The Stability of Snow Cover on Mountain Slopes

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Abstract

Avalanching stands in complicated causal relationship with snow and meteorological conditions. At present it is possible to connect these factors only in a qualitative way. For the calculation of force relations in the snow cover on mountain slopes it is necessary to know the mechanism of avalanche release. In this paper an attempt to characterize the main factors of avalanching in a quantitative way is made. The factors involved seem to be the following 4 points; the snow strength under constant load, anchorage at the circumference of a snow layer, filtration of free thaw or rain water and the slight curvature of the slope profile. A generalized stability equation of a snow layer on a mountain slope and a nomograph for corresponding calculations was proposed. The conclusion drawn here that there exist two limits of avalanche danger slope inclination has been led from the generalized stability equation (if the snow cover on the slope is homogeneous in its density and height). A plot showing the correlation between certain characteristic of the snow cover and avalanching is given.

I. Introduction

The instability of snow cover on mountain slopes is the main factor of avalanche release. Dangerous instability and consequently an avalanche release is the result of: 1) increase of snow load on mountain slopes, 2) decrease of strength properties of the snow, 3) rheological properties of snow (creep and glide).

Shearing stress in snow cover can easily be calculated, but strength properties of snow vary greatly in complicated causal relationship with meteorological conditions and cannot be calculated so easily. The most important of the conditions influencing the strength properties of snow is temperature; it seems possible to connect these factors by means of correlation analysis. It may be better to connect the main meteorological elements and snow conditions with the degree of avalanche danger, but at present it is possible only in a qualitative way. Any of the quantitative correlations between, for example, the intensity of snow-fall or snow-storms and the moment of avalanche release, are only of a localized nature, which do not yield a possibility to predict avalanching in this manner unless a series of meteorological observations is performed for several winters.

It is necessary therefore to know the mechanism of avalanche release in order to calculate force correlations provided that proper measurements in a snow cover can be accomplished.

Below an attempt to review the factors which play the main part in the mechanism of avalanche release was made based on the supposition that the strength properties of the snow are known (for instance, when periodical measurements on conditions near the site of avalanche release were previously made).

II. The Strength of Snow under Constant Load

The mechanical properties of snow are very complicated. It creeps even when the shearing stress is very small, but it shows elastic properties when the rate of force action is sufficiently large. A full description of the mechanical properties of snow has been given by means of complicated rheological models including viscous, elastic elements (Ohnishi, 1962) and a dry friction element. Under the action of forces which exceed the yield point of snow one need only to take into account the element of dry friction (Ziegler, 1963). But snow is not an ideal plastic body and the strain involved does not occur instantly. The strain rate in a snow layer, where the shearing stress has exceeded the yield point, is dependent on viscosity analogous to that of Bingham's body. And the strength properties of snow depend on that of ice. Thus, the strain rate of ice particles $\dot{\gamma}$ under constant shearing stress τ is expressed by the equation

$$\dot{\gamma} = k\tau^n , \qquad (1)$$

proposed for ice by Voitkovsky and Glen (Kalesnik, 1963). Where k is the coefficient dependent on the temperature of ice: in Voitkovsky's equation $k=B/(1+\theta)$ (B is constant, θ is the absolute value of the temperature of ice); $n>1$. The eq. (1) may also be written as follows:

$$\tau = k_1\dot{\gamma}^{n_1} , \qquad (1\text{ a})$$

where $k_1=1/\sqrt[n]{k}$, $n_1=1/n<1$. This is the equation for pseudoplastic (Wilkinson, 1960). Ice has no clearly marked yield point but conventionally the value 1.5–1.75 kg/cm^2 is assumed: at stresses exceeding these values the creep rate increases progressively and leads to destruction of an ice sample.

