of the marks. No sign of difference is seen between oiled and unoiled cases thus indicating that no condensation of water vapour takes part in the initial substantial union of ice particles. Otherwise the oil which prevents condensation of water vapour must have weakened the adhesive force  $F$ , making the marks of oiled case lie nearer to the base in Fig. 28.

### § 6 The metamorphosis of snow crystals under a temperature gradient.

The results of experiments described above in sections 1, 2 and 3 lead to the supposition that heat flowing through a layer of deposited snow is accompanied by a flow of water vapour which takes place in such a way that the vapour is transferred from one ice grain composing the snow layer to the next in the same direction as that of the heat flow. In order to verify this supposition the authors made microscopic observations on an assemblage of snow crystals through which heat was flowing (17). They filled with snow a flat box (2 cm wide, 6 cm long, 6 mm high) as shown schematically in Fig. 29 and put it on the table of a microscope in their cold room. The upper and lower sides of the box were covered with double glass plates in order to prevent heat from escaping from these sides as much as

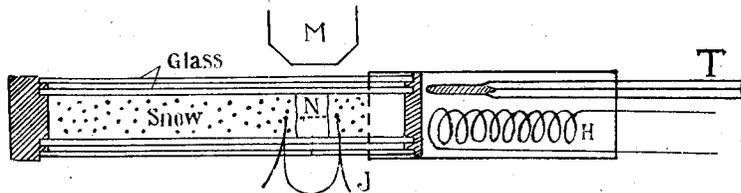


Fig. 29 Apparatus for the observation of metamorphosis of snow crystal subject to a temperature gradient.

possible. A vertical hole was made in the snow just underneath the objective lens of the microscope M. A net N made of single silk fibres was stretched in the hole and several snow crystals were put on it. An electric heater H placed at one end of the box produced a horizontal temperature gradient in the snow. A copper-constantan thermoelement J whose two joints were inserted into the snow so as to keep the hole between them was used to determine the value of temperature gradient. The alcohol thermometer T served to indicate the general temperature of the snow.

Fig. 30 of Pl. XIII shows one of the dendritic snow crystals placed on the

silk fibre net under the temperature gradient  $0.3^{\circ}\text{C}/\text{cm}$  at  $-15^{\circ}\text{C}$ . The arrow drawn in the photographs shows the direction of heat flow caused by the temperature gradient. The original crystal shown in photograph (a) changed its shape into the one shown in photograph (b) in 24 hours. Twigs facing forward the heat flow, that is, these twigs attached on the left side of the vertical branch, became thick and their outlines became angular which is strong evidence of condensation of water vapour upon them. On the other hand twigs on the right side of the branch became thin and short showing that water vapour had evaporated from their surfaces. Photograph (c) shows the appearance of the crystal after another 24 hours. All the twigs on the right side of the branch had disappeared and those on the left side became massive with distinct crystalline form.

The series of photographs in Fig. 31 of Pl. XIII shows the change in shape which occurred to one of the graupels put on the silk fibre net under the temperature gradient  $0.4^{\circ}\text{C}/\text{cm}$  at  $-15^{\circ}\text{C}$ . In 34 hours, as shown in photograph (b), a number of hexagonal prisms of ice crystal had grown on the side of the graupel facing the heat flow while its whole structure had become coarse one just as was observed when graupel was left to change in an atmosphere of uniform temperature (cf. Fig. 15 of Pl. IV). Photograph (c) shows the appearance after 10 days. The graupel has been transformed into an assemblage of large hexagonal prisms each of which must have grown in the direction facing the heat flow. It should be noticed that the graupel had shifted to the left as a whole by a distance amounting to about one third of its original diameter.

Since the snow crystals in the above two examples were not in the interior of a mass of snow but were placed on the silk net stretched in a hole made in such a mass, their behaviour must have been not wholly the same as that of snow crystals composing the actual mass of snow. But the authors think that the difference is not great for the hole was small compared to the dimensions of the mass of snow, also that the hand-to-hand delivery of water vapour by snow crystals, mentioned at the end of section 3, can be proved experimentally. The temperature gradient as large as  $0.3\sim 0.4^{\circ}\text{C}/\text{cm}$  which was used in the above experiments rarely occurs in the actual snow cover deposited on the ground, but it is not impossible that it may occur at the uppermost layer of the snow cover. As will be described in the next section dendritic snow crystals with an unsymmetrical appearance which seem to correspond to metamorphosis stage (b) or (c) of Fig. 30, Pl. XIII can be found in the naturally deposited snow cover.

The cause of the above described unsymmetrical metamorphosis of snow crystal is, of course, the difference of temperature produced between the adjacent crystals by the temperature gradient as already noted in section 3. Therefore unsymmetry becomes less distinct as the intensity of temperature gradient is reduced. Fig. 32, Pl. XIV shows metamorphosis which occurred to a dendritic snow crystal submitted to the action of a temperature gradient  $0.08^{\circ}\text{C}/\text{cm}$ . Such a gradient is realised

often in the actual snow cover. Every branch and twig of the crystal, photograph (b), Fig. 32, which had been placed under the above temperature gradient for 4 days at  $-15^{\circ}\text{C}$  is obviously of increased mass on the sides facing the heat flow.

The main part of the snow cover deposited on the ground is commonly compact and composed of granular ice particles coherent to each other forming a complicated irregular lattice. A block of such compact snow was crushed and the pieces of ice produced were put on the silk net. The series of microscopic photographs in Fig. 33, Pl. XIV shows the change which occurred to them at the temperature  $-15^{\circ}\text{C}$  and under the temperature gradient  $0.7^{\circ}\text{C}/\text{cm}$ . At a small number of points on the surface of the ice grains hoar crystals began to grow against the heat flow and, as time went on, only the larger hoar crystals survived to grow further, the smaller ones disappearing by evaporation or by being fused to the larger ones. After 30 hours the hoar crystals showed the appearance of hollow hexagonal column and they took finally, 12 days after the beginning, the cup crystal form. In this final stage the space around the hoar crystals was cleared and they were seen under the microscope sparsely distributed on the silk fibre net. These hoar crystals in the form of hexagonal column and cup crystals must be the same thing as 'depth hoar' which name SELIGMAN gave to the large hoar crystals occasionally found in the interior of deposited snow cover (21).

Photographs in Fig. 34, Pl. IX shows the case in which ice grains were placed on the silk fibre net more densely than in the above case. It should be noticed that the surface of ice grains confronting the hoar crystals growing on the neighbouring grains was shifted toward the high temperature side as clearly indicated by the variation of their position relative to the net fibres.

In the case of small temperature gradient no hoar crystal develops but the granular ice particles tend to assume crystalline form on their surfaces facing forward the heat flow. Fig. 35 of Pl. XV shows one of such cases. In 2.5 days under temperature gradient  $0.15^{\circ}\text{C}/\text{cm}$  and at temperature  $-15^{\circ}\text{C}$  the part to the left of the neck of the pebble-like ice grains shown in photograph (a) came to have polyhedral shape being enlarged in size at the same time as shown in photograph (b). Both the enlargement and the change into crystalline form of this part clearly indicate condensation of water vapour upon its surface. On the other hand the part to the right of the neck became somewhat smaller without becoming crystalline. Evaporation of water vapour must have taken place on its surface. No appreciable change in form was observed, however, on the same granule as above when it had been placed under as small a temperature gradient as  $0.08^{\circ}\text{C}/\text{cm}$  before the above experiment was made. However, careful examination showed that the surface of the granule facing the heat flow somewhat bulged out while the opposite surface somewhat retired with the result that the granule was shifted as a whole a short distance against the heat flow.

These results of experiments on the ice granules composing compact snow

shows that the hand to hand delivery of water vapour from one ice granule to the next also takes place in the compact snow when heat flows through it although the effect of heat flow on the change in form of the ice granules is not the same dependent upon the intensity of heat flow. SELIGMAN attaches importance to the role of sublimation due to the presence of temperature difference in snow cover for its metamorphosis. In the section, "The Connexion between Depth Hoar and Dry Granular Firn Snow" of Chapter III of his book (21), he states that dry granular "firn" snow is formed by the sublimation process due to temperature difference although the ice grains composing it have no external crystalline form. He describes this phenomenon in connection with the formation of dry granular "firn" snow, but, as shown in the previous paragraph, metamorphosis of ice grains due to temperature gradient without displaying crystalline form seems to be a very common phenomenon occurring in general in the interior of snow cover, being not confined to the formation of dry granular firn snow.

All the above described experiments were carried out by the use of horizontal temperature gradient, that is, by placing the flat box shown in Fig. 29 horizontally on the table of microscope. Convection current of air might occur to participate in carrying the water vapour through snow as is noted by SELIGMAN in his book. Then the box was stood vertically with the heater on the top side so that no convection current of air could take place. The snow crystals or ice granules to be observed were held between two silk fibre nets stretched vertically in the hole which was now made horizontally in the sheet of snow. The performance of the snow crystals or ice granules between the nets was the same as in the above described horizontal cases. That result shows the movement of water vapour through snow was due to diffusion phenomenon caused by the temperature gradient alone. It should be noticed that in the actual case of snow deposited on the ground the temperature gradient would be more effective than in the experimental cases shown above since the temperature of the actual deposited snow is in general higher than  $-15^{\circ}\text{C}$  which was used in the experiments for the sake of experimental convenience.

The hand to hand delivery of water vapour by the snow crystals or ice granules composing deposited snow is not the only effect of the temperature gradient. It is effective also in changing the structure of the deposited snow. A temperature gradient sufficiently intense to produce depth hoar crystal changes the structure of snow into a very fragile one. As shown in Figs. 31 and 33 the larger ice pieces have tendency to enlarge their size, under influence of temperature gradient, by attracting the smaller ones in their neighbourhood. This is in accord with the fact that ice particles composing actual snow cover become large as time goes on.

In the newly deposited snow composing the uppermost layer of the snow cover the temperature gradient is positive at night (lowering of temperature with increasing height above ground is counted as positive) and negative in day-light hours.

Therefore condensation and evaporation of water vapour take place alternatively within the period of one day on both the upper and lower parts of each snow crystal with the result that its metamorphosis is much promoted without resultant distinct unsymmetry in its form. Upper and lower parts of snow crystals subject to alternating temperature gradient became rich in mass and crystalline in form by condensation of water vapour while they are facing the heat flow, but evaporation of water vapour from their surface which follows the condensation period, when the direction of the heat flow is reversed, destroys, though not completely, their crystalline form. In this way the snow crystals composing a newly deposited layer are changed in form similarly to those put in an atmosphere of uniform temperature. But the rate of change in form of snow crystals is accelerated in the case of alternating temperature gradient since the diffusional movement of water vapour forced by it stimulates condensation and evaporation of water vapour at their surface.

Sometimes, under a layer of compact snow which is entirely composed of granular ice particles, there is found a layer of snow crystals which have suffered metamorphosis but in such a small degree that their original crystal form (for example, dendritic) is recognizable. Such case may occur under various circumstances. The following is thought to be one of them. A new snow layer was deposited so thick that the daily alternating temperature gradient failed to reach its bottom leaving its lower part to change by itself. The lower part, being not influenced by the alternating temperature gradient, delayed in metamorphosis and stays in a younger stage with layers already matured on its upper and lower sides.

As was noted in section 4 the glass box containing snow crystals to be observed must be enclosed in a thick-walled metal box in order that they might be metamorphosed in an atmosphere of uniform temperature. At first, experiments were attempted without the metal box. Angular outlines of dendritic snow crystals put on the silk fibre net began to change into round ones. But soon later the direction of change seemed to be reversed in that they began to show a tendency to return to their original angular forms as illustrated in the series of photographs in Fig. 36, Pl. XV. The crystal looks more crystalline in its form, especially at the tips of its twigs, in photograph (b) than in photograph (a), although the former shows the state 1.5 days after that of the latter. But following that, the reversal was stopped and the metamorphosis proceeded on the normal course with the result that the crystal became as shown in photograph (c). The authors are of the opinion that this unaccountable phenomenon may be due to the existence of an uncontrollable small temperature gradient in the glass box which was caused by slight non-uniformity of temperature spread throughout the interior of the thermostat. Accordingly the glass box was enclosed in the metal box in order to keep it from the influence of any non-uniformity of temperature in the thermostat. Thereafter the crystals never showed any such reversal in the direction of change. The above

occurrence proves that even the slightest temperature gradient can have an influence on the metamorphosis of snow crystals.

### §7 The temperature gradient in the actual snow cover

Four examples of temperature distribution in the snow cover are graphed in Fig. 37. Data for each of them were obtained in the snow cover deposited on the grounds of the Institute on the day and at the time inscribed in the upper left part of the figure.  $T_a$  means air temperature. Curve (a) represents the typical temperature distribution at noon. From the ground surface up to the height of 60 cm the temperature gradient had positive values,  $0.08^\circ\text{C}/\text{cm}$  near the ground and  $0.15^\circ\text{C}/\text{cm}$  at the height of 50 cm. It turned to negative at this height with its absolute value increasing towards the snow surface which was 76 cm above the ground. In the foregoing night the curve would have extended towards the snow surface without turning, maintaining the positive values which it had below the 60 cm height. It is thought that the value of temperature gradient near the snow surface would have amounted to or even exceeded  $0.2^\circ\text{C}/\text{cm}$ .

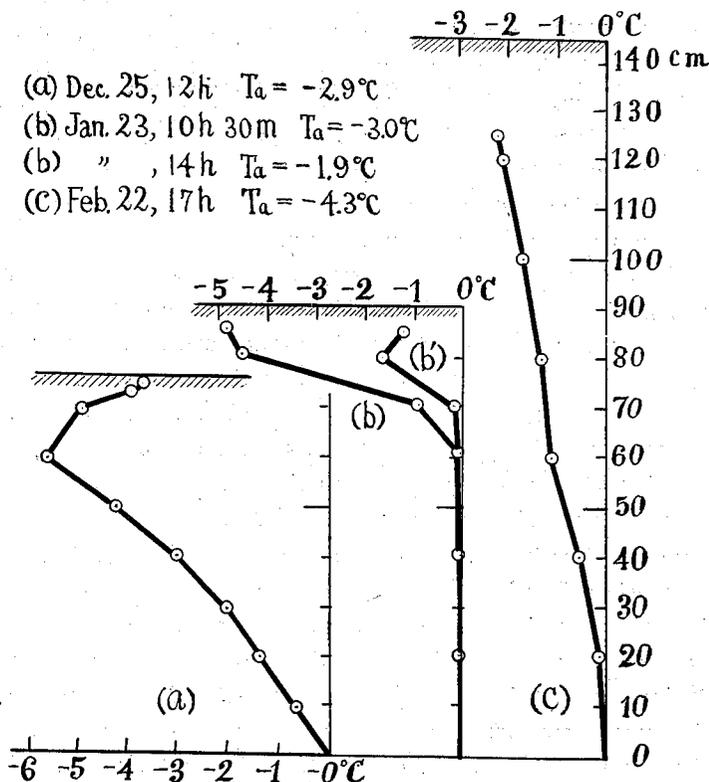


Fig. 37 Temperature distribution in the interior of snow cover (Sapporo).

