Densification of Seasonal Snow Cover*

Kenji KOJIMA
小 島 賢 治
The Institute of Low Temperature Science
Hokkaido University, Sapporo, Japan

Abstract

The theory of densification of snow, which is based on an empirical relation between the compactive viscosity factor and the density, has been applied to the numerical computations of depth-density profiles of snow cover under various conditions of mass accumulation of snow. This paper is concerned with the theoretical computations of time variation of density profiles of snow and the depth of seasonal snow cover under varying accumulation rates, where Sorge's law does not hold. The computed results are found to be in good agreement with the data observed in Sapporo and Moshiri, Hokkaido.

Densification of slightly wet snow is found to occur at nearly the same rate as that for dry snow of 0°C, while very wet snow with a water content of more than 10% is very rapidly compacted until it changes to granular snow of large grains and its viscosity reaches a considerably high value.

The compactive viscosity of wind-packed snow with very fine grains is much smaller than that for ordinary compact snow of the same density, particularly in the early stage of densification. The influence of formation of depth-hoar crystals in snow upon the densification behavior is also reported.

To examine the validity of the linear relation between the strain rate and the load, a field experiment was made by strain rate measurements of some selected snow layers subtracting or adding the snow load artificially. The results for some layers show linear relations between them within the error of measurements, while the linear relation could not be obtained for the layers very close to the surface of snow cover.

I. Introduction

Most results of numerical computations of depth-density profiles of snow which were previously reported by Kojima (1957, 1964), Benson (1959), Anderson and Benson (1963), Bader (1962), and recently Feldt and Ballard (1966) were confined merely to cases when the accumulation rates were constant with time. Consideration of time variation of the density profiles of snow under varying accumulation rate is, however, sometimes necessary particularly for discussing the densification of seasonal snow cover lasting for only several months. It is also an interesting problem to find the effect of varying accumulation rate upon the depth-density profile of snow on the ice sheets of the polar regions.

The procedures of computation or the formulae concerning densification of snow under varying accumulation rate have already been reported by Kojima (1957), Yosida (1958, 1963), and Bader (1960). But the computed results have not been reported in

* Contribution No. 799 from The Institute of Low Temperature Science.

detail. An example of change in depth-density profile with time is given in this paper by comparing the observed density profiles with those computed from the data of mass accumulation with varying rates.

Time variation of the depth of snow cover under a constant rate of accumulation of dry snow can be readily calculated. But the calculated depth sometimes deviates from the actual depth to a considerable extent because of an especially high rate of accumulation during a period in early winter. A few examples of time variation of the snow depth computed with consideration to the changing accumulation rate are given in this paper and are compared with the observed variation of the depth.

A large part of the areas with low elevations on West Honshu facing the Sea of Japan, is usually covered with deep wet snow throughout each winter period. Densification of wet snow layers was studied by measuring the rates of compaction and the free water content of wet snow layers at an area with a continuously wet snow cover.

The relations between the compactive viscosity factor and the density of wind-packed snow and, also, depth-hoar layers were observed in Sapporo.

The linear relation between strain rate and snow load was first assumed to hold in natural compaction of snow layers, unless the load is very large. A simple field experiment was made to ascertain the validity of this assumption.

II. Numerical Computation of Depth-Density Profile and Its Variation with Time

1) *Compactive viscosity factor.* Preparatory to the studies on densification of snow under natural conditions, Kojima (1954) made a series of laboratory experiments on the rheological behavior of snow under a constant uniaxial stress, and obtained a linear relation between the strain rate and the load which was less than $30\,\mathrm{g\cdot cm^{-2}}$. Such a linear relation was assumed to hold between the strain rate $\dot{\varepsilon}$ of the densification of a snow layer and the snow load σ exerted on the layer under natural field conditions. The relation may be expressed by $\sigma=\eta_C\dot{\varepsilon}$, where η_C is the compactive viscosity factor.

If a snow layer of the thickness h, at a time t, has a density ρ and the layer is compacted to have a thickness $h-\mathrm{d}h$ and a density $\rho+\mathrm{d}\rho$ in a short time interval $\mathrm{d}t$, the strain rate is

$$\dot{\varepsilon}=-\frac{1}{h}\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t}=\frac{\sigma}{\eta_C}, \tag{1}$$

taking the compactive strain to be positive and, also, considering that the mass of the layer is conserved during a period without percolation of melting water. The rate of increase in density of many layers of snow cover was measured by successive pit wall observations in Sapporo for several winter periods from 1954–55. The snow load on each layer and its variation with time were also obtained from the measured density profile at each observation. Plotting the observed values of $\log(\sigma/\dot{\varepsilon})$ of each layer against the density of the layer, the relation between the viscosity factor η_C and the density ρ for each layer of dry snow with a density range $0.1\sim0.5\,\mathrm{g\cdot cm^{-3}}$ was found to be expressed by

$$\eta_C(\rho)=\eta_{C1}\,\mathrm{e}^{k(\rho-\rho_1)}, \tag{2}$$

where η_{C1} is the viscosity factor for a definite density ρ_1 and k is a constant. For mathematical convenience eq. (2) is rewritten as

$$\eta_C(\rho) = Ce^{k\rho}, \tag{2 a}$$

where C is the value of η_{C1} when ρ_1 is reduced to zero. It was also found that k had the value 21 $g^{-1}\cdot cm^3$ for most dry compact snow layers and C was dependent not only on snow temperature but also on the initial type of snow crystals. The value of C was observed to be in a range between 0.6 and 1.6 $g\cdot day\cdot cm^{-2}$ for the snow in Sapporo, where the snow temperatures are usually in a range of $-5\sim0$°C. The viscosity~density relation which was reported by Kojima (1955, 1957, 1958) and by Yosida *et al.* (1956) is

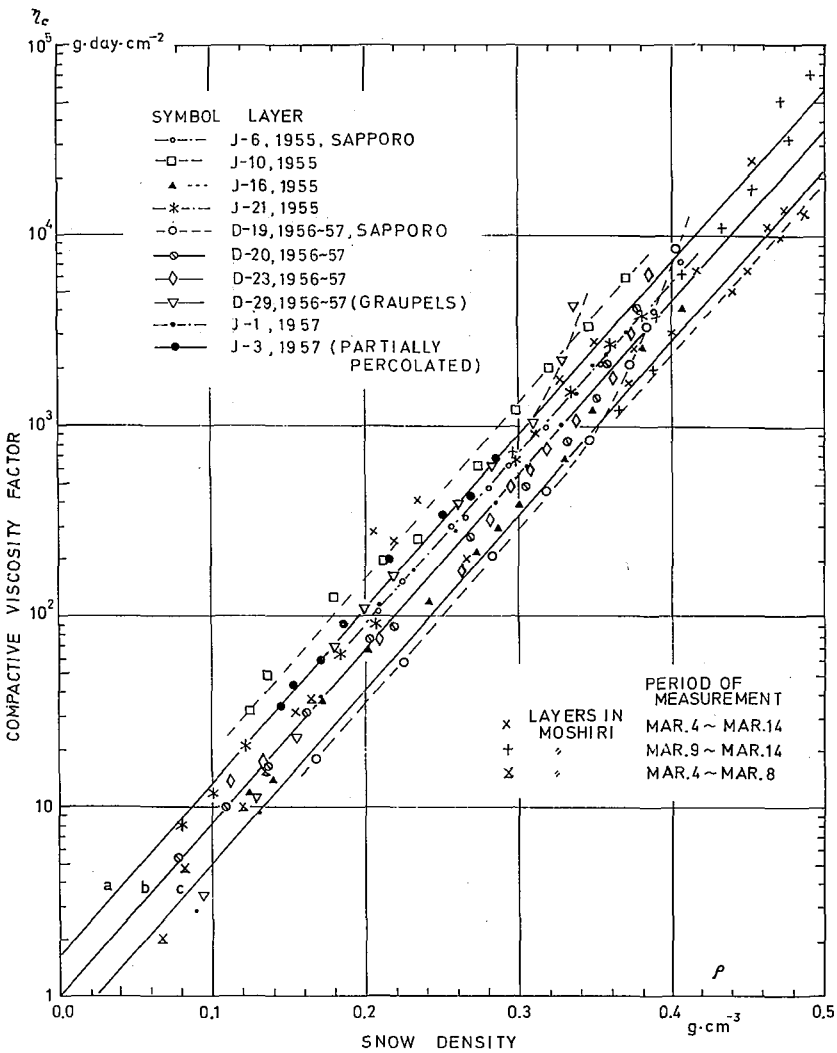

Fig. 1. The relation between compactive viscosity factor and density of snow layers in Sapporo and Moshiri, Hokkaido. The layers are of fine-gained compact snow including new snow. The snow temperatures ranges from 0 to -5°C in Sapporo and from -0.2 to -8°C for the layers in Moshiri

reproduced in Fig. 1.

