Creep of Snow and Ice

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Abstract

Constant load creep tests in uniaxial unconfined compression were performed on samples of sintered snow and bubbly polycrystalline ice. Nominal axial stresses were in the range 0.1 to 1.0 kg-wt/cm^2 for snow, and 0.5 to 20 kg-wt/cm^2 for ice. The range of temperatures investigated was from -0.5 to -34.5°C. Assuming creep to follow the Arrhenius relation, values of apparent activation energy for secondary creep under a nominal axial stress of 0.5 kg-wt/cm^2 varied from 10.7 kcal/mole for ice of density 0.83 g/cm^3 to 17.8 kcal/mole for snow of density 0.44 g/cm^3. The dependence of strain rate $\dot{\varepsilon}$ on stress σ for polycrystalline ice subjected to stresses in the range 0.5 to 20 kg-wt/cm^2 at temperatures of -4 and -10°C could best be described by a relation of the form

$$\dot{\varepsilon} = C_1\sigma + C_2\sigma^{3.5},$$

where C_1 and C_2 are constants for a given ice type. The behaviour of sintered snow is probably similar, although the tests did not go to sufficiently low stresses to confirm this supposition. It is suggested that the creep of polycrystalline ice depends on at least two distinct mechanisms in the stress range studied. Possibilities include dislocation damping for the process dominant at high stress, and drift of dislocations pinned by stress-induced order for the low stress mechanism. If each mechanism has its own characteristic activation energy, the apparent activation energy measured in creep experiments may well vary with stress level. In snow subjected to a given nominal stress, such an effect should be reflected in variation of apparent activation energy with bulk density, since true stress in the ice matrix will increase as density decreases when the nominal applied stress is fixed. The effect of bulk density on strain rate and the possibility of predicting creep rates for snow from data on creep of polycrystalline ice are discussed.

I. Introduction

The influence of temperature on the creep rate of snow does not seem to have been investigated systematically. It has been assumed, with some theoretical justification, that the temperature dependence of snow creep can be expressed by the Arrhenius function, but the values of activation energy deduced from miscellaneous observational data range widely, from about 7 to 24 kcal/mole. In an attempt to provide working relationships for the creep of high density snow, such as is encountered in ice cap construction, simple uniaxial creep tests were made. Since the creep of snow ought to be determined largely by the behaviour of its constituent ice particles, tests were also made on impermeable polycrystalline ice in the hope that creep of snow might be related to the creep of low porosity ice.

Although the data obtained probably satisfy the engineering requirements which motivated the study, they raise a number of questions concerning the creep of ice, and emphasize desirability of further study.

II. Creep Testing Methods

Since the study was concerned largely with the elucidation of physical behaviour, unconfined compression offered the most convenient test procedure; a free-standing cylinder was loaded axially in compression while the axial creep rate was measured. The other simple alternative is confined compression: a sample rigidly confined in a smooth-walled cylinder is loaded axially in compression while the axial deformation rate is measured. In principle there are more satisfactory techniques, such as a combination of hydrostatic compression and torsional shear, but they seem inordinately awkward to apply to multiple tests on fragile snow.

Although the simple approach is valid for the stated purpose, it should be recognized that the usual single viscosity parameter yielded by either confined or unconfined tests is insufficient for general application in continuum mechanics. Such a parameter may also lead to some confusion if taken as the sole basis for a general relation between viscosity and density. The significance of simple creep parameters is best illustrated by reference to linear creep theory, which appears to be reasonably valid for "low stress" problems, such as those concerning deformation in the uppermost 10 m of polar ice caps, creep in avalanche slopes, and the loading of engineering structures associated with these environments. "High stress" problems, which involve viscosity which is non-linear with stress, can at present be treated only in special cases, such as the one-dimensional simplification for densification of horizontally-bedded snow.

