Deformation of Excavations in a High Polar Névé

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Abstract

Excavations have been made in a high polar névé as part of a long-range study toward the development of a rational snow mechanics. Ten years of observed data on the deformation and computed overburden are presented for a shallow horizontal tunnel in snow, and eight years of data are given for a horizontal tunnel at a greater depth. Data on other excavations in snow are presented and the inadequacy of current theories are discussed. A minimum deformation rate is predicted for a depth of about forty meters based upon the increase of both overburden pressure and viscosity with depth.

I. Introduction

Excavations have been made in a high polar névé as part of a long-range study toward the development of a rational snow mechanics and to provide design criteria for installations made on or in the névé. The purpose of this report is two-fold: (a) to make available new data on long-term studies of deformation; and (b) to discuss current theories on deformation of tunels in snow.

The site selected (Site II) is about 360 km east of Thule, Greenland. It is at an elevation of about 2 000 m and is typical of a high polar glacier area. It has had an average annual new accumulation of about 1 m of snow, or an average of 39 cm of water equivalence, for the past several years, although the mean from 1919 to 1954 is 41.5 cm (Bader *et al.*, 1955). The mean annual temperature is about −24.4°C and only rarely does any melting take place. The snow depth-density relationship for this site is shown in Fig. 1 (Bader *et al.*, 1955) and may be expressed as

$$\gamma = 0.492 + 5.26 \times 10^{-5}\, h - (0.152)\, e^{-2.17 \times 10^{-3} h}, \qquad (1)$$

where γ is the density (g/cm³) and h the depth below the surface (cm).

A shallow tunnel, trench and deep pit were excavated during the summer of 1954 and details of the research excavations and instrumentation can be found in the above report.

Landauer (1957) reported on the first two years of data and developed a general theory to explain the deformation of a cylindrical tunnel in the névé. To provide information on a tunnel under a greater overburden, a deeper horizontal tunnel at the bottom of the deep pit was excavated during the summer of 1956. The information in this report includes the data from 1954 to 1956 as well as from 1956 through the summer of 1965.

This report is concerned with two of the four excavations: (a) a shallow horizontal

