Time to Formation of First Cracks in Ice

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Abstract

Observations have been made at two temperatures on the time to formation of the first internal cracks in columnar-grain ice subjected to a constant compressive load applied perpendicular to the long direction of the grains. Crack formation was observed to be a thermally activated process, with the time to formation of the first large crack (2 or more mm wide and 20 or more mm long) following the general law

$$t = t_0 \exp \frac{(U_0 - \alpha p)}{kT} .$$

The value of U_0, the activation energy for zero applied stress p, was found to be about 25 kcal/mole, t_0 about 4.6×10^{-17} sec and α about 1.4×10^{-20} cm^3/molecule. The amount of strain associated with the formation of the first crack was observed to increase with decreasing stress, but to be independent of temperature to within the accuracy of the experiments. The results were found to be consistent with the models for crack nucleation by pile-up of dislocations discussed by Stroh (1954) and Bullough (1964). It was observed that dislocation cracks could initiate Griffith cracks if the stress exceeded about 7 kg/cm^2, and no cracks were observed when the applied stress was less than about 5 kg/cm^2.

I. Introduction

In earlier papers (Gold, 1960, 1962) information was presented on the formation of internal cracks during the creep of columnar-grain ice under constant compressive load. A series of experiments has now been undertaken to determine the dependence of this cracking activity on stress, time, and temperature. It is the purpose of the present paper to report results of observations on the dependence of the time to formation of the first large crack on stress and temperature.

II. Experimental Procedure

The ice used in the investigation was prepared according to the method described by Gold (1960). This produces a columnar-grain ice with grain size between 1 and 6 mm. A bias in crystallographic orientation develops during freezing such that within 2 cm of the ice-air interface there is a marked preference for the basal plane to be parallel to the long direction of the columnar grains, but to have a random orientation in the plane perpendicular to that direction. Constant compressive loads, ranging in value between 4 and 20 kg/cm^2, were applied perpendicular to the long direction of the columns. The specimens were about 5×10 cm in section and 25 cm in length, with the long direction of the columns perpendicular to the 10×25 cm face.

Loads were applied to the 5×10 cm face of the specimen with a simple lever system. The loading plates in contact with the ice were made of steel, with surfaces ground to a flat "mirror" finish. Weights used for applying the load were applied or removed within about 2 sec.

Creep strain was measured over a 15.25 cm gauge length with an extensometer of variable sensitivity, the maximum being about 3×10^{-4} %. Observations have been completed at two temperatures: -9.5 ± 0.5°C and -31 ± 0.5°C.

Cracks that formed in the columnar-grain ice during creep under constant compressive load applied perpendicular to the long direction of the columns were long and narrow and usually involved only one or two grains. The long direction of the crack was in the long direction of the grain, and the plane of the crack tended to be parallel to the direction of the applied stress. Observations showed that the cracks tended to be parallel or perpendicular to the basal plane of grains in which they formed. The formation of a crack could be observed visually without difficulty by illuminating the specimen with a suitable light.

It was observed that there was a very considerable scatter in the time required for the first crack to form for a given applied load. If the load was great enough, very small cracks were observed to form during and immediately following the application of the load. This activity was followed by a period during which no cracks formed, or