On the day before the one when curves (b) and (b') were obtained, it had rained and the snow temperature had become  $0^{\circ}\text{C}$  throughout the whole snow cover which had then been 90 cm deep. But the following night chilled the snow surface and a positive temperature gradient as large as  $0.35^{\circ}\text{C}/\text{cm}$  was still to be found at 10 hr 30 min of the day of observation, as shown in curve (b). But two and a half hours later the temperature of the uppermost layer of the snow cover rose by about  $3^{\circ}\text{C}$  and it had at 14 hr. of that day the distribution represented by curve (b'). Such a rapid rise of snow temperature leads one to suppose that the temperature gradient in the range down to the depth 20 cm from the snow surface would have been much larger at the times preceding that of curve (b). It might have reached a value as large as  $1^{\circ}\text{C}/\text{cm}$ .

Curve (c) is the temperature distribution in a thick snow cover. Since snow cover attains such a great thickness as 150 cm only occasionally when the winter season is near its end in Sapporo the value of temperature gradient turned out to be less than  $0.02^{\circ}\text{C}/\text{cm}$ .

As described above the value of temperature gradient seems to lie in general in the range from somewhat less than  $0.01^{\circ}\text{C}/\text{cm}$  to somewhat greater than  $0.1^{\circ}\text{C}/\text{cm}$  in the lower and middle parts of the snow cover. In the uppermost layer about 20 cm thick in which daily temperature changes occur the temperature gradient fluctuates between large positive and negative extremes whose absolute values seem to be able to reach in special cases ones as much as  $1^{\circ}\text{C}/\text{cm}$ . Therefore the large temperature gradients used in the experiments described in the preceding section 6 are ones which can actually be found in the natural snow cover.

As is well known, the snow cover is composed of many layers each of which is composed of snow deposited in one snow fall. The two series of photographs (A) and (B) shown in Figs. 38 and 39 of Pl. XVI show the mode of change in shape of snow crystals belonging to two layers which were deposited in the latter part of January in Sapporo (22). Let the layers corresponding to the series of photographs (A) and (B) be called layer (A) and layer (B) respectively. Layer (A) was composed of dendritic snow crystals with cloud droplets frozen to their surface; it was about 20 cm thick at the end of its formation. Four days after layer (A) was formed, graupels accumulated on layer (A) to form layer (B) of 8 cm thickness.

Photograph (a) of series (A) shows the appearance of dendritic snow crystals two days after being deposited. It is seen clearly that the twigs are arranged unsymmetrically on one side of the branches with no twigs on the other side. All the branches except one have their twigs on the lower side which shows that heat had been flowing upwards for some time interval preceding the moment when the snow crystals were taken out from the snow layer. But the word 'upwards' here indicates only the direction in the photograph. It is thought that the snow crystals may have been in the reversed position in the actual snow layer, since they were taken out from it in day-light hours when heat was flowing downwards in the

uppermost layer of the snow cover. In this way dendritic snow crystals much like the crystal shown in Fig. 30, Pl. XIII modified experimentally by artificial heat flow, were found in the actual snow cover.

Photograph (b) shows the appearance of ice grains composing layer (A) 18 days after its formation. That layer carried then two snow layers of 20 cm total thickness on its top, the one being layer (B) which was deposited on it 14 days previously and the other having been deposited on layer (B) 10 days before. Therefore layer (A) had been up to this time for 18 days subjected to the influence of the alternating temperature gradient prevailing near the surface of snow cover. No trace of the form of the original dendritic snow crystals nor that of any other crystalline form is perceived on the ice grains; they seem to be in a condition corresponding to state (f) of Fig. 20, Pl. VII which reproduces a photograph of an assemblage of dendritic snow crystals placed on the silk fibre net taken after 51 day metamorphosis in a uniform temperature atmosphere. It is thought that the alternating temperature gradient accelerated the metamorphosis in the actual snow layer. The grains appeared as shown in photograph (c), Fig. 38, twenty-eight days after the formation of layer (A), when their layer was 55 cm below the surface of the snow cover. They not only had grown in size but also had become crystalline in form. It is supposed that the cause of such change in form was the positive temperature gradient of nearly constant magnitude prevailing in the middle and lower parts of the snow cover.

The graupel shown in photograph (a) of series (B), Fig. 39, Pl. XVI, had lain for 5 days in the interior of layer (B) which had been for that period the uppermost layer of the snow cover. The graupel had come to be composed of comparatively large ice particles but it showed no sign of any crystal such as hoar on its surface. The authors have not yet succeeded in finding in the actual snow cover such a graupel with hoar crystal as shown in photograph (b), Fig. 31, Pl. XIII, which was obtained by experimental procedure. Thirteen days after the formation of layer (B) when it was 20 cm below the snow surface the graupel had turned to ice grains shown in photograph (b), Fig. 39, which cannot be distinguished from those which originated from dendritic crystals. Afterwards this layer (B) followed the same course of change as followed by layer (A) and 10 days later the ice grains composing layer (B) presented crystalline appearance as shown in photograph (c).

Since the state of snow layer is determined not only by the present conditions under which it exists but also by the past conditions under which it has been placed up to the present, and since these conditions are very changeable being neither constant nor strictly periodic, the mode of change in the structure of snow layer does not always follow the same course. Such being the case it is not a rare case to find even in the uppermost layer of the snow cover ice grains showing crystalline appearance more distinctly than those do which have been for a long time subjected to the influence of the constant positive temperature gradient prevailing in the

middle part of the snow cover. Two examples of such ice grains are shown in photographs (a) and (b) of Fig. 40, Pl. XVI. One is solid ice surrounded by polyhedral planes and the other is hollow cup hoar crystal which must have been made by the same condensation process as that producing depth hoar. Depth hoars such as shown in photograph (c) of Fig. 40 are found most frequently in the lowest layer just above the ground in Sapporo. But depth hoars are not confined to the lowest layer. Those shown here were found in a layer situated 50 cm above the ground in the interior of snow cover of 130 cm thickness. The sublimation due to temperature gradient and the various phenomena caused by it in the actual snow cover are described with many illustrations by Seligman in his excellent book (21). However, as a glaciologist, he seems to have studied mainly the firnification of snow cover, which phenomenon is not important at least for such snow covers deposited on low land as those with which the present authors are concerned. As shown by the authors' experiments in the cold room and observations on the actual snow cover, the sublimation due to temperature gradient seems to be the most important agent throughout the whole course of metamorphosis of snow cover in these districts where the mean air temperature does not exceed  $0^{\circ}\text{C}$  and snow does not melt in winter.

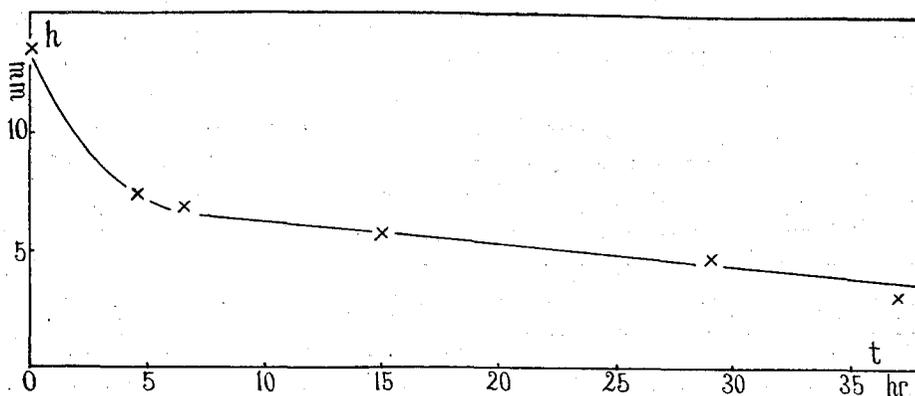


Fig. 42 Height  $h$  of a thin sheet of snow diminished with the lapse of time  $t$ .

The authors believe that another important agent having effect on the metamorphosis of snow cover is the pressure caused by its own weight. It is obvious that this pressure is the main cause of the settling of snow cover. Detailed researches on the action of pressure have not yet been made out but in Fig. 41, Pl. XVII a series of microphotographs is presented which show how snow crystals are deformed and displaced while being compressed by a weight (23). Newly fallen dendritic snow crystals were put in the narrow space made between two glass plates 2 mm apart from each other. Into the narrow space between the glass plates was inserted at their one end another glass plate which would be used afterwards as a weight to compress the snow when the first two were stood vertically. For

one or two days the plates were laid in horizontal position and then stood vertically in the thermostat of the temperature  $-5^{\circ}\text{C}$ . Otherwise, had the glass plates been stood on end immediately after the snow was put between them, the sheet of snow would have been crushed in one or two seconds. The snow sheet was initially 13.5 mm high when it was stood upright. Then its height was reduced rapidly to about half its initial height in 6 hours to be decreased hereafter at a constant small rate as shown in Fig. 42. Photographs (a), (b) and (c) in Fig. 41, Pl. XVII show, respectively, the state at first, the state when the initial rapid reduction in height was ended and the state after about 23 hours of constant rate of decrease in height. In order to bring about the deformation taking place in the structure of the snow, straight lines were drawn to form a ladder-like figure in photograph (a). A few of points on the straight lines could be distinguished by one mark or another even when the snow was deformed; these points were connected by straight lines in photographs (b) and (c). Each of the rectangles in photograph (a) was deformed into distorted rectilinear figures such as a trapezoid or pentagon. Since the rectangles were of the same order in dimension as individual snow crystal, their deformation is an evidence that the individual snow crystal itself underwent deformation additionally to displacement as a whole. In fact, the snow crystal indicated by ( $B_1$ ,  $B_2$ ,  $B_3$ ) in photograph (a) which chances to be distinctly seen also in photographs (b) and (c), had one of its branches  $B_3$  turned clockwise while the crystal as a whole was turned in the opposite direction. Moreover the turned branch  $B_3$  became curved in photograph (c) which result shows that it was under strong stress. It should be noticed that the root of its twig p had become very thin while the root of the twig q had become somewhat thick despite the very short distance between both. It is thought that water vapour which evaporated from the stressed root of twig p condensed on the unstressed root of twig q on account of the effect of stress upon vapour pressure of ice as described above in article (d) of section 5.

Seligman says that for the depth hoars to be formed it is necessary that there be a space such as cavity in which they may come into existence in the interior of snow cover. In-so-far as the present authors understand, he seems to think that the water vapour forming the hoar crystals is originated in distant places where the temperature is higher. The temperature difference existing between the spot of formation of depth hoar and the places where water vapour evaporates is the reason for the conveyance of water vapour to the former spot through the intervening snow mass. But, according to the results of the authors' experiments, the movement of water vapour through snow at any point in it is determined by the value of the temperature gradient at that point. The ice grains composing snow conveys water vapour from hand to hand at a rate which is determined by the temperature gradient prevailing where they are. In the case of depth hoars growing into a cavity in snow cover, the water vapour to form them is to be supplied by the evaporation of the wall of cavity opposite to the side on which they grow. It

needs not come from any distant places. The depth hoars can attain sizes larger than one may think, since the cavity wall confronting them retires as they grow, provided the retirement of the wall is not hindered by some cause such as the settling of snow cover. In fact large depth hoar crystals seem likely to be found within cavities which are formed in the interior of hard snow resistant to mechanical deformation. It is thought that even the narrow spaces between ice grains composing compact snow have possibilities for the growth of depth hoars if there prevails an intense temperature gradient. But the spaces are usually diminished in their size owing to the settling action of snow with the result that the growth of depth hoars is stopped at an early stage.

### § 8. Contribution of water vapour to the heat transfer in snow cover.

If water vapour evaporates from one ice surface and then condenses on another surface, the same amount of heat as that taken out from the former as latent heat of evaporation is given up to the latter surface as latent heat of condensation. Then, as long as the water vapour is produced and consumed within his field of observation, one can regard moving water vapour as carrying with it a quantity of heat. As shown in the previous sections, in the interior of snow in which a temperature gradient prevails, water vapour evaporates from one ice grain and condenses on the next one after traversing by diffusion process the narrow air gap between them. Therefore the diffusional movement of water vapour in snow caused by temperature gradient is always accompanied by transfer of heat and, since the water vapour moves only within the air filling the gaps between the ice grains, the role of water vapour can apparently be regarded to be to increase by its diffusion the heat conductivity of air. As will be shown in the following this contribution of water vapour to the apparent heat conductivity of air is of the same order of magnitude as the heat conductivity of air itself, that is, the air included in snow shows an apparent heat conductivity as large as twice that proper to the open air.

Since the air included in snow through which the water vapour diffuses is subject to a temperature gradient, the diffusion is composed of two elements. The one is the ordinary diffusion due to the gradient of concentration and the other is the thermal diffusion which is caused by the temperature gradient. Then the law of diffusion is expressed by

$$\begin{aligned} \bar{C}_1 - \bar{C}_2 &= - \frac{1}{n_{10}n_{20}} \left[ D_0 \text{grad } n_{10} + D_T \frac{1}{T} \text{grad } T \right] \\ &= - \frac{D_0}{n_{10}n_{20}} \left[ \text{grad } n_{10} + k_T \frac{1}{T} \text{grad } T \right], \end{aligned}$$

where the suffixes 1 and 2 indicate water vapour and air respectively and

$T$  = absolute temperature

$\bar{C}_1, \bar{C}_2$  = mean thermal velocities of molecule

$n_{10}, n_{20}$  = concentrations of water vapour and air (if  $n_1$  and  $n_2$  are the number

densities, and  $n = n_1 + n_2$ , then  $n_{10} = n_1/n$ ,  $n_{20} = n_2/n$ )

$D_0$  = coefficient of ordinary concentration diffusion

$D_T$  = coefficient of thermal diffusion

$k_T$  = thermal diffusion ratio. (24)

The value of coefficient  $D_0$  of ordinary diffusion of water vapour through air can be found in current physical tables, but, in so far as the authors are aware, no measurement has been made on the coefficient  $D_T$  of thermal diffusion of a mixture of air and water vapour. Fortunately, however, a general law is known to the effect that  $D_T$  depends on the concentrations  $n_{10}$ ,  $n_{20}$  of each component of any mixture in such a way that  $D_T$  becomes vanishingly small when either of the two is decreased to zero. It is obvious that the concentration  $n_{10}$  of water vapour in the air included in snow is, at its greatest, equal to the ratio of the vapour pressure of ice at  $0^\circ\text{C}$  to the atmospheric pressure, that is, 0.006 ( $= 4.56/760$ ) which is very small. Therefore the thermal diffusion is expected to be almost ineffective in the case of diffusion of water vapour in snow and no significant error will be introduced if the thermal diffusion is entirely neglected.