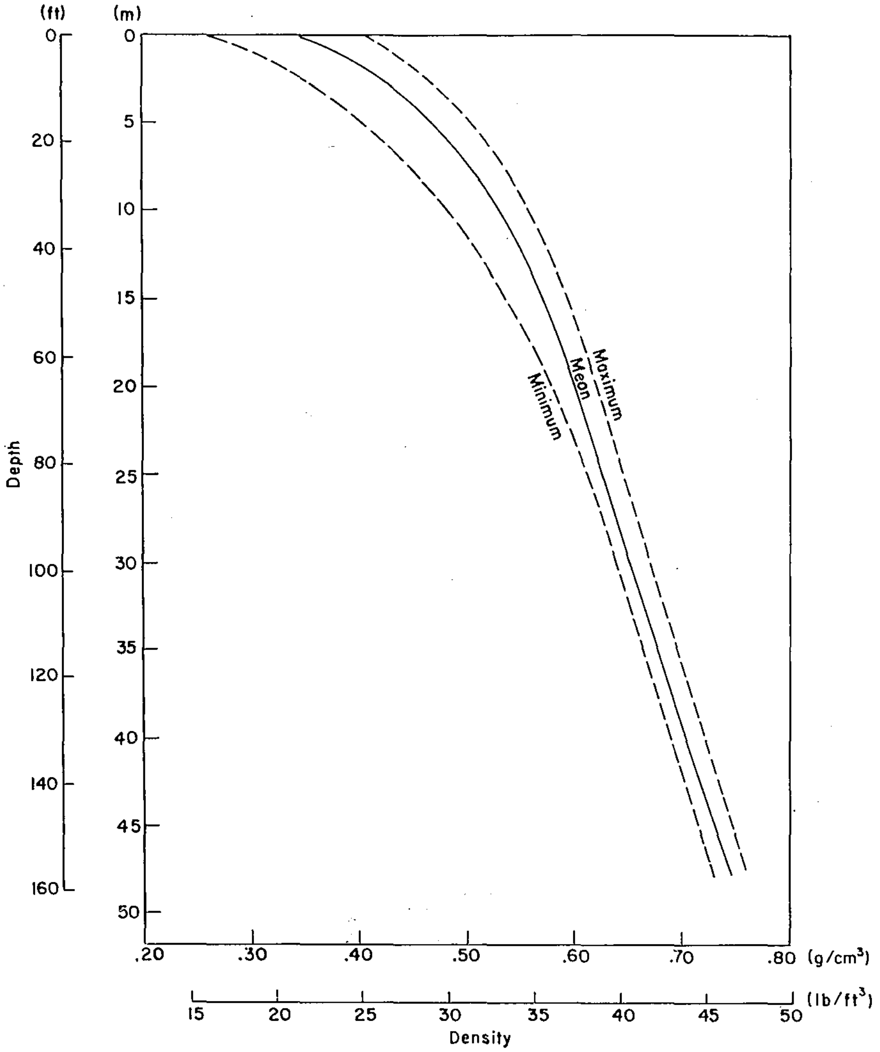

Fig. 1. Snow depth density relationship for Site II, Greenland

tunnel (centerline 6 1/2 m below the original 1954 snow surface); and (b) a deep horizontal tunnel (centerline about 30 m below the original 1956 snow surface).

II. Shallow Tunnel

A horizontal tunnel was excavated 6.1 m below the snow surface during the summer of 1954. Continuous weekly readings were taken during the first year of observation and readings were taken at least once a year thereafter except for the year 1964. The tunnel was approximately cylindrical and about 240 cm in diameter. Closure rates were determined by measuring the distance between 1/4 in. (0.6 cm) wooden dowels placed in the snow wall. The coding scheme is shown in Fig. 2 a. Some of the closure measurements are shown in Table 1.

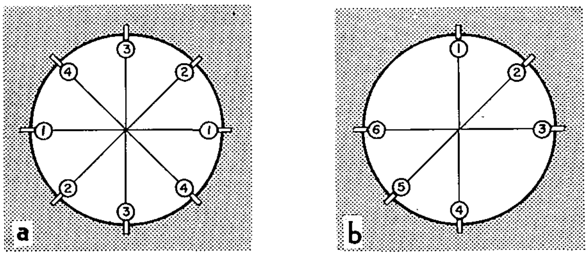

Fig. 2. a: Coding scheme for shallow tunnel,
b: Coding scheme for deep tunnel

Table 1. Shallow tunnel deformation*

	Year	Diameter (cm)	Δ (cm)	% Def/yr	Diameter (cm)	Δ (cm)	% Def/yr	$\dot{\varepsilon}$ (%/yr)
Horizontal	1954	250.1	—	—	212.8	—	—	
	1956	249.6	0.5	—	212.3	0.5	—	
	1958	250.0	−0.4	—	212.1	0.2	—	
	1960	250.2	−0.2	—	212.	.1	—	
	1963	250.0	0.2	—	212.	.0	—	
	1965	250.5	−0.5	—	211.	1.0	—	
Diagonal	1954	253.7	—	—	246.5	—	—	
	1956	246.7	7.0	1.40	234.5	12.0	2.49	
	1958	237.7	9.0	1.86	228.2	6.3	1.36	
	1960	231.1	6.6	1.41	223.6	4.6	1.02	
			Σ = 22.6	av = 1.56%/yr		Σ = 22.9	av = 1.62%/yr	
Roof to center	1954	123.5	—	—	116.5	—	—	2.35
	1956	80.5	43.0	21.08	91.	25.5	12.28	1.76
	1958	56.	24.5	17.94	77.	14.	8.33	1.01
	1960	39·5	16.5	17.28	67.3	9.7	6.71	1.30
	1963	26.	13.5	13.74	59.	8.3	4.50	0.89
	1965	17.	9.0	20.92	55.5	3.5	3.06	0.82
			Σ = 106.5	av = 17.8%/yr		Σ = 61.0	av = 6.75%/yr	
Floor to center	1954	109.	—	—	108.7	—	—	
	1956	102.5	6.5	3.08	103.7	5.0	2.35	
	1958	97.	5.5	2.76	98.7	5.0	2.47	
	1960	92.	5.0	2.65	95.	3.7	1.91	
	1963	80.	12.0	6.98	85.2	9.8	5.44	
	1965	74.	6.0	3.90	82.5	2.7	1.61	
			Σ = 35.0	av = 4.2%/yr		Σ = 26.2	av = 3.00%/yr	

* Arithmetical mean of the values obtained at F_1 and F_2.

