Wind- and Water-Drag of an Ice Floe*, **

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Abstract

A simple method for directly measuring wind-drag of an ice floe is presented together with several preliminary results, which give 0.002~0.025 cm for the roughness parameters of snow-covered upper surface of ice, and a direct method for measuring water-drag is proposed.

I. Introduction

Knowledge of wind- and water-drag of an ice floe is indispensable for the study of ice drift, yet few direct measurements of them have ever been reported. Since 1963, use has been made of flat shore ice covering the harbour of Mombetsu on Okhotsk coast of Hokkaido for measuring the force mainly exerted by wind on an ice disk. On the other hand, a direct measurement of water-drag of an ice floe has been first attempted in last winter. The both methods seem hopeful, though several improvements shall be required for them to be practical.

II. The First Method

II. 1. *Principle.* Figure 1 illustrates the method schematically. Narrow ring shaped area is cleared of ice from flat shore ice field, so that an ice disk floating in a circular pool is obtained. Pivot axles are frozen in ice at the center of the disk and at several points on the pool side. Strings with some sort of tensiometers are strung between the center axle and the side axles.

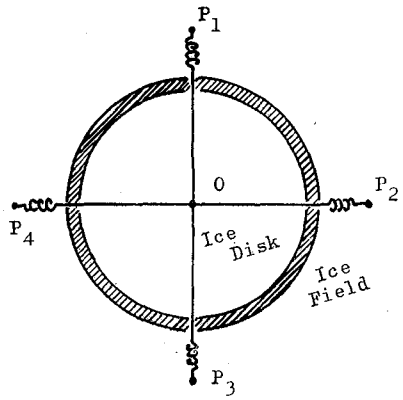

Fig. 1. Schematic diagram of the drag measurement

The vertical motion of the disk was very rapidly damped and the rotational motion was always slow in actual cases. Therefore, the motion of the disk is treated as a two dimensional motion of a point mass at the center of the disk. The equation of motion of it is given by

$$M\ddot{\boldsymbol{r}} = \sum \boldsymbol{F}_i + \boldsymbol{F}_a + \boldsymbol{F}_w , \qquad (1)$$

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where M is the mass of the disk, $\boldsymbol{r}$ the position vector of the center, $\boldsymbol{F}_i$ the force exerted by the i-th string, and $\boldsymbol{F}_a$ and $\boldsymbol{F}_w$ are horizontal components of forces exerted by air and water, respectively.

Evidently, both $\boldsymbol{F}_a$ and $\boldsymbol{F}_w$ have $\dot{\boldsymbol{r}}$ dependencies which appear as resistance to the motion. Since the forces are spatially integrated quantities, their resistant effects can be expressed in a single macroscopic form $-\mu\dot{\boldsymbol{r}}$. Separating the resistant term from $\boldsymbol{F}_a+\boldsymbol{F}_w$ as

$$\boldsymbol{F}_a+\boldsymbol{F}_w=\boldsymbol{F}_0-\mu\dot{\boldsymbol{r}}\,, \tag{2}$$

we obtain from eq. (1)

$$M\ddot{\boldsymbol{r}}=\sum\boldsymbol{F}_i+\boldsymbol{F}_0-\mu\dot{\boldsymbol{r}}\,. \tag{3}$$

The main aim is to know the dependency of the external force on macroscopic structures of wind and current systems. Averaging eq. (3) over a time interval T in which wind and current systems are macroscopically steady, we get

$$M\bar{\ddot{\boldsymbol{r}}}=\sum\bar{\boldsymbol{F}}_i+\bar{\boldsymbol{F}}_0-\mu\bar{\dot{\boldsymbol{r}}}\,, \tag{4}$$

where the bar over a symbol indicates its time average over T. Since the motion is oscillatory,

$$\bar{\dot{\boldsymbol{r}}}=0 \text{ and } \bar{\ddot{\boldsymbol{r}}}=0\,, \tag{5}$$

if T is long enough compared with the main period of the motion. Then, we have

$$\bar{\boldsymbol{F}}_0=-\sum\bar{\boldsymbol{F}}_i\,. \tag{6}$$

When the amplitude of the motion is very small compared with mean distances between the center and the side axles, $\bar{\boldsymbol{F}}_i$ can be closely approximated by

$$-\bar{\boldsymbol{F}}_i=\bar{F}_i\boldsymbol{n}_i\,, \tag{7}$$

where $\bar{F}_i$ is the time average over T of the absolute value of $\boldsymbol{F}_i$ and $\boldsymbol{n}_i$ is a unit vector from i-th side axle toward the center at an arbitrary moment. Hence, we have

$$\bar{\boldsymbol{F}}_0=\sum\bar{F}_i\boldsymbol{n}_i\,. \tag{8}$$

Thus $\bar{\boldsymbol{F}}_0$ can be computed from readings of the tensiometers.