2) *Equation of variation of snow density with time.* Substituting eq. (2 a) into eq. (1) and integrating both sides after separating the variables, the density ρ of a snow layer at a time t is given by

$$C\int_{\rho_0}^{\rho}\frac{e^{k\rho}}{\rho}\,\mathrm{d}\rho=\int_{t_n}^{t}\sigma(t)\,\mathrm{d}t\,, \tag{3}$$

where ρ_0 is the initial density and t_n is the time of deposition of the layer. The right hand side of the equation may be called the "time-integrated load" and may be denoted by $Q(t, t_n)$. Introducing the exponential integral in the left hand side, eq. (3) can be written as

$$C\left\{\mathrm{Ei}(k\rho)-\mathrm{Ei}(k\rho_0)\right\}=Q(t, t_n)\,, \tag{4}$$

where $\mathrm{Ei}(u)$ is the exponential integral defined by

$$\mathrm{Ei}(u)=\int_{-\infty}^{\infty}(\mathrm{e}^u/u)\,\mathrm{d}u\,.$$

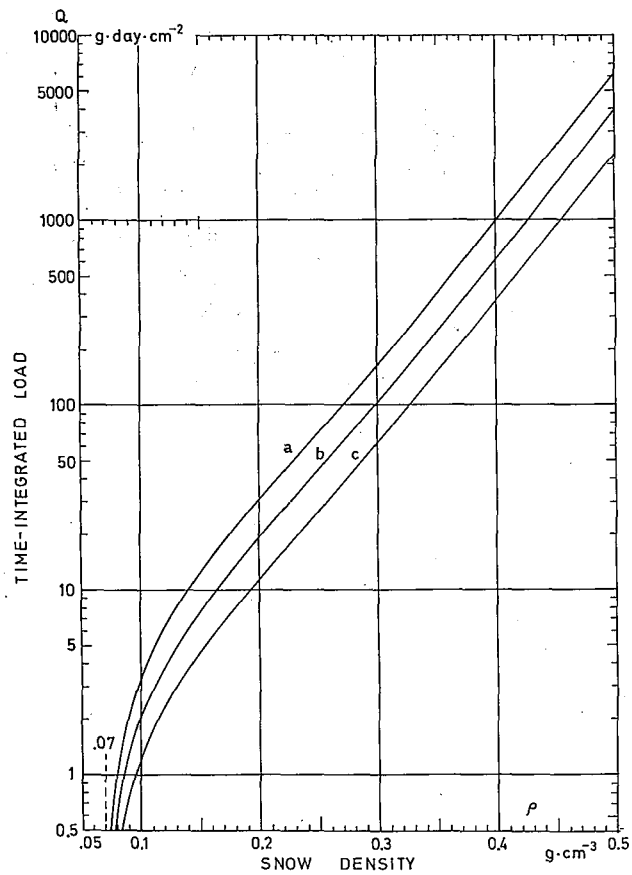

Fig. 2. The relation between snow density and time-integrated load. Three curves a, b, and c represent the relations given by eq. (4) taking C as 1.6, 1.0 and 0.6 g·day·cm^{-2} respectively with the same initial density 0.07 g·cm^{-3}

Figure 2 shows the relation between the density ρ of snow and the time-integrated load which is necessary for a snow layer to be subjected to gain the density ρ. Three curves a, b, and c express the relations for three different values of C of three straight lines in Fig. 1 and for the initial density 0.07 g·cm^{-3}. This value of ρ_0 is commonly observed in areas of Hokkaido where the wind is not very strong.

3) *Computation of depth-density profiles for varying accumulation rate.* Let the curve W(t) in Fig. 3 represent the accumulation of snow on a selected snow layer, which is named here as 0-layer. The time of deposition of the 0-layer is chosen as the origin of the time variable t. $W(t)$ is the amount of snow expressed in terms of the

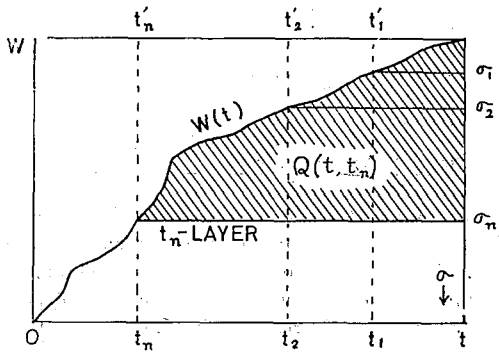

Fig. 3. Accumulation curve W(t) and time integrated load $Q(t, t_n)$ to which t_n-layer is subjected

weight accumulated on a unit horizontal area on the 0-layer from its deposition to time t. Let another layer formed at a time t_n be named t_n-layer. The weight of snow lying at the time t upon t_n-layer is expressed in Fig. 3 by the distance σ_n from the curve W(t) to the straight line marked t_n-layer. Integration of σ with time from t_n to t gives the time-integrated load, which is represented in Fig. 3 by the shaded area. This integration to obtain the value of $Q(t, t_n)$ may be easily performed graphically, by a planimeter, or other appropriate methods. Then, the density ρ_n of t_n-layer at time t can be determined by thus obtained values of Q and the $Q \sim \rho$ relation which is expressed by eq. (4) or a curve in Fig. 2, provided that a value is given to C. Applying the same procedure to other different layers such as $t_1, t_2, \cdots$ in Fig. 3, which are burdened with the load $\sigma_1, \sigma_2, \cdots$ respectively at the time t, the densities $\rho_1, \rho_2, \cdots$ of these layers at t can be obtained. In actual computations, it is convenient to use a fixed value for the initial density. As was used in the previous section, the value 0.07 g·cm^{-3} will be used as the average initial (or surface) density for the areas with comparatively weak winds and dry snow accumulation. Thus the density profile at the time is first derived as a relation between the load σ and the density ρ.

This $\sigma \sim \rho$ relation is easily transformed to a depth-density relation by numerically integrating $\frac{1}{\rho}$ with respect to σ. Since the density ρ at a depth z is defined as

$$\rho = \frac{d\sigma}{dz},$$

the depth z, where the load is σ, is given by

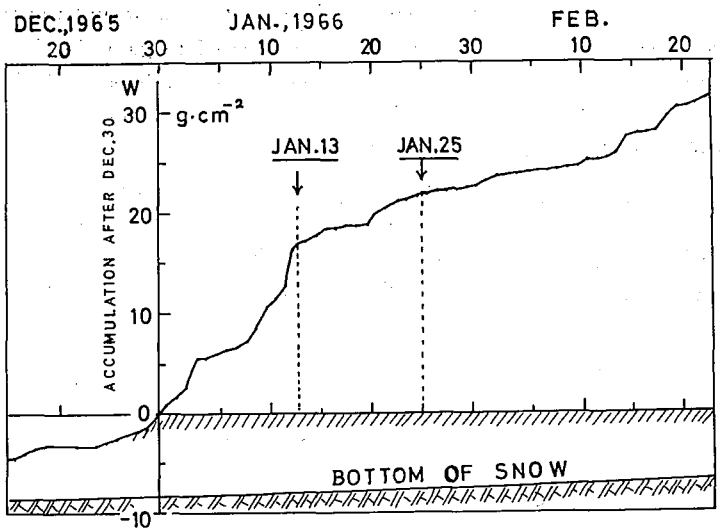

Fig. 4. Accumulation of snow in Sapporo in the winter of 1965–66. The ordinate $W(t)$ expresses the accumulation after Dec. 30, 1965. The distance between the abscissa and the line of "bottom of snow" shows the mass of snow deposited before Dec. 30, and its decrease with time means the melting of snow at the bottom

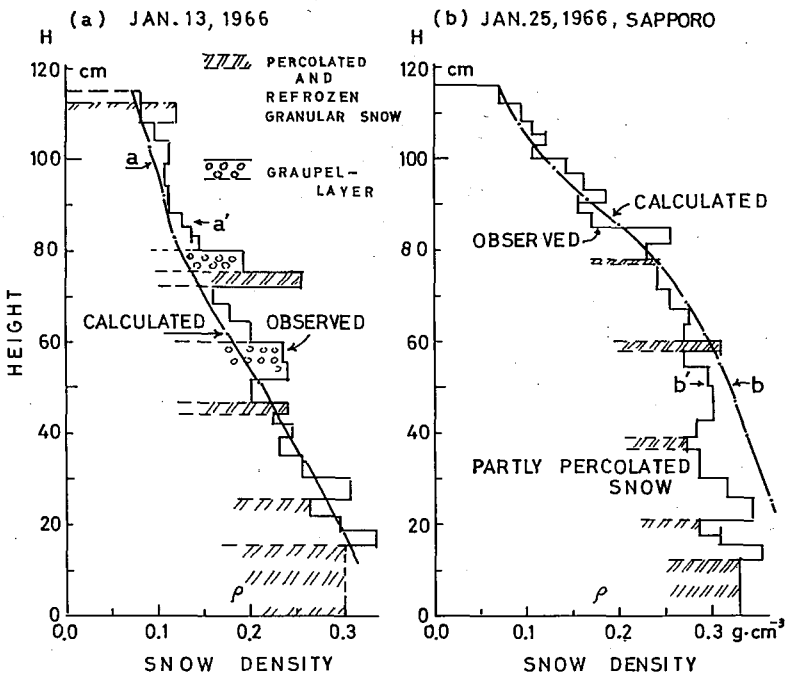

Fig. 5. Density profiles of snow in Sapporo on Jan. 13 and Jan. 25, 1966 showing their difference caused by non-linear accumulation

$$z = \int_0^{\sigma} \frac{1}{\rho} \, d\sigma \, . \tag{5}$$

This and the above obtained $\sigma \sim \rho$ relations provide the value of z for a value of ρ, namely, the depth-density relation at the time t. Variation of depth-density profile is obtained by repeating the application of the same procedure to the computations of density profiles at different times.

4) *Examples of computed density profiles and comparison with observed data.* Some examples of density profiles computed from the accumulation curves in Sapporo and Moshiri, Hokkaido are shown in this section and are compared with the observed profiles. Figure 4 shows the accumulation curve of snow in Sapporo during a period from the end of December, 1965 to the middle of February, 1966. The depth-density profiles for Jan. 13 and Jan. 25, 1966 are computed by using this accumulation curve, taking $C=1.0$ g·day·cm^{-2} and $\rho_0=0.07$ g·cm^{-3}. The computed density profiles for these different days are shown by the curves a and b in Fig. 5 and those observed on Jan. 13 and Jan. 25 are expressed by the graphs marked a′ and b′ in the same figure respectively. The depths of snow cover on these days were approximately the same, but the density profiles were quite different. The difference in densities, which should be caused by the difference in the past history of accumulation, is found to be well explained by the computed depth-density curves.