Although closure readings were taken at various cross sections, only two are presented for purpose of analysis: the deformation at the closed end wall of the tunnel, noted as G, and the average of the two central cross sections, F_1 and F_2. The shape of the end wall and the average of F_1 and F_2 are shown in Figs. 3a and 3b. It is to be expected that the shape of G and the rate of deformation would be considerably different from the middle of the tunnel due to end restraint. The cumulative vertical deformation is shown in Fig. 4. There was essentially no horizontal closure in eleven years at either the end wall or the middle of the tunnel.

Although the overburden is increasing at an average rate of 39-41 cm of water equivalence/year and hence an increase of pressure, there is a definite decrease in the vertical closure rate. This is because the snow at the tunnel level is increasing in density and the resultant increase in viscosity is greater than the increase in pressure. Even if the tunnel were not present, there would be considerable vertical strain in the lower density snow due to natural densification. This natural vertical strain/year ($\dot{\varepsilon}$) has been computed from the depth-density curve and is also given in Table 1. The decrease in distance between two points 230 cm apart (equivalent to the diameter of the tunnel) and at a depth of about 6 m, is several cm/year in a natural snow pack without a tunnel. However, $\dot{\varepsilon}$ is still considerably less than the actual observed vertical closure of the tunnel.

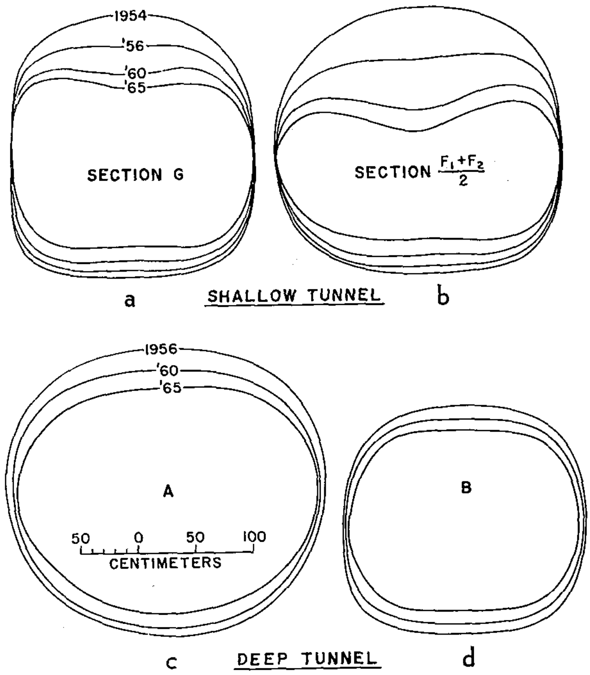

Fig. 3. Top: Deformation with time of cross sections of shallow tunnel, Bottom: Deformation with time of cross sections of deep tunnel