A word must be added about eq. (4). Separating the time dependent part from $\boldsymbol{F}_0$ as

$$\boldsymbol{F}_0=\bar{\boldsymbol{F}}_0+\Delta\boldsymbol{F}(t)\,,$$

we get

$$M\ddot{\boldsymbol{r}}=\sum\boldsymbol{F}_i+\bar{\boldsymbol{F}}_0+\Delta\boldsymbol{F}(t)-\mu\dot{\boldsymbol{r}}\,, \tag{9}$$

which represents a forced oscillation by the force $\Delta\boldsymbol{F}(t)$. The character of $\Delta\boldsymbol{F}(t)$ can be studied from the mode of oscillation which in turn can be deduced from the continuous record of tensiometers.

II. 2. *Analysis of wind.* Theories of the force exerted by fluid system on the boundary surface have been rather well developed. Here, however, we only introduce simple considerations due to Prandtl and Rossby (see any textbook on dynamic meteorology, for instance, Sutton, 1953). Consider that wind system over a wide and macroscopically flat surface is steady in such a manner that the direction of averaged wind at any point is parallel to the x-axis in the surface and that the speed of averaged wind depends only on the height, along which z-axis is chosen. In this system Reynold's

shear stress τ should be independent of z:

$$d\tau/dz = 0 . \tag{10}$$

Prandtl set the following relation between τ an u, the wind speed:

$$\tau = \rho l^2 (du/dz) | du/dz | , \tag{11}$$

where ρ is the desity of air and l is a quantity having the dimension of the length. He called the latter the mixing distance and considered that it depends only on height and surface condition. C. G. Rossby assumed that

$$l = k(z+z_0) , \tag{12}$$

where z_0, called the roughness parameter, is a quantity specifying the surface condition and k is a universal constant called Karman's constant, for which the value 0.4 is generally adopted.

Owing to eq. (10), τ becomes a constant, τ_0. From eqs. (11) and (12), we have

$$\frac{du}{dz} = \frac{1}{k}\sqrt{\frac{\tau_0}{\rho}}\,\frac{1}{z+z_0} . \tag{13}$$

Integrating eq. (13) under the boundary condition $u=0$ at $z=0$, we get

$$u = \frac{1}{k}\sqrt{\frac{\tau_0}{\rho}}\,\ln\frac{z+z_0}{z_0} . \tag{14}$$

Let u_1 and u_2 be the wind speed at heights z_1 and z_2, respectively. Then, we have

$$\frac{u_1}{u_2} = \ln\frac{z_1+z_0}{z_0}\Big/\ln\frac{z_2+z_0}{z_0} , \tag{15}$$

and

$$u_1 - u_2 = \frac{1}{k}\sqrt{\frac{\tau_0}{\rho}}\,\ln\frac{z_1+z_0}{z_2+z_0} . \tag{16}$$

Using u_1 and u_2, we obtain z_0 from eq. (15), and then τ_0 from eq. (16). The direct measurement of τ_0 is thus used to compute Karman's constant, when the wind system is such as considered by Prandtl and Rossby.

That the surface is macroscopically flat means that z_0 is very smaller than the heights where wind speed should be measured. Hence, eq. (14) can be closely approximated by

$$u \cong \frac{1}{k}\sqrt{\frac{\tau_0}{\rho}}\,(\ln z - \ln z_0) , \tag{17}$$

and z_0 and $\frac{1}{k}\sqrt{\frac{\tau_0}{\rho}}$ can be obtained graphically by plotting u versus $\ln z$.