Taking the Newtonian approximation, the equations relating stress σ and strain rate $\dot{\varepsilon}$ can be written in terms of "bulk" and "shear" viscosities, η and μ respectively, which are analogous to the bulk and shear moduli of elastic theory:

$$1/3\,\sigma_{ii}=\eta\,\dot{\varepsilon}_{ii}\,, \qquad 1/2\,\sigma'_{ij}=\mu\,\dot{\varepsilon}'_{ij}\,,$$

where $i, j=1, 2, 3$, the primes denote deviators, and η and μ are scalar if the snow or ice is assumed isotropic. Applying these to the unconfined compressive test when secondary creep is well established and making the idealizing assumption that the test sample is completely free to strain laterally, even near the ends, η and μ can be determined if the axial stress σ_z, the axial strain rate $\dot{\varepsilon}_z$, and the circumferential strain rate $\dot{\varepsilon}_\theta$ are measured:

$$\eta=\frac{\sigma_z}{3(\dot{\varepsilon}_z+2\dot{\varepsilon}_\theta)}\,, \qquad \mu=\frac{\sigma_z}{2(\dot{\varepsilon}_z-\dot{\varepsilon}_\theta)}\,.$$

In the present tests, unsuccessful attempts were made to measure $\dot{\varepsilon}_r$ (which is equal to $\dot{\varepsilon}_\theta$) using both a horizontally mounted cathetometer and a hand micrometer, and making due allowance for end effects. The only data finally recorded were σ_z and $\dot{\varepsilon}_z$, and the low stress viscosity given by their ratio was thus η_E, a viscous equivalent of Young's modulus. It is related to η and μ by

$$\eta_E=\frac{9\eta\mu}{3\eta+\mu}\,.$$

In order to apply η_E to continuum problems in snow, it would be necessary to estimate a value for μ/η or for ν_v, the viscous analogue of Poisson's ratio, by judicious interpolation between deducible limits, viz.

$$\gamma \to 0\,, \qquad \mu/\eta \to 3/2\,, \qquad \nu_v \to 0\,,$$
$$\gamma \to \gamma_{\text{ice}}\,, \qquad \mu/\eta \to 0\,, \qquad \nu_v \to 1/2\,,$$

where γ is bulk density.

In the laterally confined compressive test it is usually only possible to measure σ_z and $\dot{\varepsilon}_z$; their ratio gives a "compactive viscosity" η_c, which is directly comparable with the compactive viscosity derived from observations on the creep settlement of horizontally bedded snow layers under gravity body forces. In terms of η and μ the compactive viscosity is:

$$\eta_c = \eta + 4/3\,\mu\,.$$

Since σ_r or σ_θ would be very difficult to measure, the confined test seems unsuitable for determination of η and μ separately. However, η_c can be applied directly to some problems, while μ/η or ν_v can perhaps be estimated as mentioned above.

III. Test Procedures

Snow samples were prepared by gently grinding and sieving natural snow into cylindrical molds and compacting the aggregate under controlled conditions. The resulting snow cylinders (3.35 cm dia. $\times$7.14 cm long, finished dimensions) were sintered in an ice-saturated atmosphere at a temperature of -10°C for approximately 3 weeks, after which time mass transfer by vapour and surface diffusion was approaching completion. Ice samples were made by taking high density snow cylinders prepared in the above manner, soaking them in an ice bath, and slowly refreezing while uniform saturation was maintained. Grain size of the sintered snow ranged from less than 0.1 to 0.8 mm with a mean size of about 0.2 mm, and bulk densities of the 3 groups of samples were 0.436 ± 0.02, 0.531 ± 0.02, and 0.644 ± 0.023 g/cm^3. The ice samples contained uniformly distributed air bubbles, approximately 0.5 mm dia., and the average crystal size was approximately 0.8 mm. Bulk density of the ice was 0.832 ± 0.025 g/cm^3, so that it was quite similar to the ice formed by triaxial compression of snow at sub-freezing temperatures.

The creep tests were made by compressing cylindrical samples axially with no side restraints. All were constant load tests, which for small strain increments approximate to constant stress tests. "Low stress" tests were made with nominal axial stresses of 0.5 and 1.0 kg-wt/cm^2, using a lever system and deadweights for loading, and reading, deformation from a dial micrometer. "High stress" tests on ice were made on a Tinius-Olsen testing machine, which applied constant loads to give nominal axial stresses from 3.35 to 20 kg-wt/cm^2.

