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Air Permeability of Deposited Snow*

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Abstract

A precise air permeameter with a double-vessel sample holder of the Zwikker-Kosten type, a portable bridge-type air permeameter and a kerosene permeameter were devised for measuring the fluid permeability B of natural deposited snow. The filter velocity of fluid in the measurements, i.e., the volumetric flow rate through unit area perpendicular to the mean flow line, was up to 1.0 cm/s for either air or kerosene. The experiments conducted in this range of filter velocity confirmed the validity of Darcy's law. The values of the specific permeability B_0 , namely the fluid permeability multiplied by the viscosity of fluid that were obtained with the air permeameters agreed fairly well to those values obtained with the kerosene permeameter. Accordingly the flow of fluid subjected to the measurements could be looked upon as macroscopically laminar and viscous. Measured values of the air permeability of natural deposited snow dispersed from 3×10^{-2} to 50×10^{-2} cm \cdot s $^{-1}$ /dyne \cdot cm $^{-3}$, whereby they showed that air permeability does not clearly depend on snow density, which ranged from 0.06 to 0.51 g/cm 3 . It was shown, however, that the specific permeability B_0 of fine-grained compact snow could be satisfactorily represented by a simple formula,

$$B_0 = 0.077 d_0^2 \cdot \exp[-7.8\rho_s^*],$$

where d_0 is the "mean grain size" in cm and ρ_s^* the specific gravity of snow. This formula is reasonable, inasmuch as the dimension of B_0 is the square of length. The formula gave a qualitative interpretation in terms of grain size and density of snow as to changes in the air permeability of a particular snow layer in the course of its metamorphosis.

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I. Introduction

Deposited snow is a porous material of ice, and changes its structure—grains, ice-bonds and pores—with the lapse of time due to metamorphosis undergone during its existence. Fluid permeability is one of the most important characteristics of porous materials. Hence, the fluid permeability of deposited snow may serve as a good measure to describe its important characteristics in terms of geometrical structure, corresponding to stages of metamorphosis. Moreover, the knowledge of the fluid permeability of deposited

snow is a prerequisite to studies on a number of processes of practical concerns, in which fluid flow through deposited snow plays an essential role, for instance, in the propagation of acoustic waves over or in it, the movement of melt water therethrough, the survival of living creatures underneath, and so on.

Notwithstanding its importance, studies on the fluid permeability of deposited snow have been rather limited. One of the first systematic studies might be attributed to Bader (1939). Having measured the air permeability of a sample of artificially compressed snow at various stages of compression, he found that the air permeability K could be expressed by

$$K = anN/(N-n), \quad (\text{I. 1})$$

where n was the porosity of the sample, and a and N were constants depending on the sample used. He suggested that the constant a might be expressed by a function of grain size and grain form, but failed to try to find it out. Later Bender (1957) carried out a series of experiments in an effort to determine the dependency of K on the grain size d . His experimental finding was that K was proportional to $d^{1.63}$ for snow. His result is, however, inconsistent with dimensional analysis, since the dimension of K is the square of length. It appears that he did not take notice of this inconsistency, perhaps because he, as well as Bader, used a practical unit for pressure (cm-water equivalent), which obscured the true dimension of permeability. Recently Kuroiwa (1968) measured the kerosene permeability of natural deposited snow, but he did not try to find out the dependency of permeability on grain size.

In the above mentioned work of Bader (1939) he gave also the values of the air permeability of various kinds of natural deposited snow. However, it is suspected that these values are considerably higher than true ones, especially for soft and fragile snow such as loose new snow, depth hoar and coarse-grained snow. The reason may be a serious defect inherent to the sample holder used in his permeameter. His sample holder, a single circular tube, would not avoid destroying the part of the sample contiguous to the wall of the holder at the time it was penetrated into snow, giving rise to the leakage of air along the wall.

Intending to make precise measurement of the air permeability of deposited snow, the present author devised an air permeameter with a special sample holder. He has measured the air permeability of various kinds of natural deposited snow with an aim to seek experimentally a reasonable relation among air permeability, grain size and porosity. This paper presents the results, some of which have been published in Japanese (Ishida and Shimizu

1955, 1956: Shimizu 1956, 1960).

II. Darcy's law and permeameters

II. 1. Darcy's law

Deposited snow contains such a great number of randomly dispersed voids that its structure can be described only in statistical terms. Nevertheless, it is possible to treat the flow of a fluid through deposited snow on a macroscopic basis. The macroscopic law governing the flow of a fluid through a porous material such as deposited snow is Darcy's law. It states that the rate of flow is directly proportional to the pressure gradient causing the flow. If a volume Q of a fluid flows in t seconds through a columnar sample of deposited snow, which has parallel ends, cross sectional area A and length L , by the pressure difference Δp between both the ends, Darcy's law states that

$$u = B\Delta p/L, \quad (\text{II. 1})$$

where B is a constant and u is defined by

$$u = Q/(At). \quad (\text{II. 2})$$

The quantity u is called filter velocity and the constant B fluid permeability. The dimension in the c.g.s. system of the former is $[\text{cm}\cdot\text{s}^{-1}]$, and that of the latter $[\text{cm}^3\cdot\text{g}^{-1}\cdot\text{s}]$. The fluid permeability B gives the filter velocity, u cm/s, of the fluid through the porous medium caused by a unit pressure gradient, 1 dyne/cm³.

If (i) the resistance of porous medium, deposited snow, to the fluid flow is entirely due to viscous drag, and (ii) the fluid, air or kerosene, is inert to a porous material, ice, i.e., chemical, adsorptive, electrical, and capillary effects are absent, we may consider that the flow is purely viscous and the Darcy's law holds. In this case, we can add that filter velocity is inversely proportional to viscosity and get

$$u = (B_0/\eta)(\Delta p/L), \quad (\text{II. 3})$$

where η is the viscosity and B_0 a new constant depending only upon the porous material. The constant B_0 is called specific permeability and has the dimension of $[\text{cm}^2]$ in the c.g.s. system. B_0 corresponds to the filter velocity of a fluid, whose viscosity is 1 poise, caused by pressure gradient, 1 dyne/cm³. In other words, specific permeability is unique to a porous medium and is independent of the kind of fluid flowing through it. Comparing the relation (II. 3) with the relation (II. 1), we get

$$B_0 = B\eta. \quad (\text{II. 4})$$

Although the above treatments are for the flow of incompressible fluid, they may approximately be applied to the flow of compressible fluid such as air, if the pressure difference Δp is sufficiently small, when compared with the pressure at the exit.

II. 2. A precise air permeameter with a double-vessel type sample holder

As stated in the introduction, a single-tube sample holder has such a defect that a sample of soft or fragile snow would easily be destroyed along the wall of the tube and it results in the leakage of fluid flow thereat. To avoid such a defect, a precise air permeameter of snow was provided by use of a sample holder of the Zwickler-Kosten type (1949).

A schematic diagram of this permeameter is shown in Fig. 1. The sample holder consists of two co-axial cylindrical vessels, 20 cm and 10 cm

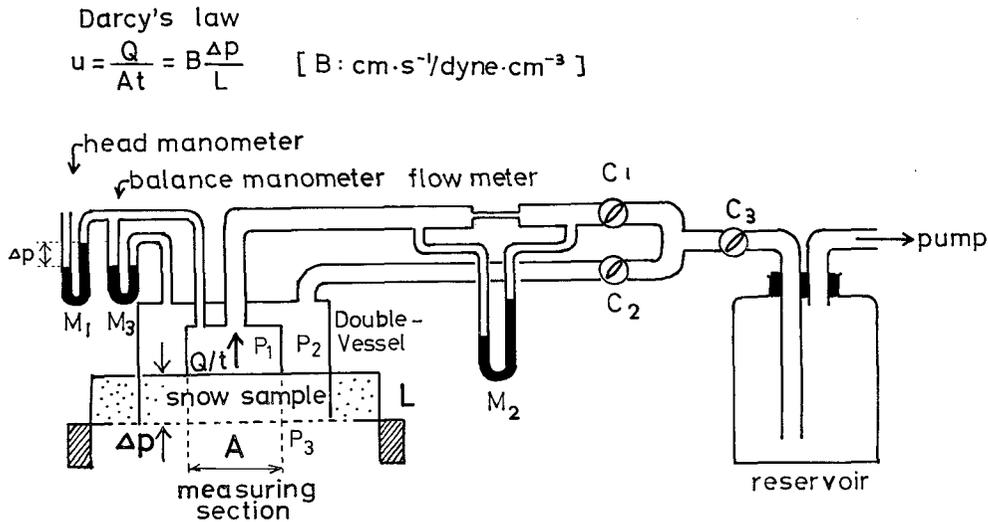


Fig. 1. Schematic diagram of the air permeameter with a double-vessel type sample holder

in diameter, respectively. The edge of the inner vessel is fixed at 5 cm up from the edge of the outer vessel. A block of snow is taken out from a snow layer in deposited snow with a box snow-sampler which has a metal-net bottom 30 cm \times 30 cm in size and a metal wall 5.5 cm in height. Then the snow sample, held in the sampler, is placed horizontally on a wooden stage and the sample holder is vertically penetrated into it down to the bottom of the sample by the aid of a guide tube. The edge of the inner vessel penetrates into the sample only 5 mm beneath the surface. Opening the stop-

cocks C_1 , C_2 and C_3 and pumping out air through the reservoir, we can make the air pressure in the inner vessel P_1 and that in the outer vessel P_2 lower than that at the bottom P_3 which is equal to the atmospheric pressure. The pressure difference between P_1 and P_3 is measured by a Göttingen-type alcohol manometer M_1 with the accuracy of 1/20 mm in height of alcohol and that between P_1 and P_2 by a sensitive alcohol manometer, U-type manometer with an inclined arm. The flow rate Q/t through the inner vessel can be read by a flow meter which consists of a capillary tube and a Göttingen-type alcohol manometer M_2 . When P_1 and P_2 are equalized by adjustments of

