On Some New Approaches to the Dynamics of Snow Avalanches

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Abstract

The report deals with some new results connected with the study of the mechanism of basic events accompanying the fall of the snow avalanche and the effect of the avalanche on different obstacles, together with a quantitative description of these effects.

The first part deals with a new mathematical schematization of the problem of movement of the avalanche down the slope and the solution of this problem. The mass of snow and air moving in the avalanche is considered as a thin layer of incompressible liquid. The forefront of the avalanche is schematised as a destruction jump of undisturbed snow located under the avalanche; the force interaction of the avalanche material with the foundation is expressed by a hydraulic scheme. The requirement for the existence of a stable solution of the evincing problem allows for a single solution describing the distribution of the basic parameters along the avalanche. The first part also describes an experimental method of measurement of parameters of a moving avalanche with the application of a surface stereophotogrammetric survey. A description of the equipment and some quantitative results of the surveys are also presented.

The second part formulates the problem of calculating pressures which are created at the obstacles with the impact of the avalanche. Here the snow-air mass of the avalanche is regarded as a compressible liquid, and an allowance is made for the wave effects in this media in the calculation of the shock. It is also shown that under certain conditions the overflow of an obstacle by an avalanche can be of a supersonic character. Two problems were solved: The problem of overflow of a wedge-line obstacle by a supersonic avalanche and the problem of a direct hit of an avalanche on an obstacle.

The third part suggests a new hydraulic-dynamical explanation of the nature of the air wave caused by the avalanche. Allowances are made inasmuch as the air wave is a semicircle vortex pushing with its ends into the slope and overtaking the avalanche when it slows down.

The fourth part describes the method of evaluation of the distance to which the avalanche is thrown, based on the results of the study of material in the cones of the avalanche debris, as well as on the results of morphometric measurements along the paths of avalanche movement and on the avalanche cones. Results of processing of data obtained in the Khibiny region are given which show the effectivity of the method.

I. Introduction

Up to the present the majority of studies on snow avalanches were descriptive, and the number of theoretical publications is limited. Recently the problem of dynamics of snow avalanches has been drawing increasing attention of physicists and mathematicians.

In 1964 the scientists of the Institute of Mechanics and of the Laboratory of Avalanche Problems of the Geography Department, Moscow State University, started their studies of the mechanism of avalanches.

The present report presents the first results of this work. The first part of the report deals with the results of research made by S. S. Grigorian, Yu. L. Yakimov, M. E. Eglit (theory) and A. V. Briukhanov (experimental method). The second part is written based on the work of I. Ye. Shurova. The third part discusses a hypothesis on the nature of the air wave suggested by Yu. L. Yakimov. The fourth part contains results of studies conducted by Yu. L. Yakimov and I. Ye. Shurova (morphometric method) and S. M. Miagkov (geographical method). The observation data on morphometry were prepared by M. Ya. Plam and S. M. Miagkov. M. Ya. Plam is also the initiator in the organization of joint activities of geographers of the Laboratory on Avalanche Problems with the specialists on mechanics of the Institute of Mechanics. This report is the result of their cooperation.

The scientific consultation on problems of mechanics was provided by S. S. Grigorian and Yu. L. Yakimov, while problems of geography were dealt with by Tushinsky.

II. The Study of Avalanche Motion

FORMULATION AND SOLUTION OF THE PROBLEM OF AVALANCHE MOTION

The most promising item in the present joint study of avalanche movement are the theories which consider an avalanche as a movement of continuous medium. One of the known work in line with this is the work by Voellmy (1955) who applied hydraulics principles to the study of avalanches. But his theory is not complete as it does not suggest a sufficiently convincing method for calculating parameters of the moving snow, for instance, the thickness of the moving snow layer according to the given characteristics of a snow cover on the slope. Generally speaking, at present no exact formulation of the problem of avalanche movement down the slope within the mechanism of a continuous medium has been presented.

This part of the report discusses the formulation and solution of the problem on the avalanche movement along the slope in a one-dimensional hydraulic scheme with an allowance for the distribution of velocities in the avalanche. Solving this problem we can determine: The velocity of the forefront, the form of the avalanche, the distribution of velocities in the avalanche.

The problem is solved based on the following assumptions:

1) The snow covering the slope with its small stresses is considered as a solid body. When the stresses reach certain limits, the snow disintegrates and behaves like a liquid. This destruction takes place at the forefront of the avalanche.

2) The change of density of the moving snow, as well as the lateral spreading is not considered.