Avalanching is possible when the ice particles in snow are broken or disaggregated; it takes place when stresses in ice exceed its yield point. If snow is subjected to stress τ_s, the stress in ice particles τ is much higher, for the force action concentrates in cross sections of ice, needles or rays of snow flakes or other parts in contact areas among ice corns (ice-bonds). It may be found after measurements in thin snow slices and simple calculations that stresses in the weakest sections or contact areas of ice particles should be around 80–500 times more than the mean stress in snow which is assumed to be a solid body. The yield point of snow thus evaluated ranges within 30–200 kg/cm^2. Indeed, avalanching is frequent when the shear stress in snow cover exceeds these figures.

Let the eq. (1) be transformed into

$$\tau - \tau_0 = \eta\omega , \qquad (2)$$

where $\tau-\tau_0$ shows an excess of shear stress τ over the yield point τ_0, η is the effective viscosity coefficient (for glide movement) and $\omega=\dot{\gamma}$ is the strain rate. Here η depends on the stress τ, but if τ is approximately constant η is constant as well.

This corresponds to the simple Bingham's rheological model. The yield point τ_0 is a variable dependent on the normal stress;

$$\tau_0 = f(\sigma)\,, \qquad \tau = f(\sigma) + \eta\omega\,. \tag{3}$$

The function $f(\sigma)$ expresses the experimental curve of the dependence of snow shearing strength τ_0 on the normal pressure σ. The latter can be approximated through Coulomb's law

$$\tau_0 = c + f\sigma\,, \tag{3 a}$$

c denotes cohesion and f the coefficient of internal friction. Substituting eq. (3 a) into eq. (2) yields

$$\tau = c + \eta\omega + f\sigma\,. \tag{4}$$

Here the term $c+\eta\omega$ is equivalent to cohesion c_e which is dependent on the cohesion c corresponding to the yield point and the glide resistance $\eta\omega$.

III. The Stability Factor

Avalanching is possible when the shearing stress τ in the weakest snow layer overcomes its shearing strength τ_s. The shearing stress

$$\tau = \gamma d \sin\alpha,$$

where γ is the specific gravity of the snow cover lying above the weakest layer, d is its thickness and α is the slope angle. The shearing strength is generally less than the ultimate strength and higher than the yield point.

If the shearing stress increases rapidly up to the ultimate strength immediate avalanching occurs; but if the shearing stress is less than the ultimate strength of the snow (but more than its yeild point) a delayed action avalanche is possible in consequence of glide which is accelerating during the passage of time. Of course this is not the single cause of avalanching, but even if the temperature and other properties of the snow were constant for some time avalanching may take place. Roch (1965) has established that avalanching is possible when the stability factor

$$s = \frac{\tau_s}{\tau}\,, \tag{5}$$

is as high as 4. This figure must be the ratio of the ultimate strength of snow to its yield point. In the USSR Lossev (1963) has experimentally ascertained that snow samples subjected to stress broke (with the lapse of time, even if the stress was 2.5–4, up to 5–8.5 times less than the ultimate strength. The same occurrence is usual in the Western Tian-Shan: avalanches frequently are known to start when the stability factor is higher than 1. For the calculations of avalanche prediction it is necessary to divide the ultimate strength C_u (normal pressure is excluded) by the so called relaxation coefficient, to obtain the value of shearing strength (cohesion) under a lasting load.

Inserting eq. (4) into eq. (5) yields the stability factor:

$$s = \frac{c_e}{\gamma d \sin\alpha} + f \operatorname{ctg}\alpha\,. \tag{6}$$

The dependence $\tau_s = f(\sigma)$ is not, generally, linear but for practical calculations one may adopt the linear approximation of τ_s.