For the low stress tests, 6 loading frames were enclosed in each of 6 constant temperature boxes. These boxes had styrofoam walls 5.6 cm thick with a plexiglass viewing panel, and constant temperature was maintained by a Micro-Set mercury regulator which actuated a heater (100 watt light bulb) and convector fan. The boxes were placed in rooms where the ambient temperature was always below the desired test temperature. Loose snow was scattered inside the boxes to saturate the air, and sample evaporation was further inhibited by sheathing the cylinders in rubber membranes. With a nominal axial stress of 0.5 kg-wt/cm^2, 3 types of snow (0.436, 0.531, 0.644 g/cm^3) were each tested at -0.5, -1, -2, -4, -10, -20 and -34.5°C; for each combination there were

6 replications, giving a total of 126 tests in this series. Additional tests on snow of 0.483 g/cm³ were made under stresses of 0.1, 0.25, 0.5 and 1.0 kg-wt/cm³ at temperatures of -1 and -10°C; made in multiples of 3, these tests totalled 24. Low stress tests on ice were made under stresses of 0.5 and 1.0 kg-wt/cm² at temperatures of -1, -2, -4, -10, -20 and -34.5°C; 3 samples were tested for each combination of conditions, giving a total of 36 tests.

For the high stress tests on ice, 3 identical sheathed samples were pressed simultaneously between the loading platens of the Tinius-Olsen machine. This procedure improved the effective load-holding capability and provided a mean result for 3 samples. Nominal axial stresses applied to the ice samples were 3.35, 5, 10, 15 and 20 kg-wt/cm², and the test temperatures were -4 and -10°C (10 runs, 30 samples tested).

Altogether 216 samples were tested, excluding those used in pilot tests to develop technique.

For ice, test durations were up to 115 days, with total axial strains in the range 4×10^{-3} to 2×10^{-2}. High stress tests on ice were continued to rupture, which occured at total axial strains of order 10^{-1}. For snow, total axial strains were in the range 5×10^{-3} to 10^{-1}, but there was no indication of tertiary creep or rupture.

Detailed observations were made during the initial period of creep, but after the first day of the tests observations were made only once a day. Transient elastic straining is not considered here, although the data are available for possible future use. It might be noted, however, that apparent values for instantaneous elastic strain are unreliable owing to slackness and flexure in the loading apparatus. The strain rates discussed in this report are logarithmic strain rates taken from the linear portions of the creep curves. Steady-state creep, in which strain increments are approximately proportional to time, developed in ice when axial strain exceeded about 4×10^{-3}; in snow axial strain was about an order of magnitude greater before steady-state creep set in, *i.e.* 2×10^{-2} to almost 10^{-1}. Log-log plots of the creep curves showed that in some cases steady-state creep never truly developed.

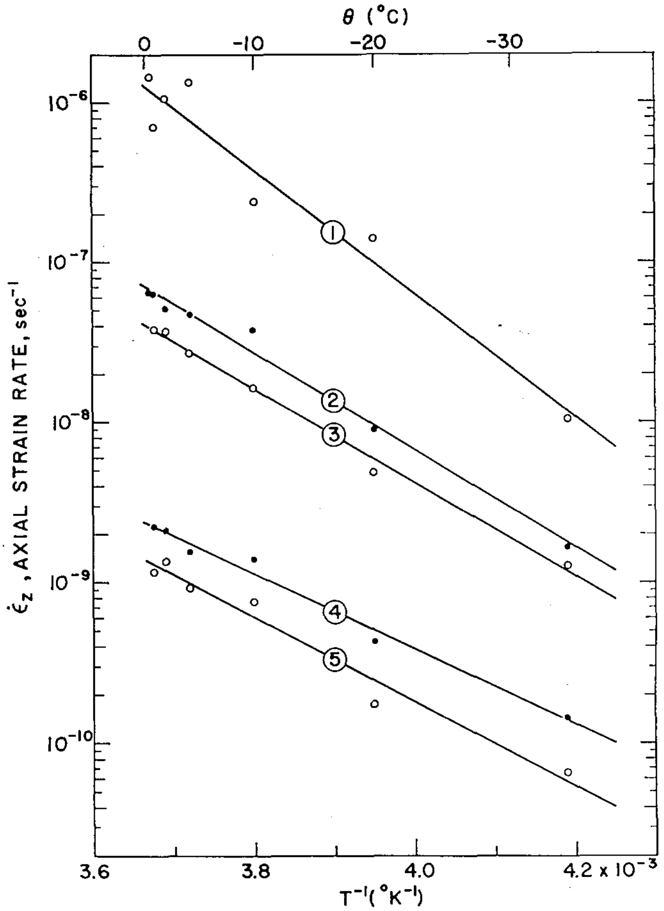
In retrospect, it appears that the present test programme might have been expedited by rapidly pre-straining the samples to about 4×10^{-3} under high stress, and thereafter relaxing the stress to the required test level.

