Ice Crystal Growth in a Temperate Glacier in Alaska*

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Abstract

The Hokkaido University Alaskan Glacier Expeditions carried out field studies on the mechanisms of ice crystal growth in the Mendenhall Glacier, Alaska in 1960 and 1964.

The 1960 expedition followed the general growth trend of the ice crystals from the glacier snow field to the terminus where many large single crystals were collected and returned to the laboratory in Hokkaido. Regional variation in orientation distribution of the *c*-axes of the ice crystals was observed in conjunction with structure of foliations. Concentration of direction of the *c*-axes of the crystals was found to occur in areas on the glacier where strong shear stresses existed. Foliated ice in such areas consisted of rows of comparatively fine grains formed by recrystallization under stress. Therefore, the strong shear stress in the glacier ice was considered as a counteracting factor for the crystal growth and it should limit the grain size probably not more than a few centimeter in diameter. The growth of the large single crystals observed at the terminus, some as large as basketballs should therefore be attributed to some accelerating mechanism occuring in a weak shear zone in the lower part of glacier.

The 1964 expedition found an exposed band of large single crystals in the center of the lower glacier under a large ice fall. The strain field around the site was measured using squares of stakes and compared to the strain field around an adjacent site composed of smaller grains. It was found that the band of large single crystals was in a zone of transition from compression at the foot of the ice fall to tension at the laterally expanding level portion of the glacier below the ice fall, whereas the area containing the smaller crystals was under compression. From this difference in the strain fields, it was concluded that the exaggerated growth of ice crystals in a temperate glacier is accelerated by the mechanism of strain annealing. The validity of this mechanism is discussed in view of some preliminary experiments in the solid growth of ice crystals.

I. Introduction

Fundamental studies of the plastic deformation of the ice crystals have developed a great deal since Dr. Nakaya's (Nakaya, 1958, 1967) very extensive work on the subject in early 1950s. Nakaya used large single crystals of ice from the Mendenhall Glacier in his experiments and most of the later studies (Butkovich and Landauer, 1959; Jellinek and Brill, 1956) of physical properties, especially of the mechanical behavior of ice single crystals made use of the same material. Small specimens may be used for mechanical tests on metals, but such tests when applied to ice require large single crystals which are difficult and time consuming to grow under laboratory conditions. The single crystals of natural ice from the Mendenhall Glacier are larger and have fewer defects than any synthetic crystals which it is now possible to grow, and they are consequently invaluable in solid state studies of ice.

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The author was stimulated by Nakaya's work to carry out extensive research on the solid state physics of ice single crystals and in 1960 led the first Hokkaido University Expedition to Alaska which brought back about 500 kg of large ice single crystals collected from ice bergs floating on Mendenhall Lake. The second Hokkaido University Alaskan Glacier Expedition in 1964 obtained about 1 000 kg of the crystals. The results of research on the mechanical properties of ice single crystals using these materials have already been presented in several publications (*cf.* Higashi, 1967 a). Both of the expeditions had additional purposes in carrying out glaciological field work on the Mendenhall and other glaciers in Alaska. The primary purpose of the field work on the Mendenhall was to investigate the growth mechanisms of large single crystals in a temperate glacier. The present paper describes the field work and proposes a mechanism for the exaggerated growth occuring in a temperate glacier.

In 1960, the general trend of ice crystal growth in the glacier was studied from various aspects. Grain size and crystal *c*-axis orientation were examined at various sites from the snow field at the top of glacier to the terminus. The velocity profile across the glacier was obtained at three different sites. Modes of flow and states of stress were deduced from various surface structural features, especially from crevasses and foliations.

Ice fabric studies of many samples at various places on the glacier disclosed that the concentration of direction of the *c*-axes of individual constituent crystals in glacier ice took place at the place of strong shear stress. Since the nearly parallel direction of the *c*-axes in adjacent crystals could not be an effective driving force for the crystal growth from the standpoint of surface energy, the strong shear stress acting in the glacier is considered as an opposing factor for the ice crystal growth. In addition, observations of micro-structure of foliations which also appeared predominantly at the place of strong shear stress revealed that bandings of small crystals were formed due to recrystallization. Thus, strong shear stress in glacier ice at marginal area near to either side banks or large medial moraines was considered as a counteracting factor against the crystal growth caused by simple annealing and the grain size might be limited, probably being not more than a few centimeter in diameter. It was therefore tentatively concluded that the growth of the large basketball-sized single crystals should be an exaggerated growth controlled by a different mechanism in a weak shear zone in the middle of the glacier.

A mechanism of strain annealing is very important in the exaggerated grain growth of certain types of metals such as aluminum, and it was expected that the same mechanism would be effective in ice crystal growth. Detailed strain tensor measurements were planned to test this hypothesis in an area where large single crystals were found. Fortunately, in 1964, a suitable area was found close to a control site containing smaller grains, in the middle of the lower Mendenhall Glacier. Strain measurements were obtained using squares of stakes. The results were analysed and are given in Table 3. These results show that strain annealing appears to be the most plausible mechanism for the exaggerated growth of ice in a temperate glacier.

II. General Characteristics and Structural Features*

The Mendenhall Glacier is one of the main glaciers which flow from the Juneau Ice Field to near the sea. The Juneau Ice Field at approximately 58°40′N–134°20′W, is nearly the southernmost point of the large nourishing snow reservoirs which range along the southern coast of Alaska (Fig. 1), and the glacier may therefore be considered a temperate glacier in the sense that it exists in a region of comparatively warm climate.

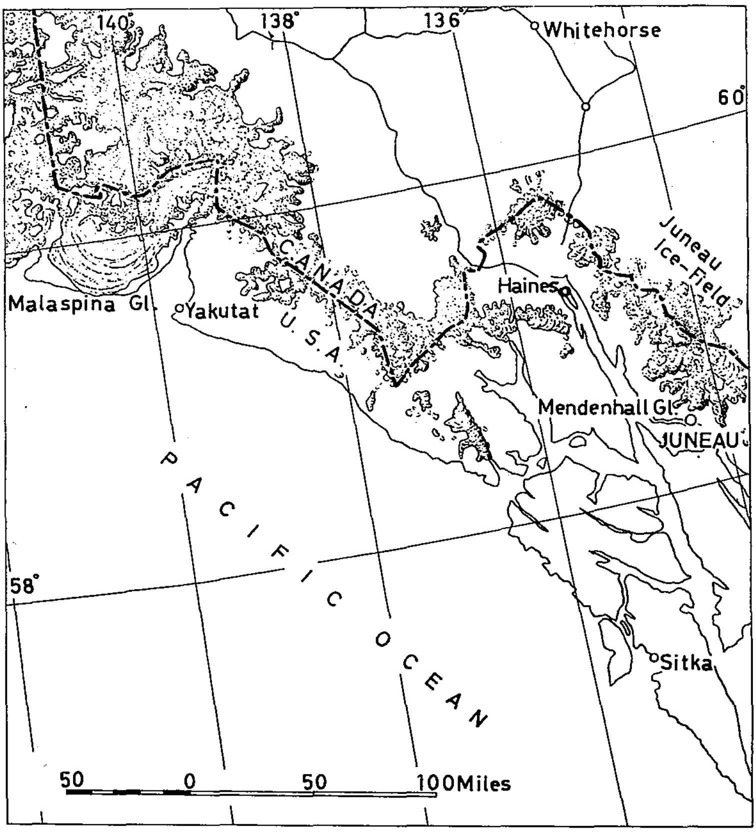


Fig. 1. Juneau Ice Field, Southeast Alaska

The glacier flows southward from the ice field for a distance of about 25 km. Its size and shape are given in Fig. 2 drawn from the United States Geological Service topographic maps and the expedition's observations. The two main tributaries entering from the left side were temporarily named the "Micki" and "Seven Sisters" glaciers and are so referred to in the text which follows. The average slope of the glacier as a whole is very gentle and as this average includes several ice falls many meters high, this slope between these falls is even more gentle.

On the northern end of the glacier, a relatively short ice divide with high, exposed nunataks, separated its nourishing reservoirs or firn from the main part of the Juneau

* Higashi, Hashimoto and others, 1961.