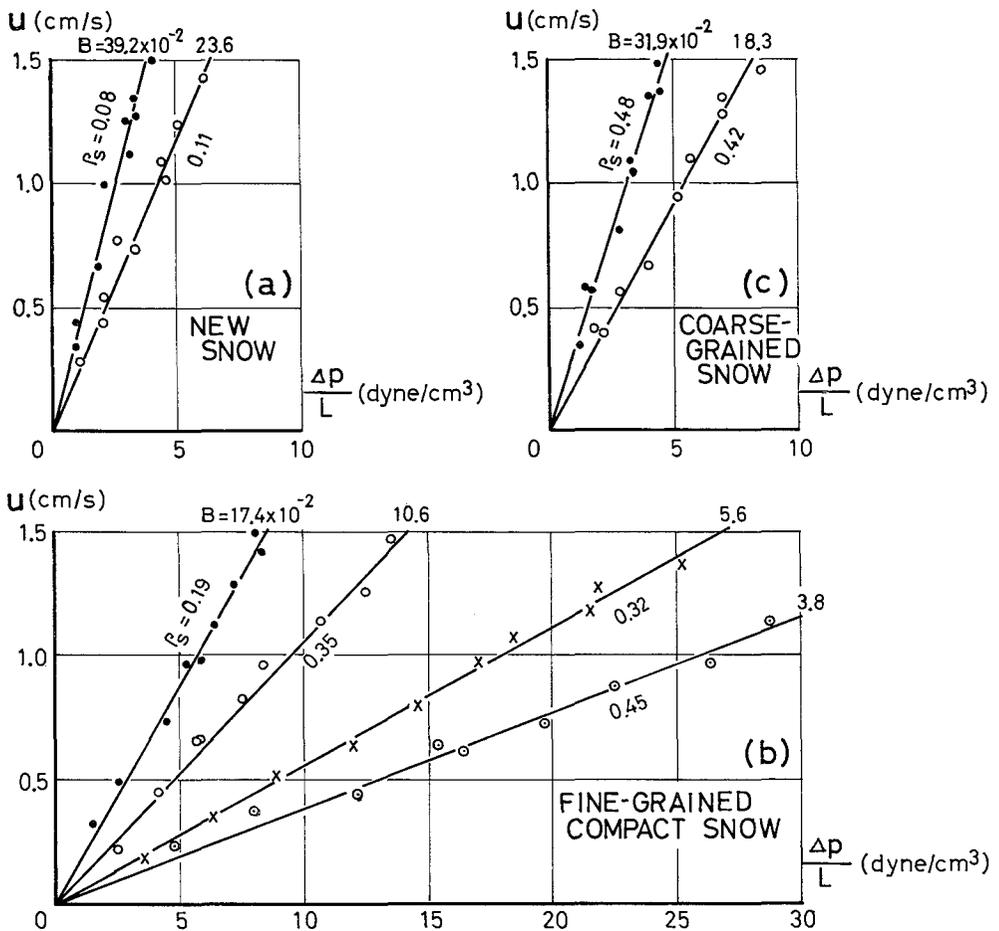


Fig. 2. Examples of air permeability measurement

(a) New snow (b) Fine-grained compact snow

(c) Coarse-grained snow

the cocks C_1 and C_2 , an air flow through the sample can be considered homogeneous except in the vicinity of the wall of the outer vessel. Under such a condition, the air flow through the inner vessel can be considered as the same as passes through an undestroyed snow sample which has the same cross-sectional area as the inner vessel. The pressure difference between P_1 and P_3 is mainly controlled by the cock C_3 in the range up to about 2 mm-alcohol or about 150 dyne/cm², which causes filter velocity in the range up to 1.5cm/s.

For the actual measurement of the air permeability of deposited snow, the pressure differences Δp in a number of several different magnitude were applied across the sample, in turn, and the corresponding filter velocities u were plotted against $\Delta p/L$. Examples of the measurements are shown in Fig. 2(a), (b) and (c). It is readily seen that Darcy's law (II. 1) holds well in the investigated range of filter velocity. Air permeability is given for the sample by the slope of the straight line by which the distribution of u vs. $\Delta p/L$ plots is represented.

II. 3. *Effect of air flow on sublimatic deformation of snow structure*

A serious objection to be considered for the use of an air permeameter is due to a possible sublimatic deformation in snow structure, either by evaporation or by condensation as a result of air flow in a snow sample during the measuring of air permeability. A series of experiments were carried out to investigate the possibility of such an effect. A snow sample was subjected to air flow, about 0.5 cm/s in filter velocity, continuously for 10 hours in a cold room. Eight measurements in total as to the air permeability of the sample were made during the continuous air flow at intervals of approximately 3 hours, as schematically shown in Fig. 3(a). The condition of room air is given also in Fig. 3(a). The results of all the measurements show a fairly good agreement, as shown in Fig. 3(b), with about 2.5% of maximum deviation from the mean and without any systematic changes in values. From the foregoing result, it may reasonably be concluded that air flow in such a magnitude does not at all affect the snow structure due to sublimation.

It may also be said from the series of experiments that the dependency of air permeability on temperature is not remarkable in the range of a temperature change of 5°C or so from a standpoint of practical concerns.

II. 4. *A portable bridge type air permeameter*

Precise measurement of the air permeability of deposited snow can be made by use of an air permeameter with a double-vessel sample holder in the

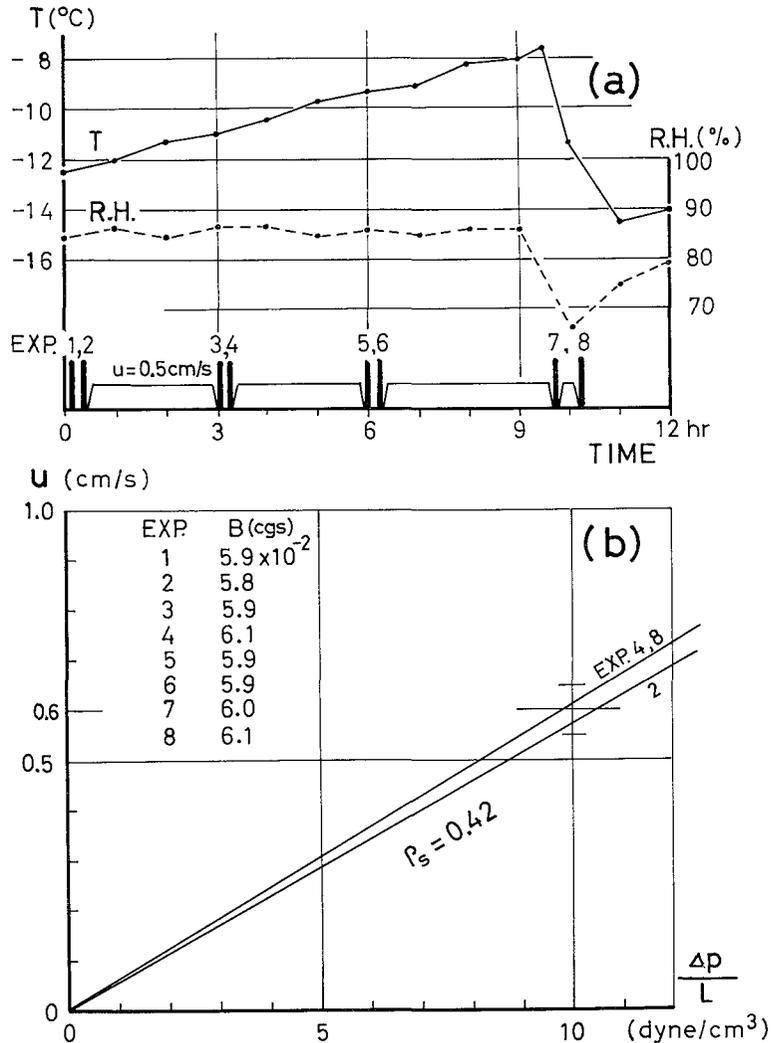


Fig. 3. Effect of air flow on sublimatic deformation of snow structure
 (a) Experimental condition (air temperature T and relative humidity R. H.)
 (b) Results of the measurements

cold laboratory. In the meantime, for field measurements, a portable air permeameter, 35 cm \times 28 cm \times 15 cm in size and 5 kg in weight, was designed. A schematic diagram of this permeameter is shown in Fig. 4 (a). This permeameter consists of two principal sections, i.e., a pressure source section and a bridge circuit section. The pressure source section consists of a hand bellows and a pressure equalizer which is composed of two rubber reservoirs R_1 and