IV. Anchorage

The stability condition $\tau \leq \tau_s$ or

$$\cos\alpha\,(\mathrm{tg}\,\alpha - f) \leq K \frac{c_e}{\gamma d}\,, \tag{7}$$

is sufficiently accurate for the "endless" layer if $K=1$. Practically it is so (with an insignificant error) when the layer has a length and width exceeding several tens to a hundred meters. The layer is generally anchored along its circumference. The anchorage should be taken into account by use of the coefficient

$$K = 1 + \frac{\Sigma P_m}{T_s} = 1 + \frac{\sigma_{cp}}{c_e}\,\frac{p \cdot d}{F}\,,$$

where P_m and T_s are forces in accordance with the circumference and the foot of the layer, σ_{cp} is the mean strength of the snow along the circumference of the layer (conventionally it may be supposed that σ_{cp} has the same order of value as c_e or, simply, $\sigma_{cp} \approx c_e$); p is the perimeter of the snow layer on the slope which has a square F. For isometrical shape of projection of the layer onto the slope the ratio $R = F/p$ (which may be considered to be a length characteristic for the geometrical configuration of the layer) is 4 times less than the diameter or the length (width) of the layer; if the layer has a width (or length) sufficiently large compared to other dimensions it is approximately 2 times less than the smallest dimension (in the projection onto the slope) of the layer and so on.

Anchorage is the cause of the snow layer being on the slope in a state of equilibrium when its stability factor calculated by means of the eq. (6) is less than 1. In general the stability factor is seldom less than 0.9, in accordance with the small influence of anchorage in many cases.

Formation of a layer of depth hoar ("swimming" snow) which has an inner friction angle less than that of the slope angle leads to dangerous stress in a snow slab, which is held in equilibrium only by anchorage. In this case the thickness of the snow slab does not appreciably affect its stability on the slope: the thicker the slab is the greater the stress in it is but on the other hand the greater is the strength of the slab as well. The stability formula (7) can be transformed into the form

$$\cos\alpha\,(\mathrm{tg}\,\alpha - f) \leq \frac{c_e}{\gamma d} + \frac{\sigma_{cp}}{\gamma}\,\frac{p}{F}\,. \tag{8}$$

If $c_e \to 0$ (when swimming snow has formed) it yields:

$$\cos\alpha\,(\mathrm{tg}\,\alpha - f) \leq \frac{\sigma_{cp}}{\gamma}\,\frac{p}{F}\,. \tag{8 a}$$

When F/p is sufficiently large (on an "endless" slope) the stability condition is reduced to $\mathrm{tg}\,\alpha \leq f$.

V. Filtration of Water

Free water in the snow layer filtrates along the impervious stratum (ice and the like) or soil (if the latter has water permeability much less than that of snow) and causes an additional force tending to push the snow layer downwards. Its action is equivalent to the decrease of the inner friction angle and increase of water equivalent or mean specific gravity of the snow layer (Moskalev, 1966):

$$f_e = \frac{1-d_0}{1+\frac{\gamma_w}{\gamma} d_0} f, \quad \gamma_e = \gamma + d_0 ,$$

where d_0 is the ratio of thickness of the aquifer (the stratum saturated with free water) to that of the whole snow layer under consideration, γ is mean specific gravity of the snow layer if there is no filtrating water, γ_w is the specific gravity of water, γ_e is the effective mean specific gravity of the snow layer having an aquifer in its foot, $f(=\mathrm{tg}\,\varphi)$ is the inner friction coefficient and φ is the inner friction angle of the snow in the foot of the layer, $f_e\,(=\mathrm{tg}\,\varphi_e)$ is the effective inner friction coefficient and φ_e is the effective friction angle of the same snow containing free water filtrating through it.

The ultimate stability equation of a snow layer lying on a water saturated stratum is as follows:

$$\gamma_e d \cos\alpha\,(\mathrm{tg}\,\alpha - f_e) = c_e , \tag{9}$$

if the anchorage is neglected, or

$$\cos\alpha\,(\mathrm{tg}\,\alpha - f_e) = K \frac{c_e}{\gamma_e d} , \tag{9 a}$$

where K takes into account forces acting on the circumference of the snow layer and so on.