Let the temperature of water vapour in the Celsius scale and the density of water vapour be denoted by  $u$  and  $\sigma$  respectively.  $u$  and  $\sigma$  are function of position  $(x, y, z)$  and of time  $t$ . But, since the time rate of change of temperature of snow is ordinarily very small,  $\partial u/\partial t$  and  $\partial \sigma/\partial t$  can be considered equal to zero. Then  $u$  and  $\sigma$  turn out to satisfy the differential equations

$$\Delta u = 0 \quad (1)$$

$$\Delta \sigma = 0 \quad (2)$$

where  $\Delta$  denote the Laplacian operator  $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ . Now let the position of a point on the surface of ice grains composing snow cover be represented by the coordinates  $(X, Y, Z)$  and let the temperature at that point be denoted by  $U(X, Y, Z)$ . If a mathematical function

$$f(x, y, z) \quad (3)$$

is a solution of differential equation (1) and satisfies the boundary condition

$$f(X, Y, Z) = U(X, Y, Z), \quad (4)$$

then  $f(x, y, z)$  represents the temperature  $u$  of the air.

If the vapour density of ice at the temperature  $U$  is denoted by  $\Sigma$ , there holds an approximate relation

$$\Sigma - \Sigma_0 = aU \quad (5)$$

provided that the absolute value of  $U$  is not too great. Here  $a$  and  $\Sigma_0$  are a constant and the vapour density of ice at  $0^\circ\text{C}$  respectively. Then the mathematical

function  $\sigma(x, y, z)$  expressing the distribution of water vapour density should satisfy differential equation (2) as well as the boundary condition

$$\sigma(X, Y, Z) - \Sigma_0 = aU(X, Y, Z), \quad (6)$$

which yields the relation

$$\sigma(x, y, z) = \Sigma_0 + af(x, y, z), \quad (7)$$

where  $f(x, y, z)$  is the same function as (3).

Formulae (3) and (7) give for the fluxes of heat flow by conduction and of water vapour flow by diffusion the expressions

$$-\mu_A \text{ grad } f(x, y, z) \quad (8)$$

$$-aD_0 \text{ grad } f(x, y, z) \quad (9)$$

respectively, where  $\mu_A$  represents the heat conductivity of air. These expressions show that both flows have the same direction and are in the same ratio of magnitude at every point in the air. Now the water vapour flow (9) carries with it a quantity of heat equal to

$$-aD_0L \text{ grad } f(x, y, z), \quad (10)$$

where  $L$  is the latent heat of evaporation or condensation of ice. Therefore the total flux of heat turns out to be the sum of (8) and (10):

$$-(\mu_A + aD_0L) \text{ grad } f(x, y, z), \quad (11)$$

which result shows that the heat conductivity of air is raised from  $\mu_A$  to  $(\mu_A + aD_0L)$  on account of diffusional movement of water vapour through air.

The magnitude of heat conductivity of air  $\mu_A$  is  $5.3 \times 10^{-5}$  cal/cm·sec·deg. The following values of  $D_0$ ,  $L$  and  $a$  at  $0^\circ\text{C}$

diffusion coefficient of water vapour through air:  $D_0 = 0.22$  cm<sup>2</sup>/sec,

latent heat of evaporation of ice:  $L = 676$  cal/gr, and

rising rate of vapour density of ice with temperature:  $a = 0.39 \times 10^{-6}$  gr/cm<sup>3</sup>·deg

yield for the product  $aD_0L$  the value  $5.8 \times 10^{-5}$  cal/cm·sec·deg which is almost the same as the value of  $\mu_A$ . Therefore it can be stated that the heat conductivity of air is doubled if it is included in the narrow gaps between ice grains composing snow cover (25).

It is now desirable to learn to what extent the diffusional movement of water vapour in snow enlarges the heat conductivity of snow itself. Although absolutely accurate knowledge of it cannot be reached owing to the complicated texture of snow, an approximate estimation can be obtained by following O. WIENER'S way of thinking which he developed concerning the problem of refractive index of a mixture as will be described in the following (26).

Let a cube of snow having unit volume be supposed, through which heat is

flowing under the action of a temperature gradient  $G$ . If the heat conductivity of snow is denoted by  $\mu$ ,  $G$  and the heat flux  $H$  are in the relation:

$$H = \mu G. \quad (1)$$

$G$  and  $H$  are nothing more than the mean values of temperature gradient and heat flux respectively which are in reality distributed in the interior of snow in a very complicated manner. Let then the following symbols represent respectively the quantities written to the right side of them:

$G_a, H_a$ : vectorial mean value of temperature gradient and heat flux distributed in the interstices filled by air between the ice grains respectively.

$G_i, H_i$ : vectorial mean value of temperature gradient and heat flux distributed in the ice texture of snow respectively.

$p, q$ : space occupied by the air and the ice in the cube of snow respectively;  $p+q=1$ , since the cube has unit volume.

$\mu_a, \mu_i$ : heat conductivities of the air included in snow and of ice respectively.

$\mu_0$ : standard heat conductivity; the heat conductivity of dry air  $\mu_A$  will be used as this in the following.

$m, m_a, m_i$ : ratios of heat conductivities to the standard heat conductivity;

$$m = \mu/\mu_0, \quad m_a = \mu_a/\mu_0, \quad m_i = \mu_i/\mu_0.$$

It follows according to the definitions of the quantities concerned that

$$G = pG_a + qG_i \quad \text{and} \quad (2)$$

$$H = pH_a + qH_i, \quad (3)$$

the latter of these equations being equivalent to

$$mG = pm_aG_a + qm_iG_i. \quad (4)$$

In general, at the interface between two matters the heat flux (strictly speaking, the component of heat flux normal to the interface) must be continuous in order that no heat may accumulate or dissipate there. Furthermore, since heat conductivity is a property of matter dependent upon the ease with which heat flows through it, the smaller the temperature gradient required to maintain a heat flux, the larger the heat conductivity of the matter. Therefore the ratio  $G_a/G_i$  must become large as  $m_a$  becomes small if  $m_i$  keeps its constant value. But the ratio  $G_a/G_i$  must depend also on the configuration and arrangement of the ice grains composing snow in addition to its dependence upon  $m_a$  and  $m_i$ . O. WIENER expressed this situation by the relation

$$\frac{G_a}{G_i} = \frac{m_i + u}{m_a + u} \quad (5)$$

and he called the number  $u$  in this formula "Formzahl". The magnitude of the

*formzahl* is determined by the configuration and arrangement of ice grains, although its numerical value is variable dependent upon as what value  $\mu_0$  is chosen as the standard heat conductivity.

Equations (2), (4) and relation (5) give

$$u = \frac{m(pm_i + qm_a) - m_a m_i}{(pm_a + qm_i) - m} \quad (6)$$

and

$$m = \frac{u(pm_a + qm_i) + m_a m_i}{(pm_i + qm_a) + u} \quad (7)$$

The value of  $m$  can be obtained from the observed heat conductivity of snow while  $p, q$  can be determined by its density. Then by equation (6) value of  $u$  can be calculated. In the table below are shown the values of heat conductivity  $\mu$  of snow calculated for different densities  $\rho$  by formula (8) of section 1

$$\log_{10} \mu = -4 + 2\rho, \quad (8)$$

which the authors obtained experimentally.  $m$ 's are ratios of  $\mu$ 's to the heat conductivity of dry air  $5.3 \times 10^{-3}$  cal/cm·sec·deg which value is chosen here as the standard heat conductivity.

$\rho$	0.1	0.2	0.3	0.4	0.5
$\mu$	1.585	2.512	3.981	6.310	$10.000 \times 10^{-4}$ cal cm·sec·deg
$m$	2.99	4.74	7.51	11.90	18.87
$u$	6.99	9.42	11.91	15.10	19.30
$m'$	1.87	3.53	6.19	10.52	17.44
$(m-m')/m$	37	26	18	12	8 %

By using the value 2 for  $m_a$  and the value 94.2 for  $m_i$  in formula (6) the values of  $u$  shown in the table are calculated.  $m_a$  is taken equal to 2 because the air included in snow acts as a medium having a heat conductivity twice as large as that of dry air as stated above. Ninety four and two-tenths is the ratio of heat conductivity of ice  $4.99 \times 10^{-3}$  cal/cm·sec·deg to that of dry air. Now that the values of the *formzahl*  $u$  are known, the imaginary value of  $m$  which the snow would show if the air included in it had the heat conductivity of dry air, can be calculated by the use of the value 1 for  $m_a$  in formula (7).  $m'$ 's shown in the table are these imaginary  $m$ 's. The difference  $m-m'$  can be considered as a measure of contribution of water vapour to the heat conductivity of snow. In the table this measure of contribution is expressed in percentage ratio to  $m$ . Snow of density 0.1 gr/cm<sup>3</sup> turns out to have its heat conductivity reduced by 37% if the diffusional movement of water vapour in it is stopped by any cause. Such a reduction of heat conductivity is large for snow of small density which contains much air in it, and becomes small as the snow density becomes great.

It should be noted that the heat conductivity of snow is not completely deter-

mined solely by the snow density even if a mean relation such as shown by the above formula (8) exists between them. Individual observed values of the heat conductivity of snow show comparatively large differences between themselves for the same density of it as shown above in Fig. 3 of section 1. These differences must be brought about by the differences in the structure of snow and the present authors think that the *formzahl* introduced by O. WIENER could be suitably used to express the latter differences.

It can easily be shown from formula (7) that  $m$  is an increasing function of  $u$ . Therefore the following value of  $m$

$$m_{\max} = pm_a + qm_i \quad (9)$$

which is obtained by putting  $u = \infty$  in formula (7) is the largest value of  $m$  which snow of a definite density can have. The structure of snow corresponding to  $u = \infty$  is such that the whole of the ice composing it takes the form of an assembly of thin columns all of which stand parallel to the direction of heat flow. Of course, such a structure is an ideal one which cannot be realised in nature. But this fact leads one to suppose that large value of  $u$  is an indication of existence of a large number of continuations of ice grains arranged almost parallel to the direction of heat flow. When  $u$  is put equal to zero in formula (7), it gives the following value of  $m$ :

$$m_{\min} = \frac{m_a m_i}{pm_i + qm_a} \quad (10)$$

which is the lowest value which  $m$  can take. This lowest value is shown by snow with such an imaginary structure that the ice composing it takes the form of plane sheets standing perpendicular to the direction of heat flow. In this case there is no continuation of ice in that direction.

In the following table are shown the maximum and minimum values of  $m$  for different densities of snow.  $m_{u, \max}$ ,  $m_{\min}$  are the values for the case  $m_a = 2$  and  $m'_{\max}$ ,  $m'_{\min}$  are those for the case  $m_a = 1$ .

$\rho$	0.1	0.2	0.3	0.4	0.5
$m_{\max}$	12.10	22.04	32.05	42.08	52.11
$m_{\min}$	2.24	2.54	2.94	3.48	4.28
$m'_{\max}$	11.12	21.25	31.37	41.52	51.61
$m'_{\min}$	1.035	1.131	1.246	1.371	1.502

### Summary

**§1. Heat conductivity of deposited snow.** A method was devised for measuring the heat conductivity of a porous substance which made it needless to insert thermometer into the substance. The method was applied to the case of a snow sample

cut out from one of the layers composing snow cover deposited on the ground. The sample was enclosed in an air-tight metal case. After the case had been kept at a temperature below  $0^{\circ}\text{C}$  as long a time as needful for the establishment of a uniform temperature distribution throughout the snow, the wall of the case was suddenly cooled to a lower temperature. The pressure  $p$  of the air contained in the snow which could be measured by a manometer connected to the metal case began to diminish to approach to a final value  $p_f$  corresponding to the lowered temperature.  $p-p_f$  changed in proportion to  $\exp(-Ct)$ . Here  $t$  is time while  $C$  is a constant dependent on the temperature conductivity of snow and on the form and dimensions of the metal case. The value of  $C$  determined by experiment allowed the temperature conductivity, and subsequently the heat conductivity, of snow to be obtained. A mean relation

$$\log_{10} \mu = -4 + 2\rho$$

was found between the heat conductivity  $\mu$  cal/cm $\cdot$ sec $\cdot$ deg and the density  $\rho$  gr/cm $^3$  of snow.

Heat conductivity was found sometimes to change during the experiment on one and the same sample of snow. The cause of such changes in heat conductivity was supposed to be the change in the structure of the snow brought about by the heat itself which was flowing in it. The heat flow must have made evaporation or condensation of water vapour take place at the surface of ice grains composing snow, inducing changes in their forms.

**§2. The diffusion of water vapour through snow cover caused by temperature difference.** The temperature of snow is generally several degrees below  $0^{\circ}\text{C}$  in the upper part of the snow cover deposited on the ground while it is kept at  $0^{\circ}\text{C}$  close to the ground surface. On account of such a temperature difference water vapour would evaporate from the ice grains composing snow cover in its lower part and condense on the ice grains near its surface. Cages made of fine metal gauze containing snow were buried in the snow cover at the three different depths and their respective weights were measured every day during the period of three weeks of winter. The cage buried in the upper part of snow cover increased its weight while a decrease in weight was observed in the cage buried most deeply. These facts show evidently that in actual fact water vapour condensed on and evaporated from snow in the upper and lower parts of the snow cover respectively.

The change in density of snow could be determined from the observed change in the weight of snow contained in the cages. It was then shown that  $4.6 \times 10^{-8}$  gr of water vapour was evaporating in the lower part of snow cover in one second and per horizontal one square centimeter. This quantity of water vapour moving upwards should cross a horizontal plane at the level 80 cm above the ground car-

rying with it a heat quantity  $q' = 3,1 \times 10^{-5}$  cal  $\text{cm}^2 \cdot \text{sec}$ . On the other hand heat conductivity of snow and temperature gradient gave  $q = 1.6 \times 10^{-5}$  cal/ $\text{cm}^2 \cdot \text{sec}$  as the quantity of heat crossing this plane. The fact that  $q$  and  $q'$  are of the same order of magnitude is considered to show that water vapour carries a considerable fraction of the heat current occurring in snow.