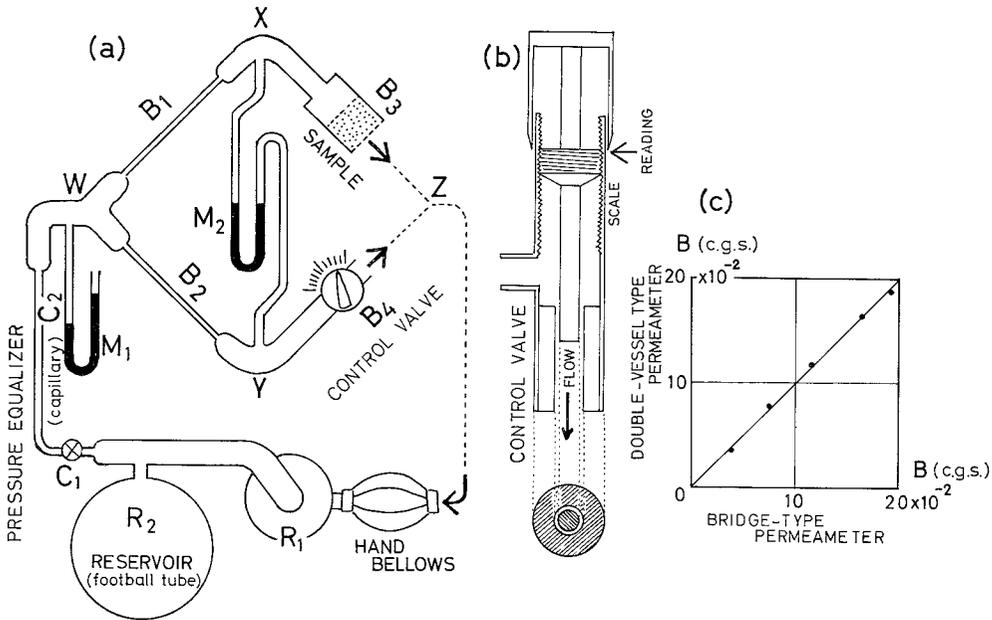


Fig. 4. Portable bridge-type air permeameter

- (a) Schematic diagram
- (b) Control valve
- (c) Comparison of air permeability measurements of fine-grained compact snow by two kinds of permeameters

R_2 , a stop cock C_1 and a capillary tube C_2 . This section can supply continuously a considerably constant pressure difference across the bridge circuit. The air pressure difference applied across the bridge circuit is measured by a manometer M_1 . The bridge circuit section consists of two capillary tubes B_1 and B_2 , a sample holder B_3 and a control valve B_4 of which outline is shown in Fig. 4(b). A snow sample is set in the arm B_3 by a single-tube sample holder. When air pressure difference is applied between W and Z by operating the pressure source section, the bridge circuit works just analogously to the Wheatstone bridge of electric circuit. Pressure difference between X and Y is measured by a manometer M_2 . The bridge can be balanced by the controlling of B_4 and the balance is detected by M_2 ; pressure difference between X and Y becomes zero when the bridge is balanced. As the air permeability of B_1 , B_2 and B_4 has already been calibrated, the air permeability of the sample B_3 can directly be read from the relation for the balance of the bridge, i. e.,

$$B_3 = B_4 \cdot B_1 / B_2. \quad (\text{II. 5})$$

As a single circular tube is used for the sample holder for this permeameter, a considerable error in measurement cannot be avoided for soft or fragile snow, as compared with measurement by the air permeameter with a double-vessel type sample holder. However, this portable air permeameter worked excellently for fine-grained compact snow, as is shown in Fig. 4(c).

II. 5. A kerosene permeameter

As was described in section II. 1., the filter velocity of a fluid through a porous medium is inversely proportional to its viscosity, if it is a purely viscous flow, and we get

$$B_0 = B\eta. \quad (\text{II. 4})$$

A kerosene permeameter was designed to investigate experimentally if such a condition and relation were actually held in the measurements.

A schematic diagram is shown in Fig. 5(a). Kerosene pumped up in the tank R_0 runs down to R_1 , then R_2 and finally to R_3 by gravity through connecting pipes. From R_3 , kerosene is pumped up again to R_0 continuously, to keep a steady circulation of kerosene through the system. The free surface of kerosene in the tanks R_0 , R_1 and R_2 is held at a constant level, respectively, by use of each drain pipe. R_0 is provided as an auxiliary reservoir to make the free surface of kerosene in R_1 very stable. A snow sample, 3 cm in diameter and 5 cm in length held in a single-tube sample holder, is placed in the connecting pipe line between R_1 and R_2 , in series with a control valve C_1 . The inner diameters of the connecting pipe and of the control valve C_1 are approximately 20 mm and the resistance thereof for kerosene flow is negligibly small, compared with that of the snow sample. The flow of kerosene between R_1 and R_2 is caused by the head H_1 and its flow rate is controlled by the resistance of the sample and that of the control valve C_1 , as H_1 is constant.

An equivalent electric circuit to the kerosene permeameter is shown in Fig. 5(b), i. e., a constant resistance of the sample, $r(\text{sample})$, and a variable resistance of the control valve, $r(C_1)$, are connected in series across an electrical potential difference of H_1 . Putting H_2 and H_3 as the electric potential drop across $r(\text{sample})$ and $r(C_1)$, respectively, we get

$$H_2 + H_3 = H_1 \text{ (const.)}, \quad (\text{II. 6})$$

$$H_2/H_3 = r(\text{sample})/r(C_1). \quad (\text{II. 7})$$

Eliminating H_3 from (II. 6) and (II. 7),

$$H_2 = H_1 r(\text{sample}) / [r(C_1) + r(\text{sample})]. \quad (\text{II. 8})$$

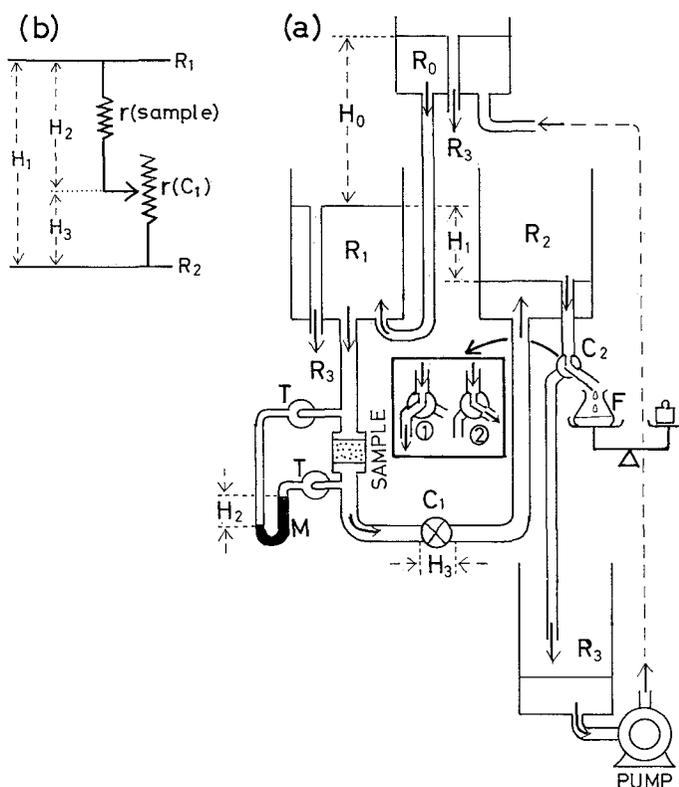


Fig. 5. Kerosene permeameter

- (a) Schematic diagram: Kerosene tanks R_0 , R_1 , R_2 , R_3 , control valve C_1 , 3-way stop cock C_2 , pressure transducer T , manometer M , measuring flask F .
 (b) Equivalent electrical circuit to the kerosene permeameter

This relation gives

$$H_2 \approx H_1, \text{ if } r(C_1) \approx 0 \text{ (as the control valve was fully opened)}$$

$$H_2 = 0, \text{ if } r(C_1) = \infty \text{ (as the control valve was closed).}$$

Therefore, we can apply a wide range of the pressure difference H_2 , from 0 up to H_1 , across the snow sample by controlling the valve C_1 .

The pressure difference across the sample H_2 is directly measured by a Göttingen-type alcohol manometer M through a pressure transducer T . A 3-way cock C_2 , located right beneath the tank R_2 , is set in the position illustrated in the figure and the net amount of the overflow of kerosene Q during t seconds is collected in a measuring flask F and thus the actual flow rate of the kerosene Q/t is directly measured.

II. 6. Viscous flow and the range of Darcy's law

A series of permeability measurements of deposited snow was carried out to investigate the flow mode of fluid through deposited snow.

First, the air permeability B_A of deposited snow was measured by the air permeameter with a double-vessel sample holder. Then, the kerosene permeability B_K was measured for the sample in succession. If the flow of both fluids was a pure "viscous flow", it would result in the following relation, namely

$$\eta_A B_A = B_0 = \eta_K B_K, \quad (\text{II. 9})$$

where η_A and η_K denote the viscosity of air and kerosene, respectively, and B_0 the specific permeability of the sample.

The results of the experiments, given in Fig. 6, show that both the air-

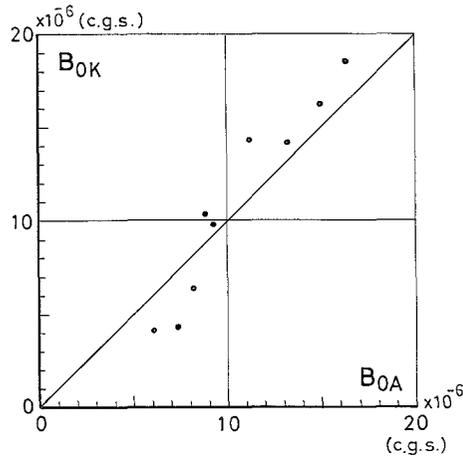


Fig. 6. Specific permeability calculated from air permeability and kerosene permeability

B_{OA} : from air permeability,

B_{OK} : from kerosene permeability

and the kerosene- flow through the snow sample can approximately be regarded as "viscous flow" in the investigated range of filter velocity.