Filtrating water acts as a lubricant which causes snow cover gliding or avalanching. Some examples of heavy avalanching in the Western Tian-Shan illustrate this statement (Moskalev, 1965).

If free water is seeping or forming in a snow layer (for instance as a result of rain or snow thawing), the equivalent inner friction angle φ_e is not constant and varies from point to point according to the thickness of the water flow within the snow layer. The full shearing force resulting from filtrating water can be calculated by use of the water balance method if the intensity of thawing or seepage from rain falling onto the slope surface is known. For these calculations one can make use of the methods given in the scientific monographs of Kuzmin (1961) and Denissov (1965).

VI. The Form of a Slope Profile

The problem of calculating stresses and forces at any point of a snow cover on an arbitrary slope is very complicated. Elementary methods of solving this problem are suitable when the slope profile is being considered as a fragment of a circle arc. In this case application of the known Swedish method of soil mechanics leads to an approximate solution (Moskalev, 1965) which may be written down in the form

$$wK_x K_r (\sin\theta - f_e \cos\theta) = c_e\ , \tag{10}$$

where $w = \gamma_e d/\cos\alpha$ is the water equivalent of the snow cover under consideration, $K_x = l/x$ is the ratio of the length of the glide arc to that of its horizontal projection, K_r is the length ratio of the radius vector drawn from the center of the arc to its gravity center (the arc having a running weight dependent on the snow load) to the radius of the arc, θ is the angle between the radius vector and the vertical line. In many cases this angle is somewhat less than the angle formed by the chord of the arc with the horizontal line. For approximate calculations, the latter angle may be taken instead of θ. The ratio K_r is rather near to 1.