IV. Apparent Activation Energy for Creep

For analysis of temperature dependence it was assumed that creep of snow and ice is a thermally activated process which follows the Arrhenius (or Boltzmann) relation derived from thermodynamic theory, *i.e.*

$$\dot{\varepsilon} = A_1 \exp\left(-\frac{Q}{RT}\right), \tag{1}$$

where $\dot{\varepsilon}$ is strain rate, A_1 is a constant for any given stress and snow type, R is the gas constant, T is absolute temperature, and Q is an activation energy for creep under the prevailing conditions. Since the range of temperatures studied was small compared with the absolute temperatures themselves, the possibility of a further temperature factor T^{-1}, suggested by dislocation climb theory, was disregarded.



	DENSITY g/cm³	NORMAL STRESS kg/cm²		APPARENT ACTIV'N ENERGY kcal/mole
1	0.436	0.5	EACH POINT MEAN OF 6	17.8
2	0.531	0.5		14.0
3	0.644	0.5		13.4
4	0.832	1.0	EACH POINT MEAN OF 3	10.7
5	0.832	0.5		11.9

Fig. 1. Variation of strain rate with temperature

In Fig. 1 the results of the temperature study are shown linearized in accordance with eq. (1). Correlation coefficients for the least squares fit of eq. (1) were in the range 0.974 to 0.997. Apparent activation energies given by the data are:

Bulk density (g/cm³)	Nominal axial stress (kg-wt/cm²)	Apparent activation energy (kcal/mole)
0.436	0.5	17.8
0.531	0.5	14.0
0.644	0.5	13.4
0.832	0.5	11.9
0.832	1.0	10.7

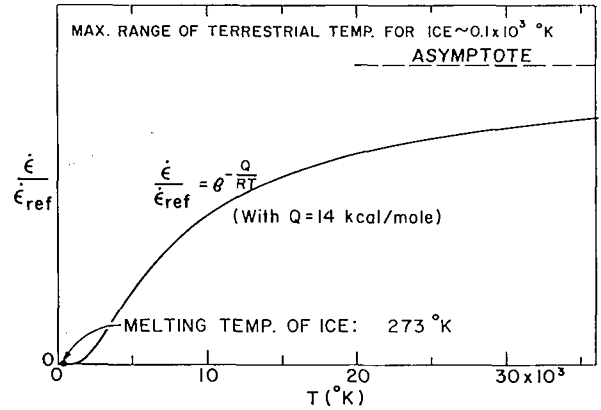


Fig. 2. *Graphic illustration of the Boltzmann or Arrhenius function*

These values of Q show considerable spread, and the limited data suggest the Q may increase as density decreases.

Although eq. (1) has a sound theoretical basis in thermodynamics and rate theory, it is not necessarily validated by fitting data for a range of T which is small compared with T itself. Regarded simply as a functional representation of a physical relation, the equation seems largely irrelevant (Fig. 2). With any reasonable value of Q ($\sim$10 kcal/mole), the inflection of the function ($T=Q/2R$) is at a temperature an order of magnitude higher than the melting point of ice, so that the particular properties of that function are simply not utilized. As a matter of fact, the present data can be linearized equally well according to the empirical relation:

$$\dot{\varepsilon} = A_2 \exp(-K_1 \theta), \tag{2}$$

where A_2 is a constant for a given stress and snow type, $-\theta$ is the Centigrade temperature, and K_1 is a rate constant for the prevailing conditions. Equation (2) also fits the probable boundary conditions for the physical phenomenon.

V. Activation Energy of Polycrystalline Ice

The mean value of Q determined for impermeable polycrystalline ice, 11.3 kcal/mole, is somewhat lower than values reported previously for creep. Disregarding the questionably high values of 31.8 and 37.8 kcal/mole found by Glen (1955) and Higashi (1959) respectively, the range of reported values is from 12 to 16.1 kcal/mole:

Investigator	Activation energy (kcal/mole)
Raraty and Tabor, 1958	12
Jellinek, 1960	14.1
Butkovich and Landauer, 1960	14.3
Jellinek and Brill, 1956	16.1

It should be pointed out that these previous tests were of relatively short duration, and the only ones to yield steady-state creep rates directly were at higher stresses than the present tests. They were also made over smaller (and higher) temperature ranges.

However, there is a weakness in the present tests in so far as total strains for low temperature tests were too small to give a complete guarantee that secondary creep was fully established.