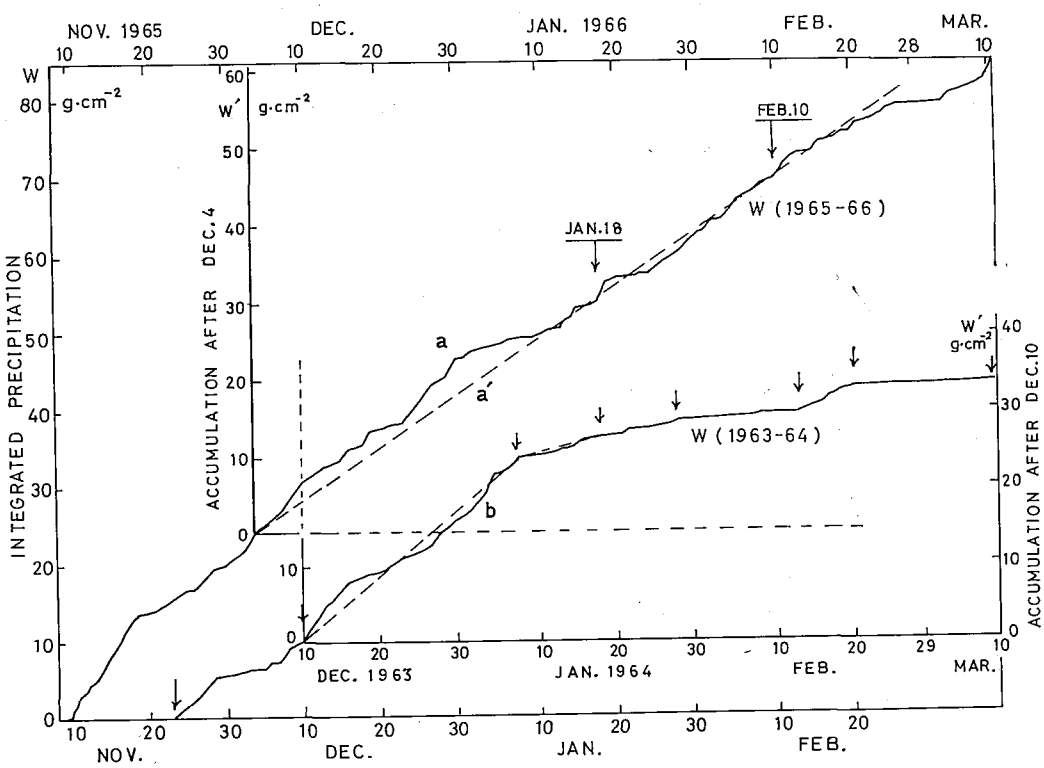

Fig. 6. Accumulation curves in Moshiri, Hokkaido in the winter of 1965–66 and 1963–64, which were obtained by successive summation of daily precipitations

The curve a in Fig. 6 shows another example of accumulation curve which is based on the data of daily precipitation observed at Moshiri in 1965–66. The accumulation curve for Moshiri is drawn by a successive summation of the amount of precipitation for each day from the beginning of the continuously lasting snow cover and plotting the integrated amount of precipitation against the date.

The density profiles on Jan. 18 and Feb. 10, 1966 are computed by using the accumulation curve a, in Fig. 6 taking $C=1.6$ g·day·cm^{-2} and $\rho_0=0.07$ g·cm^{-3}. The computed depth-density curves are shown in Fig. 7 (a) and (b) respectively. The density profiles

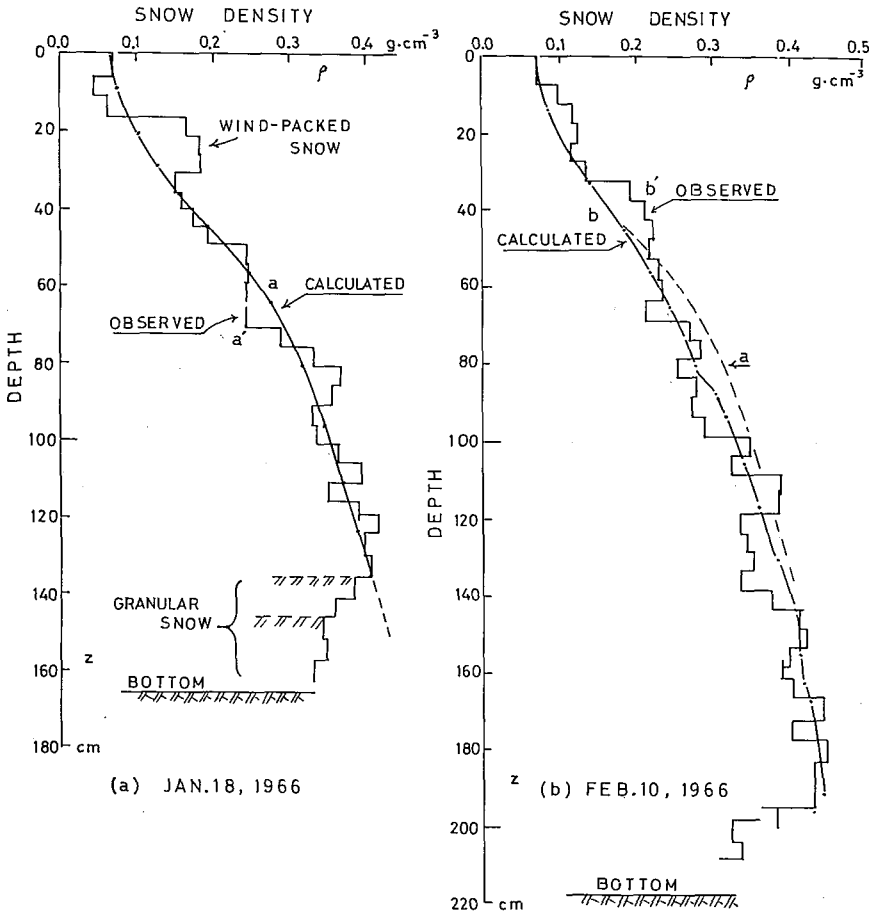

Fig. 7. Calculated and observed depth-density profiles of snow in Moshiri, Hokkaido on (a) Jan. 18, 1966 and (b) Feb. 10, 1966

which were observed on the pit walls on Jan. 18 and Feb. 10 are also shown in Fig. 7 (a) and (b) respectively. A good agreement is found between the computed and observed density profiles on each of the two different days. Further, as is expected from the approximately linear accumulation during the period we are concerned with, the observed depth-density profiles for the two days merely show a small difference between them. And the feature of this difference is also well reproduced by the computed curves.

The error of the computed density for a depth, which is related with the scattering of the plotted points in Fig. 1, may be estimated by the horizontal distance between the curves a and c in Fig. 2. Such the error is $\pm 0.026\,\mathrm{g\cdot cm^{-3}}$ for the densities larger than $0.1\,\mathrm{g\cdot cm^{-3}}$ and $\pm 0.012\,\mathrm{g\cdot cm^{-3}}$ when the density is $0.10\,\mathrm{g\cdot cm^{-3}}$ decreasing toward zero as the density approaches ρ_0, provided the actual surface density agrees with the assumed one. Another error in densities caused by the error in the assumed initial density should be significant especially for the upper layers of snow cover. This is, however, usually less than $0.03\,\mathrm{g\cdot cm^{-3}}$, if the average of surface densities observed at an area is taken as ρ_0 for calculating the compaction of snow at the area.

Temperature dependence of the constant C in eq. (3) has not been exactly determined. However, this may be approximately estimated from the results of some laboratory experiments which were conducted on the relation between the coefficient of compactive viscosity and the temperature of snow. Shinojima (1962) found the relation

$$\log\left(\frac{\eta_{C1}}{\eta_{C2}}\right) = 2.66\left(\frac{1}{T_1} - \frac{1}{T_2}\right) \times 10^3, \tag{6}$$

where η_{C1} and η_{C2} are the coefficients of compressive viscosity for the absolute snow temperatures T_1 and T_2 °K respectively. Kojima (1954) also obtained a similar but more intensive temperature dependence of compressive viscosity, that is expressed by

$$\log\left(\frac{\eta_{C1}}{\eta_{C2}}\right) = 4.55\left(\frac{1}{T_1} - \frac{1}{T_2}\right) \times 10^3, \tag{6 a}$$

for snow samples of the density $0.17\,\mathrm{g\cdot cm^{-3}}$. The result of an experiment made by Mellor and Smith (1966) gives an apparent activation energy, $17.8\,\mathrm{kcal\cdot mol^{-1}}$, for compression of snow with a density of $0.436\,\mathrm{g\cdot cm^{-3}}$. This leads to a value of $\Delta \log \eta_C / \Delta\left(\frac{1}{T}\right)$ intermediate between those for eqs. (6) and (6 a).

The difference in the values of $\log C$ which are used for the above described computations of densities in Sapporo and Moshiri is 0.20. Since the average temperature of compact snow in Sapporo in January and February, 1966 was approximately −2°C, if T_2 in eq. (6) is taken as 271°K, the value of T_1 which makes $\log(\eta_{C1}/\eta_{C2})$ equal 0.20 should be 264°K (−9°C) from eq. (6) and 266°K (−7°C) from eq. (6 a). On the other hand, the monthly average air temperature in January, 1966 was −11.3°C in Moshiri and −4.7°C in Sapporo with difference of 6.6°C. Considering these values, it may be presumable that the appropriate value of C for computing the density profiles in Moshiri should be $1.6\,\mathrm{g\cdot day\cdot cm^{-2}}$ or under.