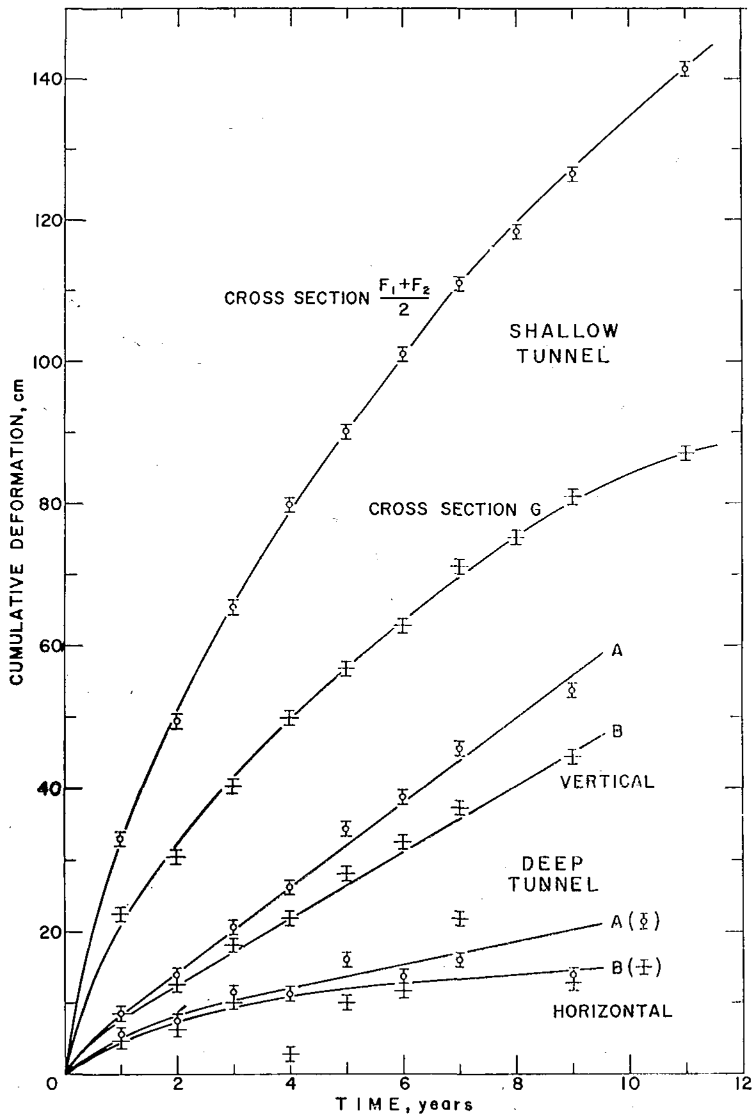

Fig. 4. Cumulative vertical deformation with time of shallow and deep tunnels

III. Deep Tunnel

A horizontal tunnel was excavated at the bottom of the 30 m deep shaft during the summer of 1956. The first and larger section of the tunnel (designated as A) varied in diameter from 247 to 276 cm. The second part of the tunnel varied from 196 to 228 cm in diameter (designated as B). One quater inch (0.6 m) wooden dowels were driven about 20 cm into the snow, with three peg pairs for each diameter: one set for horizontal, one for vertical, and one at 45°. The coding scheme is shown in Fig. 2 b.

Closure rates were determined by measuring the separation between the pegs and

Table 2. Deep tunnel deformation

	Year	Diameter at A (cm)	Δ (cm)	% Def/yr at A	Diameter at B (cm)	Δ (cm)	% Def/yr at B
Horizontal	1956	276.0	—	—	210.9	—	—
	1957	270.5	5.5	2.01	205.1	5.8	2.79
	1958	268.6	1.9	0.70	204.4	0.7	.34
	1959	266.5	2.1	0.78	202.8	1.6	.79
	1960	266.9	−0.4	−0.15	208.1	−5.3	−2.58
	1961	260.	6.9	2.62	201.	7.1	3.47
	1962	262.4	−2.4	−0.92	199.3	1.7	.85
	1963	260. *	2.4	0.92	189. *	10.3	5.31
	1965	261.	−1.0	−0.19	198.5	−9.5	−2.45
			$\Sigma=15.0$	av$=0.62=A_H$		$\Sigma=12.4$	av$=0.67=B_H$
Diagonal	1956	272.4	—	—	228.2	—	—
	1957	265.5	6.9	2.56	222.0	6.2	2.75
	1958	262.2	3.3	1.25	218.7	3.3	1.49
	1959	259.5	2.7	1.04	215.8	2.9	1.33
	1960	257.1	2.4	0.93	213.0	2.8	1.31
	1961	247.	10.1	4.09	204.	9.0	4.41
	1962	249.8	−2.8	1.13	208.	−4.0	−1.94
Vertical	1956	247.8	—	—	196.6	—	—
	1957	239.2	8.6	3.53	188.0	8.6	4.47
	1958	233.9	5.3	2.24	183.7	4.3	2.31
	1959	227.1	6.8	2.95	178.4	5.3	2.93
	1960	221.6	5.5	2.45	174.6	3.8	2.15
	1961	213.5	8.1	3.72	168.5	6.1	3.56
	1962	209.	4.5	2.13	164.0	4.5	2.71
	1963	203.	6.	2.91	159.2	4.8	2.97
	1965	194.	9.	2.26	152.	7.2	2.31
			$\Sigma=53.8$	av$=2.71=A_V$		$\Sigma=44.6$	av$=2.85=B_V$

* Computed from other measurements.

are shown in Table 2. The shape of the two tunnels is shown in Figs. 3 c and 3 d. It became obvious from the 1960 measurements that enough deformation had occurred so that the original pegs were no longer indicating the true vertical and horizontal closure, so later measurements were no longer made from just the three peg pairs, but included those made with a plumb bob as well.