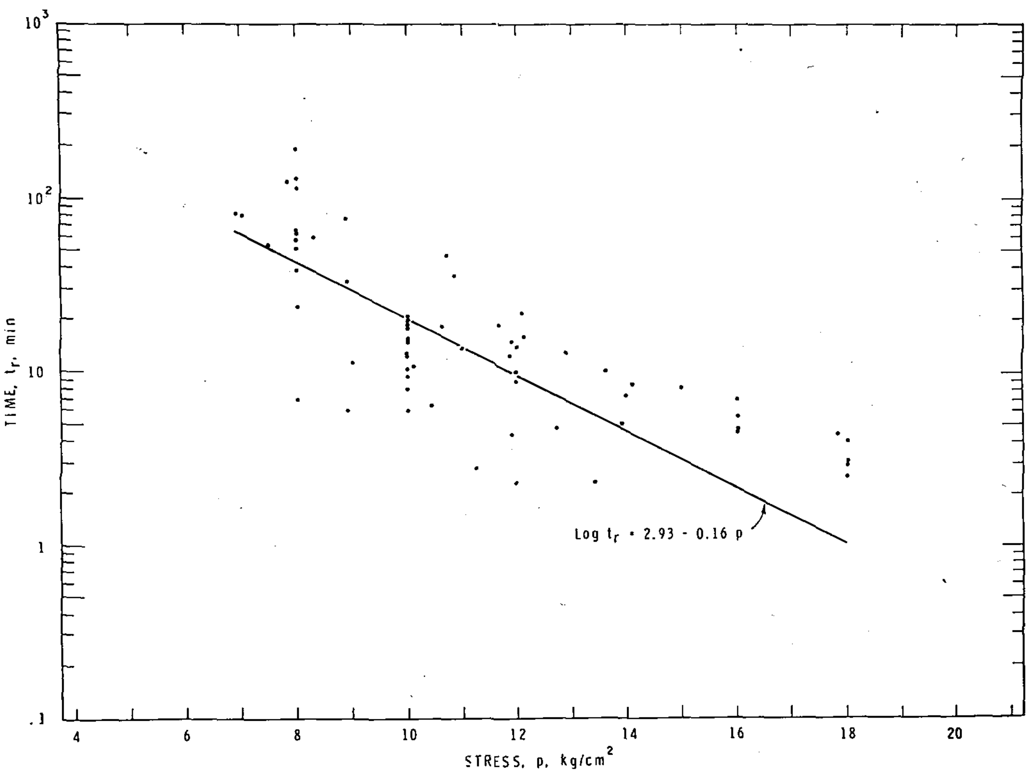

Fig. 1. Dependence on stress of the time to formation of the first large crack, temperature $= -9.5 \pm 0.5$°C

only a few that were very small. After a period of time larger cracks would form at a rate that gradually increased with time to a maximum. The formation of a large crack (greater than 2 mm wide and 2 cm long) appeared to be a reasonably well defined event. The times reported in this paper are those associated with the formation of the first of such large cracks.

III. Observations

It has been found for several materials that for the condition of constant temperature the stress dependence of the time to failure in tension is given by an empirical expression of the following form:

$$\log t_f = A + Bp, \tag{1}$$

where

t_f is time to failure, p is applied stress, A and B are constants.

The observations were analysed to see whether a similar dependence existed for the first large crack formed in ice by a compressive load.

In Fig. 1 the logarithm of the time to formation of the first large cracks observed at $-9.5°C$ is plotted against the stress. The same information for the temperature equal to $-31°C$ is given in Fig. 2. In both figures the line drawn is the least squares fit to the observations. For the tests at $-9.5°C$ the equation of the line is:

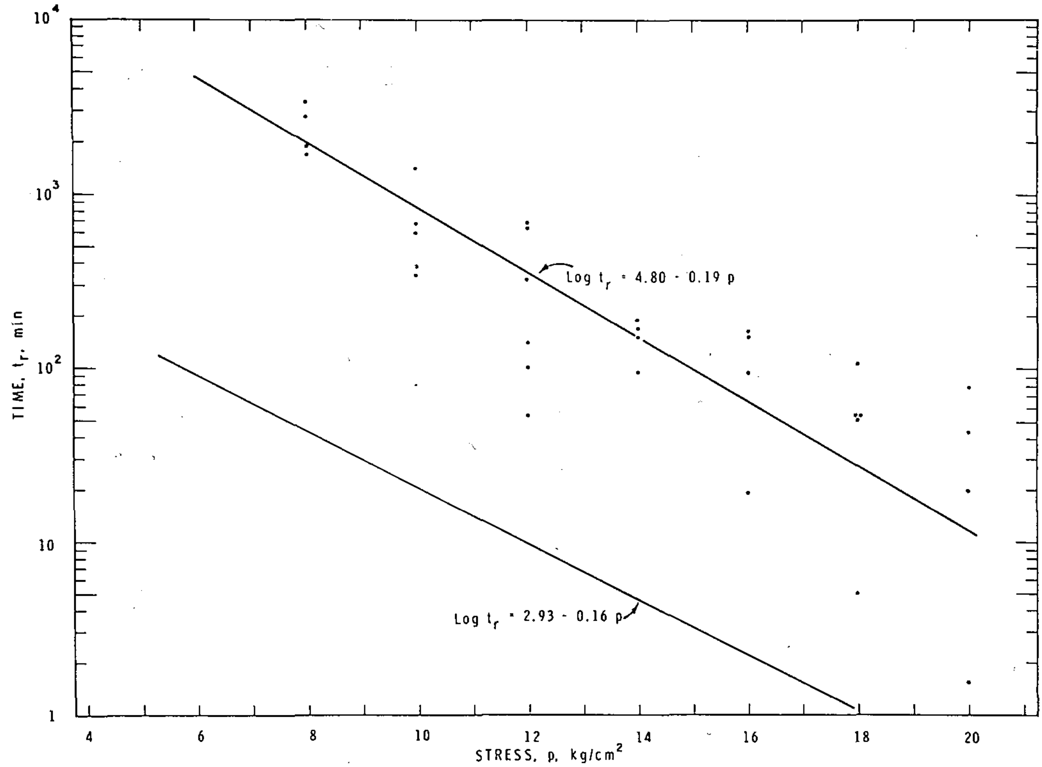

Fig. 2. Dependence on stress of the time to formation of the first large crack, temperature $= -31 \pm 0.5°C$