**§ 3. Experimental determination of coefficient  $D$  of macroscopic diffusion of water vapour through snow.** Snow was put in four cylindrical metal case with wire gauze bottom and they were piled up one upon another. The upper and lower ends of the pile were kept each at different temperatures below  $0^\circ\text{C}$  for several hours. On account of the temperature difference produced in the pile, water vapour evaporated from the snow contained in the cases located at the warmer end of the pile, and, after diffusing through it, condensed on the snow in the cases located at its colder end, causing thereby changes in the weight of each of the four cases. The coefficient  $D$  of macroscopic diffusion of water vapour through snow was then calculated from the observed change in their weight.  $D$  was found, contrary to expectation, to be four or five times as large as the coefficient  $D_0$  of diffusion of water vapour through air, being almost independent of the density of snow. This contradiction was explained by the following conception. Water vapour needs not force its way through the interspaces between the ice grains composing snow. It needs only condense on one side of an ice grain and evaporate from the other side to condense again on the side facing to it of the next grain. In this way the distance which the water vapour actually traverses by diffusion turns out to be fraction of the distance of its displacement. Such a situation makes the diffusion of water vapour through snow easier than through open air, which causes  $D$  to appear greater than  $D_0$ .

**§ 4. The metamorphosis of snow crystals in an atmosphere of uniform temperature.** For the purpose of studying how the deposited snow crystals change their form in the interior of snow cover, microscopic observations were made in the cold room of the authors' Institute on snow crystals as they underwent change in form at temperatures below  $0^\circ\text{C}$ . In order to put crystals in the state as like as possible to that in the actual snow cover, they were placed on a silk net which was stretched in a small hole made through a block of snow. The angular outline of the dendritic snow crystal became round while at the same time its branches and twigs became thin at their roots to cut themselves off finally. The form of a cross section of the branches transformed to round from its initial rectangular form towards the time when the roots of branches became somewhat slender. In the final state the crystal separated into a number of short rods or spheroids of ice. The dendritic snow crystals carrying ice droplets on their surface changed in almost the same way as above. The ice droplets gradually fused into the branches of the crystal to convert them into thick rods.

Ice droplets on the surface of snow crystal are believed to be cloud particles which were caught by the crystal while it was falling through the cloud. They are usually round in form, but occasionally hexagonal ones can be found. It was shown by experiments that round ice droplets on snow crystals became hexagonal if they were put in an atmosphere super-saturated with water vapour with respect to ice at temperature below  $0^{\circ}\text{C}$ .

The graupel, an assemblage of a large number of ice droplets, came to look like an entangled string of beads. The string grew thicker and thicker and finally the whole of the graupel displayed a structure which much resembled that of the compact snow layer itself. An assemblage of many dendritic snow crystals also reached the same final state as the graupel did. This is in accord with the fact that a compact snow layer in the actual snow cover shows the same structure from whatsoever kinds of snow crystal it has come. But the time rate of change in the form of snow crystal observed in the above experiments seemed to be much smaller than that at which snow crystals change in the actual snow cover.

It was shown by observation with polarisation microscope that crystalline directions of snow crystal were not altered by the metamorphosis.

#### § 5. Some considerations on the metamorphosis of snow crystals.

(a) *Sublimations of snow crystal as the cause of metamorphosis.* Two effects are now being considered as the possible cause of metamorphosis of snow crystals which takes place below  $0^{\circ}\text{C}$ : sublimation of snow crystal and movement of molecules along the surface of the snow crystal. The authors succeeded in preserving a snow crystal for three and a half years by keeping it in an oil film in the cold room. The crystal preserved its outline unaltered, though minute patterns on its surface disappeared. Such a long time preservation of snow crystal must have been possible because of the prohibition of sublimation by oil. Therefore the rapid metamorphosis of snow crystal in air under ordinary conditions must be caused by the process of sublimation.

(b) *Collection of foreign matters contained in snow crystal caused by its sublimation.* Dust particles which chanced to lie on the surface of a snow crystal came to unite themselves as the crystal shrank by sublimation. A dendritic snow crystal appeared stained when there was some advance in its metamorphosis. The crystal must have dissolved some foreign substance which became distributed on the crystal surface as it was being concentrated by sublimation. The roots of branches of the crystal became thin and finally cut off. The gap between the parts was bridged by a very thin string which remained for many hours. This string must have been the foreign substance left behind after the ice had evaporated.

(c) *Curvature of crystal surface as the cause of metamorphosis by sublimation.* A slight alteration made initially in the radius of a circular ice cylinder should develop into a large one owing to the variation of vapour pressure of ice due to

the change in curvature of its surface. Mathematical treatment was applied to the case when a periodic alteration was imposed on the radius of ice cylinder initially and the result of such calculation was applied to the case of thinning roots of branches of a dendritic snow crystal. That calculation gave a rate of change in radius much smaller than that observed experimentally on the actual snow crystal.

(d) *Elastic stress as the cause of metamorphosis of snow crystal.* Any stress produced in ice should raise its vapour pressure. Therefore a slight change imposed on the radius of an ice cylinder subject to a bending moment will start a subsequent change in radius because a stress is developed which is distributed variably along the cylinder due to the variation in its radius. The branches of snow crystal are being bent by the presence of their own weight and the bending moment is greatest at their roots. The rate of thinning of the roots of branches calculated mathematically turned out to be so small that it could be considered to be entirely uneffective.

The results described in this and the previous articles seem to suggest that the metamorphosis of snow crystal cannot be explained by any phenomenological conceptions. It is likely to be due to defects or faults in the internal structure of ice crystal which are produced in the course of growth of snow crystal.

(e) *Effect of elastic stress upon the metamorphosis of ice grains composing snow cover.* Although the stress is uneffective on the metamorphosis of snow crystal in such a case of small bending moment as described in the previous article, a great effect can be expected when that bending moment takes a large value. It was shown by mathematical calculations that ice grains composing a mass of snow subject to such an intense stress as just below the break-down stress, can be metamorphosed at a rate much greater than that observed on snow crystals in unstressed state.

(f) *Observation made on the root of branch of a snow crystal when it is being cut off.* The root of a branch of a snow crystal was observed under a microscope while it was being cut off by sublimation. The root became thinner and thinner and finally it was reduced to a very thin string which endured for a few hours. This string is supposed to be of the same sort as that described in article (b) of this section.

Two ice grains were put in contact at a point. In one case the contact point fused to make up complete adhesion, while in the other case a gap appeared there. The force required to separate two contacted ice spheres was measured experimentally. There happened such cases in which no force was required for separation which is in accord with the above noted observations.

**§6. The metamorphosis of snow crystals under a temperature gradient.** In order to verify the hand to hand delivery of water vapour by the ice grains com-

posing snow cover supposed in sections 1, 2 and 3, the following described experiments were performed in the laboratory cold room. A small hole was made in a sheet of snow in which a temperature gradient was made artificially. On the silk net stretched in the hole were placed many snow crystals or ice grains. In this way they were put under the action of the temperature gradient prevailing in the snow sheet. Hoar crystals developed on the side facing toward the heat flow of each snow crystal or ice grain while the opposite side withdrew in the case of an intense temperature gradient. In the case of weak temperature gradient the surface of snow crystals or ice grains on the side facing the heat flow became crystalline while the surface on the opposite side withdrew though in a degree smaller than in the former case. At any rate there could always be seen indications which showed condensation of water vapour on the side of snow crystals or ice grains facing the heat flow and evaporation of water vapour on the opposite side. These results can be regarded as evidences for the above described hand to hand delivery of water vapour being realised in the snow cover.

In the uppermost about 20 cm layer of the snow cover the direction of temperature gradient alternates within a day. It was supposed that in such a case the snow crystals would be metamorphosed in almost the same way as in the case of no temperature gradient but at a much greater rate than in the latter case. This was expected because the temperature gradient promotes the metamorphosis by accelerating the movement of water vapour, while the alternation of its direction destroys in the evaporation period the crystalline form developed on one side of the snow crystal in the condensation period.

**§ 7. The temperature gradient in the actual snow cover.** Temperatures in the snow cover observed at various depths showed that the temperature gradient in it lay generally in the range from about  $0.01^{\circ}\text{C}/\text{cm}$  to  $0.1^{\circ}\text{C}/\text{cm}$  or more in its middle and lower parts. In the uppermost layer of the snow cover where the direction of temperature gradient alternated, its magnitude could amount to a value as large as  $1^{\circ}\text{C}/\text{cm}$ . Therefore the temperature gradients of such intensities as used in the experiments described in the previous section were realised in the actual snow cover.

Such a dendritic snow crystal with branches carrying twigs only on one side of them as was produced experimentally by the artificial temperature gradient was found in the uppermost layer of the snow cover. Compact snow which composed the main part of the snow cover consisted of ice grains with more or less crystalline edges or planes. This finding is in accord with the experimental one that ice grains came to show polyhedral appearance when put under the influence of a temperature gradient of such an intensity as found in the main part of the snow cover.

The temperature gradient is regarded more satisfactorily as the cause of diffusion of water vapour through snow cover than the difference of temperature existing between separate two points, because the diffusion is caused by the movement of water vapour from one ice grain to the next one at a rate determined by the

temperature gradient existent at that spot.

§ 8. **Contribution of water vapour to the heat transfer in snow cover.** The water vapour evaporating from one ice grain and condensing on the next one carries with it a quantity of heat. Therefore the air between ice grains composing snow cover can be regarded as conveying this quantity of heat in addition to the heat flow due to thermal conduction. It was shown that this situation lead to the finding that the heat conductivity of air appeared to be double that proper to it.

The effect of the apparent increase of heat conductivity of air on the heat conductivity of snow was estimated by means of the theory which Otto WIENER had developed concerning the problem of refractive index of mixture. It was shown that heat conductivity of snow of density  $0.1 \text{ gr/cm}^3$  would be diminished by 37% if the movement of water vapour in its interior were stopped.

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"T. K." in the following is the abbreviation for "Teion-Kagaku" (Low Temperature Science), a scientific publication written in Japanese, issued by the Institute of Low Temperature Science, Hokkaido University, Sapporo, Japan.

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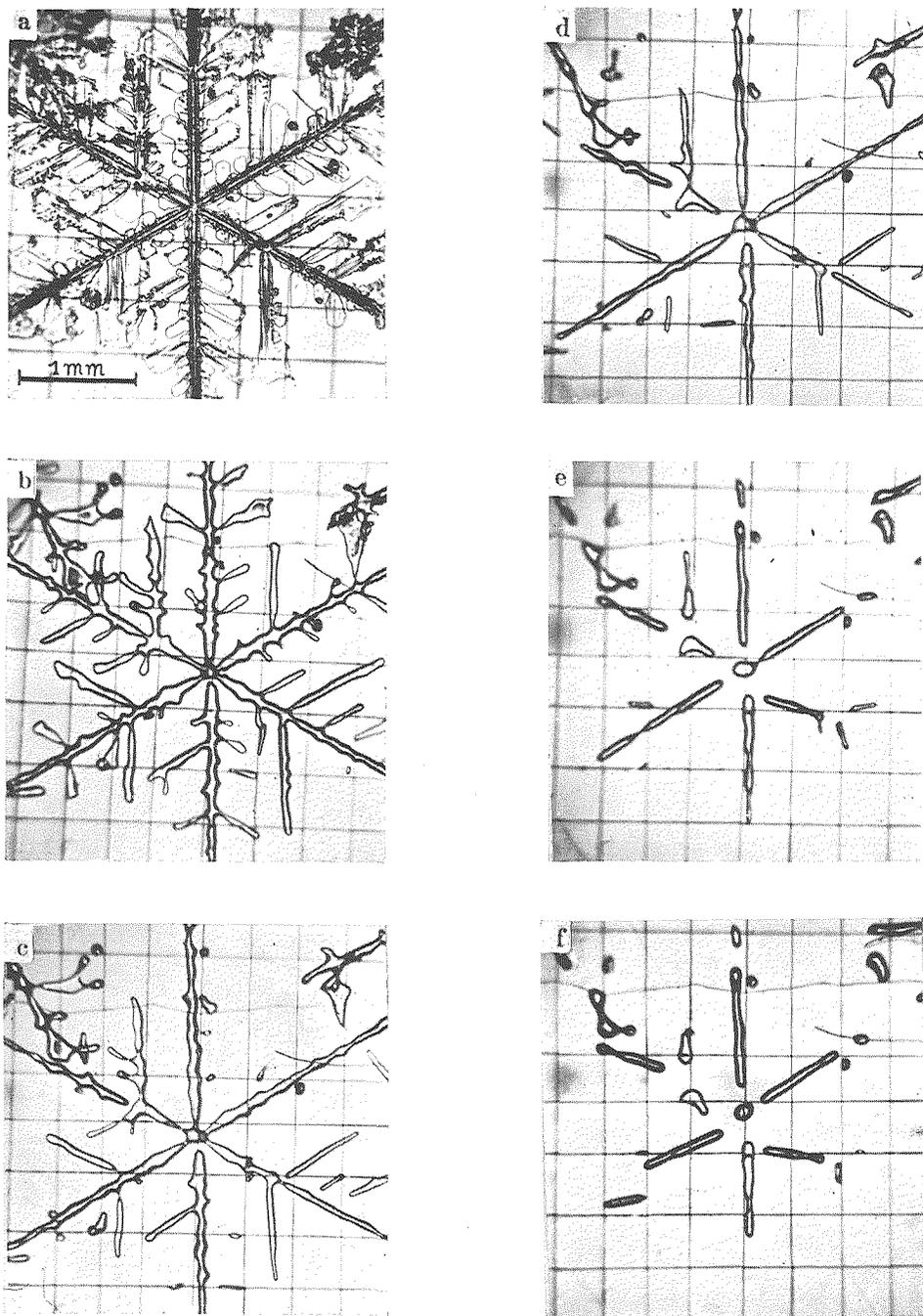


Fig. 8 Metamorphosis of a dendritic snow crystal in an atmosphere of uniform temperature  $-7^{\circ}\text{C}$ . (a) original crystal, (b) after 2 days, (c) after 10 days, (d) after 15 days, (e) after 31 days, (f) after 36 days.