Even though the detailed creep mechanisms of ice are yet to be firmly established, the assumption of thermal activation requires that there should be some correspondence between the creep activation energy and the activation energy for self-diffusion. Measurements on the rate of tritium diffusion through single crystals at different temperatures give activation energies for self-diffusion of 13.5 kcal/mole (Dengel and Riehl, 1963), 14.4 kcal/mole (Blicks *et al.*, 1966), and 14.2 kcal/mole (Ramseier, personal communication).

In many metals the activation energy for self diffusion (in kcal/mole) is approximately 0.038 times the absolute melting temperature, which for ice would be about 10.4 kcal/mole.

As a matter of interest, activation energies of ice for mechanical and dielectric relaxation, and also for proton magnetic resonance, are between 13 and 14 kcal/mole, while activation energies for electrical conductivity are between 11 and 12 kcal/mole.

In view of the fact that apparent values of Q are found here to vary with snow density, it seems reasonable to enquire into the factors which might affect the apparent activation energy of ice. Two possibilities suggest themselves. The first is that chemical contamination affects the temperature dependence of ice creep: this has actually been demonstrated by Raraty and Tabor (1958), who showed that ice contaminated with 1% sodium chloride behaved like pure ice below the eutectic temperature of the salt, but exhibited a marked increase in the temperature dependence of creep at temperatures above the eutectic. The second possibility is that apparent activation energy varies with the stress level, an effect which has not previously been reported.

All the snow types tested were sintered aggregates of equant ice grains, and since the tests were made at constant nominal stress (load divided by total sample cross-section), true stresses in the ice matrix obviously varied with density. Hence it is conceivable that the observed change of activation energy with snow density, if real, might reflect a dependence of the apparent activation energy of ice on stress (or strain rate). The reality of activation energy variations with density is supported by scraps of evidence outside the present tests: an earlier engineering study (Mellor and Hendrickson, 1965) hinted at such an effect, Ramseier (personal communication) noticed variations of Q with density in field creep tests, and Yosida *et al.* (1956) obtained values of 20.8 and 23.8 kcal/mole for transient creep of low density (0.17 and 0.25 g/cm^3) snow.

The experimental programme was not designed to test the constancy of Q for ice through a wide stress range, but there is a possibility that some evidence can be gathered from the measurements of strain rate as a function of stress.

VI. Strain Rate of Ice as a Function of Stress

In Fig. 3 strain rate of ice is plotted against stress for two different temperatures, -4 and -10°C. Displaced scales have been added in order to express axial stress σ_z and axial strain rates $\dot{\varepsilon}_z$ in terms of shear, assuming that the test samples behave as ideally stressed elements. Defining octahedral shear stresses τ_o and octahedral strain rates $\dot{\varepsilon}_o$ by

$$\tau_o^2 = 1/2\, \sigma'_{ij}\sigma'_{ij} \text{ and } \dot{\varepsilon}_o^2 = 1/2\, \dot{\varepsilon}'_{ij}\dot{\varepsilon}'_{ij}\,,$$

where σ'_{ij} and $\dot{\varepsilon}'_{ij}$ are deviator stress and deviator strain rate components respectively (i, j=1, 2, 3), then

$$\tau_o = \frac{\sigma_z}{\sqrt{3}} \text{ and } \dot{\varepsilon}_o = \frac{\sqrt{3}}{2}\,\dot{\varepsilon}_z\,,$$

assuming the ice to be incompressible and isotropic.

Results for the two test temperatures are consistent and in broad agreement with previous findings. At high stress ($\tau_o \sim 10^7$ dyne/cm^2), strain rate is approximately proportional to the third or fourth power of stress, in reasonable agreement with earlier data (Glen, 1955; Steinemann, 1959; Butkovich and Landauer, 1959; Mellor, 1959). At low stress ($\tau_o \sim 5\times10^5$ dyne/cm^2), strain rate is more nearly linearly proportional to stress. The data of Fig. 3 are bolstered by 6 sets of data for σ_z=0.5 and 1.0 kg-wt/cm^2 at temperatures −1, −2, −4, −10, −20 and −34.5°C; assuming a power dependence of strain rate on stress for the small stress range, these data yield a mean exponent of 0.93. Linear viscosity at low stress ($\tau_o \sim 10^4$ to 10^5 dyne/cm^2) was demonstrated previously by Butkovich and Landauer (1960).