Then the equation of motion (assuming the quasi-stationary character of movement) in the coordinate system, connected with the front of the moving avalanche, can be given as follows:

$$u \frac{\mathrm{d}u}{\mathrm{d}x} = -\frac{g}{kh_0}\left[K_0^2 - \frac{h_0}{h}(w-v)^2\right] - g\frac{\mathrm{d}h}{\mathrm{d}x}\cos\psi, \tag{1}$$

and the continuity equation is:

$$uh = wh_0\,, \tag{2}$$

where $u=w-v$, v is the absolute velocity of snow particles, w is the velocity of the forefront of the avalanche, x is the coordinate up the slope; h_0 is the thickness of snow covering the slope, h is the thickness of the moving layer, g is the acceleration of gravity, ψ is the tilt angle of slope to the horizon, k is the coefficient determining the force of hydraulic resistance with the turbulent motion of the snow, K_0^2 is connected with the gravity forces and, probably, with additional resisting forces not associated with the velocity (Fig. 1).

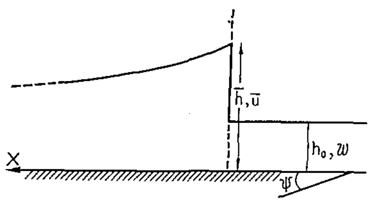


Fig. 1. The scheme of the moving avalanche

In general ψ, h_0 and K_0 are the known functions of x; for simplicity we shall consider ψ, h_0 and K_0 here as constant.

The boundary conditions for this equation are the conditions of conservation of mass and momentum at the forefront of the avalanche, which can be reduced to one condition for $\bar{h}$, *i.e.* the thickness of the moving layer at the forefront:

$$(\bar{h}-h_0)\,[\bar{h}\,(\bar{h}-h_0)-2h_0^2M_0^2] = \frac{2h_0\bar{h}P_*}{\rho g \cos\psi}\,, \tag{3}$$

where $M_0=w\cdot(gh\cos\psi)^{-\frac{1}{2}}$ is the "Much number of the free stream", P_* is the instant compression strength for snow, ρ is the density of snow.

From eq. (3) we can see, in particular, that with $P_*\neq 0$ at the forefront a jump always appears (*i.e.* $\bar{h}\neq h_0$).

The coefficients of the eqs. (1), (2) and (3) depend on the velocity of the front of the avalanche w. At all w satisfying the condition $w^2 \geq \frac{27}{4} K_0^2$ there are solutions of eqs. (1), (2) and (3), in which h and u rapidly tend to become constant with removal from the forefront of the avalanche.

If the movement of the avalanche is not supplemented by any outer agents then it is natural to expect an additional condition. This condition provides a stationary movement to the forefront, in such a way that the snow velocity u in the back part of the avalanche (with $x\to\infty$) becomes greater or equal to the velocity of propagation of small surface waves, *i.e.*,

$$u_\infty \geq u_{\text{crit.}} = \sqrt{gh_{\text{crit.}}\cos\psi}\ .$$

In this case the events in the tail of the avalanche cannot affect the movement of the core of the avalanche or its forefront. Actually this condition coincides with the condition when $h=h_{\text{crit.}}$ is the root of the equation:

$$K_0^2 - \frac{h_0 w^2}{h^3}(h-h_0)^2 = 0\,. \tag{4}$$

This allows for a distinguishment from a great number of solutions corresponding to different w, a solution that will give a single value of w, the form of the avalanche and the distribution of velocity at various points of the avalanche from the forefront to the point $x = x_{crit.}$, where $u = u_{crit.}$ and $h = h_{crit.}$. Beyond that point the solution can be continued with different methods; however, all these continuations but one are linearlly unstable.

In the only stable solution with $x \to \infty$ the "supersonic" regime is carried out. This solution for $\eta = h(x)/h_{crit.}$, shall be obtained in a closed form:

$$\eta - \bar{\eta} + \ln\left(\frac{\eta - \eta_1}{\bar{\eta} - \eta_1}\right)^{\alpha_1} \left(\frac{\eta - \eta_2}{\bar{\eta} - \eta_2}\right)^{\alpha_2} = -\lambda \frac{x}{x_{crit.}}, \tag{5}$$

where

$$\bar{\eta} = \frac{\bar{h}}{h_{crit.}}, \quad \eta_1 = \frac{h_1}{h_{crit.}}, \quad \eta_2 = \frac{h_2}{h_{crit.}}, \quad \alpha_1 = \frac{1+\eta_1+\eta_1^2}{\eta_1-\eta_2}, \quad \alpha_2 = \frac{1+\eta_2+\eta_2^2}{\eta_2-\eta_1},$$

$$\lambda = \frac{K_0^2}{k h_0 \cos\psi},$$

where h_1, and h_2 are the roots of the eq. (4).