$$\log t_r = 2.93 - 0.16p, \tag{2}$$
$$\text{standard deviation} = 0.32,$$

where t_r is the time to the formation of the first large crack; and at $-31°C$,

$$\log t_r = 4.80 - 0.19p, \tag{3}$$
$$\text{standard deviation} = 0.42.$$

Both lines are drawn in Fig. 2 for comparison.

In calculating the least squares fit at $-9.5°C$, the observations at 16 and 18 kg/cm^2 were omitted as the average of the logarithms of the times associated with these stresses appeared to be significantly greater than would have been expected from the observations made at lower stresses. The specimens used for these tests were cut from only two blocks of ice, both of which had an average grain size of less than 3 mm. If these observations are included, the equation for the least squares fit becomes

$$\log t_r = 2.60 - 0.13p.$$

The logarithm of the creep strain e in percent associated with the formation of the first crack is plotted against the logarithm of the stress for $-9.5°C$ in Fig. 3, and for $-31°C$ in Fig. 4. Again, the line drawn is the least squares fit to the observations. At $-9.5°C$:

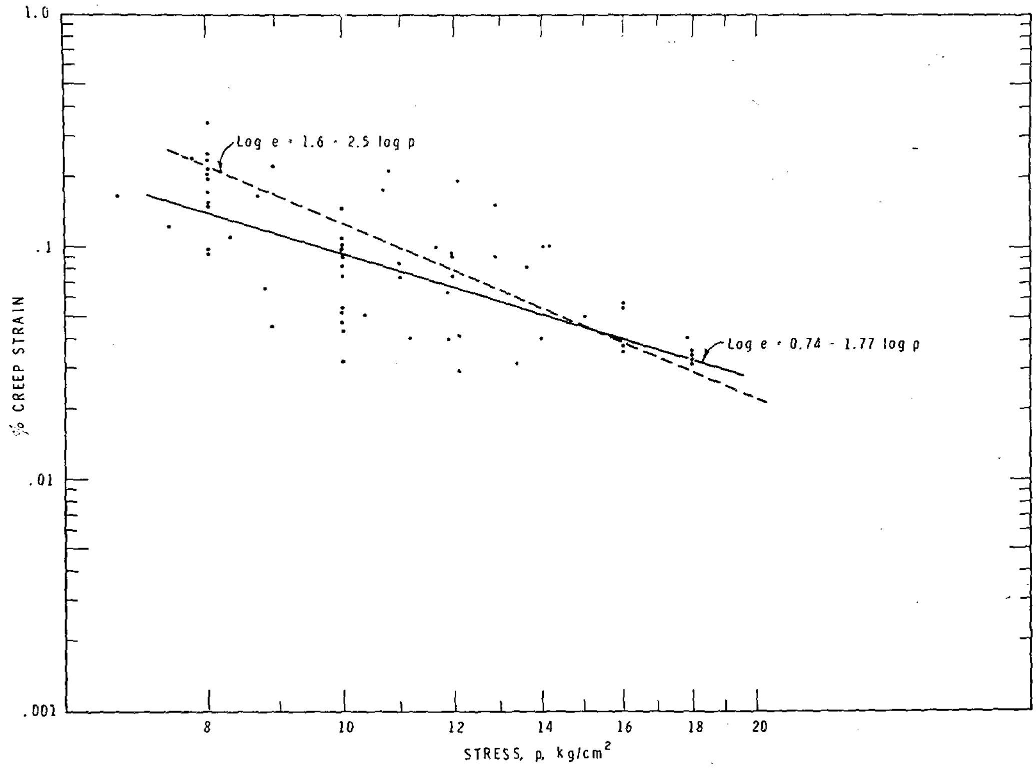

Fig. 3. Dependence on stress of the amount of creep strain to the formation of the first large crack, temperature $= -9.5°C$