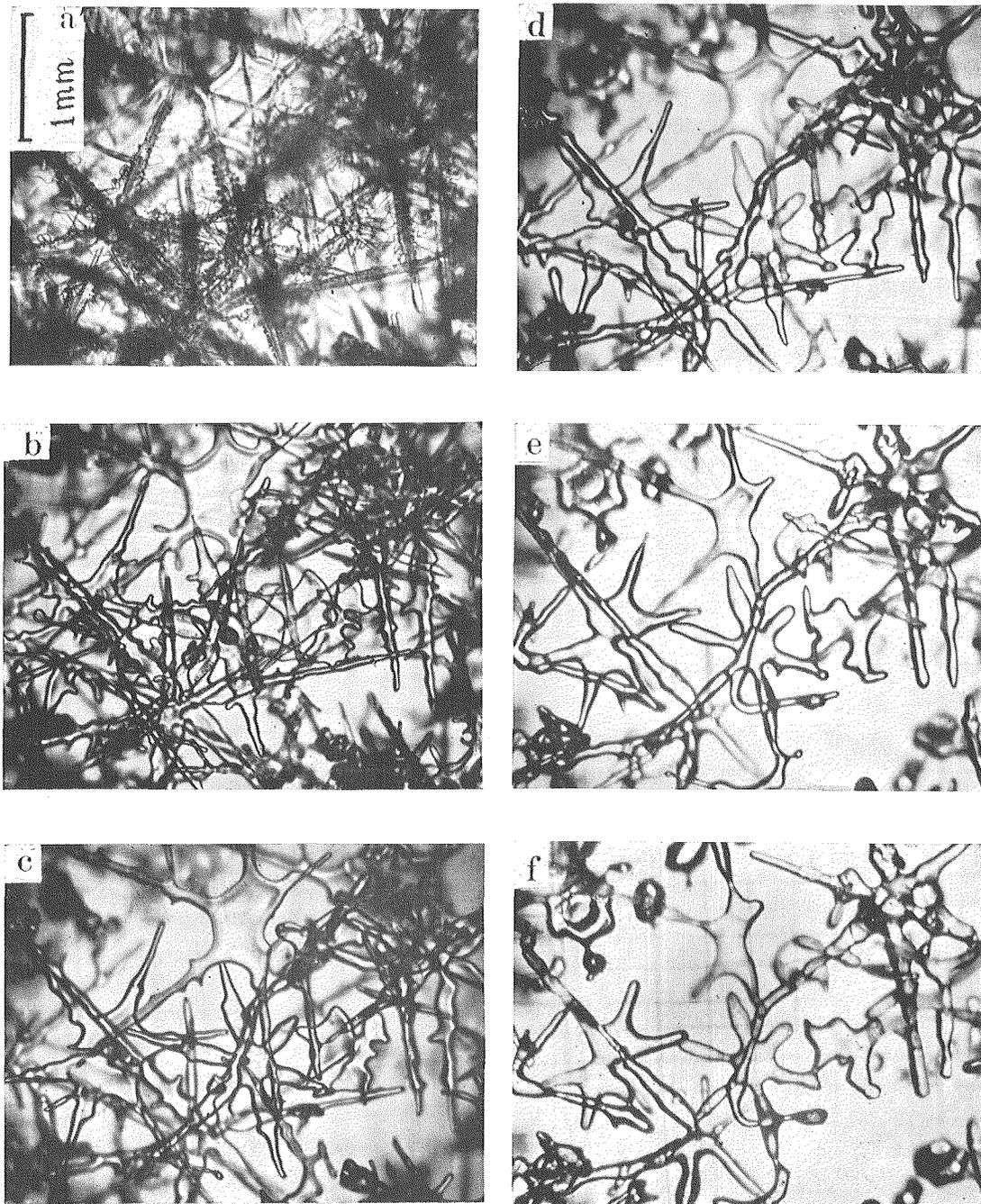


Fig. 19 Metamorphosis of an assemblage of dendritic crystals in an atmosphere of uniform temperature  $-15^{\circ}\text{C}\sim-20^{\circ}\text{C}$ . (a) after 1 hour, (b) after 4 days, (c) after 10 days, (d) after 15 days, (e) after 30 days, (f) after 51 days.

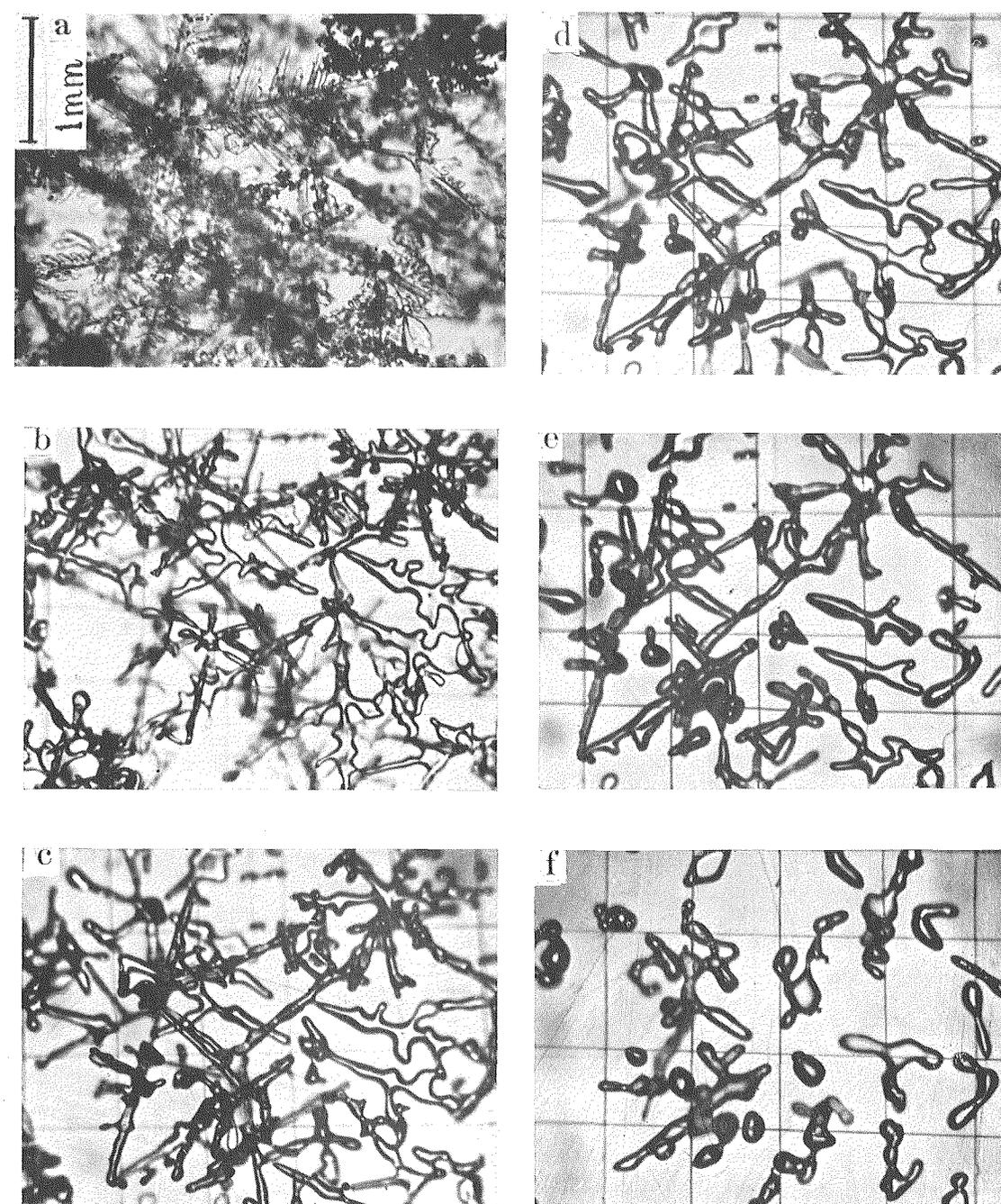
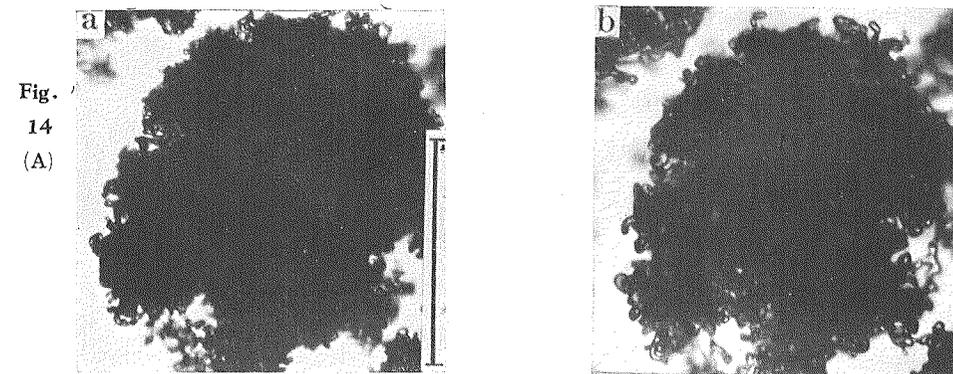
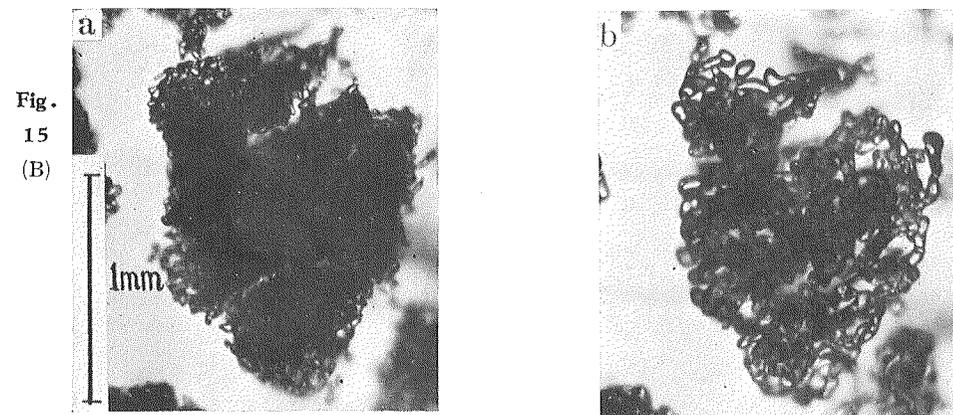
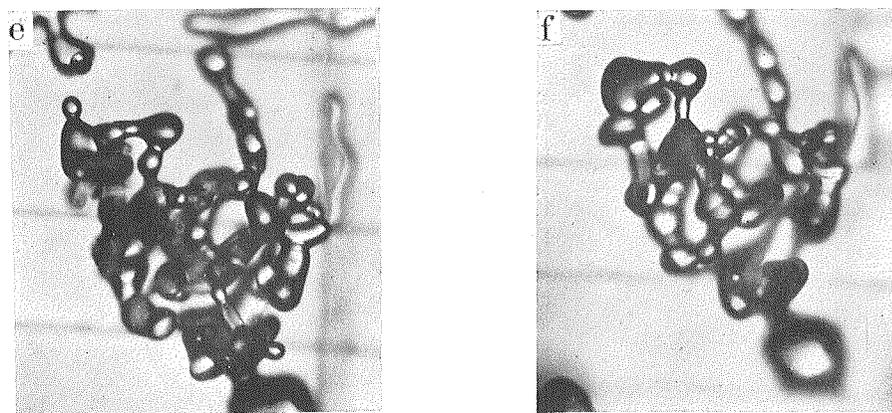


Fig. 20 Metamorphosis of an assemblage of dendritic snow crystals in an atmosphere of uniform temperature  $-6^{\circ}\text{C}\sim-7^{\circ}\text{C}$ . Time of photographs is the same as in Fig. 19.

Fig.  
14  
(A)Fig.  
15  
(B)

Metamorphosis of graupel in an atmosphere of uniform temperature.

Fig. 14  $-15^{\circ}\text{C}$ . (a) original crystal, (b) after 2 days, (c) after 5 days, (d) after 10 days.