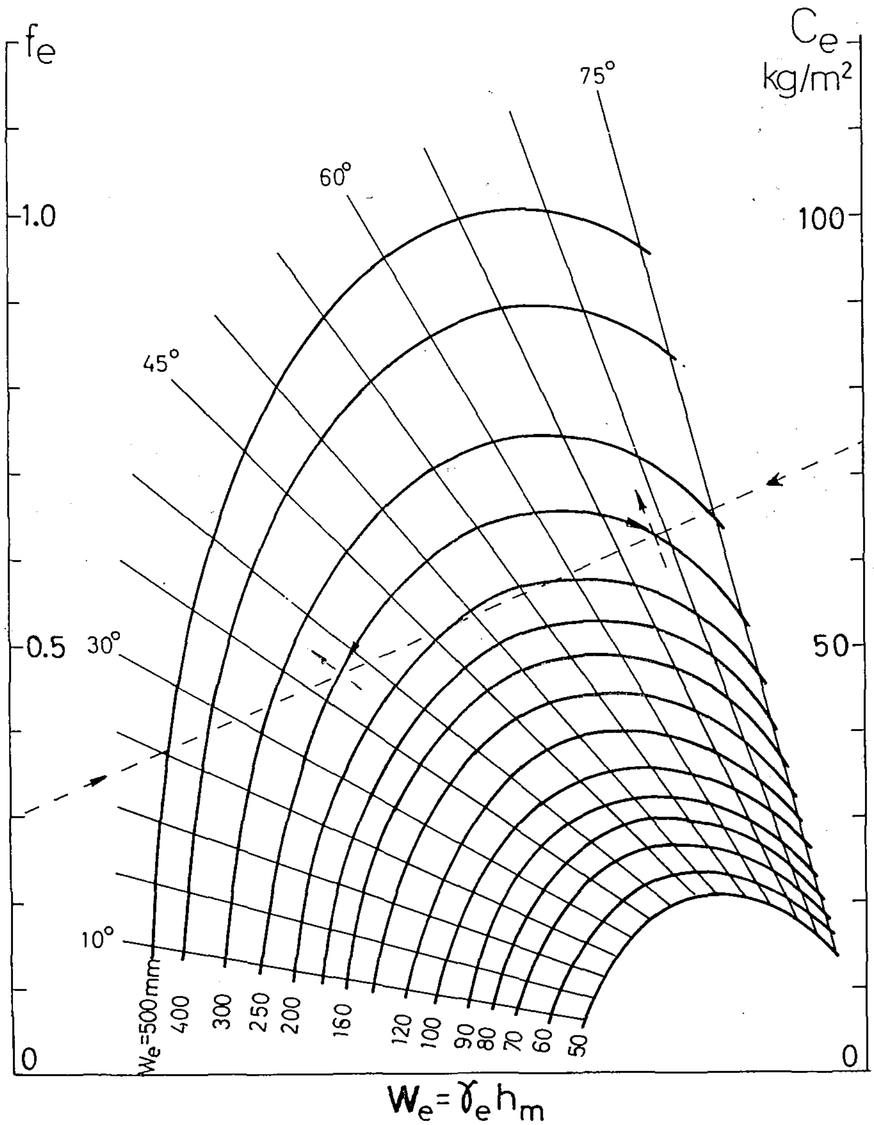

Fig. 1. The nomograph for calculations of the slope angle of instability of snow cover

Similar correlations were obtained for a snow layer which lies on a concave or convex slope which is regarded approximately to be a part of a sphere surface.

VII. The Generalized Stability Equation

The equation for the snow layer which is in a state of ultimate equilibrium is approximated as follows:

$$\cos^2 \alpha_m (\mathrm{tg}\, \alpha_m - f_e) = K \frac{c_e}{w_e}\,, \qquad (11)$$

where α_m is roughly the mean angle of the slope and the factor K includes the coefficients K_x and K_r;

$$K \approx \frac{1}{K_x K_r}\left(1+\frac{\sigma_{cp}}{c_e}\frac{pd}{F}\right).$$

$w_e = \gamma_e h_m$; h_m is the mean height of snow cover ($h_m = d_m/\cos\alpha$).

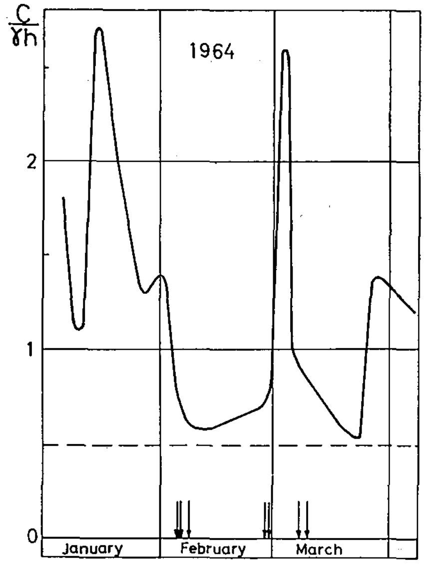

Fig. 2. The plot showing correlations between the ratio $c/\gamma h \approx K \cdot c_e/w_e$ and avalanching in the region of the Western Tian-Shan (the onset of avalanche release are pointed out with arrows)

Equation (11) has two roots if the ratio Kc_e/w_e is less than 0.5. It has only one root if the ratio is equal to 0.5 and has no intrinsic roots if the ratio is more than 0.5 ($\varphi_e = 0$ was coventionally adopted). The first case takes place when a group of differently inclined (from α_{m_1} to α_{m_2}) slopes is subjected to avalanche danger, the second case corresponds to avalanche danger which exists on slopes of certain uniform inclination and the last case corresponds to the state of avalanche security.

After corresponding calculations (the nomograph of the eq. (11) constructed by the author is given), one is to draw a graph showing the course of the ratio Kc_e/w_e in the course of time; if the ratio decreases to 0.5 or less avalanche danger exsists, and the danger increases with the decrease in this value. From the trend of the curve one can predict avalanche danger ahead of time.

VIII. Conclusion

An analysis of the main factors in the mechanism of avalanche release was attempted.

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