The average cumulative deformation of the two sizes of the tunnels as a function of time is shown in Fig. 4 together with the deformation for the shallow tunnel. There would be a vertical strain even if the excavations were not there, due to natural densification of the snow because of the overburden. This may be computed from depth-density-time relationships. In the case of the large and small diameter tunnels it would be a diminishing of vertical diameter height by 12.8 and 10.2 cm respectively for the period of nine years.

Although Fig. 4 shows a slight flattening of the deformation-time curves, if constant deformation is assumed for the time period being considered, the following ratios are of interest:

$$\frac{A_V}{A_H} = \frac{2.71}{0.62} = 4.37, \tag{2}$$

$$\frac{B_V}{B_H} = \frac{2.85}{0.67} = 4.26, \tag{3}$$

$$\frac{A_V + B_V}{A_H + B_H} = \frac{2.71 + 2.85}{0.62 + 0.67} = 4.31. \tag{4}$$

When the closure due to natural densification is subtracted from the vertical closure then

$$\frac{A'_V}{A_H} = \frac{2.06}{0.62} = 3.33, \tag{2$'$}$$

$$\frac{B'_V}{B_H} = \frac{2.18}{0.67} = 3.26, \tag{3$'$}$$

$$\frac{A'_V + B'_V}{A_H + B_H} = \frac{2.06 + 2.18}{0.62 + 0.67} = 3.29. \tag{4$'$}$$

Several conclusions may be reached: (a) the total vertical closure is about four times greater than the total horizontal closure; (b) the vertical closure due to the excavation is about three times greater than the total horizontal closure; (c) there is a slight slowing down of the closure rate; and (d) there is a greater total amount of closure in the larger tunnel, but the percentage of closure is slightly greater for the smaller tunnel.

IV. Theoretical

It is assumed that Sorge's Law (Bader, 1954) is valid: *i.e.*, that the density at a particular depth is independent of time. The depth-time of deposition relationship may be expressed as

$$\sigma_h = At = \int_0^h \gamma \, \mathrm{d}h, \tag{5}$$

where σ_h is the vertical pressure (g/cm^2) at a level deposited t years ago, and A is the annual accumulation (water equivalence). The depth of the tunnels from the surface increases every year and the vertical strain rate $\dot{\varepsilon}_h$ is

$$\dot{\varepsilon}_h = \frac{1}{\delta h} \frac{\mathrm{d}}{\mathrm{d}t} (\delta h) = \frac{-A}{\gamma^2} \frac{\mathrm{d}\gamma}{\mathrm{d}h}. \tag{6}$$

Landauer (1957) treated the case of a tunnel in snow by replacing the elastic parameters in a long thick-walled, compressible elastic tube with viscous parameters. He obtained the equation:

$$-\dot{\varepsilon}_a = \frac{(m-2)\sigma_h}{2(m-1)\eta} \left(\cos^2\theta + \cos\theta + \frac{1}{m-2} \right), \tag{7}$$

where $-\dot{\varepsilon}_a$ is the radial closure rate, η is the snow viscosity and m a plastic analogue of Poisson's ratio. His theoretical values for vertical closures were considerably higher than those actually observed. Mellor (1964) has pointed out the wide variations in the values of η as reported by various investigators, as well as noting that $\dot{\varepsilon}$ is proportional to σ only at low stresses. Although the substitution of viscous parameters in an equation developed for elastic conditions might be permissible in some instances, in the case considered here where large deformations take place, the compatibility conditions no longer hold and eq. (7) is incorrect.