In attempting to describe the dependence of strain rate on stress for ice, three functional forms have been considered; they are (i) a simple power relation, (ii) a hyperbolic sine function, and (iii) a linear relation plus a power function. One of these, the simple power law, can immediately be relegated to the role of a convenient empiricism for limited stress ranges, since it is incompatible with the data. The second, the hyperbolic sine relation, deserves more serious consideration, for in addition to its inherent empirical suitability it has a certain theoretical rationality.

In early attempts to provide a physical explanation for creep, Eyring and Kauzmann applied theory of reaction kinetics, assuming that creep occurs by atomic diffusion (see, for example, Kennedy, 1962). It can be argued that if the normal height of the energy barrier H between two lattice locations is biased by stress such that the barrier heights become $(H-\Delta H)$ and $(H+\Delta H)$ in forward and reverse directions, the net rate at which atoms surmount the barrier in the forward direction is proportional to

$$\exp\left(-\frac{H-\Delta H}{RT}\right)-\exp\left(-\frac{H+\Delta H}{RT}\right), \quad i.e.\ \ 2\exp\left(-\frac{H}{RT}\right)\sinh\left(\frac{\Delta H}{RT}\right).$$

If it is further assumed that ΔH is proportional to the biasing stress σ, then this argument predicts that creep velocity will be proportional to sinh σ at any given temperature.

The present data were tested against the function

$$\dot{\varepsilon} = \frac{1}{\eta_1}\,\sigma_* \sinh\frac{\sigma}{\sigma_*}\,, \tag{3}$$

where η_1 is a viscosity coefficient and σ_* is a constant with the dimensions of stress. η_1 was estimated from the low stress portions of curves fitted by inspection (when $\sigma \ll \sigma_*$, $\dot{\varepsilon} \approx \sigma/\eta_1$), while a first estimate of σ_* was made from the slope of the high stress portions of the curves (when $\sigma \gg \sigma_*$, $\mathrm{d}\dot{\varepsilon}/\mathrm{d}\sigma \approx (1/\eta_1)\exp(\sigma/\sigma_*)$). The estimated values of η_1 and σ_*

were then adjusted by trial. The best fitting curves were not very satisfactory, since the curvature of the function was too acute.

The third type of relation applied to the stress dependence of ice creep is

$$\dot{\varepsilon} = C_1\sigma + C_2\sigma^n , \tag{4}$$

where C_1 and C_2 are constants for a given ice type and temperature. Butkovich and Landauer (1959) tested this form against rather scattered data with $n=3$ (thus in effect approximating the hyperbolic sine), but concluded that a simple power function gave a better fit. Meier (1959) proposed eq. (4) in its general form, and postulated that ice creeps by two distinct mechanisms. For these mechanisms he suggested dislocation climb, which requires that $n=4.5$, and some undetermined type of grain boundary flow leading to Newtonian behavior.