II. 3. *Actual procedures.* The axles are first dug in ice. After they have been completely frozen, the ring shaped area is cleared of ice with the aid of a chain saw. Geometries of actual measurements are given below:

Year	Diameter and mass of the disk		Width of the ring	Distance between the center and side axles	Number of side axles
1963	600 cm	7.8×10^6 g	30 cm	440 cm	3
1965	800	1.4×10^7	30	650	2 to 4
1966	600	6.1×10^6	30	450	4

In 1963 and 1965, spring balances of 4 kg-wt/10 cm were mainly used as tensiometers, which composed a part of the linkage between the center and each side axle. The rest part of the linkage consisted of 1 mmϕ string of soft iron. Readings were made manually at every five or ten seconds. In 1966, electro-load-cells were used as tensiometers and tension was recorded continuously by an oscillograph. In this case, the spring balance was used as a part of the linkage, so that the linkage became a soft elastic system. (Since we adopted, as a main part of the linkage, soft-iron string which is very easy to creep, we had to maintain the linkage rather soft in order to ensure its elasticity. If we use steel string instead of soft-iron string, the linkage can be chosen more rigid.)

Wind speed was measured with small cup anemometers at heights 16, 36, 76 and 156 cm in 1963 and 20, 40, 80, 160 and 320 cm in 1965 and 1966, and wind direction was measured by means of a wind-vane at height 100 cm.

II. 4. *Results.* Several results are given below:

Year and run-number	Date Feb.	Time	Wind: Speed at 156 or 160 cm* (m/s)	Wind: Direction at 100 cm*	$-\sum F_i n_i$: Direction† (degree)	$-\sum F_i n_i$: (g-wt)	τ_0 (dyne/cm²)	z_0 (cm)	k
63-# 1	14	15. 30–15. 40	3.4	NW–NNW	335	160	0.55		
# 2		15. 50–16. 00	3.2		313	160	0.55		
65-# 1	20	14. 50–15. 00	3.0	SE–SEE	130	225	0.45	0.0025	0.70
# 2		15. 10–15. 15	2.9	S	180	213	0.42	0.002	0.68
# 3		15. 40–15. 50	3.9	SW	122	370	0.73	0.025	0.53
# 4	21	10. 20–10. 30	8.0	SW	134	947	1.9	0.02	0.43
# 7		13. 30–13. 35	9.5		116	714	1.4		
# 8	22	11. 15–11. 20	7.1	W	270	1 186	2.4	0.009	0.56
# 10		13. 45–13. 50	6.7	NW	301	1 290	2.6	0.007	0.68
66-# 4	26		5.4?		128	1 475	4.5		
# 5			4.7		179	447	1.5		

* The height above the surface of sea ice.
† The angle measured from North in NESW-wise.

The wind data in 1965 are rather closely approximated by the logarithmic law, eq. (17), as seen in Fig. 2. Assuming that the force in the above table were solely due to wind, or in other words, neglecting contributions to the force from weak tidal currents which were supposed less than 1 cm/s, and using values of wind speed at 20 and 160 cm, we get values of z_0 and k shown in the above table. The values of z_0 are of the order of those obtained for arctic ice floes (Untersteiner and Badgley, 1965), while the values for k are somewhat larger than the accepted value 0.4. The reason for this deviation is not clear.

Concerning the use of eq. (9), only one example shall be given. Figure 3 shows readings of a tensiometer in the measurement 63-# 2, and Fig. 4 the result of harmonic analysis of it. One can easily see three peaks at periods, 35.3, 27.3 and 23.1 sec. The proper periods of the system are calculated from the geometry and the elastic constants of the linkages as 23.0 and 22.3 sec (see Suzuki, 1964). The oscillation of period 35.3