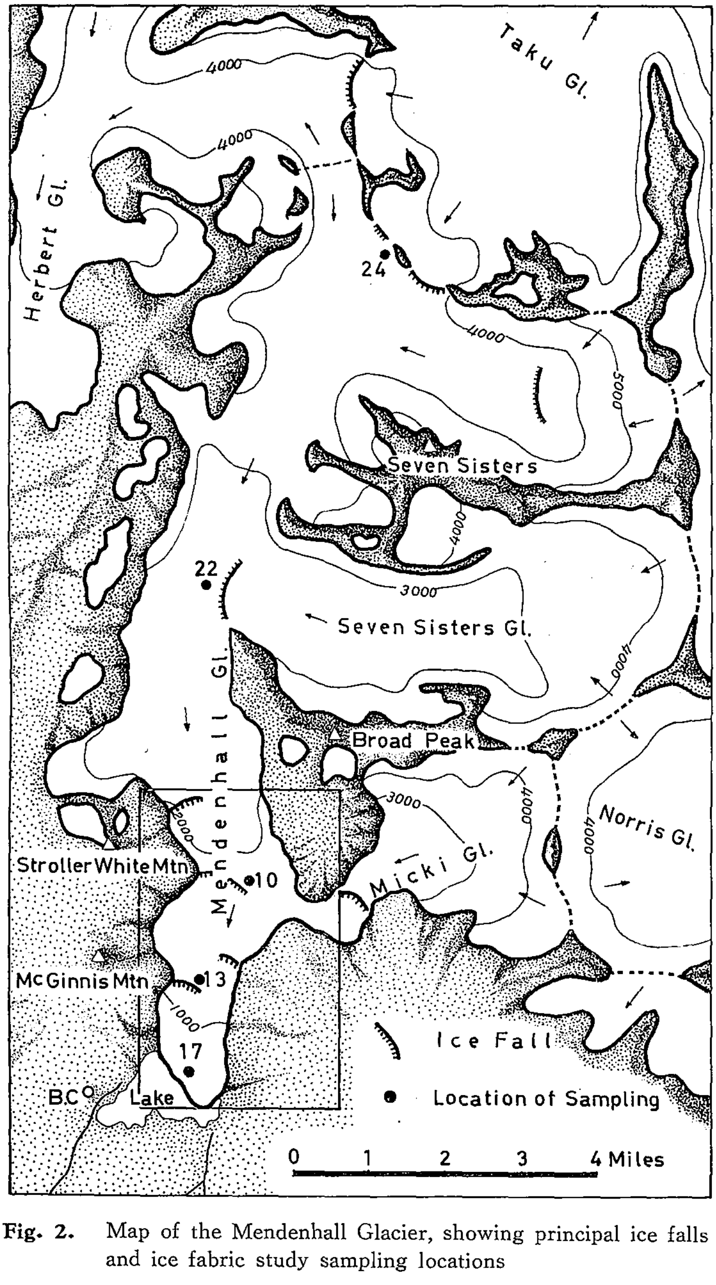


Fig. 2. Map of the Mendenhall Glacier, showing principal ice falls and ice fabric study sampling locations

Ice Field (dotted line at the top of Fig. 2). The lower limit of the firn area was determined to be at an altitude of approximately 2 500 ft (800 m), which is the altitude at which the Seven Sisters Glacier flows in to the Mendenhall. Therefore, the total area of the Seven Sisters Glacier actively nourishes the Mendenhall whereas the Micki does not much. Actually, the Seven Sisters seems to be more active than the main stream of the Mendenhall which originate mainly from upper reservoir behind the peaks of

Seven Sisters, because the rate of retreat of snout has been much less in the left (east) side than in the right (west) side (Higashi, 1967 b).

Structural features which produce various patterns on the glacier surface may be useful indicators of the general mode of glacier flow although they are sometimes too complex to be understood. These structural features were cerefully observed over the entire glacier and in particular detail in the half way up the glacier. Above approximately 2 000 ft (700 m), the snow cover on the ice made it impossible to observe the structural features. Aerial photographs provided by U. S. G. S. complimented the lack of information to some extent. Structural features are generally classified as stratifications, foliations, ogives or crevasses. Medial moraines may also be included. A brief description of the occurrence of these features on the Mendenhall Glacier follows.

a) *Medial moraines*

As is shown in Fig. 3, a photograph looking down the glacier from the west ridge of Broad Peak, the medial moraines are not well developed. The most remarkable is the one which lies at the junction of the Mendenhall and Seven Sisters Glaciers, and which appears to divide the flow into two parts. Another medial moraine starts from the junction with the Micki Glacier and diminishes toward the left because of the overall retreat of the glacier ice. Several very weak medial moraines may be seen on the right side of the glacier. The upper ends of these weak moraines vanish into small ponds where they join small separated tributaries coming in from the right side. The general features of the moraines and disposition of ice falls and other features are shown in

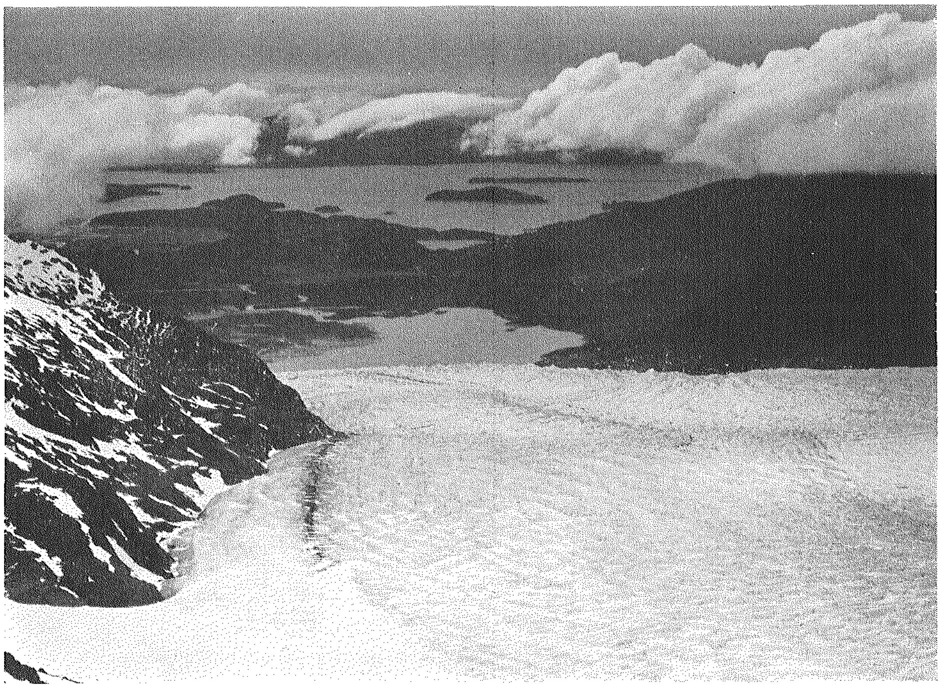


Fig. 3. Lower part of the Mendenhall Glacier, looking south from the west ridge of Broad Peak, showing Mendenhall Lake (center) and medial moraines