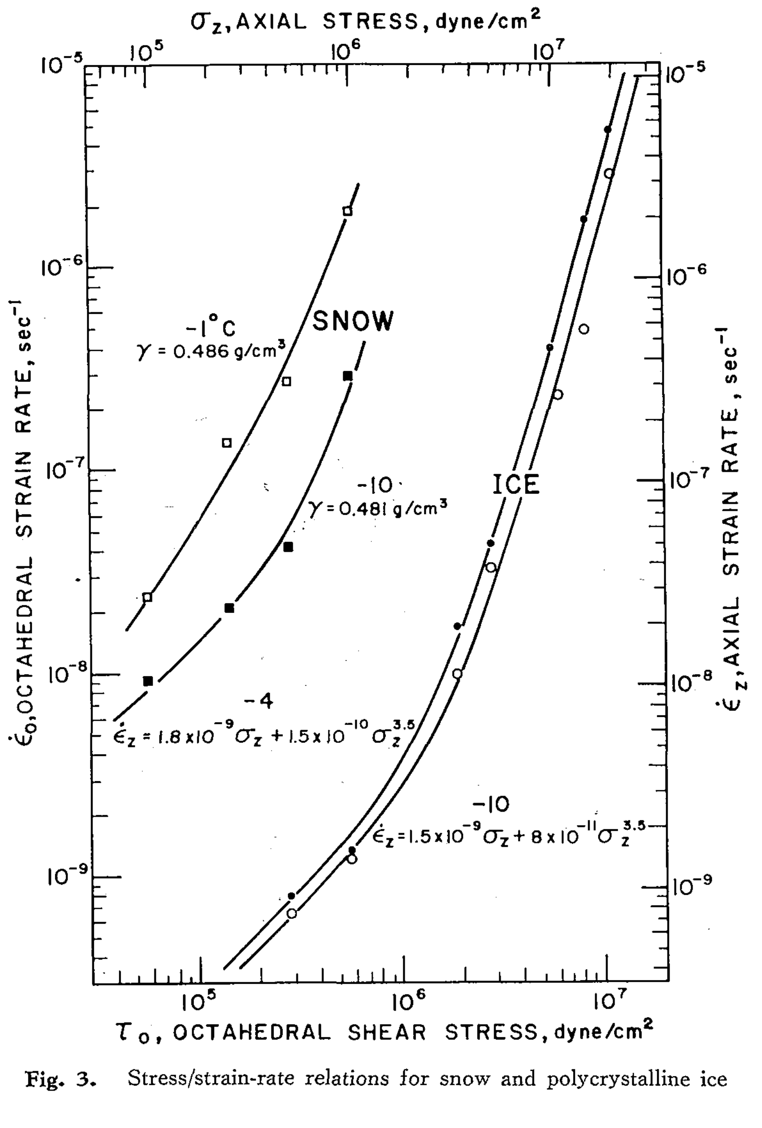


Fig. 3. Stress/strain-rate relations for snow and polycrystalline ice

Meier's idea seems reasonable, although it is still difficult to decide what the physical creep mechanisms might be. Weertman's (1957) dislocation climb mechanism leads to $n=4.5$, while another mechanism suggested in the same paper, movement of dislocation lines in a Peierls stress field, leads to $n=2.5$. Still another mechanism, dislocation damping, gives $n=3$ (Weertman, 1962). Tegart (1964), reviewing the problem of ice creep, favoured Weertman's value $n=2.5$ and added a suggestion of $n=2$ from consideration of non-basal slip in the crystal. These last values were for temperatures above $-65°C$; at very low temperatures Tegart expected the dislocation climb mechanism to operate. No explanation for Newtonian creep at low stress has been offered, although it is conceivable that if dislocations were pinned by stress-induced order, and no dislocation multiplication occurred, dislocation drift at low stress might lead to the observed behaviour (Weertman and Weertman, 1964; Weertman, personal discussion).

The best fit of eq. (4) to the data (for positive coefficients) was obtained with $n=3.5$. The resulting relationships were:

For $-4°C$ $$\dot{\varepsilon}_z = 1.8\times10^{-9}\,\sigma_z + 1.5\times10^{-10}\,\sigma_z^{3.5}, \tag{4 a}$$

For $-10°C$ $$\dot{\varepsilon}_z = 1.5\times10^{-9}\,\sigma_z + 8\times10^{-11}\,\sigma_z^{3.5}, \tag{4 b}$$

where $\dot{\varepsilon}_z$ is in sec^{-1} and σ_z is in bars.

These results tend to support the view that there are at least two distinct creep mechanisms operating in the stress range studied. At the highest stresses the dominant mechanism is one which gives creep velocities proportional to stress raised to the power 3.5; this behaviour is in reasonable accord with Weertman's suggested dislocation damping mechanism.

If two different creep processes exist, each with its own characteristic activation energy, then there may well be a change in the apparent activation energy as stress varies and alters the relative contribution of each process to the gross creep. The results in Fig. 3 are actually consistent with an increase of apparent activation energy with stress, but since there are only two values of the temperature parameter the effect is far from proved.

Additional tests were planned so as to check whether the apparent activation energy for high stress (12 kg-wt/cm² axial) creep was significantly different from the value obtained from the low stress (0.5 to 1.0 kg-wt/cm² axial) tests. This programme has been beset with practical difficulties, but preliminary indications are that tests in the temperature range -3.5 to $-35°C$ give an activation energy of 15–16 kcal/mole.

VII. Variation of Creep Rate with Density

For practical purposes the results shown in Fig. 1 can be utilized to gain some idea of the variation of creep rate with snow density. In Fig. 4, points taken from the regression lines of Fig. 1 are plotted; in spite of the limited data, no attempt has been made to suppress the "step" of the resulting semi-logarithmic plot, since there is mounting evidence that such a step is a real feature of compressive creep data for the stress magnitude considered (see, for example, Mellor, 1964, p. 49). Although the limits of the step in Fig. 4 are obviously ill-defined, it might be noted that it lies between 0.53 g/cm³, which is close to the practical maximum density for close packing of equant grains, and