Darcy's law does hold only for the range of the laminar flow of a fluid. Generally, this range is defined in terms of the Reynold's number which is defined as

$$\begin{aligned} R_e &= U\delta\rho/\eta \\ &= U\delta/\nu, \end{aligned} \quad (\text{II. 10})$$

where U is fluid velocity, δ diameter associated with the porous medium, ρ

fluid density, η fluid viscosity and ν dynamic viscosity of the fluid. For fluid flow through a porous medium, it may be practical and convenient to define a special Reynold's number R_e^* , taking filter velocity u instead of actual velocity U , namely

$$R_e^* = u\delta/\nu. \tag{II. 11}$$

It was experimentally confirmed that Darcy's law holds for kerosene flow through deposited snow in the range of filter velocity up to 1 cm/s, at least, as shown in Fig. 7. (For the case of air flow, the reading of the flow meter fluctuated violently as u exceeded the value, approximately 1.5 cm/s, due to a turbulence in the vicinity of the capillary tube of the flow meter, even if

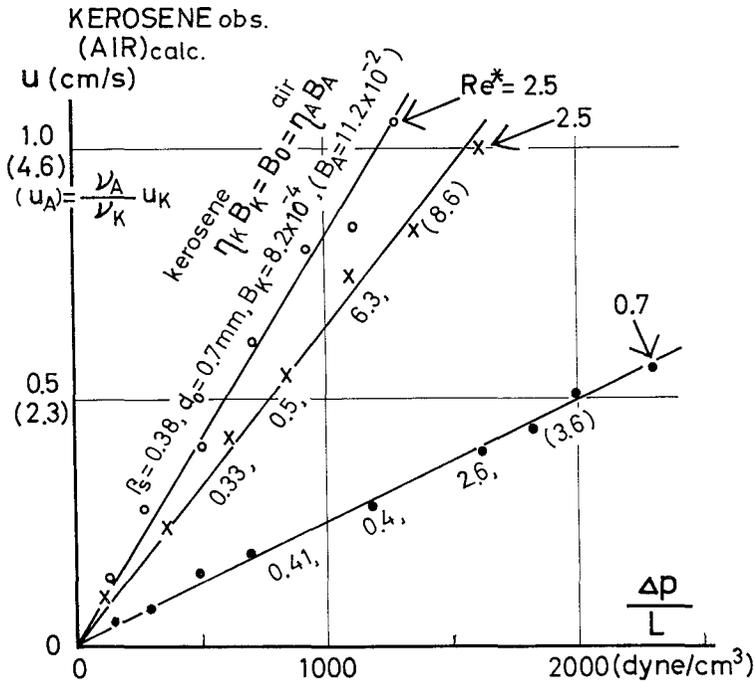


Fig. 7. Kerosene permeability of deposited snow

air flow through the sample was still laminar.) By the relation (II. 11), we can estimate the filter velocity of air u_A corresponding to that of kerosene u_K which has the same mode of flow through deposited snow, namely

$$u_A = u_K(\nu_A/\nu_K), \tag{II. 12}$$

where ν_A (=12.9 cS) and ν_K (=2.8 cS) are the dynamic viscosity of air and kerosene at -13°C , respectively. Using this relation, it can be said that

Darcy's law will hold for the range of filter velocity up to 4.7 cm/s, at least, for air flow through deposited snow.

III. Air permeability of natural deposited snow

III. 1. *Distribution of values of air permeability of natural deposited snow*

The air permeability of various kinds of natural deposited snow has been measured since 1955. The distribution of the air permeability and density of natural deposited snow, grouped by the Classification of Snow by the Japanese Society of Snow and Ice, is given in Fig. 8(a):

(i) New snow: Newly deposited snow is generally very light, loose and soft; sometimes the original crystal shapes of snow remain therein for a period from a few hours up to a couple of days. Deposited snow in such a stage is named "new snow". The density of new snow ranges approximately from 0.05 to 0.16 g/cm³ and the air permeability approximately from 20 to 50 × 10⁻² cm⁴/dyne·s.

(ii) Fine-grained compact snow: By densification and metamorphosis, new snow changes its structure with the lapse of time, even without undergoing snow melt at all. The snow crystals in deposited snow turn into fine granules connected by ice bonds with each other, resulting in a fine network of ice with very low porosity and very fine pore structure. It shows a high degree of mechanical strength as a whole. This is named "fine-grained compact snow". The major part of deposited snow in Hokkaido and in Alpine regions of Japan, during the coldest time in winter, consists of fine-grained compact snow. Its density ranges approximately from 0.15 to 0.50 g/cm³ and air permeability approximately from 3 to 22 × 10⁻² cm⁴/dyne·s.

(iii) Coarse-grained snow: By the progress of warm metamorphosis of deposited snow, fine-grained compact snow turns into "coarse-grained snow", composed of coarse snow grains and big pore structure. Coarse-grained snow exists generally at the bottom of deposited snow in winter and all the deposited snow turns into coarse-grained snow in spring. Its density ranges approximately from 0.35 to 0.52 g/cm³ and air permeability approximately from 15 to 33 × 10⁻² cm⁴/dyne·s.

However, a simple relation between air permeability and density could not be found even among each grouping of snow.

Bader (1939) measured the values of the air permeability of various kinds of natural deposited snow. He defined his permeability K by use of an equation

$$Q = K(qn/100)(\Delta h/L), \quad (\text{III. 1})$$

where Q is volume rate of flow in cm³/s, q cross section of the sample in

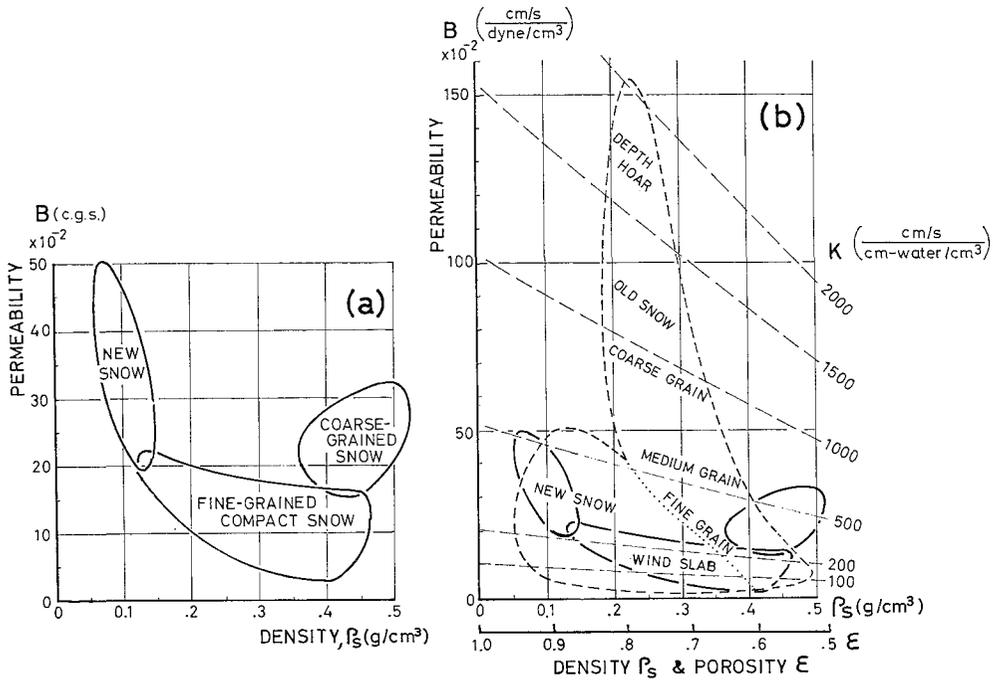


Fig. 8. Distribution of air permeability of various kinds of natural deposited snow

- (a) by Ishida and Shimizu
- (b) Comparison with Bader's observation
(Bader—broken lines; Ishida-Shimizu—solid lines)

cm^2 , n porosity of the sample in %, Δh pressure drop in cm of water column and L length of the sample in cm. Equation (III. 1) leads to

$$Q/q = u = K \varepsilon (\Delta h / L), \tag{III. 2}$$

where $\varepsilon = n/100$. Comparing the equation (III. 2) with (II. 1), the difference of the physical meanings of B and K becomes clear. Namely, K is the fluid permeability referred to the apparent linear pore velocity. Moreover, considering the difference of unit for pressure, the relation between B and K is given as

$$B = (\varepsilon / 980) K. \tag{III. 3}$$

Distributions of the air permeability of various kinds of natural deposited snow obtained by Bader and by the author were compared in Fig. 8 (b). Owing to the relation (III. 3), K axis is slightly curved and inclined in the B - ρ_s diagram, as shown by broken lines in the figure. As a result, the following

is mentioned. Bader obtained an extraordinarily high valued domain of air permeability for depth hoar and coarse-grained snow in the medium porosity range of his investigation. As stated in Introduction and Section II. 2., this value was probably caused by the sample holder. Because, depth hoar, for example, has so fragile structure that it easily collapses even by a light touch with a finger tip. Hence it seems almost impossible to have a depth hoar sample in a single-tube sample holder without disturbing its structure.