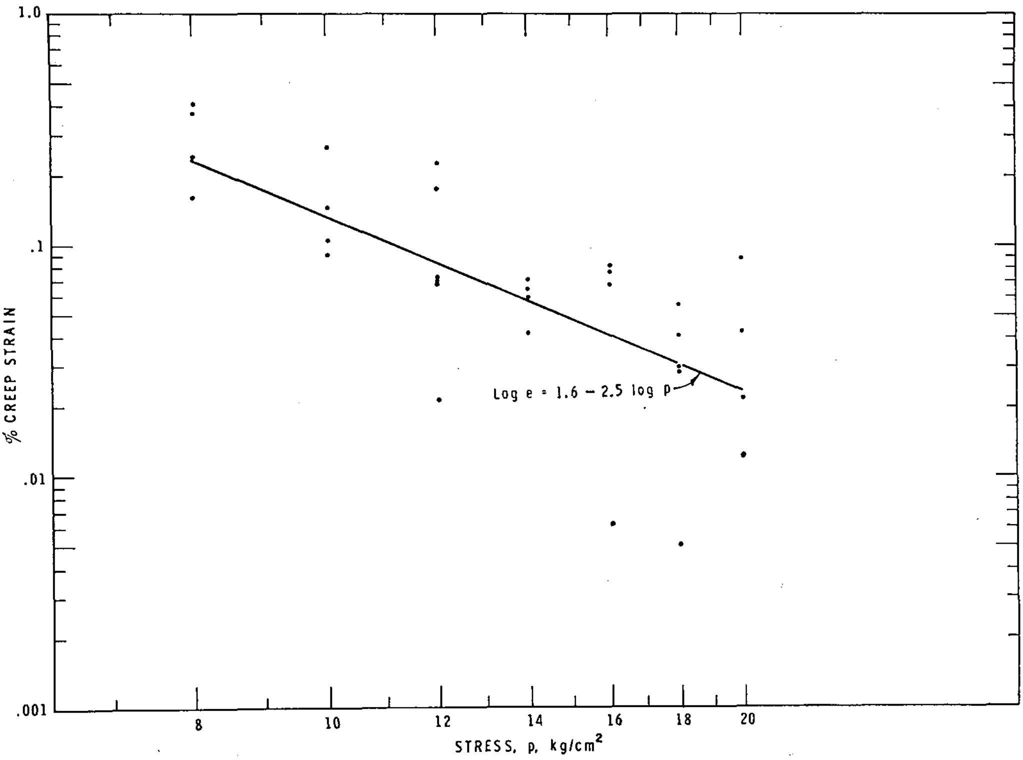

Fig. 4. Dependence on stress of the amount of creep strain to the formation of the first large crack, temperature $=-31°C$

$$\log e = 0.74 - 1.77 \log p\,, \tag{4}$$
$$\text{standard deviation} = 0.23\,.$$

If the points for $p=16$ and $18\ kg/cm^2$ are included,

$$\log e = 0.90 - 1.91 \log p\,.$$

At $-31°C$:

$$\log e = 1.6 - 2.5 \log p\,, \tag{5}$$
$$\text{standard deviation} = 0.33\,.$$

In Fig. 3 the best fit lines for -9.5 and $-31°C$ are presented for comparison.

In Fig. 5 is presented on normal probability paper the frequency distribution of ($\log t_r - \log \bar{t}_r$) for the observations at $-9.5°C$, where t_r is the observed time to formation of the first large crack for a given experiment, and $\bar{t}_r$ is the time for the same stress determined from eq. (1). A similar plot is given in Fig. 6 for ($\log e_r - \log \bar{e}_r$) for $-9.5°C$, where e_r is the observed creep strain at the time of formation of a crack, and $\bar{e}_r$ is the value calculated from eq. (4).

The straight line drawn in each figure was calculated assuming a normal distribution for the observations and standard deviations given with eqs. (2) and (4), respectively. These figures indicate that the observations had a random distribution about the calculated best fit line. A plot of ($\log t_r - \log \bar{t}_r$) and ($\log e_r - \log \bar{e}_r$) against grain size indicated