III. Variation of Depth of Snow Cover

1) *Variation of snow-depth under a constant accumulation rate*

As was previously reported by Kojima (1957) and by Yosida (1958, 1963), the depth H of snow cover at the time t-days after the beginning of a continuous accumulation of a constant rate A may be given by

$$H(t) = A\int_0^t \frac{1}{\rho(t)}\,\mathrm{d}t, \tag{7}$$

if no liquid precipitation occurs and no melting takes place during the t days. Since

the density of any t_n-layer at time t under these conditions is given by

$$C\left\{\mathrm{Ei}(k\rho)-\mathrm{Ei}(k\rho_0)\right\}=Q(t,t_n)=\frac{A}{2}(t-t_n)^2\,, \tag{8}$$

the integral of eq. (7) is easily computed by numerically integrating $\frac{1}{\rho}$ with respect to t, following the equation

$$C\left\{\mathrm{Ei}(k\rho)-\mathrm{Ei}(k\rho_0)\right\}=\frac{A}{2}t^2\,. \tag{8 a}$$

The time variation of the depth of snow cover at Moshiri was computed, considering the accumulation rate to be approximately constant and using eqs. (7) and (8 a), for the period from Dec. 4, 1965 to the end of February, 1966. Since there were occasional rain falls and considerable melting before and after this period, Dec. 4 was chosen as the origin of time t for the calculation by eq. (7). Then the computed depths of snow deposited after Dec. 4 were added to the thickness of the layer of granular snow formed before Dec. 4. This pre-thickness was 38 cm on Dec. 4, 30 cm on Jan. 18, and 21 cm on Mar. 11. The accumulation rate A is given as 0.68 g·cm^{-2}·day^{-1}, which is the slope

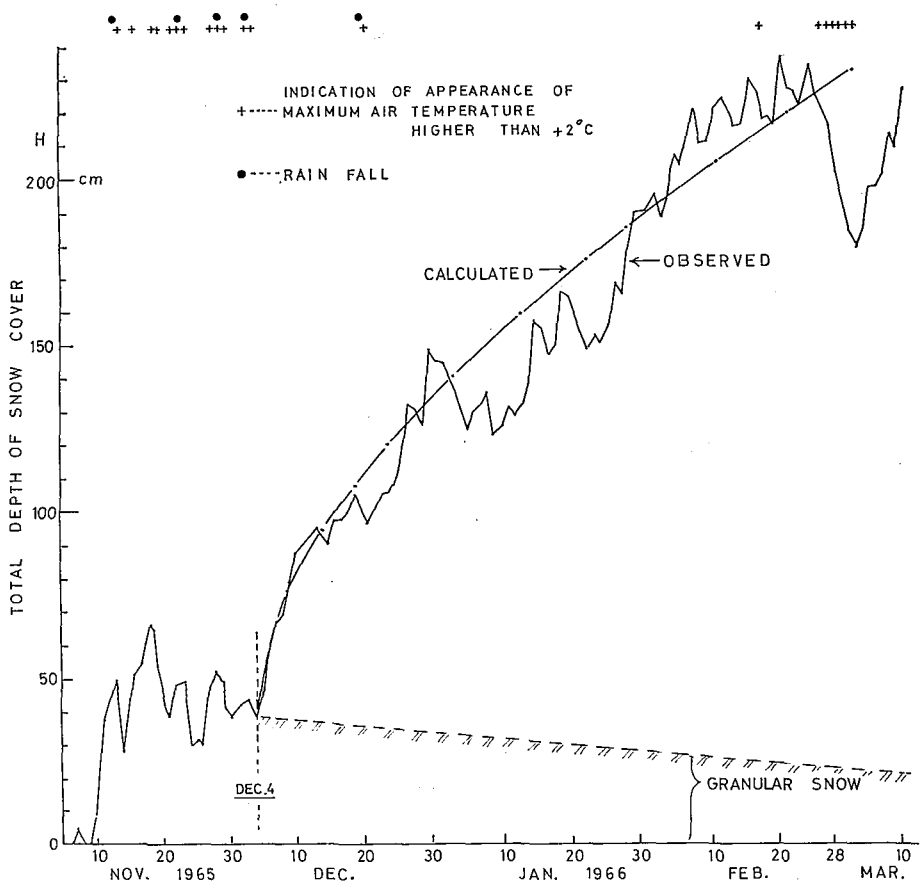

Fig. 8. Calculated and observed variations of snow depth in Moshiri in 1965-66. A linear accumulation expressed by a straight line a′ in Fig. 6 is used in the calculation

of the straight line a′ in Fig. 6. The thus computed depth-time curve and the observed variation of snow depth are shown in Fig. 8. The general tendency of increase in the observed snow depth was found to be approximately represented by the depth-time curve based on calculation. Some deviations of the calculated curve from the observed depths may be caused by the following reasons: 1) the temperature was sometimes rather high before the middle of January, but such a variation in temperature was not taken into account for the calculated curve, 2) actual accumulation was not exactly linear, and 3) the accumulation curve a in Fig. 6 may have some errors which come from the error of measurements of daily precipitation.

2) *Variation of snow-depth under varying accumulation rate*

Another accumulation curve b in Fig. 6 represents the integrated precipitation in Moshiri after Nov. 21, 1963. The rate of accumulation was rather high before Jan. 7, 1964, but thereafter it decreased with a remarkable change. An approximate linear accumulation is considered only during each of several short periods as shown by the straight lines crossing the curve b in Fig. 6, where the short vertical arrows indicate the boundaries of the periods. The computation of time variation of the depth of snow which was accumulated after Dec. 10 was made using eq. (7) for only the first period from Dec. 10 to Jan. 7 and the depths thereafter were obtained by the following procedure. To calculate the depth on Jan. 18, 1964, for instance, the total thickness of snow cover is divided into three parts. The lowest part is the snow accumulated before Dec. 10, the second is the layer deposited during the period from Dec. 10 to Jan. 7, and the third is the upper part formed after Jan. 7. The thickness of the last part is calculated by the method for linear accumulation assumed for the period from Jan. 7 to

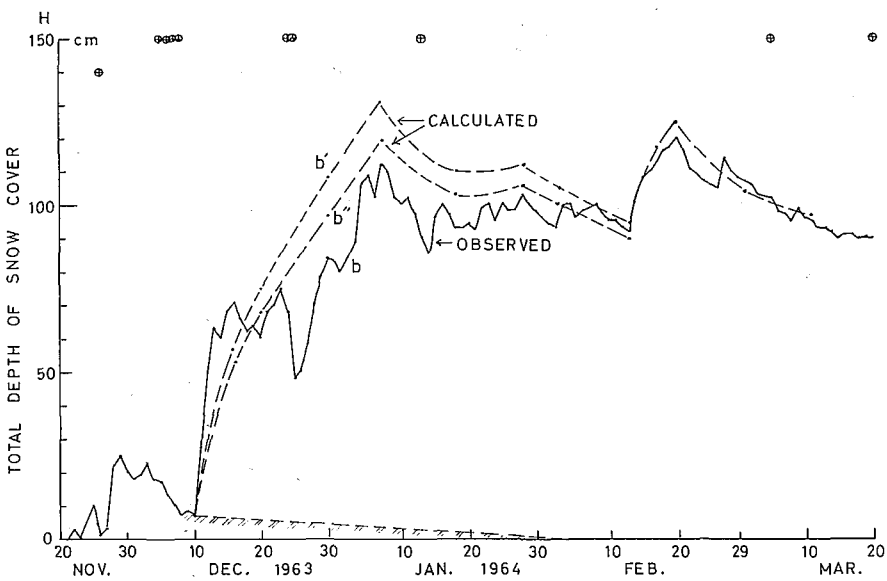

Fig. 9. Calculated and observed variations of snow depth in Moshiri in 1963–64. Varying accumulation rate is considered for the calculation. Each circle with a cross at the top of the figure shows the day with the maximum air temperature higher than +2°C

Jan. 18. To obtain the thickness of the second part on Jan. 18, the load-density relation of this part is first computed and then this is transformed to the relation between the load and the depth below the upper boundary of the layer using eq. (5). The thickness of these two layers is added to that of the old granular snow formed before Dec. 10 to obtain the total depth at that time. The rate of melting of snow at the bottom of the snow cover is assumed to be 0.13 cm·day^{-1} of snow. The dashed curves b′ and b″ in Fig. 9 represent thus calculated variations of snow depth for the values of C, 1.6 and 1.0 g·day·cm^{-2} respectively.