III. 2. *Metamorphosis of deposited snow and change of its air permeability*

To investigate a relation between the metamorphosis of deposited snow and the change of its air permeability, a series of observations was carried out. The density, air permeability and grain size of snow were measured on several particular snow layers in deposited snow with 10-day intervals approximately through a winter. Fig. 9 gives schematic diagram of the observations: The dates of observations, stratigraphy of deposited snow and selected snow layers are given in Fig. 9. Each snow layer in deposited snow was named, for example, as D-9 and J-8, which identify the snow layers set on December

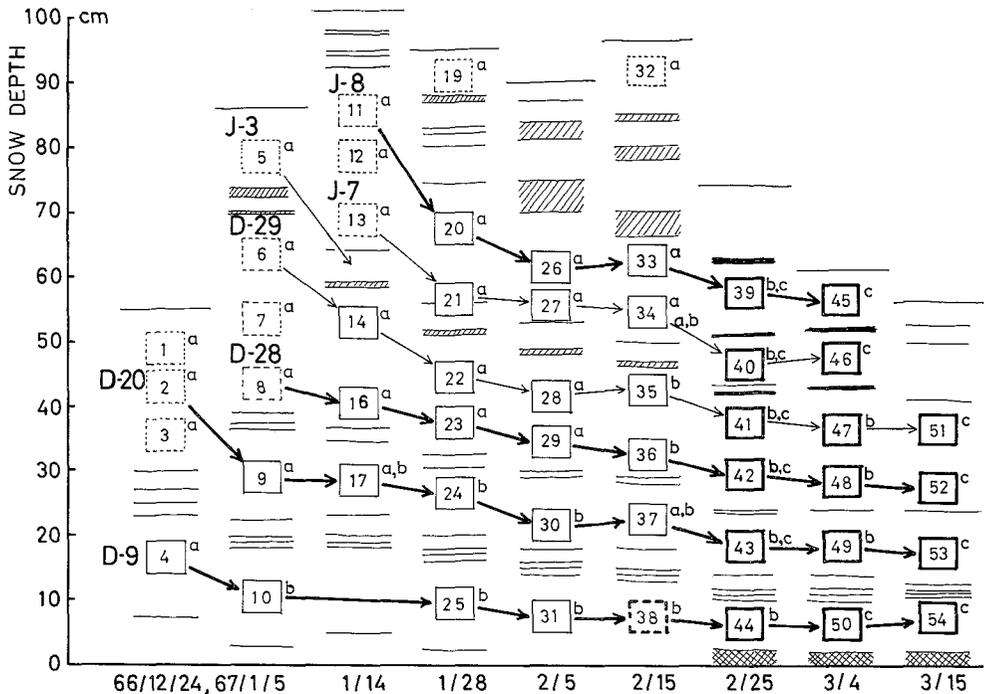


Fig. 9. Observation of metamorphosis and change of air permeability of natural deposited snow, stratigraphy and sampling

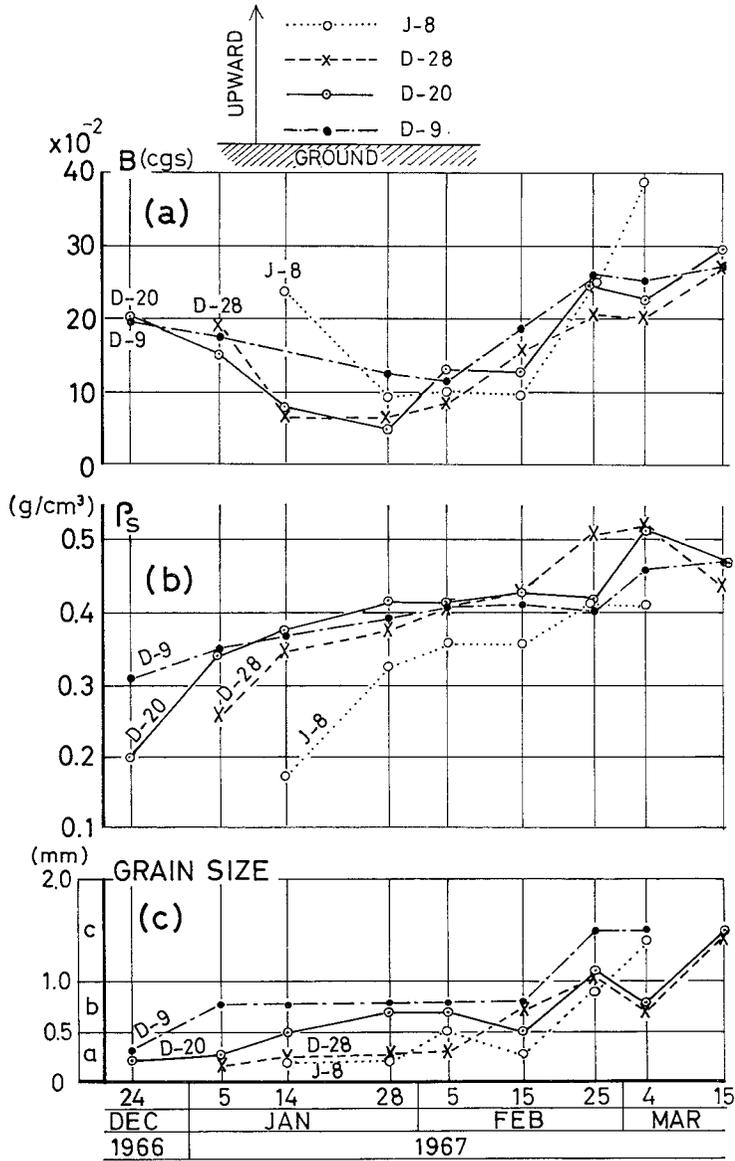


Fig. 10. Metamorphosis of deposited snow and change of air permeability
 (a) Air permeability
 (b) Density
 (c) Grain size (according to the International Classification of Snow)

9 and January 8, respectively.

Changes of the air permeability, density and mean grain size of the layers D-9, D-20, D-28 and J-8 are given in Fig. 10. As is readily seen, the air permeability of each snow layer decreases once and then increases with the lapse of time while the density and grain size of those layers increase monotonically. This fact also may imply that air permeability can serve as an important quantity for the description of conditions of deposited snow.

The change of the air permeability B with the snow density ρ_s of each snow layer was plotted in Fig. 11. As a general tendency, B decreased gradually with an increase of ρ_s in the range in which ρ_s was smaller than 0.42 g/cm^3 and then B increased abruptly without a big change in ρ_s . It

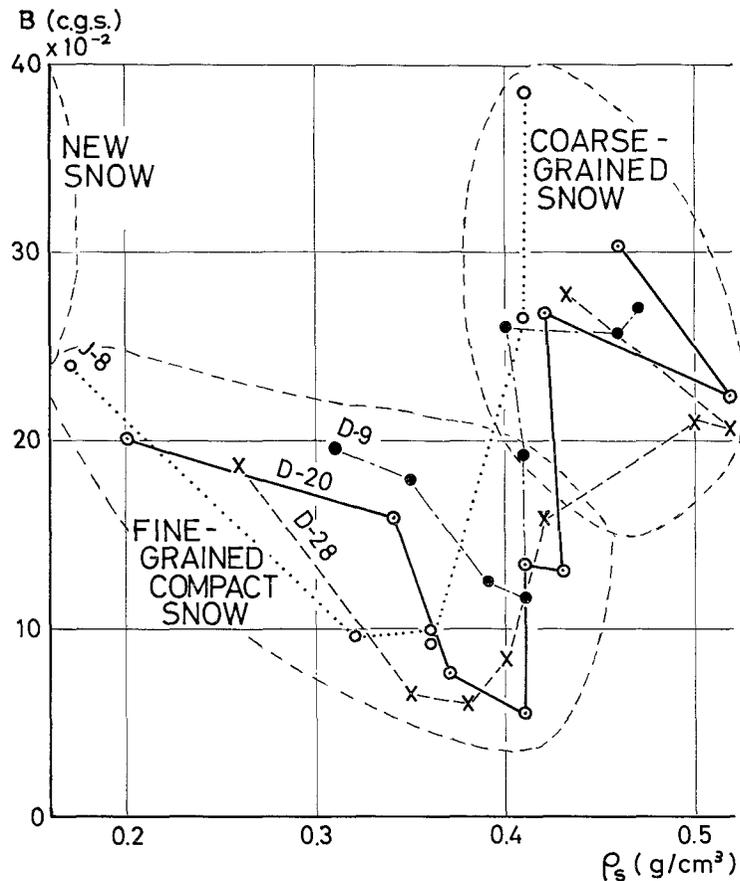


Fig. 11. Changes of air permeability and density of snow layers caused by metamorphosis

may imply that there exist two different kinds of structural change of deposited snow depending upon its density range.

A general tendency of the change of the air permeability of a snow layer in regard with snow density is schematically shown in Fig. 12, making use of photographs of the thin sections of snow. It is qualitatively seen that the air permeability B decreases with a decrease of porosity and pore size from (a) to (c), and then B increases with an increase of pore size from (c) to (f), while snow density increases monotonically.

IV. Air permeability and structure of fine-grained compact snow

IV. 1. General remarks

The definition of the specific permeability B_0 of a porous material implies by itself that B_0 depends only on the geometrical structure of the material. In other words, B_0 can be deducible theoretically from the geometrical structure of the porous material, at least in principle. As stated at the beginning of Section II. 1., the structure of snow is so complicated that it can only be described in statistical terms. Hence, a theory to deduce B_0 should be of a statistical nature. There have been no such theories that satisfactorily deduce B_0 . What one should do at the present time is to search experimentally for the possible dependency of B_0 on structural variables, that is, those variables relevant to the geometrical structure of the porous material which is of a statistical nature.