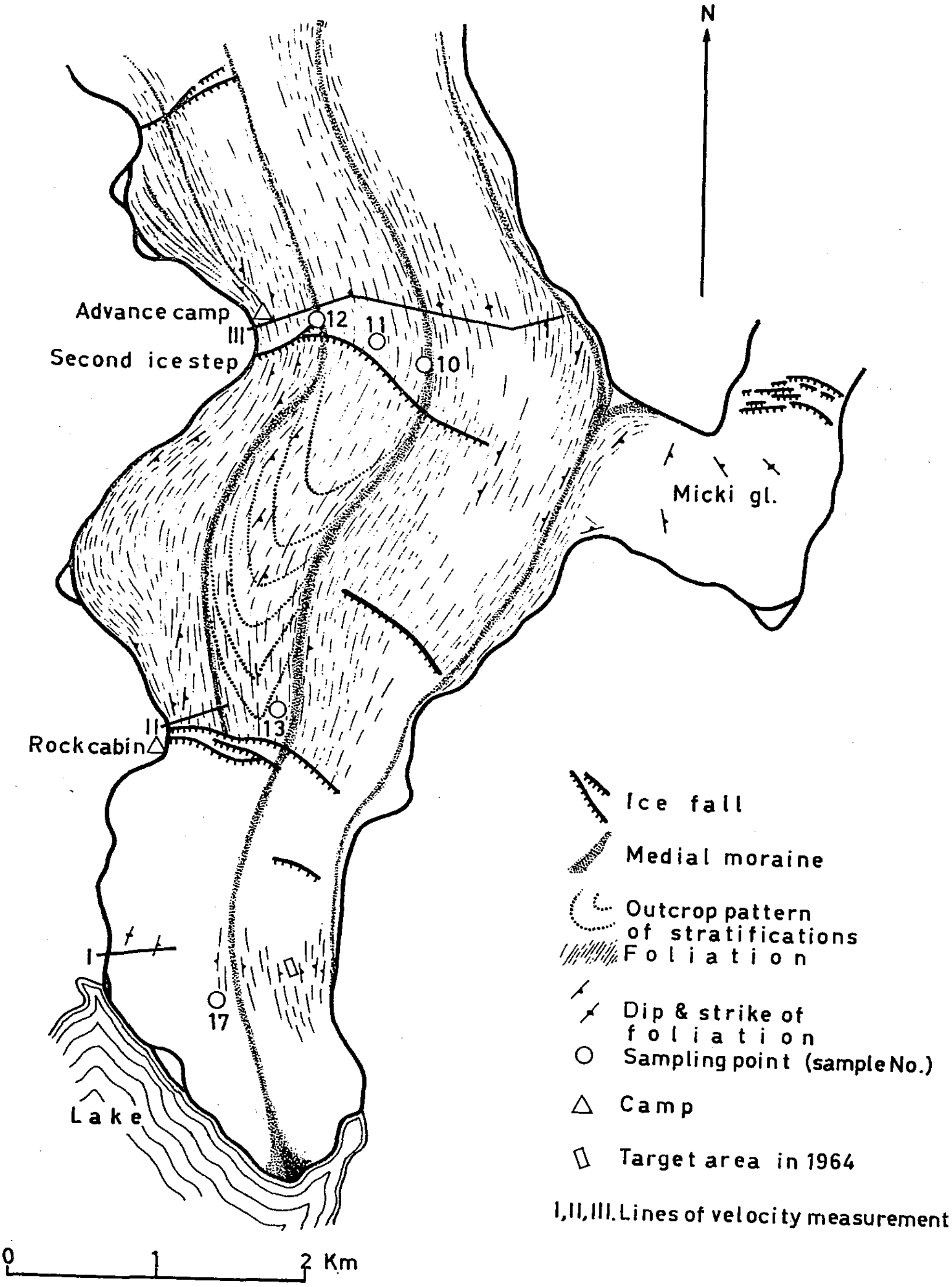


Fig. 4. Structural features and study areas, lower Mendenhall Glacier

Fig. 4, which is an enlargement of the terminal area delimited in Fig. 2.

b) *Stratification*

Stratification is defined as a sedimentary feature characterized by alternate layers of dirty ice which were originally lying flat. The dirty layers are formed by concentrations of dust particles contained in the ice or drifted to it during the summer from bare rocks near the firn area. Stratification may be easily recognized on the faces of the ice cliffs and in the walls of the crevasses from a point about mid-course in the Mendenhall down to a prominent ice fall near the point called "Rock Cabin", but no sedimentary features were observable below that point.

On fresh exposures obtained by scraping, the uppermost layer of each bed was found to be composed of blue coarse-grained granular ice 10 to 50 cm thick. These layers were formed during summer by percolation and refreezing of melt water from firn snow. The overall thickness of the beds was usually 1.5 to 2.0 m. Assuming the specific gravity of compact, water soaked, surface snow to be 0.5 g/cm^3 in the firn area, and that of glacier ice to be 0.9 g/cm^3, the thickness corresponds to an annual accumulation of snow of 2.6 to 3.6 m, a water equivalent of 1.35 to 1.8 m.

Although the stratifications appeared grossly flat, closer examination showed that the bedding was often severely compressed into steep, complex folds with ptygmatic or isoclinal habits. The limbs of the folds usually conformed to the general disposition of the foliation. An example of such folded stratification is shown in Fig. 5. These minor but characteristic, fluctuations are probably formed during glacial flow by differential slippage along the plane of foliation. The three-dimentional folding of the lines of

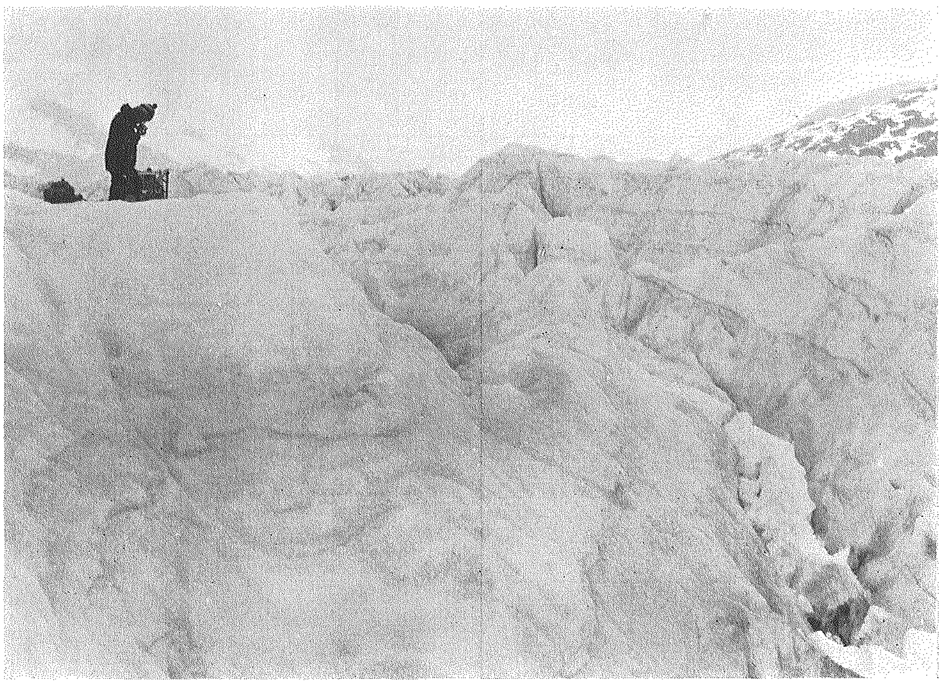
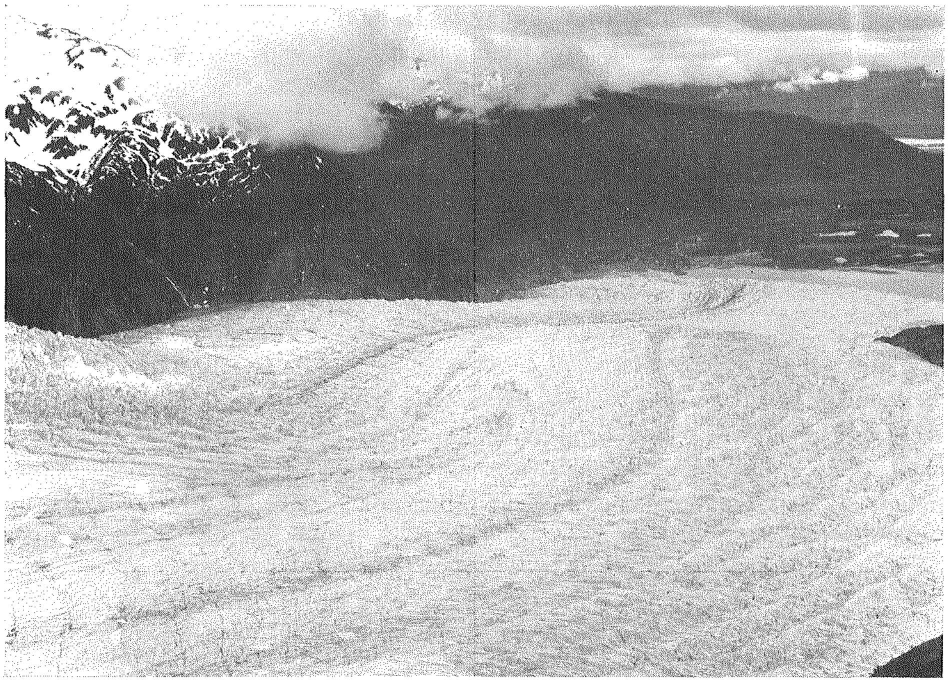