Time variation of the depth of snow in Sapporo observed during January and February in 1966 is shown by a thick graph in Fig. 10 and the computed depths are shown

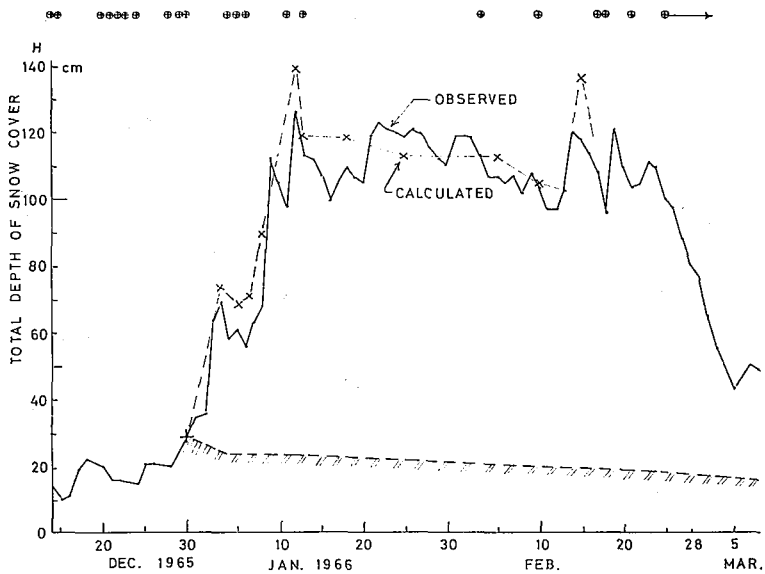

Fig. 10. Variation of depth of snow cover in Sapporo in the winter of 1965–66, showing a comparison between the calculated depths and the observed data. Each circle with a cross at the top of the figure shows the day with the maximum air temperature higher than +2°C. The arrow attached on the last circle means that such a warm condition appeared everyday after Feb. 25

by crossed marks and dashed lines in the same figure. The computation is based on the accumulation curve in Fig. 4 and the curve b in Fig. 2 for which $C=1.0$ g·day·cm^{-2} and $\rho_0=0.07$ g·cm^{-3}. No linear accumulation is considered in this case except within each day. The characteristics of time variation of the depth of snow for this case of non-linear accumulation is well expressed by the computed result.

IV. Densification of Wet Snow

Densification of wet snow was studied at Shiozawa, Niigata Prefecture, where the snow cover, which is 1~2 m deep in February, is usually wet throughout the winter except for some layers, during a day or two after their deposition.

If a snow layer of fine grains is very wet, it is compacted very rapidly until the texture changes to that of granular snow with coarse grains. Then the wet granular snow is compacted at a very much lower rate than that for dry compact snow with a medium density. On the other hand, if the content of liquid water in a snow layer does not exceed 5%, the effect of existence of liquid water upon the metamorphism of snow grains is not very remarkable and the snow maintains the feature of compact snow of fine grains for many days.

Figure 11 (a) shows the decrease in thickness h and the increase in density ρ of such a slightly wet snow layer. The upper curve ρ_w shows the increase in the observed

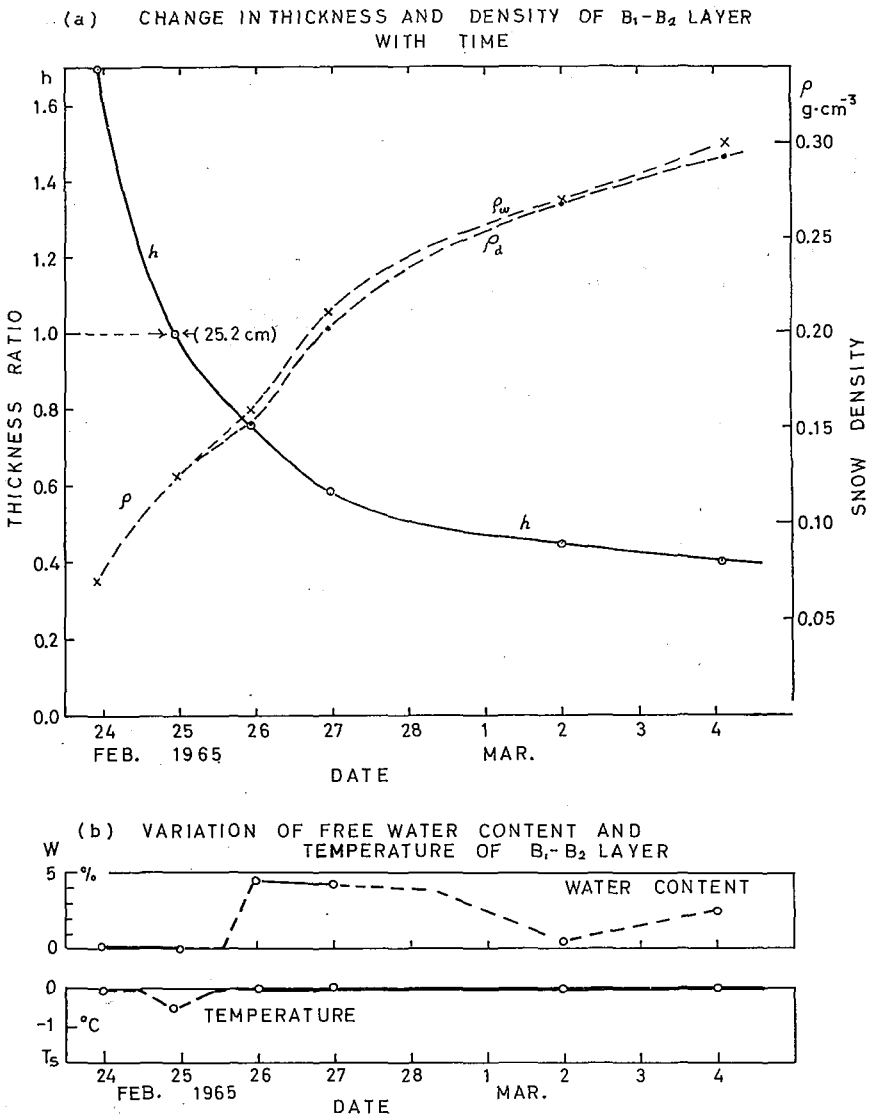

Fig. 11. Time variation of the features of a slightly wet snow layer. (a), thickness h and density ρ; (b), free water content W and snow temperature T_s

gross density including liquid water, and the lower curve ρ_d represents the density of ice network which is obtained by subtracting the contribution of free water from the gross density. Several time markers $B_1, B_2, \cdots$, for the convenience of pit wall observations, were made by scattering colored powder on the snow surface, and the snow lying between B_1 and B_2 is named B_1–B_2 layer. The variation of the free water content and the snow temperature of B_1–B_2 layer are shown in Fig. 11 (b). The free water content was measured by a calorimetric method developed by Yosida (1959).

Figure 12 shows some other examples of variation of the density of wet snow layers. The thick curves show the increase in ρ_d of F–23B and F–24A layers which

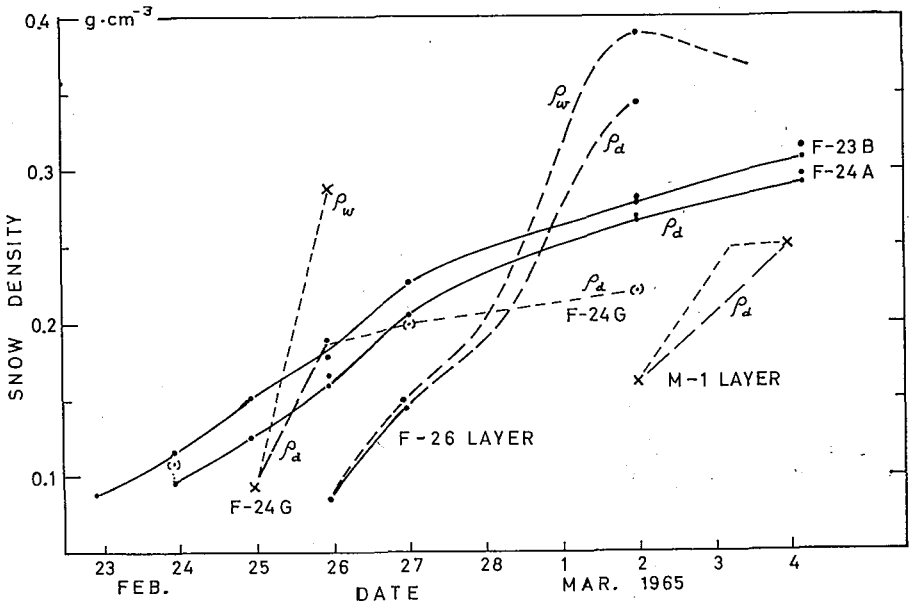

Fig. 12. Variation of the density of wet snow layers. Solid curves show the increase in density of slightly wet snow and dashed lines show the change in density of snow layers which are getting very wet

are about 3 cm thick layers and are portions of B_1–B_2 layer. The values of ρ_w for each layer are also plotted but are not connected by a curve. Variation in the densities ρ_w and ρ_d of very wet snow, F–24G and F–26 layers, are shown by dashed curves in Fig. 12.

The maximum free water contents of these layers were 35 and 12% respectively which changed to coarse grained wet snow. The third and the fourth values of ρ_d for F–24G layers, plotted in Fig. 12, were determined by the second observed value of ρ_d and the thicknesses observed still later.