IV. 2. Porosity

The most fundamental structural variable of a porous material is total porosity. It is defined by the fraction of the bulk volume of the material occupied by all kinds of voids. The mass of air in voids being neglected, the following relation is evident:

$$\rho_s V_s = \rho_i V_i, \quad (\text{IV. 1})$$

where ρ_s is the bulk density of the porous material (snow), ρ_i the density of solid (ice) constituting the matrix, V_s and V_i the volume of the porous material and the solid, respectively. The total porosity ε is then given by

$$\varepsilon = (V_s - V_i) / V_s = 1 - \rho_s / \rho_i. \quad (\text{IV. 2})$$

Naturally, the voids which contribute to fluid flow through the porous material are such as connect both the ends of the material across which the pressure difference to cause fluid flow is applied. Such voids are called to be "effective". "Effective porosity" is then defined by the fraction of the

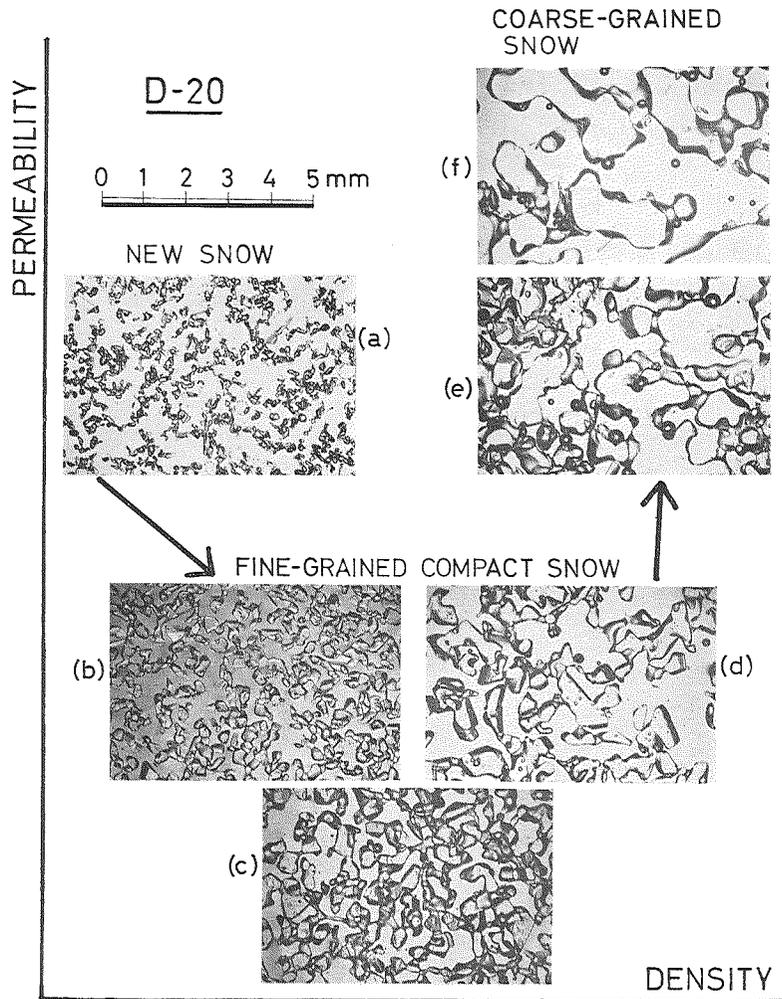


Fig. 12. Schematic illustration of metamorphosis and change of air permeability of a snow layer, D-20

bulk volume of the porous material occupied by all effective voids.

For the case of natural deposited snow, whose density is up to 0.52 g/cm^3 , almost all voids are effective, as is presumed from the photographs of the thin sections of such snow in Fig. 12. Hence, the effective porosity of deposited snow is almost equal to total porosity. Hereafter in this section it is assumed that effective porosity is represented by the relation (IV. 2).

IV. 3. Mean grain size

The geometrical structure of a block of snow taken from a layer of deposited

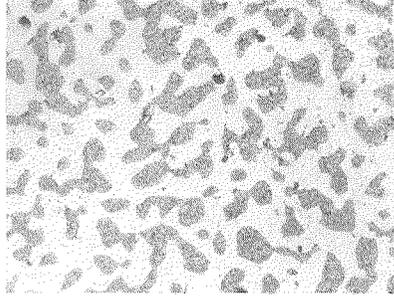


Fig. 13. Cross section of a deposited snow
 Dark portion: snow grain. White
 portion: solid aniline (*i. e.* air void)

snow is considerably homogeneous in a statistical sense, especially for the case of fine-grained compact snow, as is observed by many authors (Shimizu 1967, 1968; especially Narita 1969). That means any cross-section of a snow block can be looked on as a representative one. Let the volume of a snow block be V_s , that of ice in it V_i , the bulk density of the block ρ_s and that of ice ρ_i ($=0.917 \text{ g/cm}^3$). These quantities satisfy the relation (IV. 1). Let the cross-sectional area of the snow block be S_s , and that occupied by ice on the cross-section S_i . From the above mentioned statistical homogeneity, the following relation holds:

$$S_i/S_s = V_i/V_s = \rho_s/\rho_i. \quad (\text{IV. 3})$$

The space occupied by ice in a block of snow appears as a grain in a cross-section of the block, as shown in Fig. 13. Let the number of grains in the cross-section be N , then

$$s_i = S_i/N = (S_s/N)(\rho_s/\rho_i) \quad (\text{IV. 4})$$

is the mean area of the grains.

The snow structure of low density may be considered as a random assemblage of ice grains of various size supporting each other through light contacts. With an increase in density, the snow structure gradually turns into a three-dimensional network of ice cords whose thickness varies from point to point. The further increase of density converts the role of ice and voids, *i. e.*, the snow structure turns into a three-dimensional network of voids, then into a random assemblage of isolated voids. The structure of natural deposited snow corresponds to the first and second stages. (Structure of the old firn and the glacier ice may correspond to the third and final stages, respectively.) Therefore, the grains appeared in a cross-section of a snow

block from natural deposited snow are the cross-sections of ice grains or of ice cords, depending upon the case.

The quantity s_i defined above is the mean of such cross-sections. The square root of s_i is then a measure of the mean size of the ice grains or the mean thickness of the ice cords. In order to see a relation between the square root of s_i and the three-dimensional grain size, we will consider a particle model of the deposited snow which is an assemblage of randomly packed ice spheres of uniform diameter d_0 , making a bulk density ρ_s . In a cross-section of the particle model of deposited snow, there appear a number of circular cross-sections of the uniform spheres in various diameters ranging up to d_0 . From the randomness of packing, the mean diameter \bar{d}_0 of those circular cross-sections can be computed by

$$\pi \bar{d}_0^2 d_0 = \frac{2}{3} \pi d_0^3, \quad (\text{IV. 5})$$

or

$$d_0 = \bar{d}_0 \sqrt{3/2}. \quad (\text{IV. 6})$$

The quantity s_i in the relation (IV. 4) is equal to $(\pi/4)\bar{d}_0^2$. Hence, from the relations (IV. 4) and (IV. 6), we get

$$d_0 = \sqrt{6\rho_s/n\rho_i}, \quad (\text{IV. 7})$$

where $n (=N/S_s)$ is the number of the "grains" appeared in the cross-section of a unit area.

In order to make things intuitive, in the following section, we will use d_0 , which will be hereafter called the "mean grain size", instead of the square root of s_i .

IV. 4. Procedure of counting n

Various techniques of making a cross section of snow have been developed (Bader 1939, Kojima 1960, Fuchs 1955, Shimizu 1958, Kinoshita and Wakahama 1959, Narita 1969). Here, the Narita's method, which is a modification of Kinoshita-Wakahama's method, will be summarized briefly.

(i) Dip a snow sample in ice-saturated liquid aniline ($\text{C}_6\text{H}_5\text{NH}_2$) at temperatures of $-5 \sim -10^\circ\text{C}$. After aniline has filled up voids of the sample completely, cool the sample down to $-20 \sim -30^\circ\text{C}$. Then aniline in the voids will solidify without any remarkable change in volume, whereby a very rigid aniline block of the sample is obtained. Make a smooth cross-section of the block with a microtome or a planer.

(ii) Sprinkle fine dyeing-powder on the smoothed cross-section of the

block with a soft writing brush. Since the dyeing powder adheres only to ice, ice grains in clear shape can easily be obtained, as shown in Fig. 13. Then, take a microphotograph of the cross-section and count the number n .

In the actual counting of n , a complicated-shaped grain having m remarkable constrictions was counted as $m + 1$ grains.

IV. 5. Dependency of the specific permeability of fine-grained compact snow on the "mean grain size"

It is evident that the specific permeability B_0 depends on the effective porosity which is almost equal to the total porosity ϵ for ordinary natural deposited snow. In order to eliminate an effect on specific permeability by effective porosity and to find out clearly an effect on the same by the "mean grain size" d_0 , we classified all the data, for which the "mean grain size" had been measured, into six equi-density groups (corresponding to the equi-(effective) porosity groups, in accordance with the relation (IV. 2)), A, B, C, D, E and F, as shown in Fig. 14. Then, B_0/η ($=B$) was plotted against d_0 by groups, as shown in Fig. 15(a) to (f). From the figures it is clearly seen that B_0 was proportional to d_0^2 , if snow density (therefore, the effective porosity of deposited snow) was constant. This result is very consistent, since the dimension of B_0 is the square of length as mentioned in Section II. 1. Now, we will go back to search for the dependency of B_0 on effective porosity (or density).