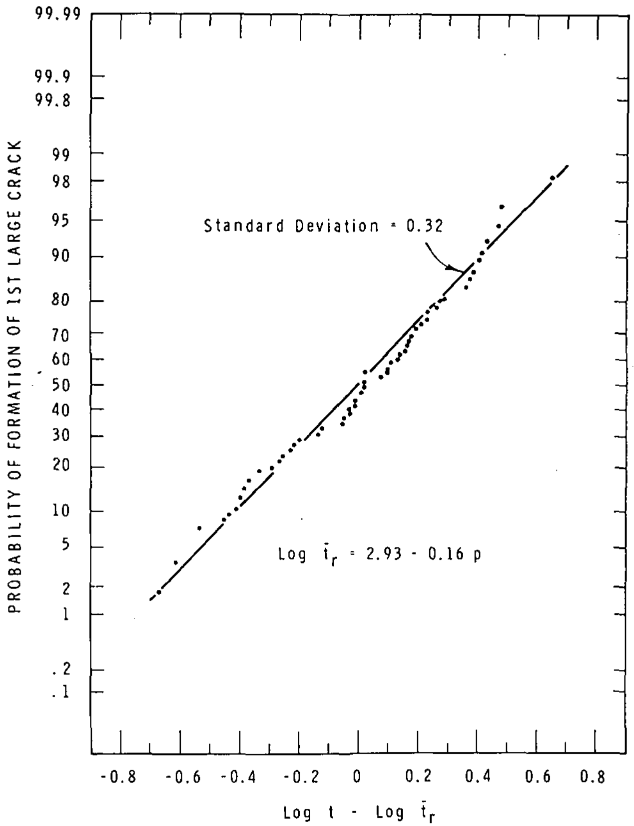

Fig. 5. Probability that $\log t_r - \log \bar{t}_r$ at time of first large crack exceeds given value

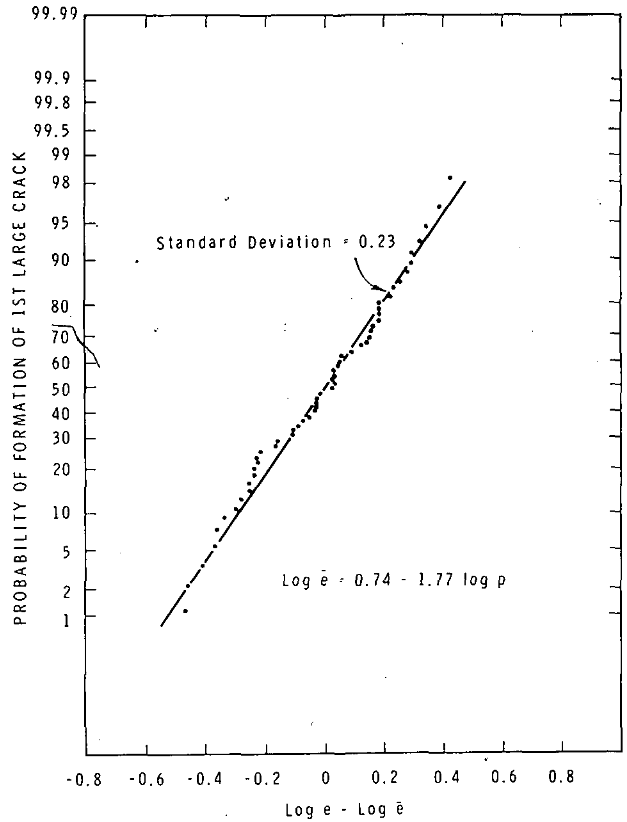

Fig. 6. Probability that $\log e - \log \bar{e}$ at the time of the first large crack is less than a given value $T = -9.5°C$

that these quantities did not depend significantly on grain size for the range of sizes associated with the tests.

IV. Discussion

Zhurkov and his co-workers (1958, 1960) and Sanfirovo *et al.* (1963) have shown for a number of metals and alloys that failure is a thermally activated process described by the general law

$$t_f = t_0 \exp \frac{(U_0 - \alpha p)}{kT}, \tag{6}$$

where

t_f = time to failure for an applied tensile stress p,
T = temperature in °K,
U_0, t_0 and α = constants that define the strength of the material,
k = Boltzmann's constant.

U_0 and t_0 have been observed to be independent of temperature and amount of creep strain; Sanfirova *et al.* (1963) and Zhurkov *et al.* (1963) have obtained evidence that α depends on the state of the structure.

Figures 1 and 2 indicate that for the range of stress and temperature of the experiments the time to formation of the first large crack in ice appears to satisfy eq. (5). If it is assumed that this is so, then from eqs. (1) and (6)

$$2.30\,A = \ln t_0 + \frac{U_0}{kT},$$

$$2.30\,B = -\frac{\alpha}{kT}.$$

Values for U_0, α and t_0 calculated from eqs. (2) and (3) are given in Table 1.