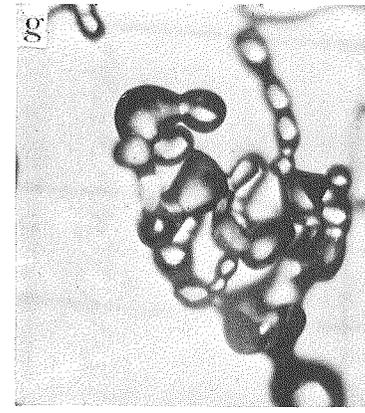
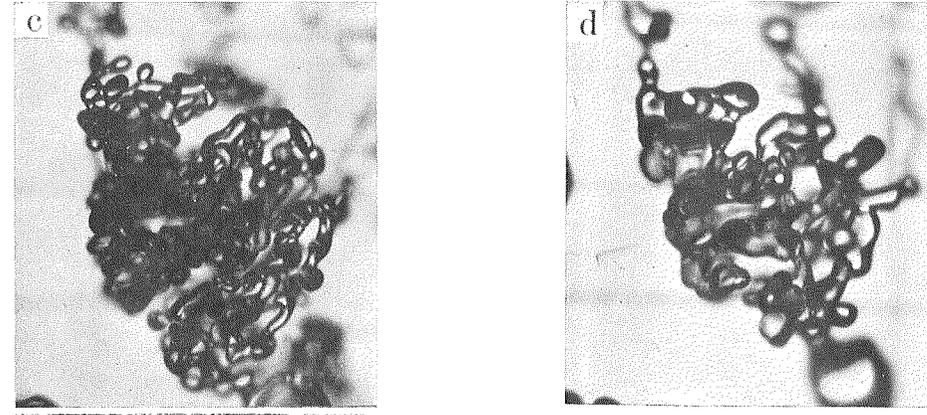
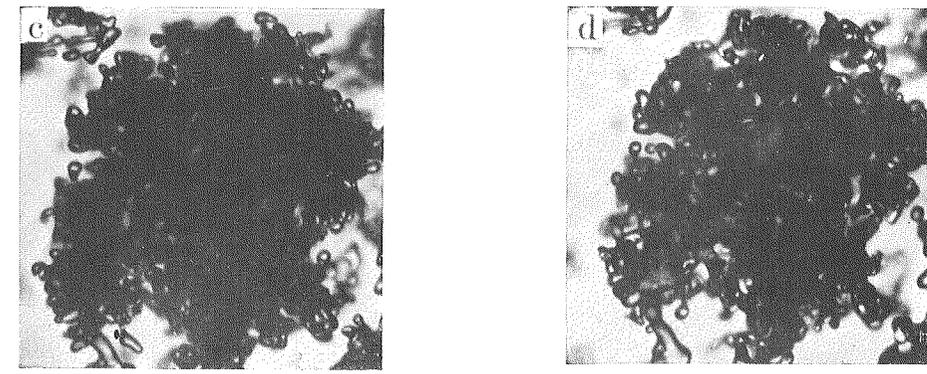
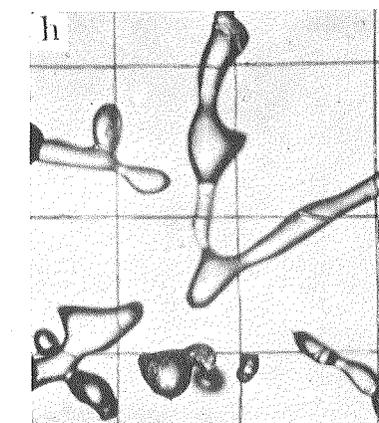
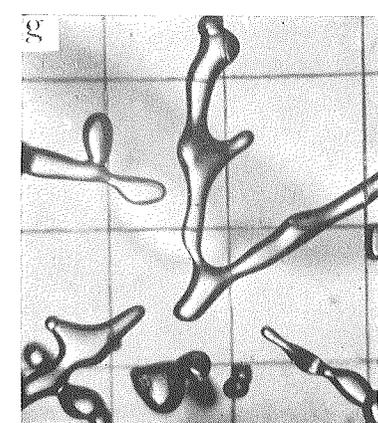
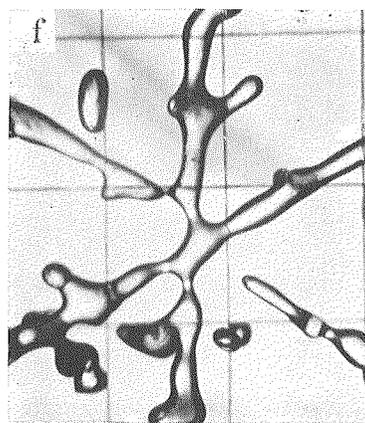
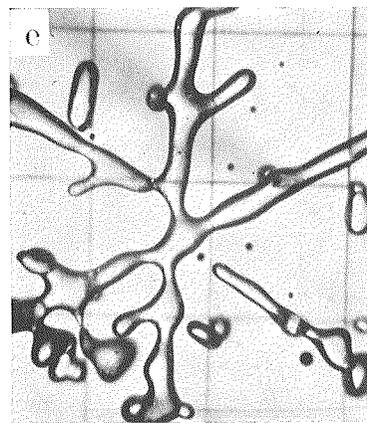
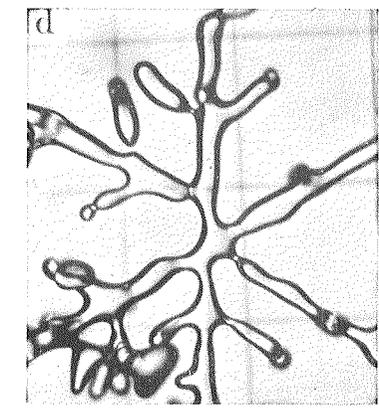
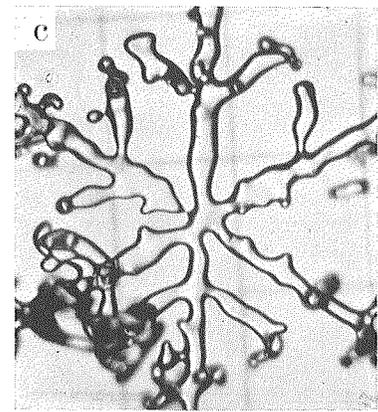
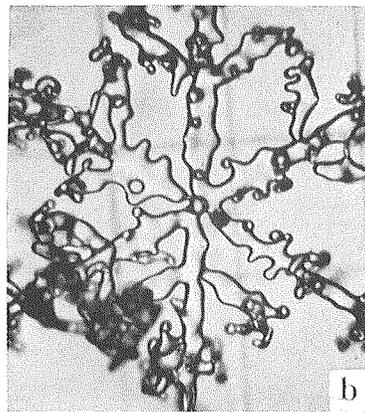
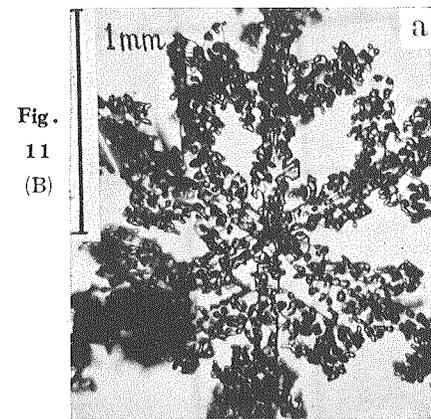
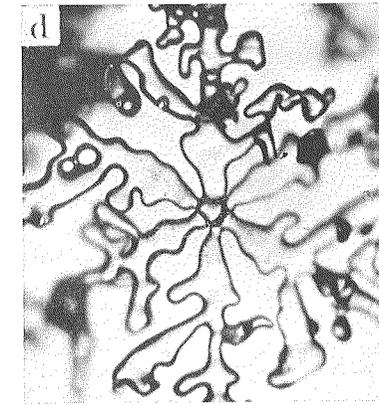
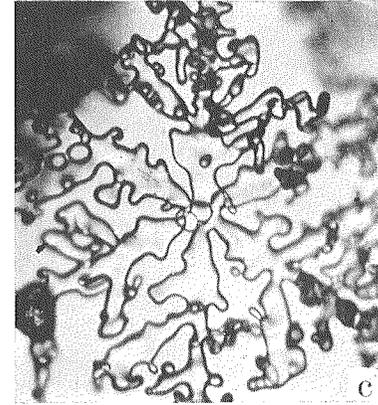
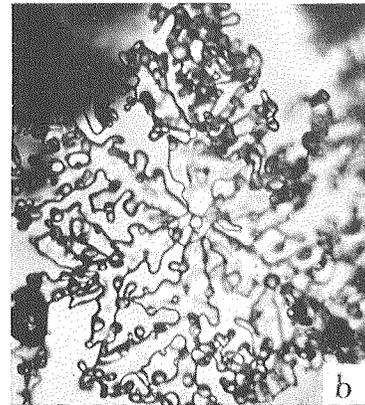
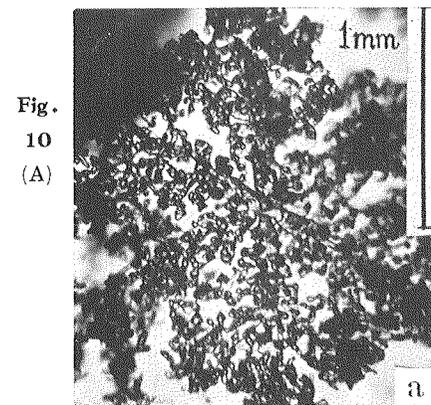


Fig. 15  $-6^{\circ}\text{C}$ , except the period from 7th day to 28th (corresponding to photographs from (d) to (f)) during which the temperature was kept at  $-0.1^{\circ}\text{C}$ . (a) original crystal, (b) after 2 days, (c) after 5 days, (d) after 10 days, (e) after 15 days, (f) after 28 days, (g) after 49 days.



Metamorphosis in an atmosphere of uniform temperature of dendritic snow crystal with ice droplets frozen to it.

Fig. 10  $-15^{\circ}\text{C}$ . (a) original crystal, (b) after 2 days, (c) after 5 days, (d) after 10 days.

Fig. 11  $-6^{\circ}\text{C}$ , except the period from 7th day to 28th (corresponding to photographs from (d) to (g)) during which the temperature was kept at  $-0.1^{\circ}\text{C}$ . (a) original crystal, (b) after 2 days, (c) after 5 days, (d) after 10 days, (e) after 15 days, (f) after 21 days, (g) after 28 days, (h) after 49 days.

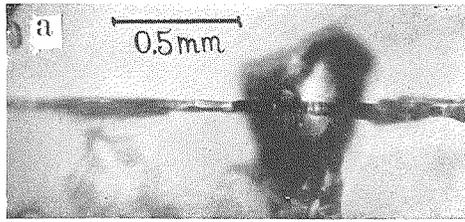


Fig. 9 Side view of a branch of dendritic snow crystal as it undergoes metamorphosis in an atmosphere of uniform temperature  $-15^{\circ}\text{C} \sim -20^{\circ}\text{C}$ . (a) original crystal, (b) after 18 days.

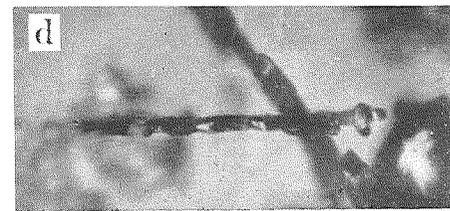
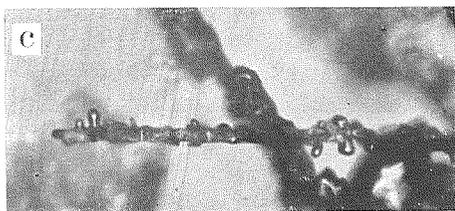
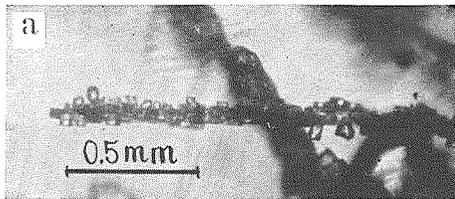
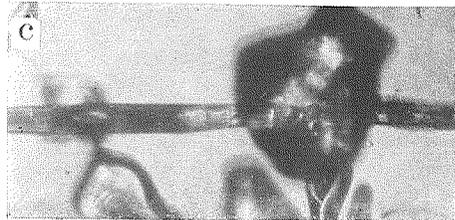


Fig. 12 Side view of a branch of dendritic hoar crystal onto which ice droplets were frozen artificially. They were hexagonal prisms initially (a). They changed their shape as shown by the sequence of photographs in an atmosphere of uniform temperature. (b) shows the state 5 hours after (a). (c) - (e) show subsequent changes.  $-20^{\circ}\text{C}$ .

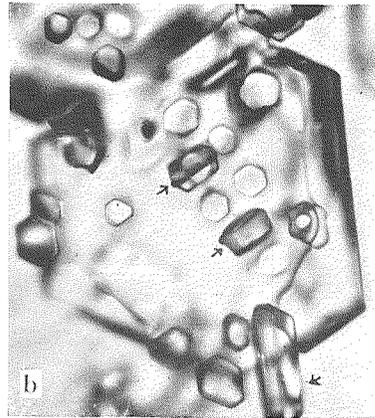
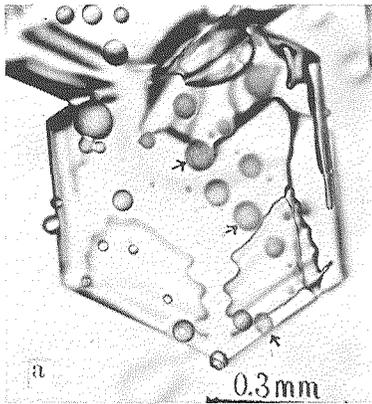


Fig. 13 (a) Water droplets frozen to a hoar crystal. (b) Water vapour evaporating from a snow mass weakly heated condensed on them to cause them to turn into hexagonal prisms.

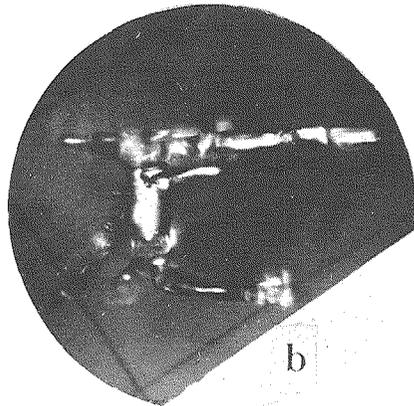
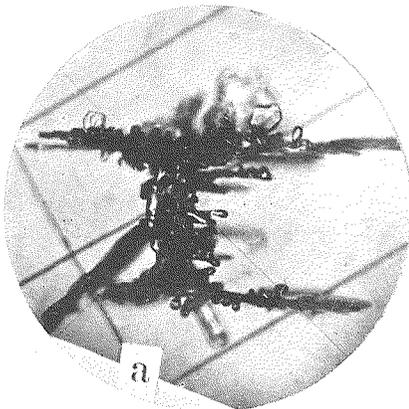


Fig. 18

Cryralline direction of a snow crystal is not altered by metamorphosis.

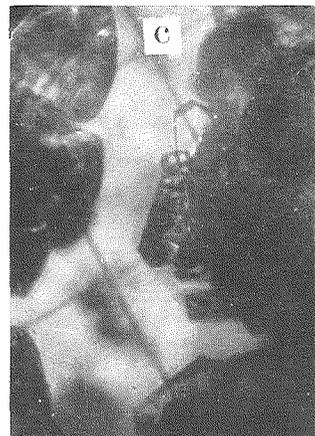
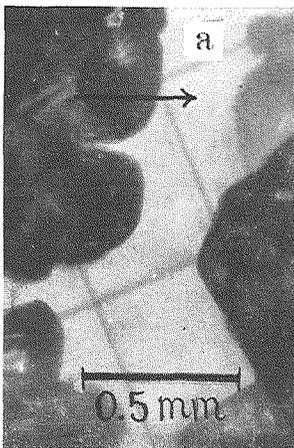


Fig. 34 The surface of ice grains on the left side of the photographs withdrew on account of evaporation of water vapour from it.

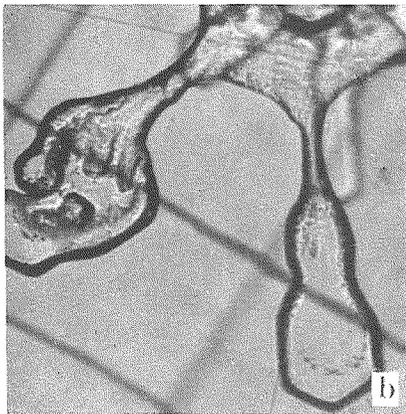
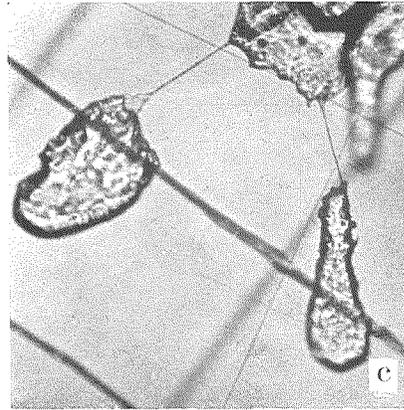
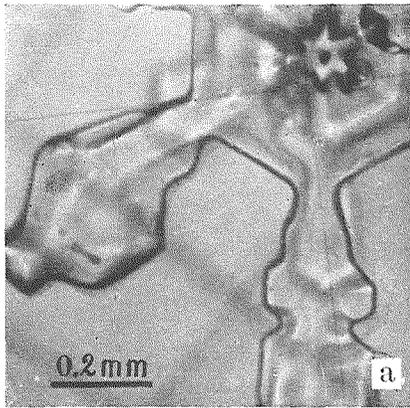


Fig. 23 A snow crystal (a) became stained as it was metamorphosed by sublimation (b). Finally the dissolved substance united to strings (c).