The compactive viscosity factors of these wet snow layers are plotted against the dry-density ρ_d in Fig. 13. The viscosity-density relation for B_1–B_2 layer is determined from the curve h in Fig. 11 (a) and are shown by small solid circles in Fig. 13. The two long straight lines marked S-a and S-c in Fig. 13 represent the upper and the lower limits of the relations for dry compact snow in Sapporo, where the temperature of such snow is usually in a range of 0～−5°C. Comparing these lines with the relations for

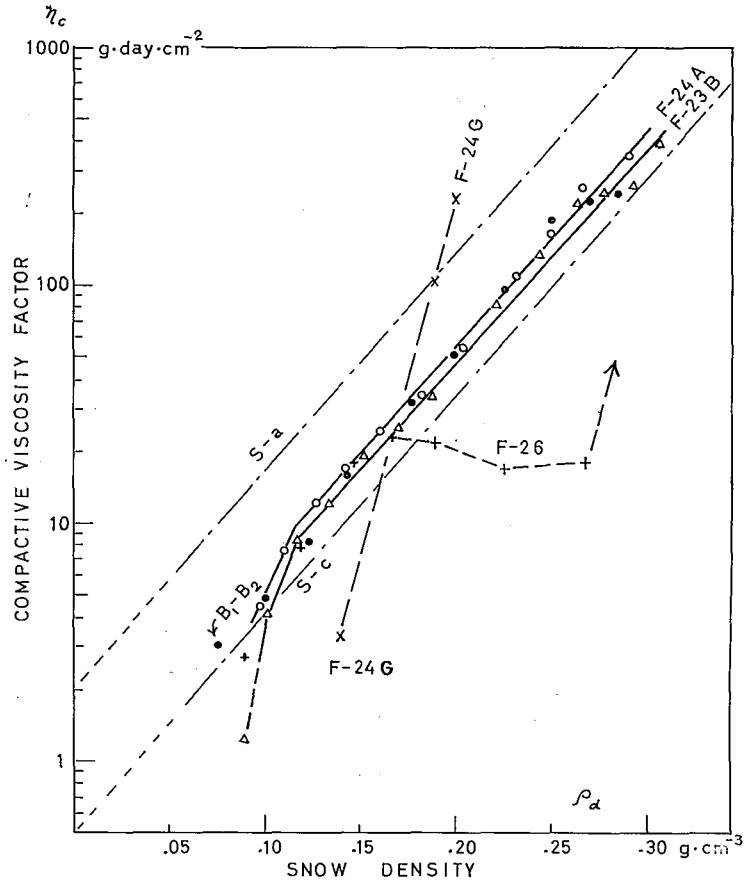

Fig. 13. The relation between the compactive viscosity factor and density of wet snow layers. The density means the dry-density of ice-network composing wet snow

the wet snow layers, it was found that the viscosity-density relation for slightly wet snow is approximately the same as that for dry compact snow with the temperature at 0°C. The dashed lines marked F-24G and F-26 in Fig. 13 represent the values of (load/strain-rate) for very wet layers in relation to the density ρ_d.

It was also found that the coarse grained wet snow has a much higher viscosity factor than that for dry compact snow of the temperature close to 0°C, if compared for the same density.

V. Compactive Viscosity of Wind-Packed Snow and Depth-Hoar Layers

1) *Wind-packed snow.* When a snow layer is formed under the action of a comparatively strong wind, the initial density of newly deposited snow is in a range of 0.10~0.15 g·cm^{-3} in flat plain areas in Hokkaido. The relations between the compactive viscosity factor and the density of two wind-packed snow layers in Sapporo are shown by the straight lines marked a and b in Fig. 14. It was found that the viscosity factors

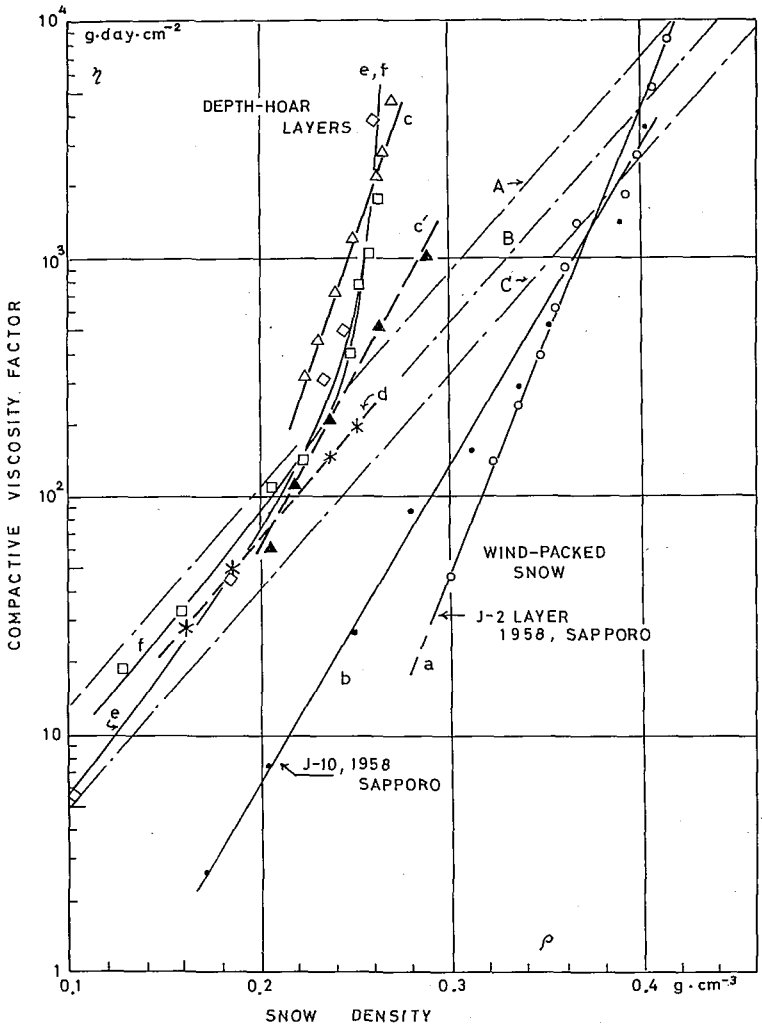

Fig. 14. The viscosity-density relations for depth-hoar layers and wind-packed snow layers

of these layers were very low as compared with those for ordinary snow with the same density, and it was noted that the viscosity factors increase more rapidly with respect to density than the viscosity of an ordinary compact snow layer. The reason for this may be explained as follows: the wind-packed snow contains a great number of small fragments of snow crystals within the network consisting of larger grains and the fragments contribute to the comparatively large density but are not effective in increasing the viscosity until the compaction proceeds to some extent.

The maximum of ten-minute averages of the wind speed during the formation of the layer for curve a in Fig. 14 was 28 m·sec^{-1} at a height of 10 m above the ground, which is quite unusual in Sapporo.

2) *Depth-hoar layers.* The influence of formation of depth-hoar crystals in snow upon the viscosity factor and the hardness of the snow was studied under field conditions

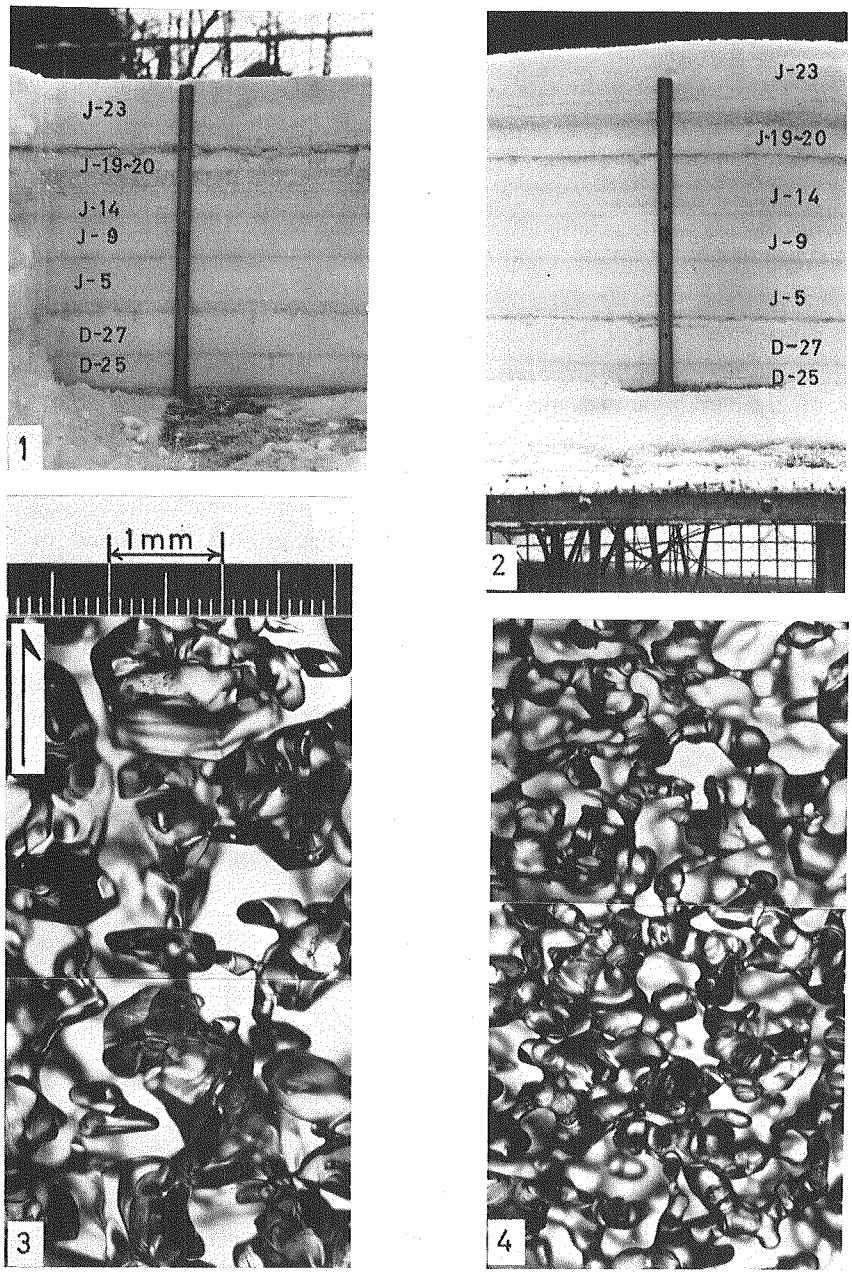

Fig. 15. 1 : Vertical pit wall of snow deposited on the ground in Sapporo. (Photographed on Jan. 24, 1959)

2 : Vertically cut wall of snow deposited on the wire net 1.7 m above the ground. (Photographed on Jan. 24, 1959)

3 : Micro-photograph of a thin-section vertically cut from D-27 layer on the ground. The vertical direction is indicated by an arrow in the photograph. (Sampled and photographed on Jan. 31, 1959) (× 15)

4 : Vertical thin-section of D-27 layer on the net. (Sampled and photographed on Jan. 31, 1959) (× 15)

and was previously reported by Kojima (1956, 1959) partly in Japanese.