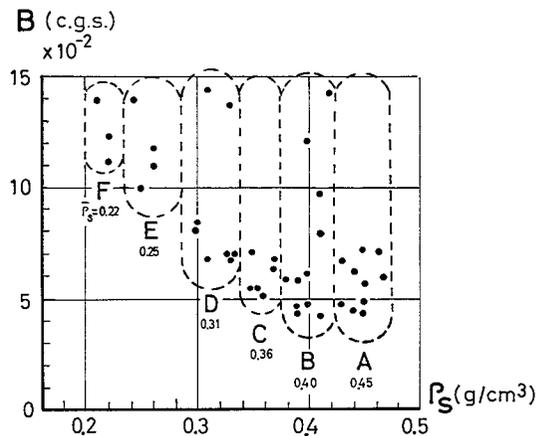


Fig. 14. Equi-density groups of fine-grained compact snow

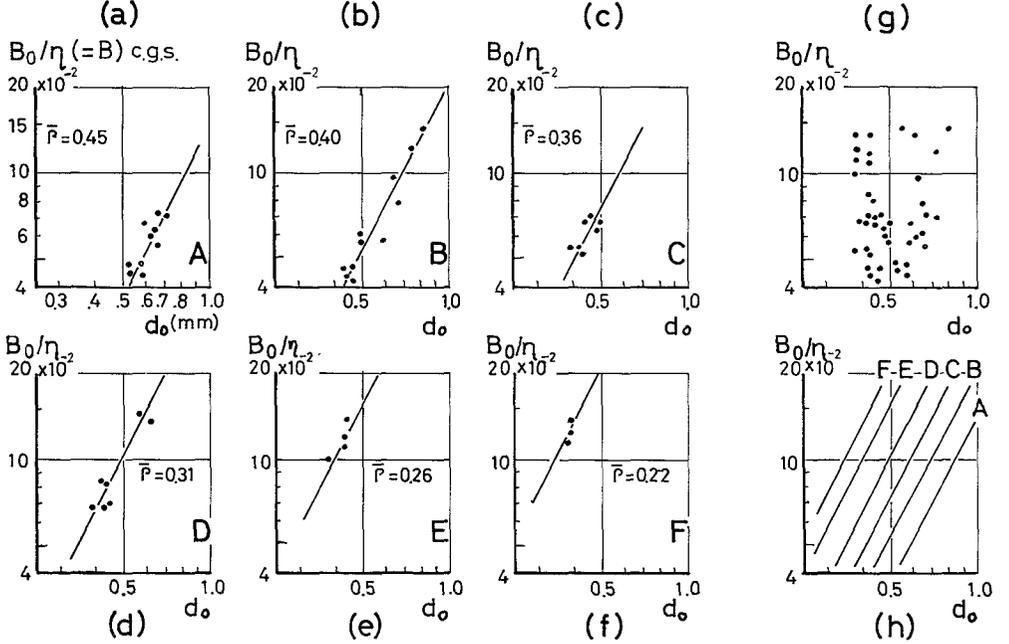


Fig. 15. Dependency of the specific permeability B_0 of fine-grained compact snow on the "mean grain size" d_0

IV. 6. *Dependency of the specific permeability of fine-grained compact snow on effective porosity (or specific gravity)*

Though effective porosity is primarily related to fluid flow through a porous material, we will use the specific gravity ρ_s^* as an independent variable, because it is more familiar to snow investigators than effective porosity is, and both of the quantities are related by the simple relation (IV. 2) at least for ordinary natural deposited snow.

In order to find out the effect of "effective porosity" on specific permeability, the values of B_0/d_0^2 , for a constant value of d_0 , read from the lines in Fig. 15, were plotted against the specific gravity ρ_s^* , in Fig. 16. It is clearly seen from the figure that six points, A, B, ... F, which correspond to the intersections of the axis, $d_0 = 0.05$ cm, and equi-density lines in Fig. 15, respectively, lay very closely on a straight line. Thus, a simple experimental formula was obtained for the porosity function and then specific permeability in terms of the specific gravity ρ_s^* , namely

$$B_0/d_0^2 = 0.077 \cdot \exp[-7.8\rho_s^*], \quad (\text{IV. 8})$$

or

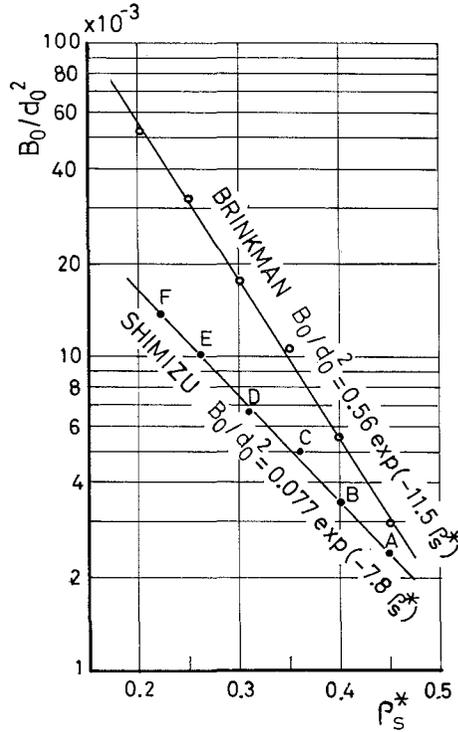


Fig. 16. Porosity function B_0/d_0^2 of specific permeability

$$B_0 = 0.077 d_0^2 \cdot \exp[-7.8 \rho_s^*]. \quad (\text{IV. 9})$$

where d_0 is in cm.

V. Discussion

V. 1. Other independent structural variables than the mean grain size and porosity

Beside the mean grain size and effective porosity which appeared in the empirical formula (IV. 9), various authors have proposed many other independent structural variables which may affect the specific permeability of a porous material. Some examples are the shape, surface roughness, angularity, relative orientation of grains (all in the case when the matrix is regarded as an assemblage of grains) and the tortuosity of pipes (in the case when void space is regarded as an assemblage of pipes).

In the deduction of the empirical formula (IV. 9), measured values showed somewhat a small degree of dispersion on each stage of the analysis (see Figs.

15 and 16). This does not necessarily mean that other independent structural variables than the "mean grain size" and specific gravity (i. e. porosity) played a marginal role in determining the permeability of compact snow. Rather, it means that those other variables affect the permeability of various samples of fine-grained compact snow in a similar manner. In other words, those variables take almost a constant value respectively for all the snow categorized under fine-grained deposited snow.

V. 2. Comparison of the relation (IV. 9) with the Kuroiwa's results

Kuroiwa (1968) carried out a series of measurements of the kerosene permeability k of natural deposited snow and artificially compressed snow with kerosene permeameters of the falling head type. Because of the difference in the unit used for measuring pressure, the numerical values of B_K in the present paper is related to those of his k by

$$B_K = k/784. \quad (\text{V. 1})$$

He plotted measured values of $\log k$ against total porosity (see Fig. 15 in his paper). The plotted points for artificially compressed snow (the same sample being compressed successively) lay very closely on a straight line, but those for natural deposited snow dispersed very widely, perhaps because he didn't classify his data by grain size. For the latter case, however, he boldly gave a straight line which might show a general tendency of the dependency of permeability on total porosity. In terms of the specific permeability B_0 and the specific gravity ρ_s^* , his two lines were represented by

$$B_0 = C_1 \cdot \exp[-7\rho_s^*] \quad \text{for natural deposited snow} \\ (0.14 < \rho_s^* < 0.42) \quad (\text{V. 2})$$

$$B_0 = C_2 \exp[-21\rho_s^*] \quad \text{for artificially compressed snow} \\ (0.41 < \rho_s^* < 0.87) \quad (\text{V. 3})$$

Since the precise value of the viscosity of kerosene used by him was not known, the values of two constants C_1 and C_2 could not be determined. But, the coefficient of ρ_s^* in the exponent in (V. 2) agrees surprisingly well to that in (IV. 9), in spite of the rough way of obtaining (V. 2).

The difference of the coefficient of ρ_s^* in the exponents in (V. 2) (or (IV. 9)) and that in (V. 3) suggests that there may be some fundamental changes in the mechanism of flow through snow at its specific gravity being about 0.41.

V. 3. Comparison of the relation (IV. 9) with Brinkman's theory

In the stationary state of the flow of a fluid through a column of a

porous material, the pressure difference of the fluid between two opposite ends of the column divided by the cross sectional area of the column is equal to the drag force of the fluid on the inner surface of the porous material, if the inertia force of the fluid is neglected. Hence, the permeability of the material can be obtained if drag force for a given filter velocity of the fluid is calculated. Drag force, however, has been calculated only for very simple cases such that the material is regarded as a bundle of parallel pipes (see Section V. 5). As stated in the previous section some statistical considerations are necessary for calculating the drag force exerted on a porous material, based on more or less realistic structure.

Brinkman (1949) deduced the specific permeability of an assemblage of uniform spheres in diameter d_0 , and gave the following equation,

$$B_0 = (d_0^2/72)[3 + 4/(1-\epsilon) - 3\sqrt{8/(1-\epsilon)} - 3]. \quad (\text{V. 4})$$

The relation can be very closely approximated by

$$B_0 = 0.56 d_0^2 \cdot \exp[-11.5 \rho_s^*]. \quad (\text{V. 5})$$

The marked discrepancies of the coefficients in the relations (V. 5) and (IV. 9) should be noted.

V. 4. *Metamorphosis of natural deposited snow and change of its permeability*

On the basis of the formula (IV. 9), the curves of B against ρ_s were given in Fig. 17, where the mean grain size d_0 was taken as a parameter. Changes of the density and permeability of natural deposited snow due to its metamorphosis may be schematically represented by the dotted line A-B-C in the figure. The area enclosed by broken lines in the figure is a possible region for actual fine-grained compact snow. The process A to B corresponds to the densification of fine-grained compact snow usually accompanied with a gradual increase in the mean grain size and it corresponds also to the fact that the specific gravity of the snow ρ_s^* in the relation (IV. 9) has a predominant effect on the specific permeability B_0 for this process. The process B to C corresponds to metamorphosis from fine-grained compact snow to coarse-grained snow where a remarkable increase in the mean grain size is characteristic, and it corresponds also to the fact that the mean grain size d_0 in the relation (IV. 9) has a predominant effect on the specific permeability B_0 for this process.