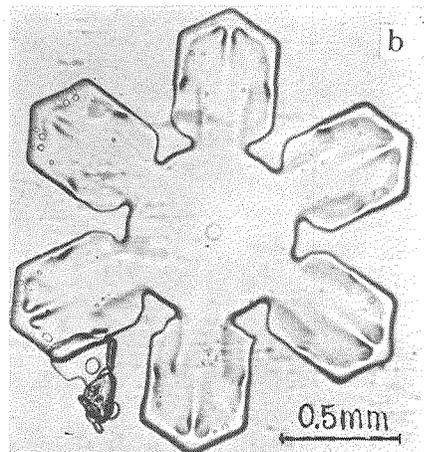
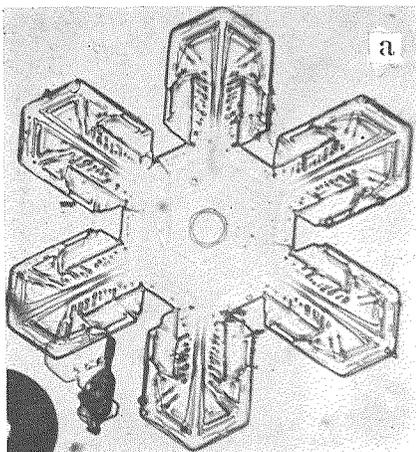
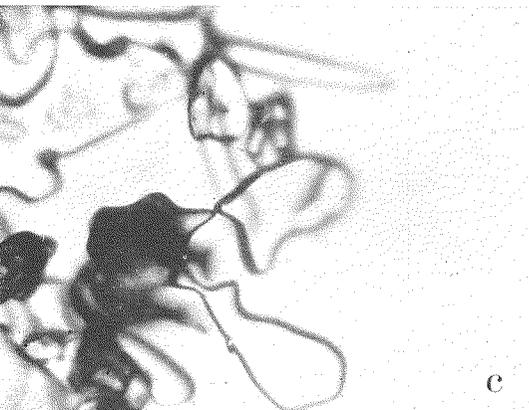
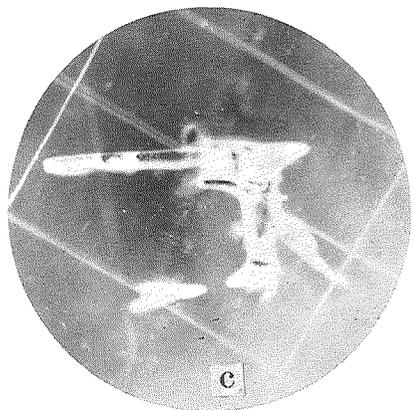
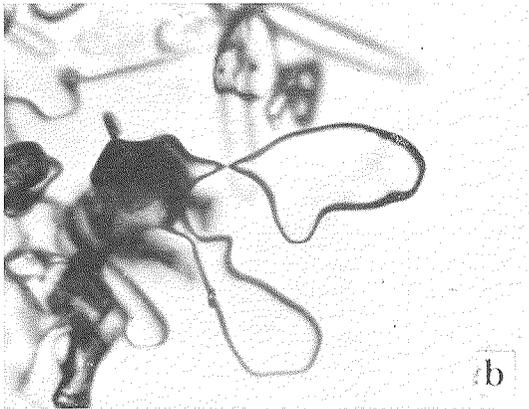
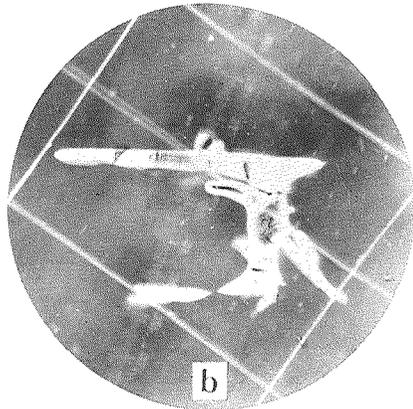
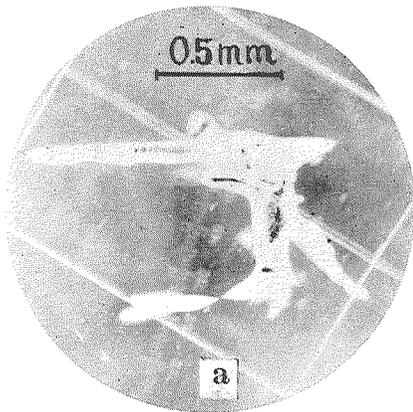


Fig. 21 A snow crystal (a) took the appearance of (b) after having been preserved in oil at temperature  $-25^{\circ}\text{C}$  for three and a half years. The change from (a) to (b) corresponds to 20 hour metamorphosis in air at the same temperature.



Thinning of the branches of snow crystal at their roots.

Fig. 24 (left) (b) 20 minutes after (a), (c) 16 hours after (b).

Fig. 25 (right) (b) 41 hours after (a), (c) 6 hours after (b).

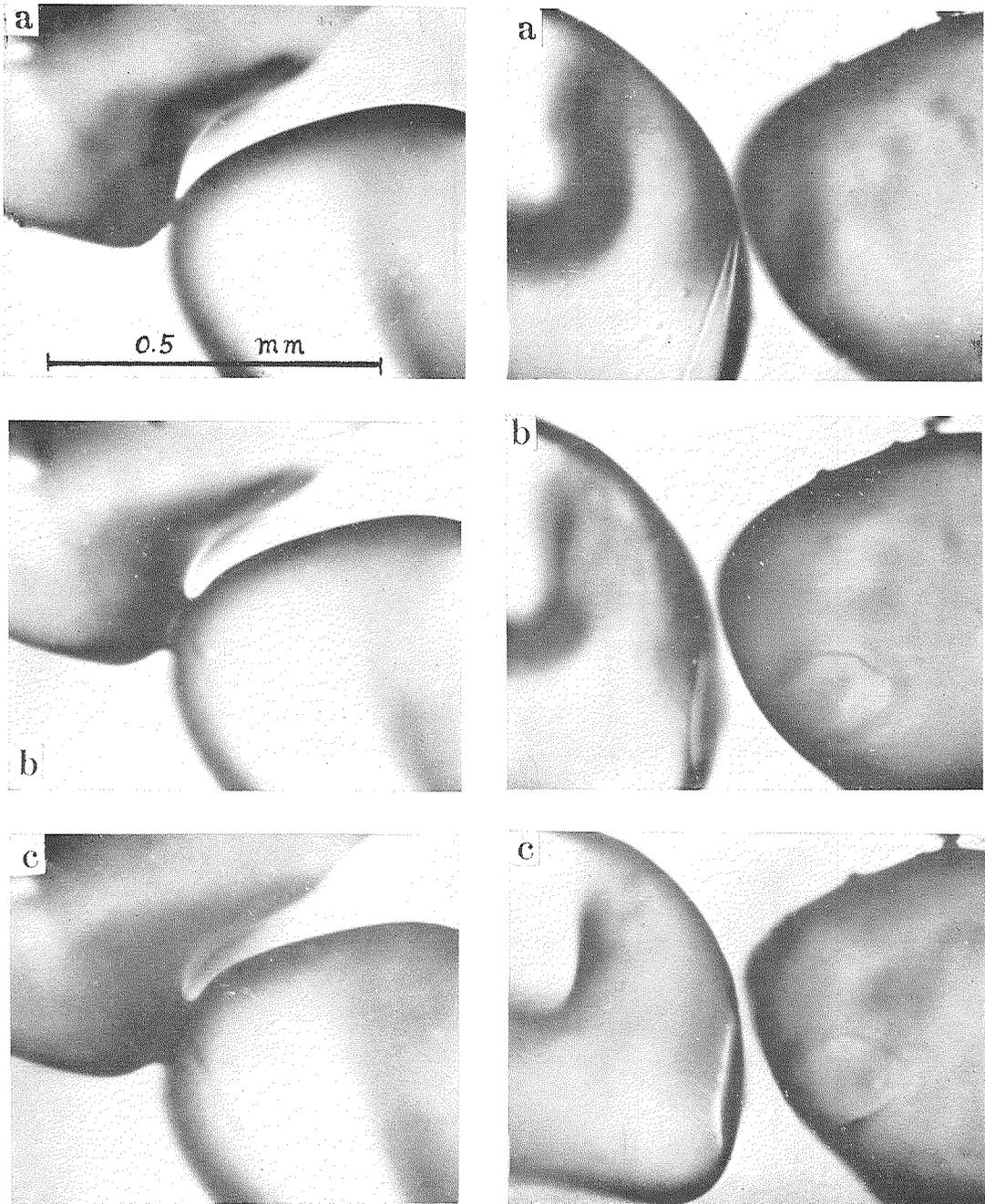
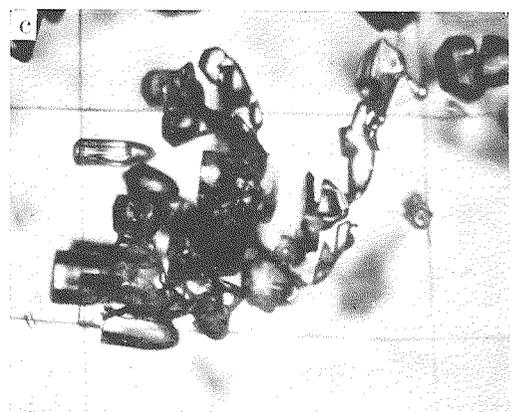
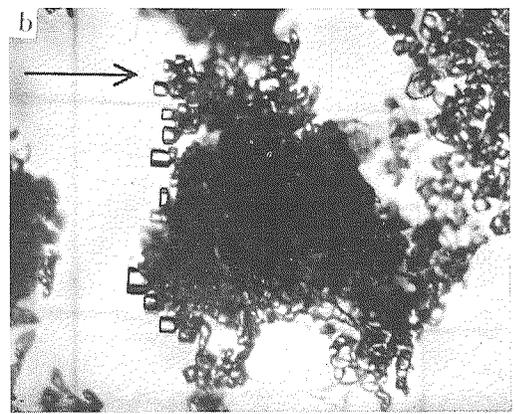
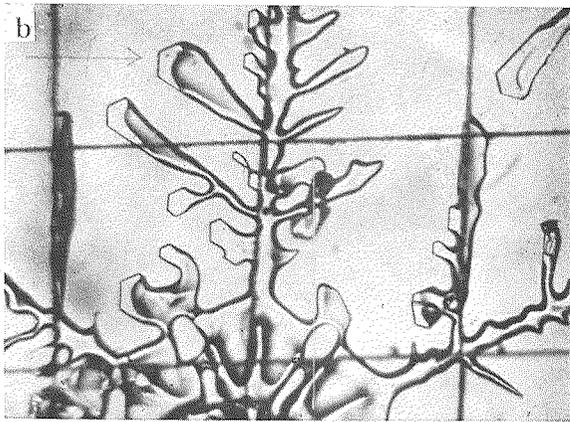
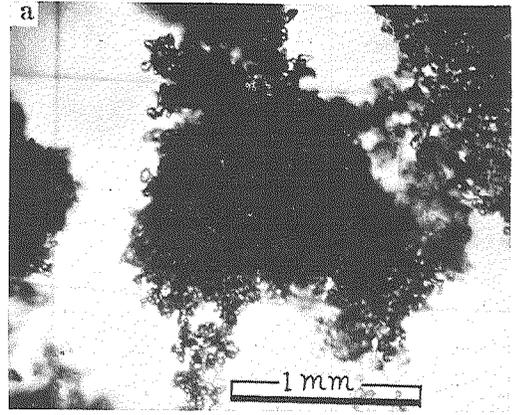
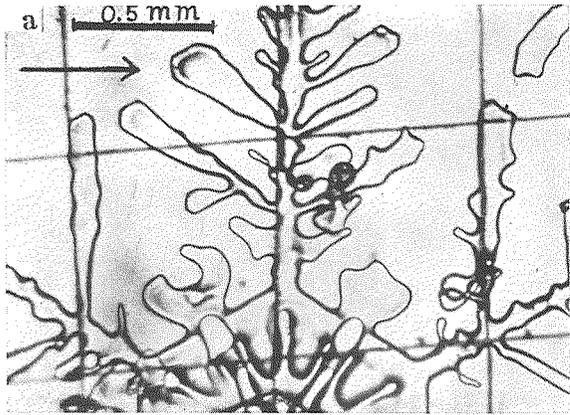


Fig. 26 and Fig. 27 Two ice grains touching at a point. In one case (Fig. 26. left) the contact point was fused while in the other (Fig. 27. right) it was separated apart. In both cases the temperature was kept at  $-13^{\circ}\text{C}$  for the first 22 hours and then it was raised to  $-6^{\circ}\text{C}$  to be kept at this value thereafter. (a) 1 hour after the contact, (b) after 45 hours, (c) after 69 hours in the case of Fig. 26 and after 15 days in the case of Fig. 27.



Metamorphosis of snow crystal subjected to a temperature gradient. The arrow shows the direction of heat flow caused by the temperature gradient.  
**Fig. 30** (left) Dendritic snow crystal;  $-15^{\circ}\text{C}$ ,  $0.3^{\circ}\text{C}/\text{cm}$ ; (a) original crystal, (b) after 24 hours, (c) after 48 hours.  
**Fig. 31** (right) Graupel;  $-15^{\circ}\text{C}$ ,  $0.4^{\circ}\text{C}/\text{cm}$ ; (a) original graupel, (b) after 34 hours, (c) after 10 days.

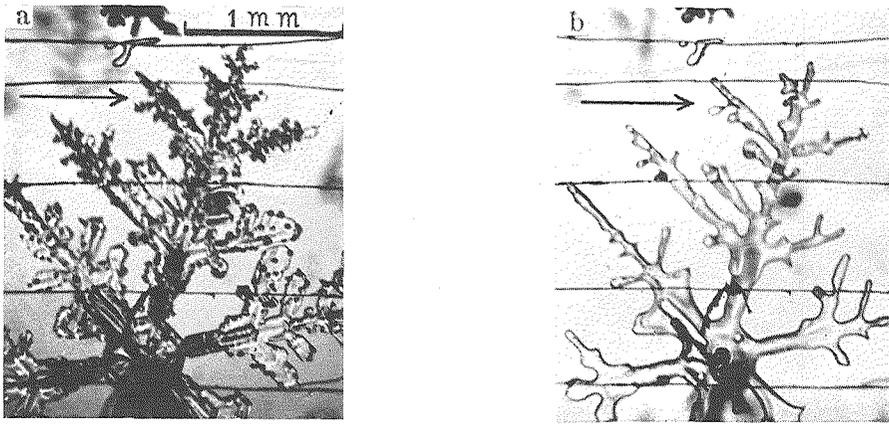


Fig. 32 Dendritic snow crystals subjected to a weak temperature gradient  $0.08^{\circ}\text{C}/\text{cm}$  at  $-15^{\circ}\text{C}$ ; (b) 4 days later than (a).