Table 1. Calculated values for α, U_0 and t_0 in $t_r = t_0 \exp \frac{(U_0 - \alpha p)}{kT}$

Temperature °C	α cm³/molecule	U_0 erg/molecule	U_0 kcal/mole	t_0 sec
−9.5	1.38×10^{-20}	17.66×10^{-13}	25.4	4.6×10^{-17}
−31.0	1.49×10^{-20}			

The value for U_0, the activation energy for $p=0$, was found to be equal to 25.4 kcal/mole. If the observations for 16 and 18 kg/cm² at −9.5°C are included, it is equal to about 30 kcal/mole. The value obtained for α was about 1.4×10^{-20} sec. It must be recognized, however, that these results are of a preliminary nature only because of the scatter in the observations and the fact that measurements were made at only two temperatures.

It is of interest that according to the result shown in Figs. 3 and 4 the amount of creep strain to the formation of the first large crack for a given stress is independent of temperature to within the accuracy of the experiments. This indicates that formation of a crack nucleus of critical size depends primarily on the amount of creep strain, that is, on geometrical factors and not on factors controlling the rate at which the deformation occurs.

Earlier observations (Gold, 1960) had shown that for the first 2 to 3 % creep strain of columnar-grain ice with the basal plane of grains tending to be parallel to the long direction of the columns, the deformation was almost wholly two dimensional; the creep strain in the long direction of the grains was much less than perpendicular to that direction. The average creep rate in the direction of the applied stress, therefore, should be equal but of opposite sign to that in the perpendicular direction. Since it is much more difficult to cause slip to occur on non-basal planes of ice than on basal planes, and since the dependence of the creep rate of ice on stress is the same for both tension and compression (Steinemann, 1954), it would be expected that those grains with their basal planes parallel or perpendicular to the applied stress would be subject to a transverse tensile stress whose maximum value would be about equal to the applied stress. It is, in fact, in grains with this general orientation that cracks are usually observed to form.

The orientation of the basal planes with respect to the grain boundaries and the

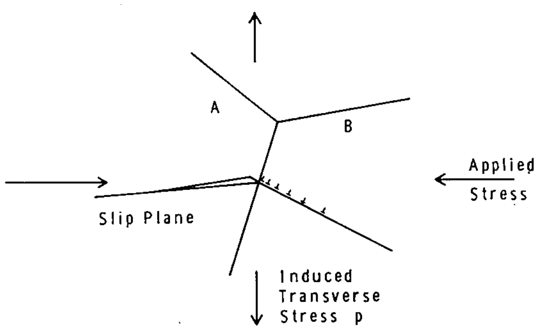

Fig. 7 a. Nucleation of crack in grain A by pile-up of dislocations at grain boundary in grain B

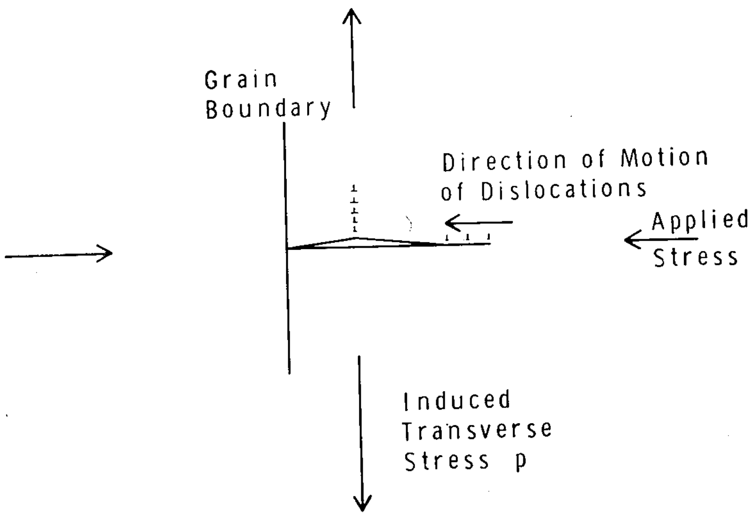

Fig. 7 b. Nucleation of crack due to pile-up of dislocations at grain boundary applied stress in the present experiments would appear to be most suitable for the nucleation of cracks by the pile-up of pure edge dislocations after the manner discussed by Stroh (1954, 1957) and Bullough (1964). Consider the situation shown in Fig. 7 a, where the crack in grain A has been nucleated due to the stress imposed by the pile-up of dislocations at the boundary in B. A possible example of such a crack is shown in