The compaction properties of snow deposited on the ground were compared with those of the snow deposited on a broad net of iron wire stretched taut to cover a cage, at a height of 1.7 m above the ground with a surface area of 2.7×3.6 m. Since the vertical sides of the cage are also made of iron nets, the snow deposited on the cage is cooled by the cold air from both above and below. The temperature gradient within the snow on the net is, therefore, always smaller than that within the snow accumulated on the ground. The weather conditions at Sapporo in January, 1959 were such that numerous depth-hoar crystals were produced in the lower layers of snow on the ground, while no depth-hoar apeared in the snow deposited on the iron net. Photograph 1 and 2 in Fig. 15 show the stratigraphy on the vertical wall of snow on the ground and that of snow on the net. Photographs 3 and 4 in Fig. 15 show the microphotographs of vertical thin sections cut, on Jan. 31, from D–27 layer formed on Dec. 27, 1958 on the ground and D–27 layer on the net respectively. The difference in the texture of snow on the ground and that on the net was found to be very remarkable.

The compactive strain rate of D–27 layer on the ground was found to be much diminished as the depth-hoar crystals developed within the layer as compared with that for D–27 layer on the net. This lead to comparatively high values of compactive viscosity of the depth-hoar layer. The relations between the compactive viscosity factor and the density of D–27 layer on the ground and of D–27 layer on the net are represented by the graphs c and c′ in Fig. 14 respectively. The straight line marked d in the figure shows the viscosity-density relation for J–14 layer on the ground. No hoar crystals were produced within this layer because it was in the level where the temperature gradient did not continue to exist in the same direction. And the viscosity-density relation was found to be the same as that of ordinary compact snow.

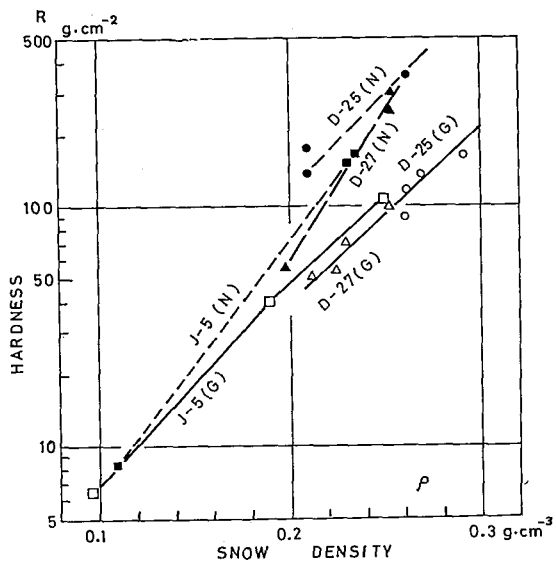

Fig. 16. The relation between hardness and density of snow, showing a comparison between depth-hoar layers and compact snow of fine grains

Since the temperature of snow on the ground was always higher than that of snow on the net, the observed viscosities were reduced to the values at −5°C by using the viscosity-temperature relation of eq. (6 a). The viscosity factors plotted in Fig. 14 for these layers are thus corrected values. The curves e and f in Fig. 14 represent the viscosity-density relations for two depth-hoar layers observed in Sapporo in 1956. These results show that the depth-hoar layers have a much greater viscosity against a gradual compaction than a fine grained compact snow has, with respect to the same density.

Regarding the hardness of snow against a rapidly increasing stress of compaction, such as an impulsive force of breaking snow, the depth-hoar layers D–27 and J–5 on the ground were found to have much smaller values than those on the net had. This is shown in Fig. 16 by the graphs of hardness-density relations for these layers. The hardness of another depth-hoar layer, which was observed in Sapporo in 1956, was also found to be 1/4∼1/5 as small as that of a fine grained compact snow layer of the same aging.

VI. A Field Experiment on the Relation between Strain Rate and Snow Load

It is first assumed that the strain rate of densification of a snow layer is proportional to the load exerted by the weight of overlying snow. A field experiment was made to examine whether this simple creep law applies exactly to the densification of snow under natural conditions.

The snow load on some selected layers were artificially subtracted from or added to some portions of snow cover as illustrated in Fig. 17. Such a modification of snow was made on one side of a trench, which was dug in the snow. The trench was 10 m long, 2 m wide, and 1 m deep. In the area marked A in Fig. 17, an upper part of the

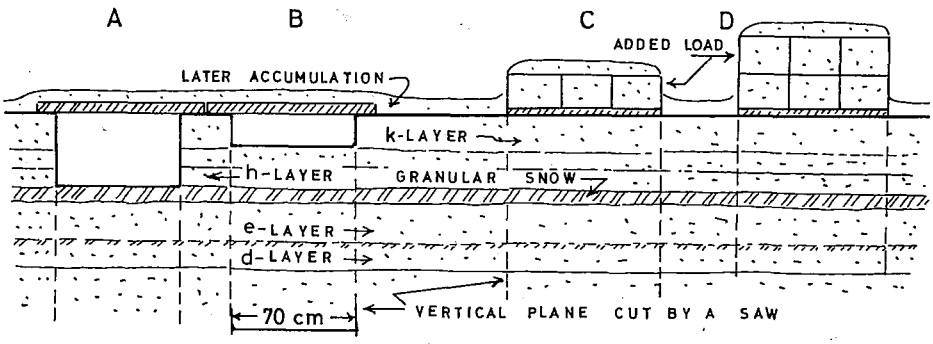

Fig. 17. Artificial subtraction and addition of snow load in the field experiment in Moshiri done in order to observe the relation between strain rate and snow load

snow cover, 35 cm deep and 70×70 cm wide, was carefully removed keeping the upper surface of the remaining snow flat and horizontal. The removed snow from the next section B had a thickness of 15 cm and the same area as that of A. The trench wall of the test site was covered again by piling up snow blocks, and the holes in A and B were covered by polystyrene-foam plates of 5 cm thickness. To add an extra snow load, at a test section such as C in Fig. 17, several snow blocks with the same thickness

were placed on a plate of polystyrene foam which was laid on the natural surface. The densities of several selected layers, such as d-, e-, h-, and k-layers indicated in Fig. 17 were measured before and after the test period. The strain rates of compaction of the layers are obtained from the observed increases in density of the layers. The amount of subtracted snow load and added load for each section, the period of strain measurement, and the data for a layer h, as an example, are listed in Table 1.

The strain rates for the layers h, e, and d are found to be proportional to the loads on these layers within the error of measurements as shown in Fig. 18 (a), where the vertical line on each plotted point represents the possible error in the strain rate. But a linear relation between the strain rate and the load was not found for the k-layer as shown in Fig. 18 (b). The above mentioned results were obtained at Moshiri in March, 1966.

Another experiment was made in the same way at Sapporo in 1964 as a trial attempt, where only a part of the snow cover was modified by removing an upper layer with a thickness of 30 cm. The increases in the density in 10 days were measured for

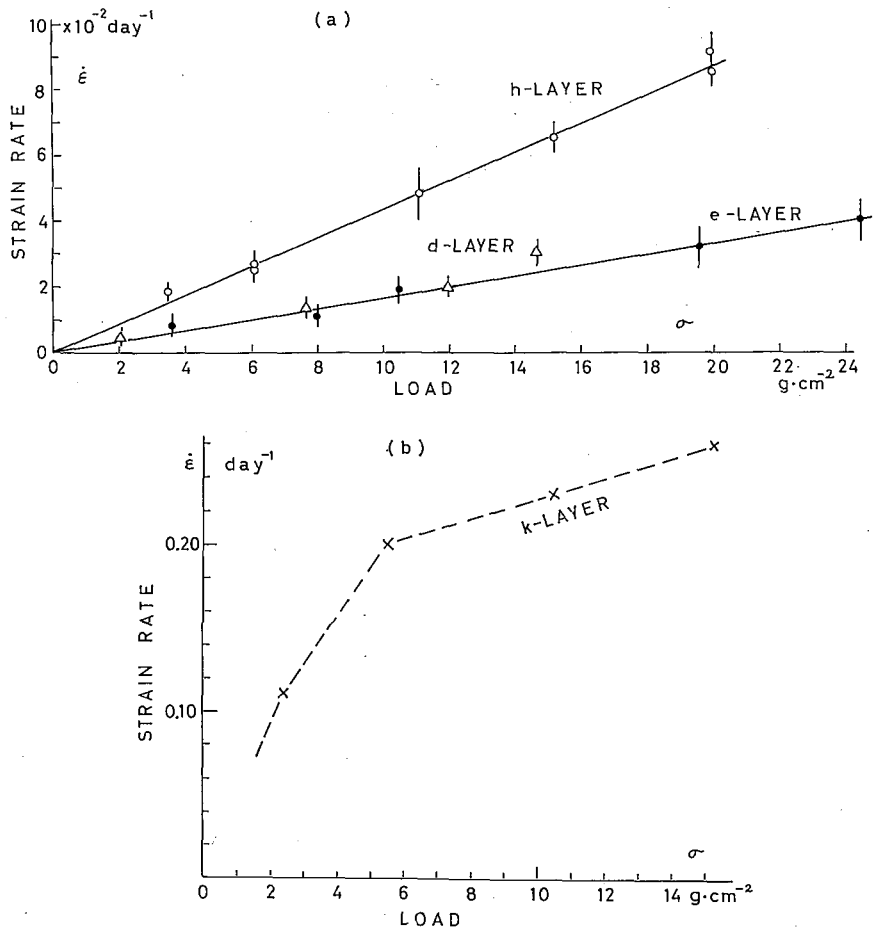

Fig. 18. The relations between strain rate and load, which were obtained by a field experiment in Moshiri, Hokkaido

Table 1. Subtracted and added snow load, period of measurements, and the data for h-layer

		A	B	C	D	E
Subtracted load	g·cm^{-2}	6.9	2.5			
Added load	g·cm^{-2}			5.4	9.0	13.1
Period	days	4.8	4.8	2.3	3.0	2.0

Data for h-layer

		B	Natural		C	D	E
Average load	g·cm^{-2}	3.5 (const.)	6.1		11.1	15.2	20.0
Temperature	°C		−1.5∼−0.5				
Initial density	g·cm^{-2}	0.196	0.196	0.218	0.218	0.217	0.249
Density increment	g·cm^{-2}	0.022	0.039	0.012	0.022	0.030	0.043
Time interval	days	4.8	4.8	2.0	2.0	2.0	2.0
Strain rate		0.019*		0.027	0.048	0.065	0.085

* Reduced to the value for the same period as others.