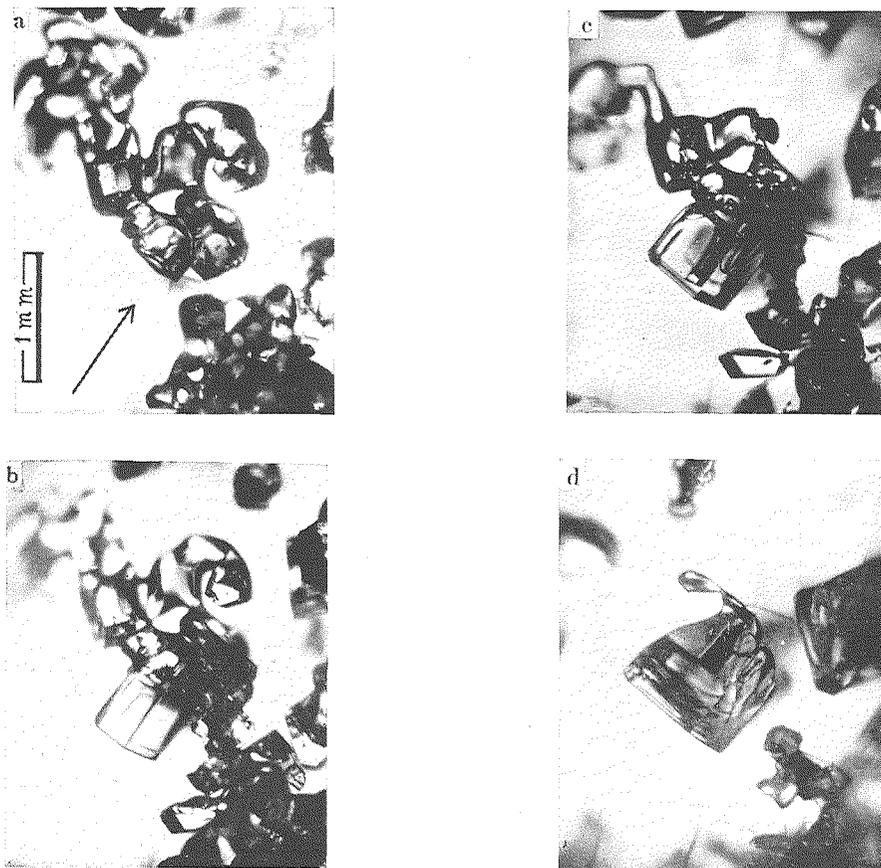
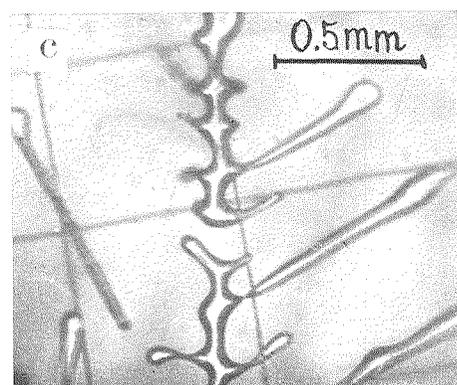
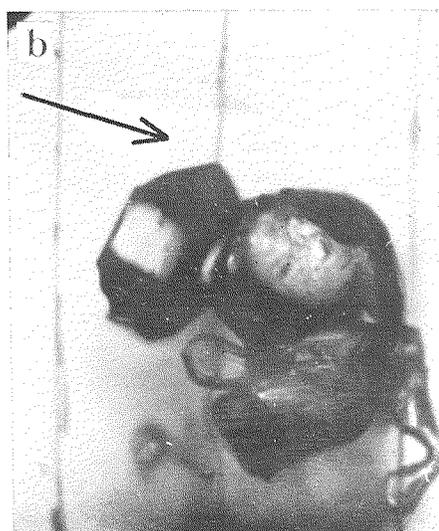
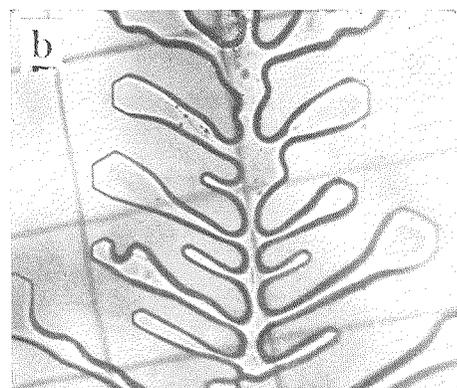
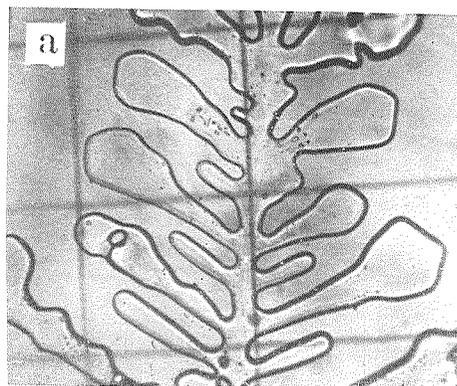
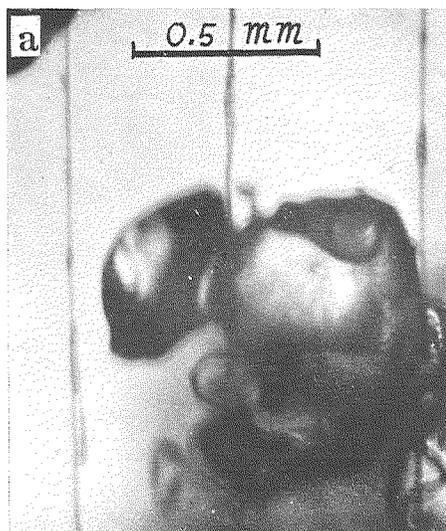


Fig. 33 Development of hoar crystals on the ice grains by an intense temperature gradient  $0.7^{\circ}\text{C}/\text{cm}$  at  $-15^{\circ}\text{C}$ . (a) after 3 hours, (b) after 3 days, (c) after 7 days, (d) after  $12\frac{1}{2}$  days.



**Fig. 35** (two photographs on the left) Round surface of the left part of ice grain altered to polyhedral one.  $-15^{\circ}\text{C}$ ,  $0.15^{\circ}\text{C}/\text{cm}$ , 57 hours.

**Fig. 36** (three photographs on the right) Metamorphosis of a dendritic snow crystal kept in a glass box. Accidental temperature gradient turned the outline of the snow crystal more crystalline at one stage of its metamorphosis as shown by photographs (a) and (b). (a) after 3 days, (b) after 4.5 days, (c) after 18 days.  $-15^{\circ}\text{C}\sim-20^{\circ}\text{C}$ .

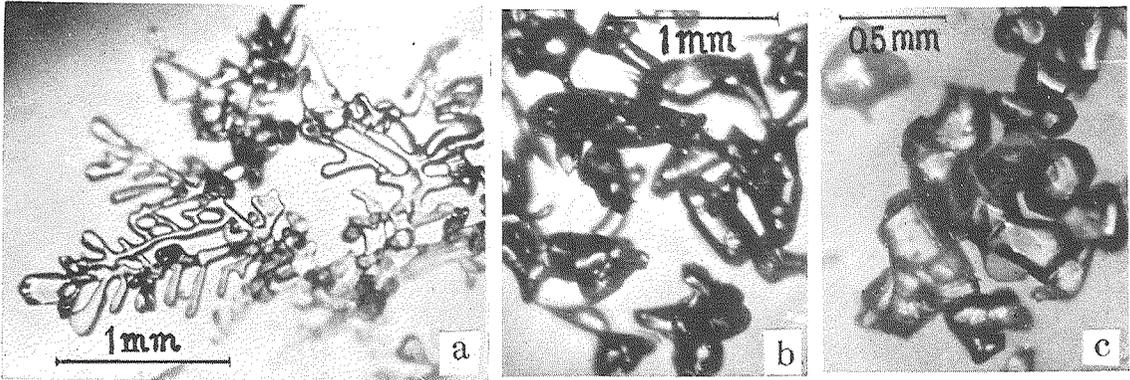


Fig. 38 Series (A). Ice grains in a snow layer deposited as an assembly of dendritic snow crystals. (a) 2 days after deposition. Twigs are found only on one side of the branches of crystal. (b) 18 days " , (c) 28 days " .

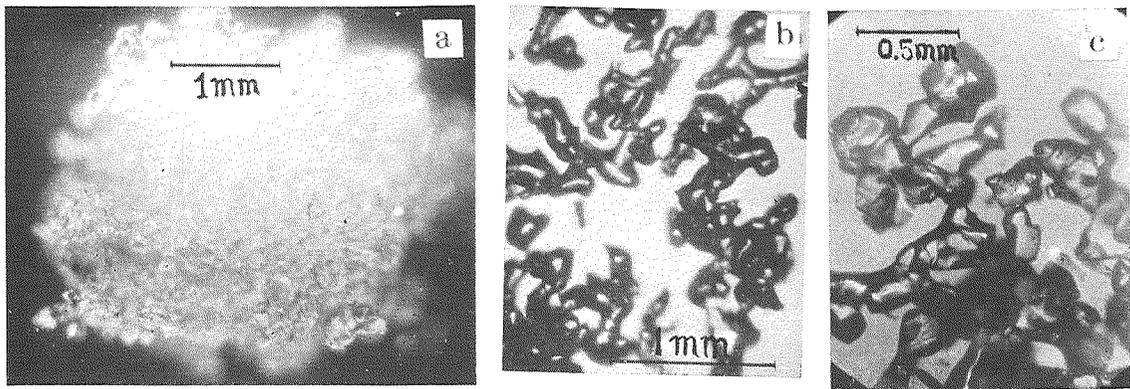


Fig. 39 Series (B). Ice grains in a snow layer deposited as an assembly of graupels. (a) 5 days after deposition, (b) 13 days " , (c) 23 days " .

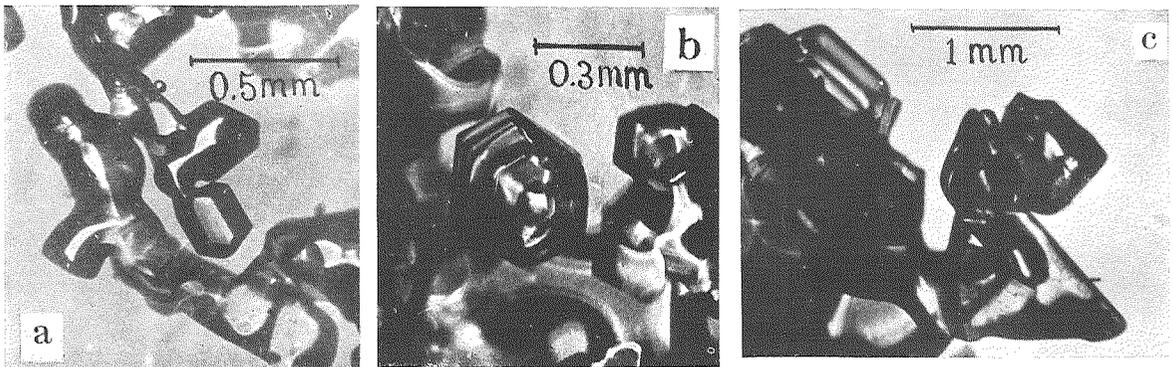


Fig. 40 (a), (b): Ice grains with crystalline appearance found in the upper layer of snow cover. (c): Depth hoars found in a snow layer 50 cm above the ground level.

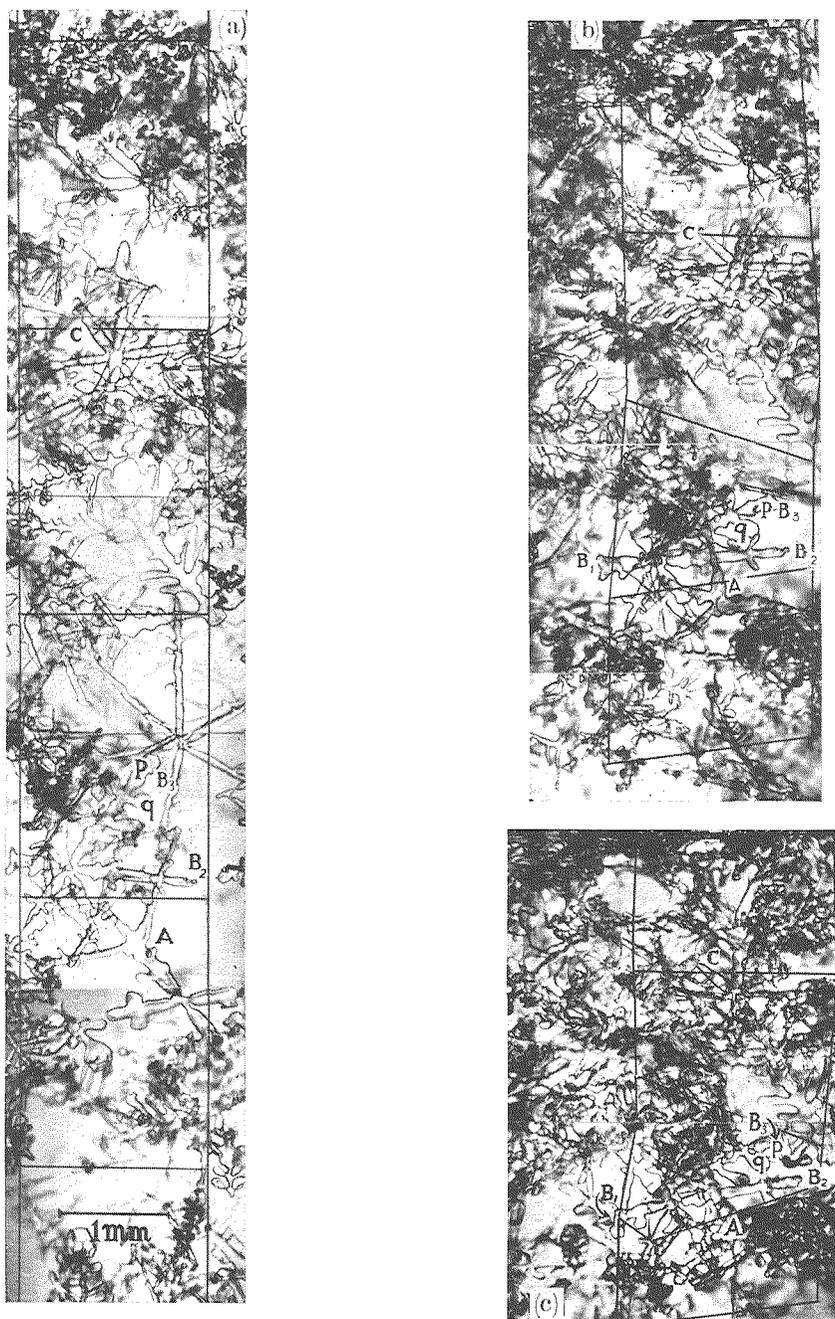


Fig. 41 An assembly of dendritic snow crystals subjected to compressive force.  $-5^{\circ}\text{C}$ . (b) 6.5 hours after (a), (c) 22.5 hours after (b).