Results of actual measurements of the change of the permeability of various snow layers are shown in Fig. 18. The turning point B was clearly observed for every layer. The density of snow at the point B ranged from

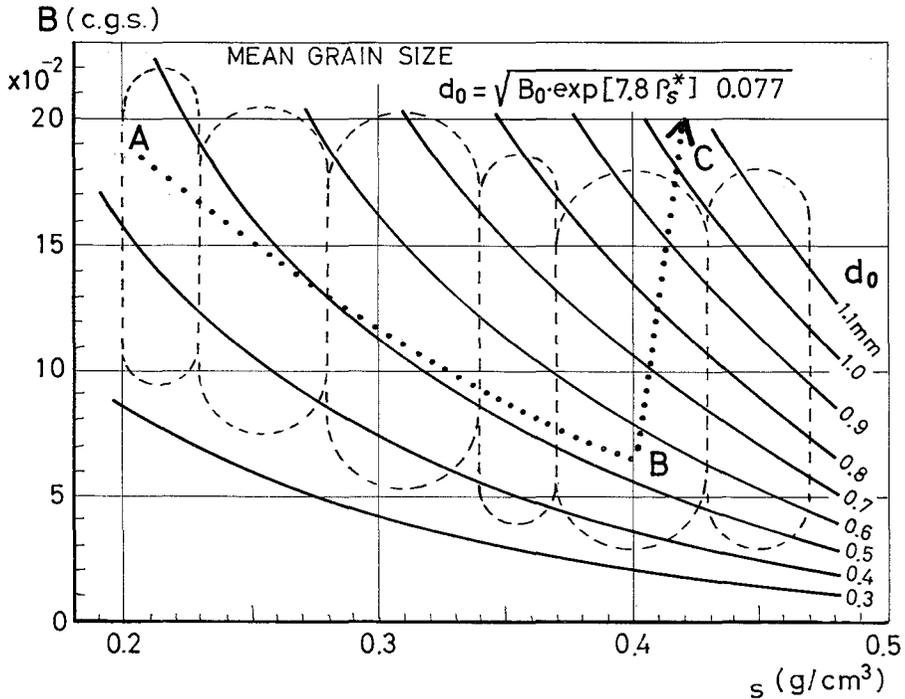


Fig. 17. The "mean grain size" d_0 of fine-grained compact snow and a schematical path of metamorphosis

0.36 to 0.42 g/cm³. Some layers showed a remarkable decrease in permeability prior to reaching the point B, suggesting a decrease in the mean grain size. This was possibly caused by local unhomogeneity of natural deposited snow.

V. 5. *Interpretation of measured permeability on a pipe model (snow structure regarded as a bundle of pipes)*

If pores of a porous material consist of parallel straight circular pipes, the permeability of the material in a parallel direction to the axis of the pipes can be easily calculated.

The volume rate Q of flow of a fluid having viscosity η through a straight circular pipe of radius a_0 and length L is given by the well-known law of Hagen-Poiseuille,

$$Q = (\pi a_0^4 / 8\eta) (\Delta p / L), \quad (\text{V. 6})$$

where Δp is the pressure difference between both the ends of the pipe.

Assume that pores of a porous material consist of n such pipes per unit

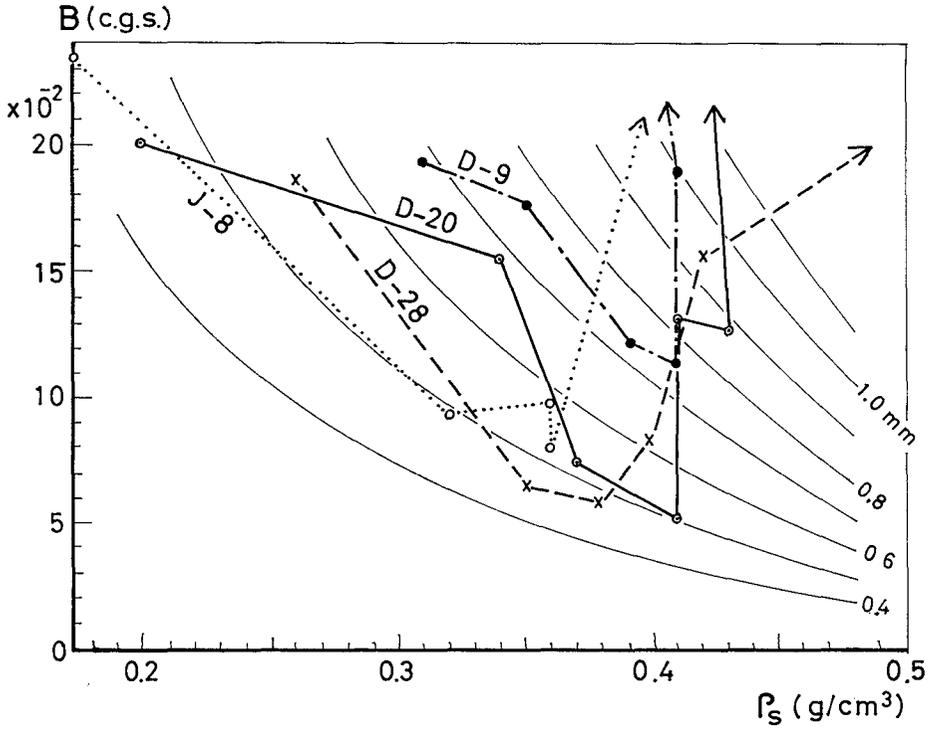


Fig. 18. Metamorphosis of fine-grained compact snow

area of the material. The volume rate Q_n of the fluid flow through a unit area of the material is given by

$$Q_n = (n\pi a_0^4 / 8\eta) (\Delta p / L). \quad (\text{V. 7})$$

From the relations (V. 7) and (II. 1), the fluid permeability B of the material is given by

$$B = n\pi a_0^4 / 8\eta. \quad (\text{V. 8})$$

The porosity ε of the material is obviously given by

$$\varepsilon = n\pi a_0^2. \quad (\text{V. 9})$$

Substituting (V. 9) into (V. 8), one gets

$$B = \varepsilon a_0^2 / 8\eta \quad (\text{V. 10})$$

and

$$D_0 = 2a_0 = \sqrt{32\eta B / \varepsilon}, \quad (\text{V. 11})$$

where D_0 is the diameter of the pipe.

On the basis of the relation (V. 9), the matrix of the material being assumed to be ice and the fluid to be air, the curves of B against density were plotted in Fig. 19, where D_0 was taken as a parameter. The area enclosed by a broken line in the figure indicates a possible region for actual fine-grained compact snow. The figure shows that, if the structure of fine-grained deposited snow is regarded as such as described above, the diameter of the pipe ranges from 0.2 to 0.4 mm.

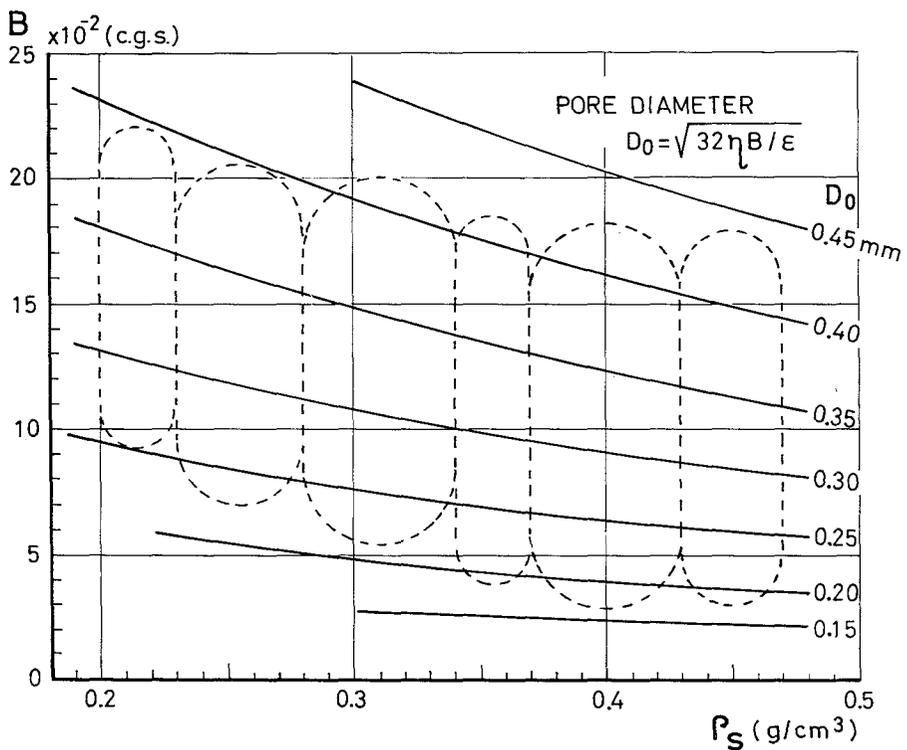


Fig. 19. The diameter D_0 of the pipes equivalent to fine-grained compact snow

Wakahama (1968) measured the height of water sucked into a block of snow due to capillary effect. The measured heights correspond to the suction of pipes in diameter ranging from 0.3 to 0.5 mm. The close agreement of his results to the results of the foregoing analysis is to be noted.

VI. Concluding Remarks

The air- and kerosene-permeability of natural deposited snow was measured with three kinds of permeameters. First, the distribution of the values of air permeability was obtained as to various kinds of natural deposited snow in Hokkaido. Then, the change of the air permeability of a snow layer, caused by the metamorphosis of snow, was investigated through a winter. Finally, a structural analysis of natural deposited snow was attempted by the aid of fluid permeability, whereby a simple empirical formula of the specific permeability B_0 for fine-grained compact snow was obtained in terms of the "mean grain size" d_0 and the "effective porosity" (or specific gravity ρ_s^*). It was reasonably concluded that other independent structural variables than d_0 and ρ_s^* (or ε) would take almost constant values for all kinds of fine-grained compact snow. For other kinds of snow, i. e., new snow and coarse-grained snow, however, they may take other values than those for fine-grained compact snow, resulting in different values of the coefficients of d_0 and ρ_s^* , in the formula. Hence, further measurements of the permeability of other kinds of natural deposited snow are necessary for the next stage of investigations. As was frequently stated in the previous sections, the geometrical structure of a porous material such as deposited snow is extremely complicated and can possibly be described only in statistical terms. It is therefore expected that a further investigation on the permeability of deposited snow, either natural or artificially compressed, should be pursued with due consideration to statistical treatment.

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