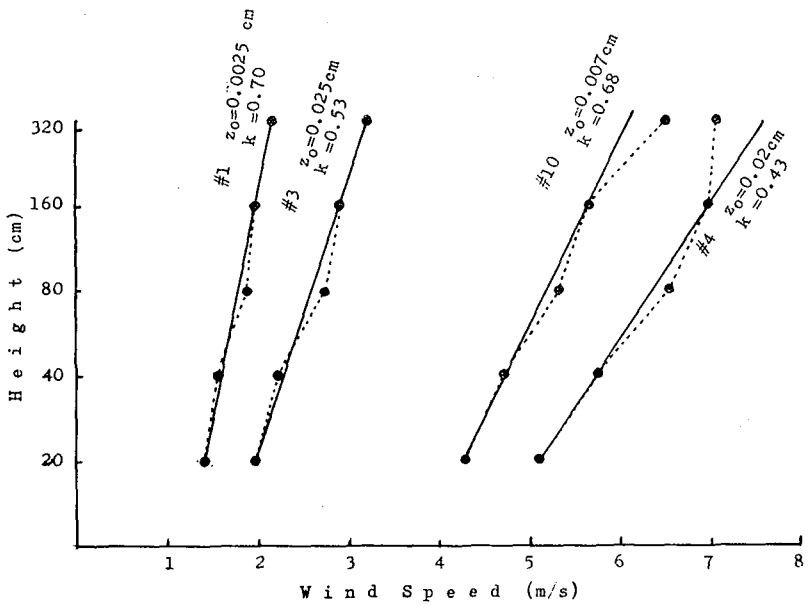

Fig. 2. Wind profiles in 1965

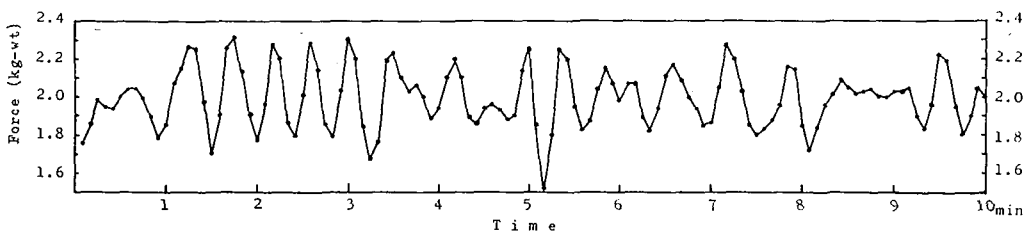

Fig. 3. One example of tension readings in 63-#2

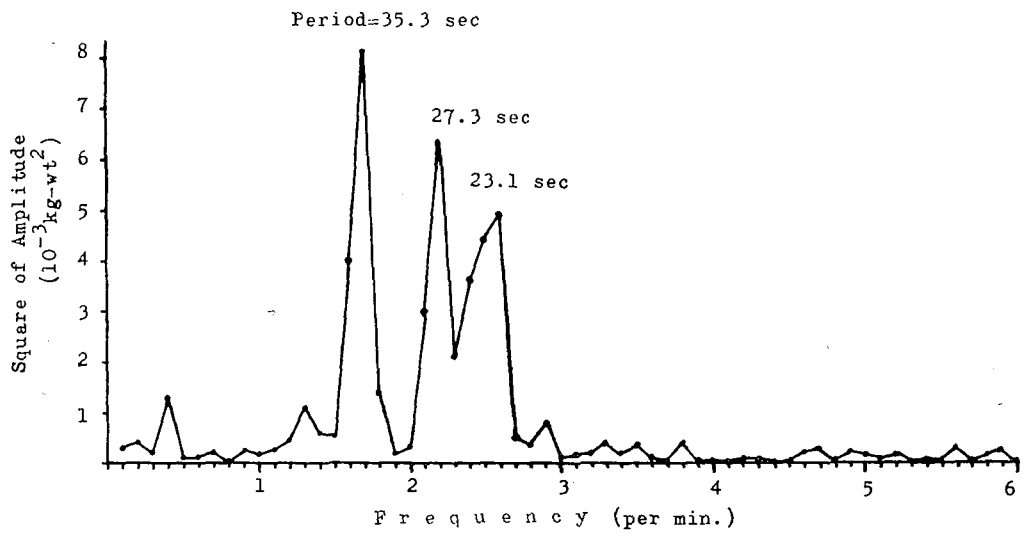

Fig. 4. Harmonic analysis of the tension readings shown in Fig. 3

sec is very probably due to a mode of wind, its gustiness.

III. Water Drag Measurement

The first method may be used for measuring water drag when contributions from current is large. But a more direct method was adopted in last winter for measuring water drag of an ice floe. The method consists in pulling an ice floe by a boat and measuring the speed of the boat and the tension of the pulling wire. A 50 HP moter boat and an electro-load-cell of capacity 500 kg-wt were used for pulling and for measuring tension, respectively. But owing to the lack of appropriate recording devices, no consistent data were obtained.

References

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