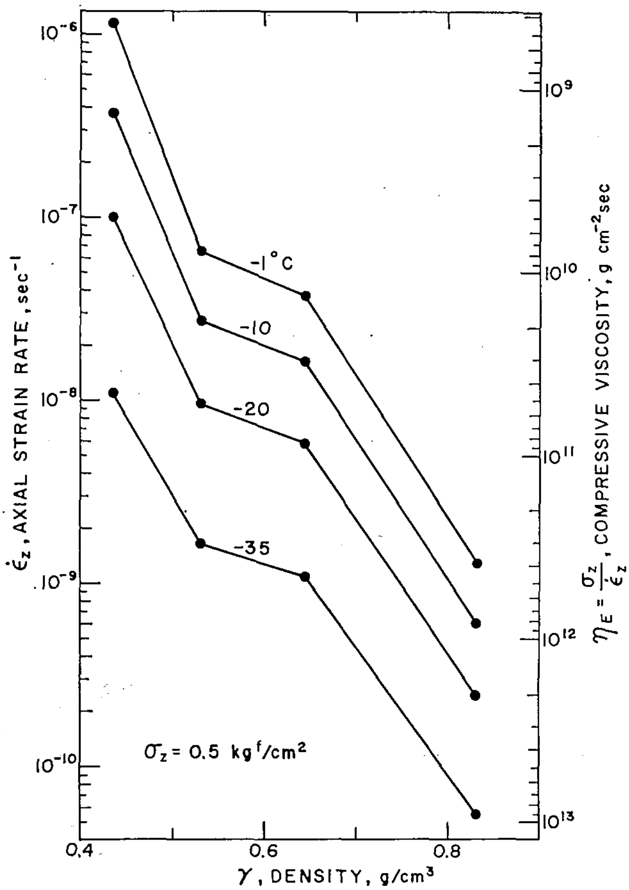


Fig. 4. Variation of strain-rate and compressive viscosity with density

0.65 g/cm^3, which is about the theoretical maximum density for close packed spheres. Considering the parallelism of the lines for the lowest density section of the plot, there seems to be a possibility that the lower density limit of the step might decrease somewhat with decreasing temperature.

In seeking an understanding of creep variation with snow density it is first recognized that creep of the constituent ice grains and their bonds is the controlling factor. Above the maximum density for close packing ice grains must be deformed, while below that density it becomes easier to strain the snow by translating grains relative to each other. If snow is tested under a fixed and moderate nominal stress (*i.e.*, load per unit sample area), assumed to be low enough for straining to be Newtonian, there will be some density below which the effective stress in the ice matrix exceeds the limit for Newtonian behaviour; strong variation of creep rate with density might be expected in this range.

In attempting to relate the creep of snow to known creep properties for ice, it seems simplest to consider first the high density range in which snow grains are closely packed and the general structure does not vary too drastically. The most obvious line of argument is that the effective stress in the ice matrix of the snow varies in proportion to the amount of ice in any given plane when a fixed nominal stress is applied to the sample. Thus the effective stress σ_e might be related to the nominal stress σ_z in the form

$$\frac{\sigma_e}{\sigma_z}=a\left(\frac{1}{1-n}\right) \text{ or } \frac{\sigma_e}{\sigma_z}=a(1+r), \tag{5}$$

where n is bulk porosity, r is void ratio, and a is a coefficient which may or may not be constant. Some additional deductions can be made from the known variations of strength with density (Ballard and Feldt, 1966; Mellor and Smith, 1965), and from consideration of limits and boundary conditions. With these and other refinements, such as incorporation of a stress concentration factor in a, several suggestions for the variation of σ_e with porosity or void ratio can be advanced, chiefly variants of eq. (5) or of the form exp (br). It would then seem possible to use eq. (4 b) and the $-10°C$ line of Fig. 4 to test the simple hypotheses. However, it was found that all of the stress-porosity relations which were tried led to a variation of creep rate which was far too weak in the high density range. Only by invoking high effective stresses, and hence non-linear creep of the ice, could the observed rate of variation be reproduced; no rational argument to support the necessary stress-porosity functions could be found for the high density range, although it seems likely that non-linear creep becomes a significant factor in low density snow.

The only known theoretical treatment of this problem (Feldt and Ballard, 1966) deals with Newtonian creep under lateral confinement for the density range 0.41 to 0.60 g/cm^3. It appears to explain density dependence in the "step" range quite adequately, but it is not applicable to either high or low density ranges. It seems highly desirable to pursue further the questions raised above.

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