Ôura (1958, 1959) reported the deformation of a snow cave 1 m high, 1 m wide and about 5 m long in a 2 m deep snow cover. He treated the problem as a two-dimensional case of a viscous material and considered the roof of the cave as a plate. Although he had some success in comparing the observed roof deformation with this theoretical model, his model is not considered to be appropriate for tunnels at some depth below the snow surface.

A similar deep tunnel, 2 m in diameter and 6 m long, was excavated at the 30 m level in the fall of 1957 at Wilkes Station, Antarctica. Cameron (1961) has reported on the first two years of measurement and noted that, as in Greenland, the observed strain rate was considerably greater than that calculated by Landauer's formula.

Ramseier (1962) has reported on the two year closure of the snow mine at the South Pole Station. He also found that the vertical closure decreased with depth down to 30 m (the maximum depth of the mine), but there was a slight decrease with depth of the horizontal closure. He obtained the empirical relationship for vertical closure

$$C_V = (725 - 4.85\,H) \times 10^{-4}\,, \tag{8}$$

and for lateral closure

$$C_L = (150 - 1.15\,H) \times 10^{-4}\,, \tag{9}$$

where H is the depth in feet and C is closure in inches/month. No attempt was made to make a theoretical analysis.

Gow (1963) has reported on the closure rate of a deep drill hole in Antarctica. He found an increase in closure rate with time and suggested that this might be due to the recrystallization of the ice around the bore hole. When he used the expression developed by Nye (1953)

$$S = \left(\frac{\sigma}{nB}\right)^n, \tag{10}$$

where S is the rate of contraction, σ the hydrostatic stress and n and B are constants, he found that n varied from 3.1 to 6.0 with time, and also that it varied slightly with σ for the same year. It has been assumed that ice may be treated as a hydrostatic material.

V. Conclusions

It is of considerable practical importance to be able to predict the closure rate of excavations in snow and ice. Although empirical relations seem to be quite well established for ice, the complex problem of a varying density snow with a varying overburden

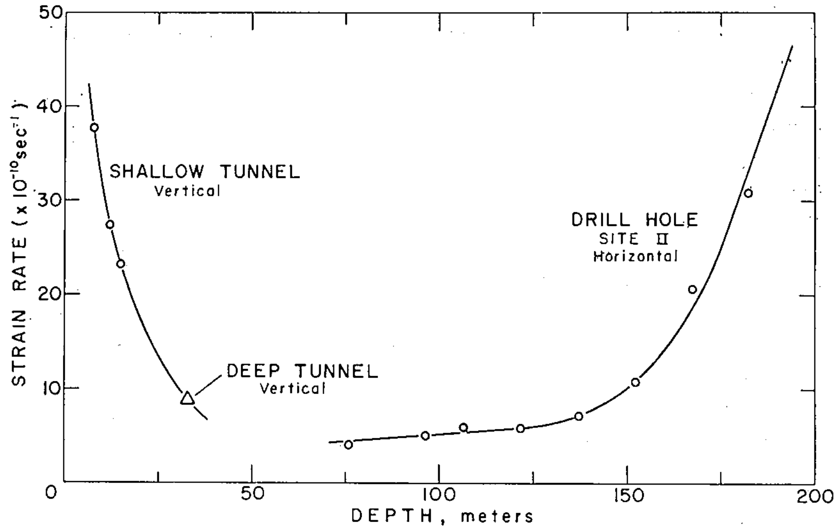

Fig. 5. Vertical strain rate with depth for the shallow tunnel, deep tunnel and drill hole at Site II, Greenland

has not been satisfactorily solved. It is further complicated by the large deformations that take place and the unsymmetrical shapes which the tunnels begin to take. A plot of the vertical strain rate with depth for the two tunnels is shown in Fig. 5. In addition, the horizontal strain rate for a deep drill hole at Site II is also shown (Hansen and Landauer, 1958). The interesting conclusion may be drawn that there is a depth at which minimum closure takes place. This was first suggested by Bender and Abele (1959) and also proposed by Cameron (1961). It is estimated that this is in range of 50 to 75 m below the surface in the zone where snow is changing to ice. To provide additional information at this interesting depth, an inclined shaft about 300 m long with a maximum depth of 100 m was completed during the summer of 1965 on the Greenland Ice Cap and observations will be made over a period of years.

Acknowledgmets

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