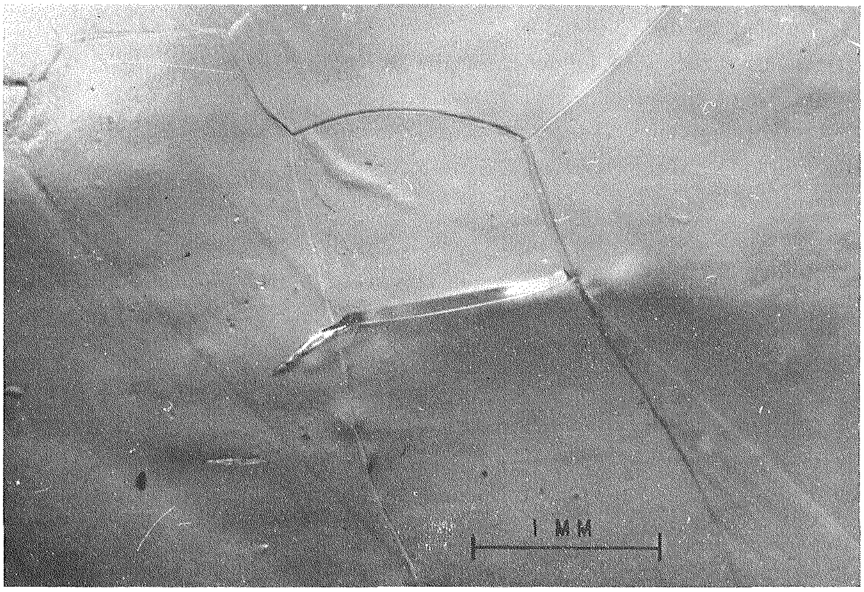

Fig. 8 a. Possible example of mechanism illustrated in Fig. 7 a.

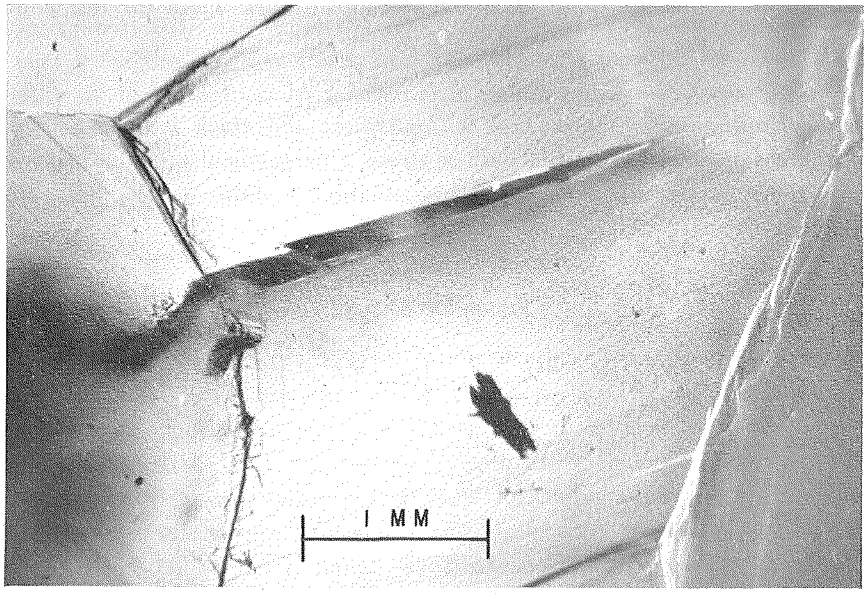

Fig. 8 b. Possible example of mechanism illustrated in Fig. 7 b. Note crack perpendicular to slip planes in grain on left-hand side

Fig. 8 a. Stroh gives for the energy of the crack per unit of length perpendicular to the figure:

$$W=\frac{n^2b^2G}{4\pi(1-\sigma)}\ln\frac{4L}{a}-\frac{pnba}{2}-\frac{p^2\pi a^2(1-\sigma)}{8G}+2a\gamma\,, \tag{7}$$

where

b=length of the Burgers vector,
n=number of dislocations,
L=effective radius of influence of the dislocations ($L\gg a$),
a=width of the crack,
G=rigidity modulus,
σ=Poisson's ratio,
γ=surface energy,
p=tensile stress perpendicular to the plane in which the crack forms.