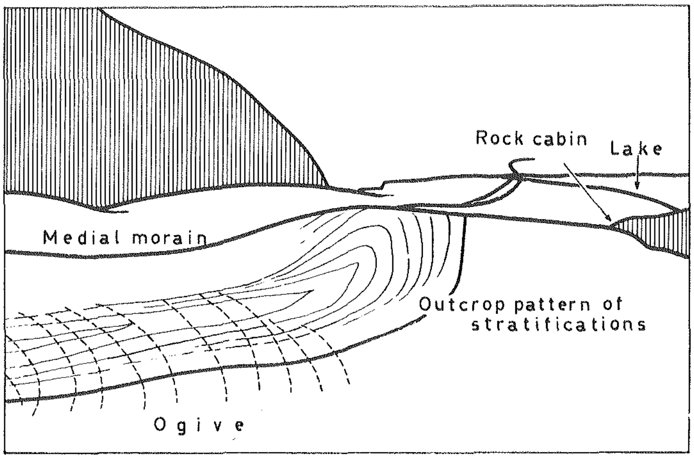
Fig. 5. Stratification, appeared on crevasse wall, middle of the glacier stream

stratification shows that the direction of the differential movement must vary from place to place and/or from time to time.

When these stratifications emerge at the inclined surface of the ablation zone of the glacier, they usually form outcrop patterns which are tongue-shaped and pointing down-glacier, or are characterized by regular dispositions of dirt bands in hyperbolic curves



(A)



(B)

Fig. 6. (A), Outcrop pattern of stratification below the Second Ice Step. Photograph taken from the east ridge of Stroller White Mountain. (B), Drawing for clarifying the pattern on the photograph (A)

on their surface. Photographs taken by the U.S.G.S. in 1948 show beautiful outcrop patterns on the upper glacier near the firn area, but in 1960, the snow cover lingered late into July, obscuring the upper reaches. However, a faint outcrop pattern may be discerned below the second ice step in Fig. 6 (A) taken on a cloudy day from approximately 2 500 ft high on the east ridge of Stroller White Mountain. Figure 6 (B) is included to help in finding the pattern on the photograph of Fig. 6 (A). This pattern is shown in the center of Fig. 4, too.

c) *Foliation*

Foliation is a secondary, compact, layered structure characterized by steeply dipping, alternating bands of granular, blue ice and bubbly, white ice. These bands were observed to consist of layers of coarse and fine-grained ice. Marked foliation developed in marginal areas and near medial moraines. In these areas, the thickness of the individual foliations varied from a few to a little more than 10 cm. Figure 7 shows foliations at the margin of the glacier near the advance camp.

Since the foliation runs nearly parallel to the direction of glacier flow in the main stream of the glacier, and is most strongly developed in the marginal areas and near the medial moraine, it must be associated with the strong shearing stress to which the ice at such areas is subjected. Laboratory experiments on recrystallization of coarse-grained or single crystals of ice under strong shear stress conducted by Shumsky (1958) and Rigsby (1960) also support this theory of the formation mechanism of foliation. If this interpretation is valid, it may be assumed that under the shearing stress produced by differential movement of ice in the middle reaches of the glacier,

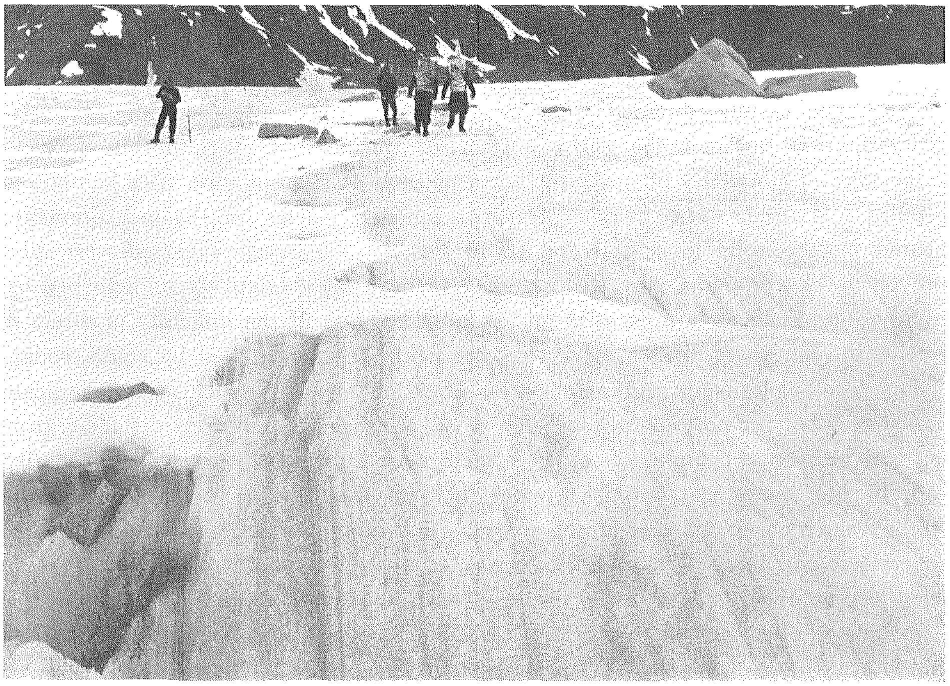


Fig. 7. Foliations at margin near advance camp

the foliation plane is a large-scale slip plane in glacier ice. As will be discussed later, the foliation was taken as an indicater of the slip plane of the glacier ice as a whole at locations where samples were taken for ice fabric studies. When the party traversed the glacier, the dip and strike of the foliation were measured and recorded at many places and the results are incorporated in the map in Fig. 4.

d) *Ogives and crevasses*

Ogives are semi-circles of alternating dark and light bands which transverse the flow direction of a glacier. There are well developed ogives on the Mendenhall, just below the prominent, "Rock Cabin Ice Fall". Other ogives are observed on the Micki Glacier, and some less developed ones below the second ice step. Since these ogives were not well developed either in amplitude or color contrast, and are not directly related to the problem of ice crystal growth, further description would serve no useful purpose.

Beautiful crevasse patterns were observed well up in the middle reaches of the Mendenhall. These patterns are typical illustrations of Nye's theory (Nye, 1952) of formation of crevasses based upon plasticity theory. Although crevasses may give some indication of the overall stress conditions in a glacier, they are not directly related to ice crystal growth, and further description is again omitted.

III. Factors Related to Grain Growth on the Glacier

a) *Surface velocity of glacier movement*

Measurements of the rate of ice movement are necessary and important to any investigation of glacier dynamics, but because of our limited time on the glacier (May 25–July 14, 1960), the velocity measurements given here are restricted to the surface only, and for the same reason, only show tendencies at certain locations during the summer. The results obtained were insufficient to allow interpretation of horizontal velocity distribution over the entire surface or the seasonal variation in the velocity. For the sake of simplicity of measurement a method of triangulation with bamboo marking poles was used. The lines of traverses on the glacier for the measurements are indicated by the solid lines I, II and III in Fig. 4. The areas traversed were all comparatively level where the flow was assumed to have been relatively undisturbed by any abrupt motion such as is present at an ice fall. Because of the difficulty of direct measurement, the length of the base lines on the bank was determined by stadia rods.