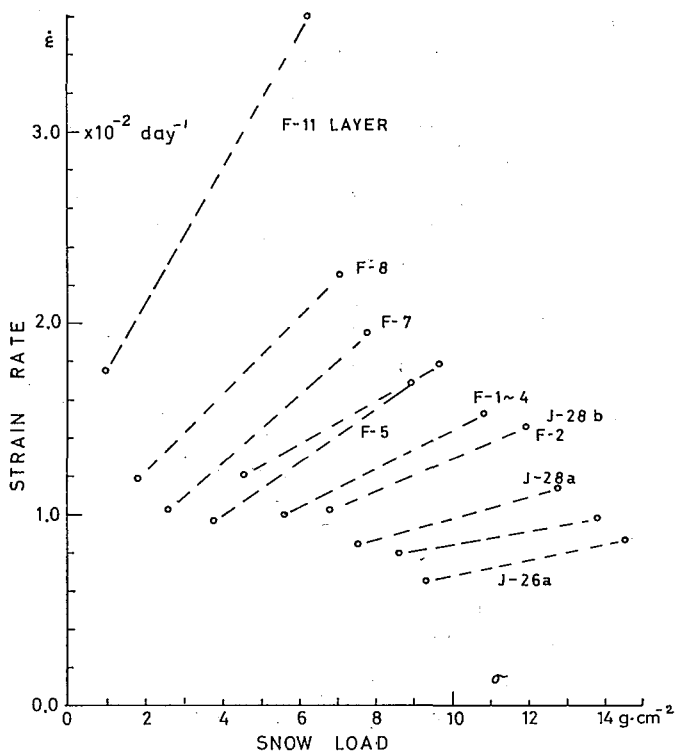

Fig. 19. Strain rates versus snow loads which were observed by a field experiment in Sapporo

several layers at unmodified sites and those from which the overlying snow load was partially subtracted and kept constant. The observed strain rates are plotted against the loads, which are constant with time for the layers of modified area and 10 day averages for the layers in the natural area, as shown in Fig. 19.

The results suggest that the rate of compactive strain of snow might be related not only to the load but also to another factor which might be especially effective for young snow layers in the upper portion of a snow cover. The temperature of a layer at the modified site and in a natural site had no significant difference.

VII. Discussions

Some results of observations in Sapporo show that the compactive viscosity of a new snow layer, which is obtained by $\sigma/\dot{\varepsilon}$, is sometimes much lower than that given by eq. (2 a) which holds in the viscosity-density relation for later stage densification of the layer. In other words, the relation between $\log \eta_c$ and the density is, for some cases, not linear when the density is very small. The reason may be considered to be partly in the effect of cyclic variation of snow temperatures which take place in the upper part of snow cover. If there is a non-linear relation between the strain rate of new snow and the load, as stated in the previous section, the observed value of $\sigma/\dot{\varepsilon}$ may not provide the viscosity factor in an exact sense. And, if so, it may be possible that $\sigma/\dot{\varepsilon}$ for a new snow layer has smaller values than those expected from the $\eta_C \sim \rho$ relation for the range of larger density of the layer.

The effect of cyclic variation of snow temperatures upon the compactive strain rate was discussed by Bader (1962) and, also, by Kojima (1964) concerning the rate of densification of upper layers of snow in Antarctica. However, the effect has not been ascertained by actual measurements of variation of snow temperature and the rate of strain.

All the computations for density profiles and the related phenomena, which are described in Section 2 and 3, are based on the empirical viscosity-density relation, where the viscosity of snow is dealt with as a function of snow density alone. It was, however, experimentally shown by Kinosita (1960) that the viscosity of snow is also dependent on the strain rate. Further, the viscosity of snow is considered to increase as the sintering of snow grains proceeds even if the density remains constant. Further investigation may be required to determine how to introduce the effects of these factors into the theory to make more precise explanations of densification phenomena of snow. It may be said, however, that the above described theory with an empirical viscosity-density relation is rather convenient to be applied for an approximate estimation of densification phenomena from a given condition of snow accumulation and vice versa because of its simplicity in the mathematical expressions.

Widely scattered values of the compactive viscosity factor for various types of snow in Figs. 1, 13, and 14 seem to be in an arrangement in the order of grain size or any similar quantity expressing the microscopic texture of snow. Studies on the viscosity of snow in connection to its microscopic texture, such as, for instance, Nakaya's experiment in Greenland (Nakaya and Kuroiwa, 1966) and a theoretical study by Feldt and Ballard (1966), may be of great importance.

Acknowledgments

A number of density measurements at Moshiri were made in the yard of the forest ranger's office of the Experimental Forest of the Hokkaido University. The meteorological data, which were routinely observed there, were also very useful. The observation on densification of wet snow was made at the Snow Research Station in Shiozawa, a branch of the Technical Research Institute of the Japanese National Railways. The author expresses his thanks to Dr. Mikio Shoda, the chief of the station, and to other people at the station who helped the author in his work.

The suggestions and encouragements given by Prof. Hirobumi Ôura, Prof. Zyungo Yosida, and the late Prof. Ukichiro Nakaya of the Hokkaido University during the studies on this field were especially helpful.

References

1) ANDERSON, Don L. and BENSON, C. S. 1963 Densification and diagenesis of snow. *In* Ice and Snow (W. D. KINGERY, *ed.*), M.I.T. Press, Cambridge, Mass., 391–411.
2) BADER, H. 1960 Theory of densification of dry snow on high polar glaciers, 1. *SIPRE Res. Rept.*, **69**, 1–8.
3) BADER, H. 1962 Theory of densification of dry snow on high polar glaciers, 2. *CRREL Res. Rept.*, **108**, 1–18.
4) BENSON, C. S. 1959 Physical investigations on the snow and firn of northwest Greenland 1952, 1953, and 1954. *SIPRE Res. Rept.*, **26**, 1–62.
5) FELDT, E. D. and BALLARD, G. E. 1966 A theory of the consolidation of snow. *J. Glaciol.*, **6**, 145–157.
6) KINOSITA, S. 1960 The relation between the deformation velocity of snow and the types of its deformation. *Low Temp. Sci.*, **A 19**, 135–146.*
7) KOJIMA, K. 1954 Visco-elastic properties of snow. *Low Temp. Sci.*, **A 12**, 1–13.*
8) KOJIMA, K. 1955 Viscous compression of natural snow-layer, I. *Low Temp. Sci.*, **A 14**, 77–94.*
9) KOJIMA, K. 1956 Viscous compression of natural snow-layer, II. *Low Temp. Sci.*, **A 15**, 117–136.*
10) KOJIMA, K. 1957 Viscous compression of natural snow layers, III. *Low Temp. Sci.*, **A 16**, 167–196.*
11) KOJIMA, K. 1958 Viscous compression of natural snow layers, IV. *Low Temp. Sci.*, **A 17**, 53–64.*
12) KOJIMA, K. 1959 The influence of temperature gradient upon the grain texture, settling rate, and brittleness of snow. *Low Temp. Sci.*, **A 18**, 29–46.*
13) KOJIMA, K. 1964 Densification of snow in Antarctica. *In* Antarctic Snow and Ice Studies (M. MELLOR, *ed.*), Antarctic Res. Ser. 2, Amer. Geophys. Union, Washington, D. C., 157–218.
14) MELLOR, M. and SMITH, J. H. 1966 Creep of snow and ice. *In* Physics of Snow and Ice, Part 2 (H. ÔURA, *ed.*), Inst. Low Temp. Sci., Sapporo, 843–855.
15) NAKAYA, U. and KUROIWA, D. 1966 Physical properties and internal structure of Greenland snow. *In* Physics of Snow and Ice, Part 2 (H. ÔURA, *ed.*), Inst. Low Temp. Sci., Sapporo, 953–971.
16) SHINOJIMA, K. 1962 Study on the visco-elastic deformation of deposited snow. *Railway Tech. Res. Rept., Japanese National Railways*, **328**, 1–56. (In Japanese).

17) YOSIDA, Z. *et al.* 1956 Physical properties of deposited snow, II. *Contr. Low Temp. Sci.*, **9**, 1–81.

18) YOSIDA, Z. *et al.* 1958 Physical properties of deposited snow, IV. *Contr. Low Temp. Sci.*, **13**, 55–100.

19) YOSIDA, Z. 1959 A Calorimeter for measuring the free water content of wet snow. *Low Temp. Sci.*, **A 18**, 17–28.*

20) YOSIDA, Z. 1963 Physical properties of snow. *In* Ice and Snow (W. D. KINGERY, *ed.*), M. I. T. Press, Cambridge, Mass., 485–527.

* In Japanese with English summary.