The energy has a stationary value when $\partial W/\partial a=0$, from which one obtains for the crack width:

$$a=\frac{4G}{p^2\pi(1-\sigma)}\left\{\gamma-\frac{nbp}{4}\pm\gamma\left[1-\frac{nbp}{2\gamma}\right]^{1/2}\right\}. \tag{8}$$

This indicates that there are two possible values for the width. The value associated with the negative sign is a stable width. For $nb\ll 2\gamma/p$, $a=n^2b^2G/8\pi(1-\sigma)\gamma$; and is the equilibrium width of a crack formed by a pile-up of edge dislocations of strength nb when the transverse stress p can be ignored. The value associated with the positive sign is an unstable width, and for $nb=0$ corresponds to the width of nucleus required for the formation of a Griffith crack due to stress p (Griffith, 1921; Stroh, 1954). When $nb=2\gamma/p$ the two roots are equal, and the crack formed by the dislocation pile-up is just large enough to act as a Griffith crack. The critical crack width for this condition is

$$a_{\text{crit}}=\frac{n^2b^2G}{2\pi(1-\sigma)\gamma}=\frac{2G\gamma}{\pi(1-\sigma)p^2}. \tag{9}$$

A second situation is shown in Fig. 7 b, where the crack is formed in the same plane as the assumed pile-up, again with a stress p perpendicular to the plane in which the crack forms. A possible example of this situation is shown in Fig. 8 b. According to Bullough (1964)

$$W=\frac{n^2b^2G}{4\pi(1-\sigma)}\ln\frac{4L}{a}-\frac{\pi a^2(1-\sigma)p^2}{8G}+2a\gamma\,, \tag{10}$$

and for $\partial W/\partial a=0$

$$a=\frac{4G}{p^2\pi(1-\sigma)}\left\{\gamma\pm\left[\gamma^2-\frac{n^2b^2p^2}{16}\right]^{1/2}\right\}, \tag{11}$$

The two roots are now equal when

$$nb=\frac{4\gamma}{p}, \tag{12}$$

and

$$a_{\text{crit}}=\frac{n^2b^2G}{4\pi(1-\sigma)}=\frac{4G\gamma}{\pi(1-\sigma)p^2}. \tag{13}$$

In the present experiments the formation of a Griffith crack does not lead to failure of the specimen, but rather to the relaxation of the transverse stress, p. At a grain with a relatively large resistance to deformation the stress p would act on an area of width about equal to the grain diameter. The formation of a Griffith crack would involve, therefore, only one or two grains, particularly during the initial stages of deformation. It was observed that at $-9.5°C$ the formation of such large cracks occurred only for stresses greater than about $7\,kg/cm^2 = 6.86\times10^6\,dyn/cm^2$. For stresses less than this amount the cracks were small, with widths that appeared to be a millimeter or less. For stress less than about $5\,kg/cm^2 = 4.90\,dyn/cm^2$ no cracks were observed to form. In both cases the creep strain was significantly greater than that required for the formation of a large crack according to Fig. 3.

If it is assumed that

$\gamma = 105\,erg/cm^2$ (Hesstvedt, 1964),

$G/1-\sigma = 4\times10^{10}\,dyn/cm^2$ (Gold, 1958),

$p = 6.86\times10^6\,dyn/cm^2$ (equal but opposite in sign to the applied compressive load),

then substituting these values into eq. (9) gives

$$a_{crit} = 5.7\times10^{-2}\,cm\,,$$

for the case shown in Fig. 7 a.

Substituting these values into eq. (13) gives $a_{crit} = 11.4\times10^{-2}\,cm$ for the case shown in Fig. 7 b. These values are in reasonable agreement with the observations. The corresponding values for nb are $3.1\times10^{-5}\,cm$. If it is assumed that the length of the Burgers vector for ice, b, is $4.5\times10^{-8}\,cm$ then the corresponding values for n are 680 and 1 360.

The fact that the observations are in reasonable agreement with theory (when the energy associated with the formation of a unit area of crack is assumed equal to the surface energy for the ice-vapour interface) indicates that there is little or no plastic flow associated with the formation of a crack. A similar observation was made for cracks formed in ice by the application of a thermal shock (Gold, 1963). This indicates that for the stress situations studied it is easier for non-uniform internal stresses to nucleate and propagate cracks parallel and perpendicular to the basal plane than to cause significant slip on non-basal planes.

V. Conclusions

This study has indicated that the formation of cracks in ice is a thermally activated process. The amount of creep strain that occurs to the formation of the first large crack for a given applied compressive stress appears to be largely independent of temperature. This suggests that the process or processes controlling creep behavior are associated also with the nucleation of cracks. The characteristics of the initial part of the crack-forming process are consistent with dislocation models of crack nucleation discussed by Stroh (1957) and Bullough (1964). Dislocation cracks can nucleate Griffith cracks when the maximum applied shear stress (assumed equal to one-half of the applied compressive load) is about $3.43\times10^6\,dyn/cm^2$. Internal cracks do not form when the maximum applied shear stress is less than about 2.5×10^6 dyn/cm^2.

Acknowledgments

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