The results of the measurements are shown in Fig. 8, in which vectors indicate the actual movement for periods of about ten days as measured at various distances from the side. The horizontal components of these vectors are indicated by arrows. For clarity, the length of the vector is ten times greater than the scale of the distance. It is obvious that the velocity is much less at the margins and near the medial moraine. The maximum velocity on line III in midglacier reached about 70 cm/day. When the velocity is plotted versus the distance from the side of the glacier, there is a parabolic relation which indicates an abrupt increase of velocity in the marginal areas; this tendency strongly indicate the existence of strong shear in such areas. The triangulatation method used also allows measurements of vertical components of the velocity. Although the

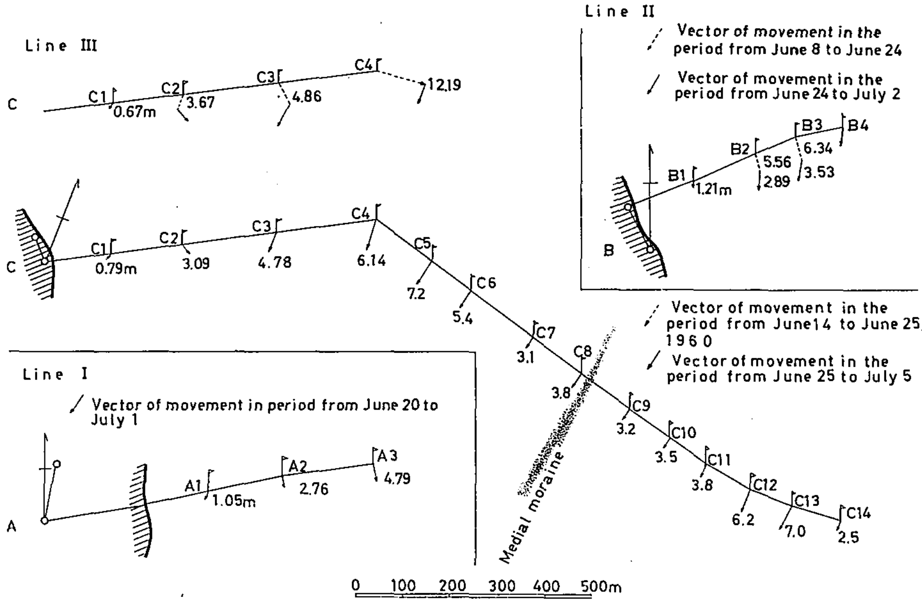


Fig. 8. *Velocity of ice movement at three transverse lines*

actual value for the vertical component was comparatively small and therefore includes large relative errors, it did agree in order of magnitude with the decrease of the surface level from ablation during the time the measurements were taken.

b) *Ice fabric study and grain size*

Ice fabric studies are one of the primary requirements in investigations of mechanisms of crystal growth in glacier ice. As has already been pointed out by various investigators such as Rigsby (1960), the orientation of the *c* or optic axes of glacier ice crystals tend to concentrate in a particular few directions with reference to the surface or to the foliation. This tendency is particularly noted in temperate glaciers, although it is also observed in polar glaciers. If the driving force for crystal growth in solid state is attributed to the difference in surface energy among adjacent single crystals contacting each other with different orientations, this tendency must be an opposing factor for the crystal growth. In this reasoning, the ice fabric studies were carried out to see in what extent the concentration of *c*-axes of crystals occured on the glacier.

About 30 samples were collected from various points on the glacier, and the location of some of these sampling areas are indicated in Figs. 2 and 4. Samples from areas numbered 24, 22, 10, 13, and 17 (Figs. 2 and 4) constitute a series collected approximately along the center line from the upper to lower glacier. Sample areas numbered 10, 11, and 12 (Fig. 4) were used to compare the tendency across the glacier at the height of the advance camp. Before any sample was removed, the orientation of the foliation passing through the sample was measured with a clinometer. The samples were returned to the advance camp, occasionally by helicopter, and examined in a snow cave using an universal stage especially designed for ice fabric studies. At this altitude, there was little melting of the samples. Thin sections ($\approx$2 mm) were made by melting the samples with a flat-bottomed cooking pan partially filled with warm water. In order to correlate any preferred orientation of c-axes with the foliation, the orientation of the foliation was noted on each specimen.

Fabric diagrams were obtained by plotting the c-axes on the lower hemisphere of a Schmidt equal-area projection net, and the results are given in Fig. 9. Geographical directions are ignored in this figure, though the diagrams represent horizontal sections of ice specimens, because the primary concern is to show the relation between the c-axes and the foliation planes which are indicated by curved lines crossing each diagram. Five density grades are given for the concentration of c-axes in 1% of the area of the Schmidt net.

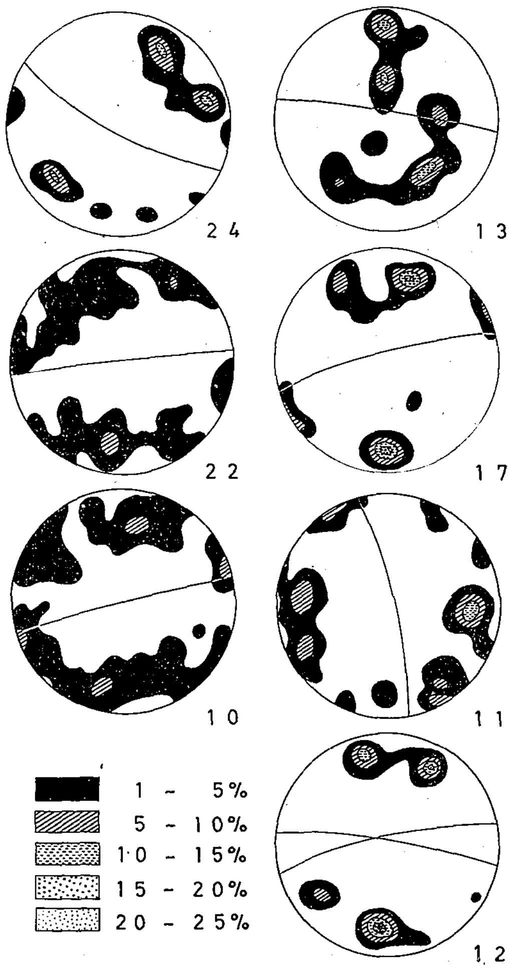


Fig. 9. Fabric diagrams for principal ice samples. See Figs. 2 and 4 for location

As has been previously noted by other investigators, every diagram showing perferred orientation has three or four maxima. Kamb (1959) treated the phenomena both from his own observations on the Blue Glacier and from experimental data on stressed ice crystals obtained by several workers; he concluded that the four maxima originated in a process of recrystallization such as occurs in glacier ice subjected to shear stress. As Rigsby (1960) has already observed in ice crystals in various temperate glaciers, the diagrams here show that the center of the cluster of maximal c-axes coincides roughly with the pole of the foliation plane. Therefore, if the foliation planes are considered as macroscopic shearing planes produced by differential movement of ice in glacier flow, there is a tendency for the c-axes of ice crystals to orientate in a direction normal to the maximum shear plane of the glacier. In other words, the planes of easy glide, or the basal planes, in ice crystals orientate so as to coincide with the maximum shear plane in the glacier.

Comparison of diagrams 22, 10, 13, and 17 in Fig. 9 reveals that the tendency described above becomes stronger lower on the glacier. Diagram 24 is an exception to this in that there is a strong preferred orientation even though the sample was taken higher on the glacier. A possible explanation for this is presented later. Further comparison of 10, 11 and 12 in Fig. 9 reveals that there is very strong preferred orientation at the margin and very weak preference at the center of the glacier. This is clear evidence for a theory that preferred orientation of c-axes of ice crystals is a result of strong shearing stress acting upon the glacier ice. The previously noted preferred orientation in the direction of the down-ward flow is probably explained by the length of time the crystals were subjected to the shearing stress even though the amount of shear was relatively small.

Determination of the mean size of the grains in various samples was greatly affected by the large numbers of fine grains in the foliation layers, but the mean diameter of the coarser grains increased substanitally down the glacier, varying from about 1 cm at the area No. 22, and $2\sim3$ cm at No. 10, to $4\sim5$ cm at No. 17. It may therefore be concluded that grain size increases with age of ice. The shape of the grains, especially of the finer ones, is somewhat elongated in the direction of movement. This can be seen in Fig. 10, which is a photograph of a thin section of ice crystals at the foliation taken under crossed polaroids. The photograph shows beautifully the structure of foliation, alternate bandings of finer and coarser grained ice crystals. It can be clearly seen that, when the ice was subjected to strong shear stress, coarser grains recrystallized into many finer ones were in rows running parallel to the plane of shearing.

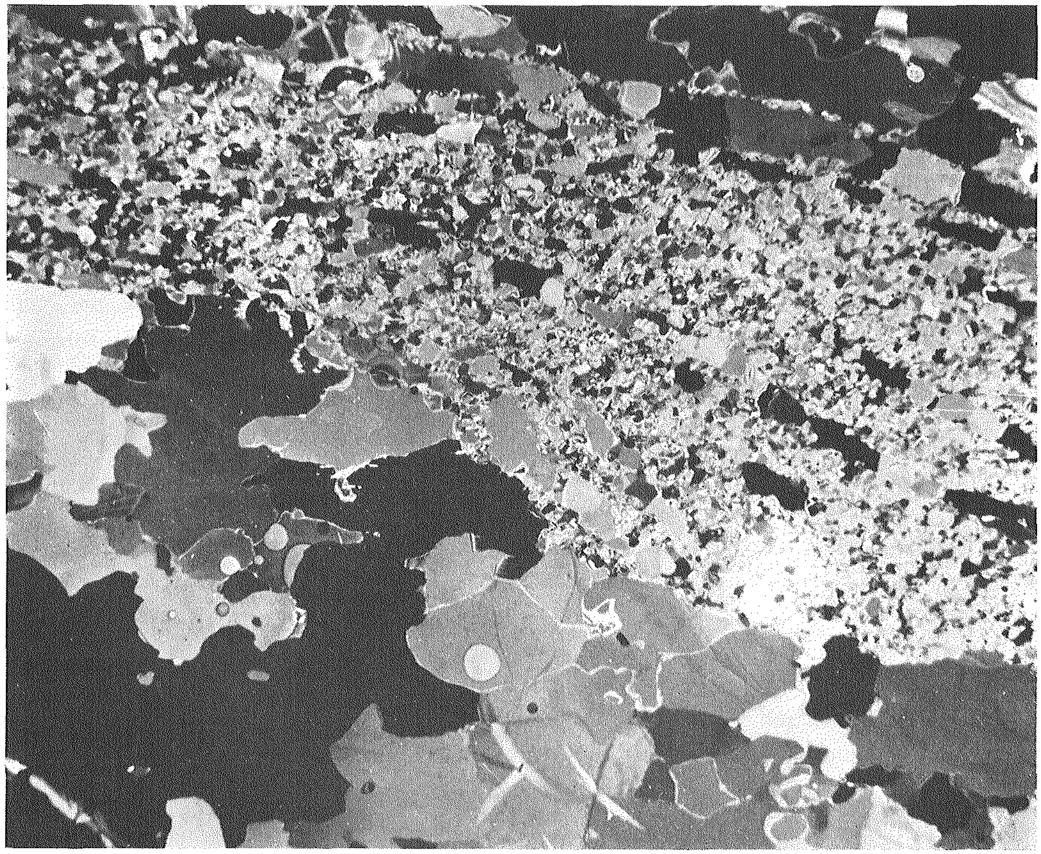


Fig. 10. Photograph under crossed polaroids of thin section of foliated ice. Note alternating layers of small and large grains, and elongation. ($\times1$)

IV. Strain Measurements

Although the general tendency of ice crystal growth in the glacier was indicated by observations and measurements at various sites from the upper to the lower end of the glacier, the 1960 expedition could not find the origin of the large ice single crystals which were collected from icebergs on Mendenhall Lake. It was simply assumed that these icebergs containing rows of large single crystals had come from the central portion of the left of the glacier snout which had been observed to be most actively calving. The conclusion drawn from the ice fabric study in conjunction with the foliation patterns was that the strong shear stress acting in marginal area either near to the side banks or to the large medial moraines act as a counteracting factor for crystal growth. From this reason, it was presumed that the large single crystals were probably formed in center of the glacial stream where the shear stress is less because there is less differential movement.

The Mendenhall Glacier is actually divided into two main streams by the central medial moraine. Since the right (west) side of the glacier is inactive and retreating, it is unlikely to produce calving icebergs and therefore the large single crystals collected on the lake must originate from central portion of the left (east) side stream. However, no such origin was discovered in 1960 by crossing this part of the glacier. If we consider that the ice crystals in the central portion of the left side stream passed through ice-falls on the way down, it becomes obvious that the most rapid growth must occur in the lower part of the glacier. Growth from a diameter of a few centimeter to that of a basketball in a rather short period of time must certainly qualify as exaggerated growth in crystal growth terminology and it was assumed that the growth mechanism was probably strain annealing. If this is a real mechanism, many large single crystals should be found on the left side of the glacier a little below the last ice-fall. It was expected that at this place the compressive stress just at the foot of ice-fall be released and probably transformed to tensile stress by lateral expansion of the glacier bed.

As a result of these conjectures, one of the primary objectives of the 1964 expedition was to find such an area abundant with large single crystals in an expected area on the glacier and to measure the strain-rate tensor there. A traverse on the glacier disclosed just such an area covering $100 \times 50\,m^2$ on the slope of a ridge about 500 m below the last ice fall and 300 m from the east bank (target area, Fig. 4). The crystals were very large; $10\sim15$ cm in diameters. Aerial view of this target area is shown in a photograph of Fig. 11.

In this area, the slope of the glacier becomes gentle and the glacier widens out so that it may be considered a transition zone from compressive to tensile stress in glacier ice. Away from this ridge, the grain size gradually diminished to about $4\sim5$ cm in diameter at a distance of 100 m in all directions. This fact makes this area an excellent place for comparison studies of states of stress in areas of coarse and fine grained ice.

Nye's improved method for determining strain-rate tensor on a glacier (Nye, 1959) was used in this investigation. Lines of strain-rate tensor measurements were established from aerial and surface observation of the glacier and are shown in Fig. 12. The series C_1, C_2 and C_3 covers a straight line along the ridge on which the large single crystals

Fig. 11. Aerial view of the target area for strain-rate tensor measurements. Lines C and F are approximate. Taken from helicopter, July 6, 1964

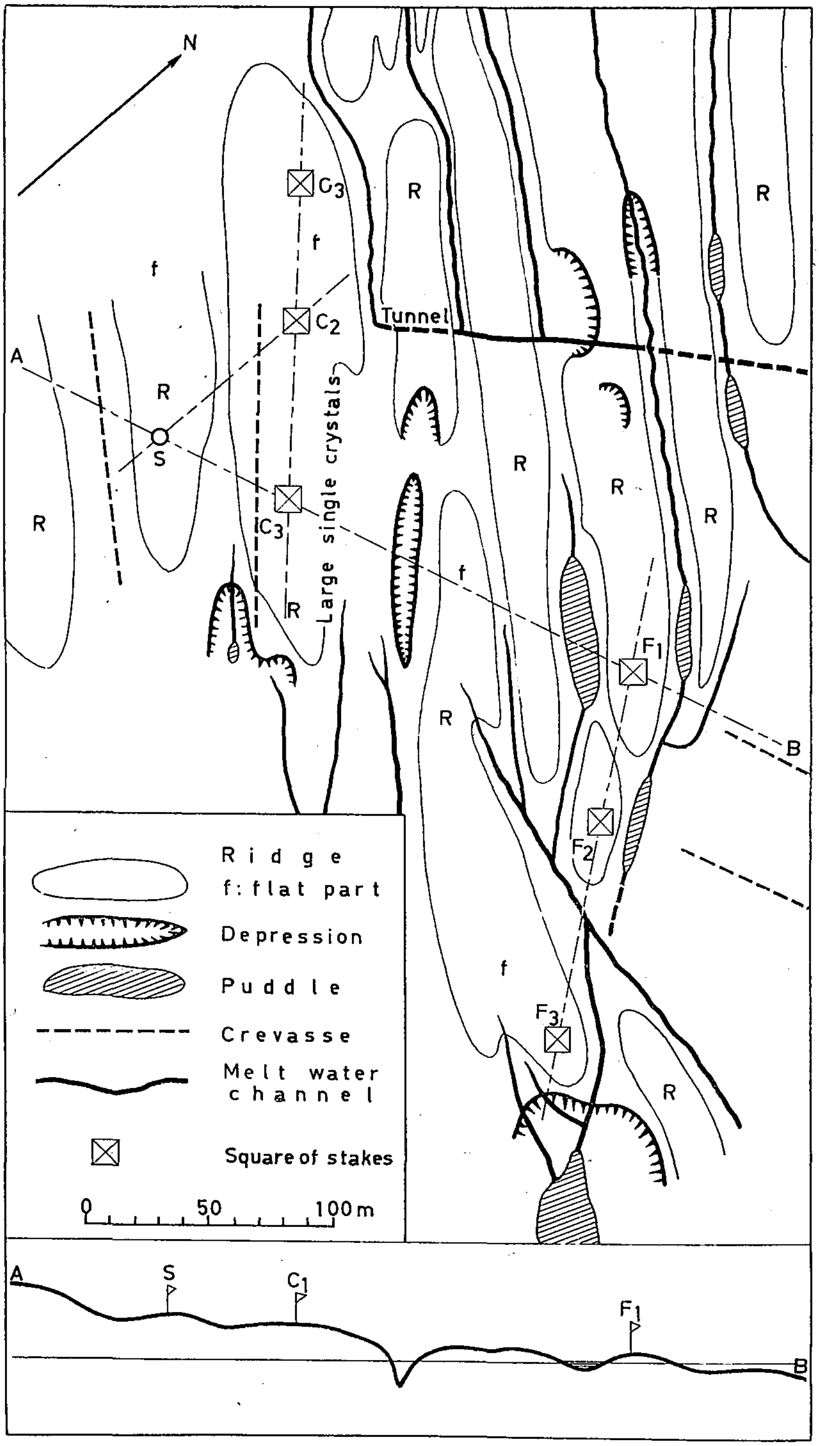


Fig. 12. Topographical map of target area, showing disposition of stake systems. Cross section along line AB at the bottom is schematic

were exposed, and that F_1, F_2 and F_3 is almost parallel to the C series in an area of fine grains. The points C_1, C_2, C_3 and F_1, F_2, F_3 all indicate key stakes in the center of a square of stakes. The initial distance from the center to each corner of the square was 8.00 m for C and 10.00 m for F. The initial direction of diagonals was laid out NS and EW (Magnetic) using a theodolite. Though the area was comparatively flat, errors in taping on the surface of the ice resulted unequalness of the length of the sides when they were measured after half of the diagonals had been determined. The discrepancy between the actual length of the sides and the theoretical values (C: 11.31 m; F: 14.14 m) may be seen in Table 1 which shows the measured values for the initial and final lengths of each sides and the diagonals. The maximum error was estimated to be approximately 1%. The time interval between measurements was 17 days (June 23–July 10, 1964) and the bamboo stakes were reset twice during this interval to prevent tilting or falling as a result of surface ablation.

Table 1. Measured values for the deformation of each square

Square	Quantity*	a_1	a_2	b_1	b_2	c_1	c_2	d_1	d_2
	l_1	8.00	8.00	11.23	11.395	8.00	8.00	11.315	11.24
C_1	l_2	8.06	8.10	11.44	11.49	8.28	7.95	11.38	11.42
	$\dot{\varepsilon}$	0.16045	0.26676	0.39784	0.17835	0.73872	−0.13464	0.12291	0.34118
	l_1	8.00	8.00	11.325	11.415	8.00	8.00	11.200	11.305
C_2	l_2	8.00	8.05	11.40	11.445	8.045	8.03	11.235	11.37
	$\dot{\varepsilon}$	0.00000	0.01610	0.01483	0.0548	0.01265	0.0786	0.06572	0.01292
	l_1	8.00	8.00	11.19	11.325	8.00	8.00	11.385	11.28
C_3	l_2	7.955	8.15	11.16	11.30	7.94	7.975	11.47	11.34
	$\dot{\varepsilon}$	−0.12114	0.39894	−0.05766	−0.04752	−0.1617	−0.06720	0.1597	0.1139
	l_1	10.00	10.00	14.225	14.20	10.00	10.00	14.26	14.20
F_1	l_2	10.00	10.01	14.10	14.02	9.94	10.015	14.27	14.15
	$\dot{\varepsilon}$	0.00000	0.02146	−0.18950	−0.27394	−0.12923	0.03218	0.01503	−0.07574
	l_1	10.00	10.00	14.18	13.99	10.00	10.00	14.31	14.105
F_2	l_2	9.97	9.96	14.18	14.00	10.02	9.97	14.20	14.02
	$\dot{\varepsilon}$	−0.06428	−0.08603	0.00000	0.01522	0.04292	−0.06428	−0.16580	−0.13005
	l_1	10.00	10.00	14.01	14.06	10.00	10.00	14.25	14.23
F_3	l_2	9.97	9.90	14.015	13.91	10.00	9.90	14.22	14.15
	$\dot{\varepsilon}$	−0.06453	−0.21584	0.07653	−0.23037	0.00000	−0.21584	−0.04524	−0.27339

* Units: l_1, l_2, m; $\dot{\varepsilon}$, yr^{-1}

The results are given in Table 1, in which the calculated values for the strain-rate, $\dot{\varepsilon}$ are included for each direction. Since Nye's method (Nye, 1959) was used to calculate the strain-rate tensor, there is no need to repeat the details of practical procedure of calculation here.

Values for $\dot{\varepsilon}$ were obtained for $\theta = 0$, 45, 90, and 135° (measured clockwise from magnetic north or oz in Fig. 13) using the average of the two values for *a*, *b*, *c*, and *d* in Table 1. The consistency of the data was checked by evaluating $(\dot{\varepsilon}_0+\dot{\varepsilon}_{90})-(\dot{\varepsilon}_{45}+\dot{\varepsilon}_{135})$

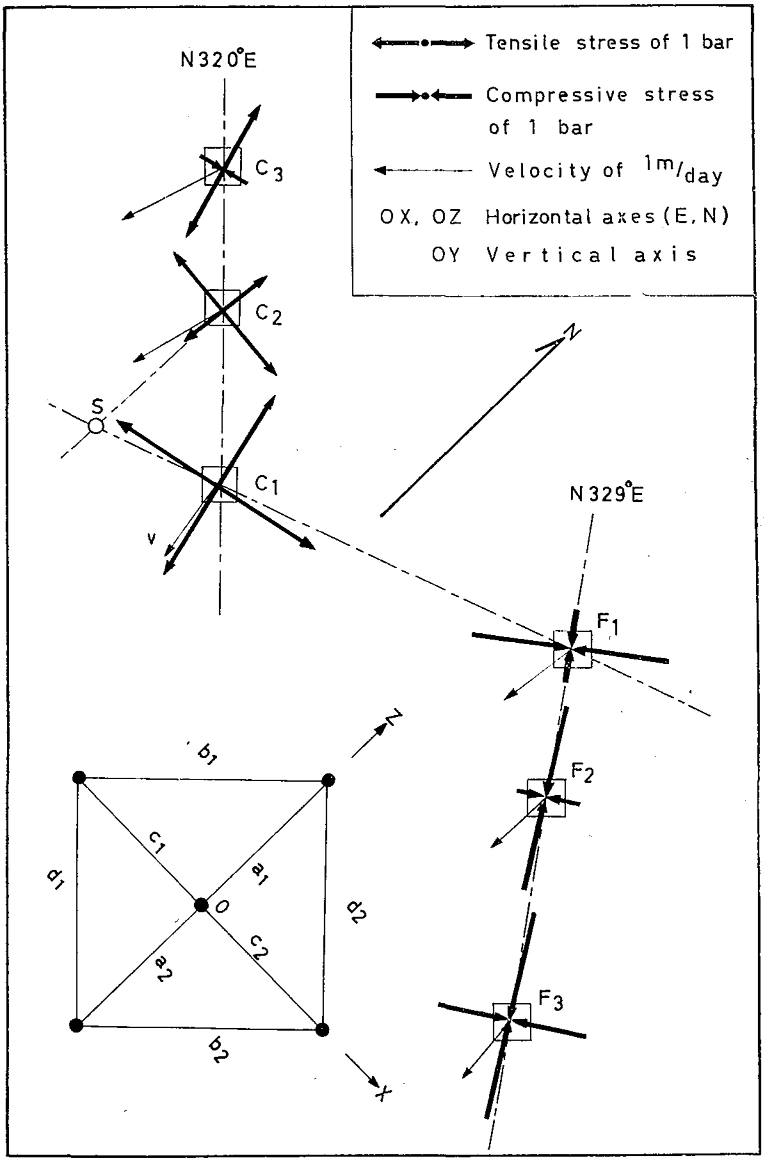


Fig. 13. Direction and magnitude of the principal stresses and horizontal components of velocity in target area. Compare with Fig. 12

and the absolute value of this quantity did not exceed 0.06 except in the case of F_1 where it was 0.223. The small values obtained prove the reliability of data. The strain-rate components, $\dot{\varepsilon}_x$, $\dot{\varepsilon}_{zx}$ and $\dot{\varepsilon}_z$ were calculated for each square from $\dot{\varepsilon}_0$, $\dot{\varepsilon}_{45}$, $\dot{\varepsilon}_{90}$, and $\dot{\varepsilon}_{135}$ and are given in Table 2, which also includes the standard error.

The principal strain rates in the horizontal or XZ plane ($\dot{\varepsilon}_1$ and $\dot{\varepsilon}_3$) were obtained using the values from Table 2 and Mohr's circle. The principal strain-rate, $\dot{\varepsilon}_2$, corresponding to the perpendicular axis OY (Fig. 13) was obtained by $\dot{\varepsilon}_2=\dot{\varepsilon}_y=-(\dot{\varepsilon}_x+\dot{\varepsilon}_z)$, assuming a constant volume for each element of ice. These results are given in Table 3.

Table 2. Strain-rate components (yr^{-1})

Square	$\dot{\varepsilon}_x$	$\dot{\varepsilon}_{xz}$	$\dot{\varepsilon}_z$	Standard error
C_1	0.3044	0.0255	0.2160	0.0041
C_2	0.0507	−0.0023	0.0132	0.0089
C_3	−0.0995	−0.0947	0.1538	0.0259
F_1	−0.1046	−0.1007	−0.0453	0.0968
F_2	−0.1050	0.0412	−0.1372	0.0236
F_3	−0.0009	0.0646	−0.4129	0.0051

Table 3. Principal strain-rates and stresses

Square	$\dot{\varepsilon}_1$	$\dot{\varepsilon}_2$	$\dot{\varepsilon}_3$	ϕ	$\dot{e}$	τ	$\tau/\dot{e}$	σ_1'	σ_2'	σ_3'	σ_1	σ_3	σ
C_1	+0.311	−0.520	+0.209	$\dot{\varepsilon}_3$ N 15°W	+0.453	1.306	2.882	+0.896	−1.50	+0.602	+2.40	+2.10	+1.50
C_2	+0.054	−0.064	+0.013	$\dot{\varepsilon}_3$ N 3.4°E	+0.0585	0.804	13.75	+0.700	−0.88	+0.18	+1.58	+1.06	+0.88
C_3	+0.186	−0.055	−0.131	$\dot{\varepsilon}_1$ N18.6°W	+0.165	1.027	6.223	+1.154	−0.342	−0.818	+1.50	−0.47	+0.34
F_1	+0.030	+0.150	−0.180	$\dot{\varepsilon}_1$ N 37°W	+0.167	1.030	6.176	+0.182	+0.926	−1.109	−0.744	−2.04	−0.926
F_2	+0.027	+0.113	−0.141	$\dot{\varepsilon}_3$ N33.7°W	+0.129	0.969	7.504	+0.20	+0.85	−1.05	−0.65	−1.91	−0.85
F_3	−0.077	+0.242	−0.166	$\dot{\varepsilon}_3$ N34.3°W	+0.215	1.093	5.10	−0.39	+1.23	−0.84	−1.62	−2.08	−1.23

Units: Strain-rate, yr^{-1}; Stresses, bars

In this table, the angle between OZ and the principal axes closest to it are indicated by ϕ. The principal axis is that of the algebraically greater principal strain-rate, $\dot{\varepsilon}_1$, when $\dot{\varepsilon}_z > \dot{\varepsilon}_x$ and the algebraically lesser principal strain-rate, $\dot{\varepsilon}_3$, when $\dot{\varepsilon}_z < \dot{\varepsilon}_x$.

The results of the stress calculations given in Table 3, are based upon the reciprocal form for the flow law of ice given by Glen (1955) as follows.

$$\tau = 1.577\, \dot{e}^{0.238}\,,$$

where τ is an effective shear stress and $\dot{e}$ is an effective strain-rate. The principal stress deviators, σ_1', σ_2' and σ_3', are given as follows:

$$\sigma_1' = \frac{\tau}{\dot{e}}\,\dot{\varepsilon}_1\,, \qquad \sigma_2' = \frac{\tau}{\dot{e}}\,\dot{\varepsilon}_2\,, \qquad \sigma' = \frac{\tau}{\dot{e}}\,\dot{\varepsilon}_3\,,$$

and the principal components of stress, σ_1 and σ_3 (putting $\sigma_2=0$ for the direction perpendicular to the surface) are obtained by

$$\sigma_1 = 2\sigma_1' + \sigma_3'\,, \qquad \sigma_3 = \sigma_1' + 2\sigma_3'\,.$$

The values of the hydrostatic pressure $\sigma = 1/3\,(\sigma_1 + \sigma_3)$ and the principal stresses σ_1 and σ_3 are given in the table.

In Fig. 13, the magnitude and direction of the principal stresses are indicated by heavy arrows, and the velocity of each key stake by light arrows. The position of the key stakes are those given for June 23 and the principal stresses are for the mean survey date July 1. The remarkable feature of the stress distribution in this area is that all stresses on C are tensile with the exception of σ_3 at C_3 which is slightly compressive. All stresses on F are compressive.

V. Mechanisms of Glacier Ice Crystal Growth

The stress distribution obtained in the target area on the surface of the glacier indicates that transition from compressive to tensile stress is associated with exaggerated growth of ice single crystals. Since it is quite obvious that glacier ice below an ice fall is subjected to a strong longitudinal compressive stress, it may easily be assumed from the topography of the target area which extended about 500 m below the last ice fall, that the northern end of the area was still in compressive stress. Therefore, the stress is changing from compression to tension on line C in this area and this change means that individual crystals are gradually being released from compression and subjected to tension. This release from strain should give rise to the strain annealing process of crystal growth.

On the other hand, compressive principal stresses at every point on line F indicates that this area, in which no exaggerated crystal growth was observed, is in compression. It may, therefore, be concluded that the large single crystals found on line C were grown by the mechanism of strain annealing. The details of the strain annealing mechanism in ice crystal growth are not yet clear, although some preliminary experiments have been conducted by Rigsby (1960) and recently by the present author (Higashi and Haga).

Since crystal size decreased up-stream approaching the average about 100 m above C_3, (Fig. 12) crystal growth is probably a very rapid process, occuring in a distance of about 200 m. If the average velocity here is assumed to be as shown in Fig. 13, approximately 1 m/day, then this distance corresponds to about 200 days. If ice crystals of a mean grain size of 5 cm in diameter grow to that of 15 cm in 200 days, the growth rate is about 0.5 mm/day which is reasonable growth rate for strain annealing. It is almost the same order of magnitude obtained in the author's preliminary laboratory experiments and about 10 times that of simple annealing at $-0.2°C$. Rigsby gives the same order of magnitude for his annealing experiments (Rigsby, 1960, Figs. 12 and 16). Therefore, the exaggerated growth occured in a distance about 200 m on the glacier is not too rapid viewed in the light of experimental data.

If this mechanism for rapid crystal growth by strain annealing is real, exaggerated growth should occur not only in the specific target area but in other area in the lower part of the glacier where conditions are favorable for the release of strain. The large single crystals found in some of the icebergs on Mendenhall Lake are produced in such areas on the glacier. And the fact that the number of large single crystals which appear in the lake varies from year to year may possibly be explained by changes in stresses or other conditions in those areas where they are produced. Comparatively large single crystals of ice found in an ice-fall on the upper glacier (Sample No. 24) may be also attributed to a locally operating strain annealing mechanism. Temperature profiles obtained by Miller (1958) at high camp on the Taku Glacier show that below a depth of 65 ft in the névé and ice, the temperature is not subjected to seasonal variation and remains close to 0°C at all times. It may be assumed from this that simple annealing should occur throughout the entire glacier flow and that this process should produce larger crystals than those actually observed in the lower part of the Mendenhall.

The observed growth rate, from snow flakes in the upper basin to diameters of 4～5 cm on the lower glacier, appears slow in comparison to experimentally obtained growth rates of grains in polycrystalline ice at temperatures near the melting point. It appears, therefore, that the process of simple annealing is counteracted by shearing stresses in the glacier which produces small recrystallized grains.

There are two mechanisms for ice crystal growth in a temperate glacier; the one is the simple annealing which governs the general growth trend throughout the glacier flow, and the other is the strain annealing which explains the growth of some basketball-sized large single crystals in a lower glacier. Crystal size attained by the simple annealing mechanism is limited only a few centimeters in diameter by the counteraction of recrytallization caused by shearing stresses in glacier flow. Therefore, it needs the strain annealing to have extraordinary large single crystals.

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