Then we have

$$w = c_0 \sqrt{1 + \frac{K_0}{c_0}}, \qquad h_{crit.} = h_0 \left(1 + \frac{K_0}{c_0}\right),$$

$$x_{crit.} = \frac{h_{crit.}}{\lambda} \left[\bar{\eta} - 1 - \ln\left(\frac{1-\eta_1}{\bar{\eta}-\eta_1}\right)^{\alpha_1}\right],$$

where $c_0 = \sqrt{g h_0 \cos\psi}$ is the velocity of the "sound" of the free stream.

Thus, if we know the properties of the snow cover, its thickness and the profile of the slope, then by solving eq. (5) we can immediately determine the longitudinal profile of an avalanche, distribution of velocities in the avalanche and movement velocity of the front. Some solutions are given on Fig. 2.

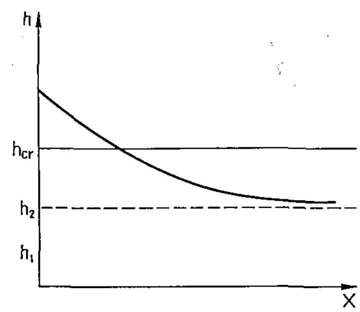


Fig. 2. Longitudinal profile of the moving avalanche

EXPERIMENTAL RESEARCH OF AVALANCHE MOVEMENT BY STEREOPHOTOGRAMMETRIC METHODS

The first working model of equipment for stereo-survey of avalanches, constructed by A. V. Briukhanov, was based on two photographic cameras AΦA-39 in the Laboratory of Aerophotomethods of the Geography Department, Moscow State University, with Dr. I. R. Zaitov as Chief of the Laboratory.

Figure 3 shows the general view of the set of the operating model of the stereo-cameras for photographing of avalanches. The equipment has the following technical parameters:—

size of photograph: $7 \times 8\ \text{cm}^2$,
objective "Uran 27" with $f_{k1} = 100.32$ mm
and $f_{k2} = 100.62$ mm,
relative aperture is 1 : 2.5–1.22,
resolvine power in the center (according to instructions) is 45.0 lines/mm,
focal-plane shutter with exposure time of 1/700; 1/1 400; 1/1 800,
accuracy of synchronisation of the operations of shutters (based on special research) is 1/1 000 sec,

the smallest interval between exposure is 2.0 sec,
accuracy of operation of orientation device is 4″,
the value of scale of horizon levels is 10″.

An oscillograph ПО-4 was used during the survey for precise determination of intervals between exposures with tape recordings of impulses from shutters at the moment of exposure and from the time recorded working at 50 Hz frequency. The accuracy of determination of intervals between exposures by interpretation of the oscillogram was 0.01 sec.

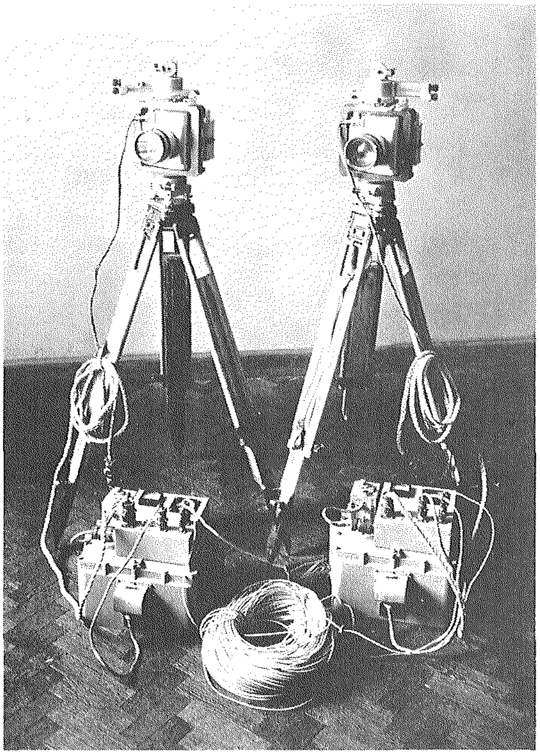


Fig. 3. A set of stereocameras

Theoretical calculations of the expected errors in the determination of separate parameters of avalanche movement were carried out with the assistance of the indicated equipment on the basis of the error theory of surface stereophotogrammetric survey, yielded the following results in the case of a survey with $Y_{\max}=1\,000$ m and $B=100$ m.

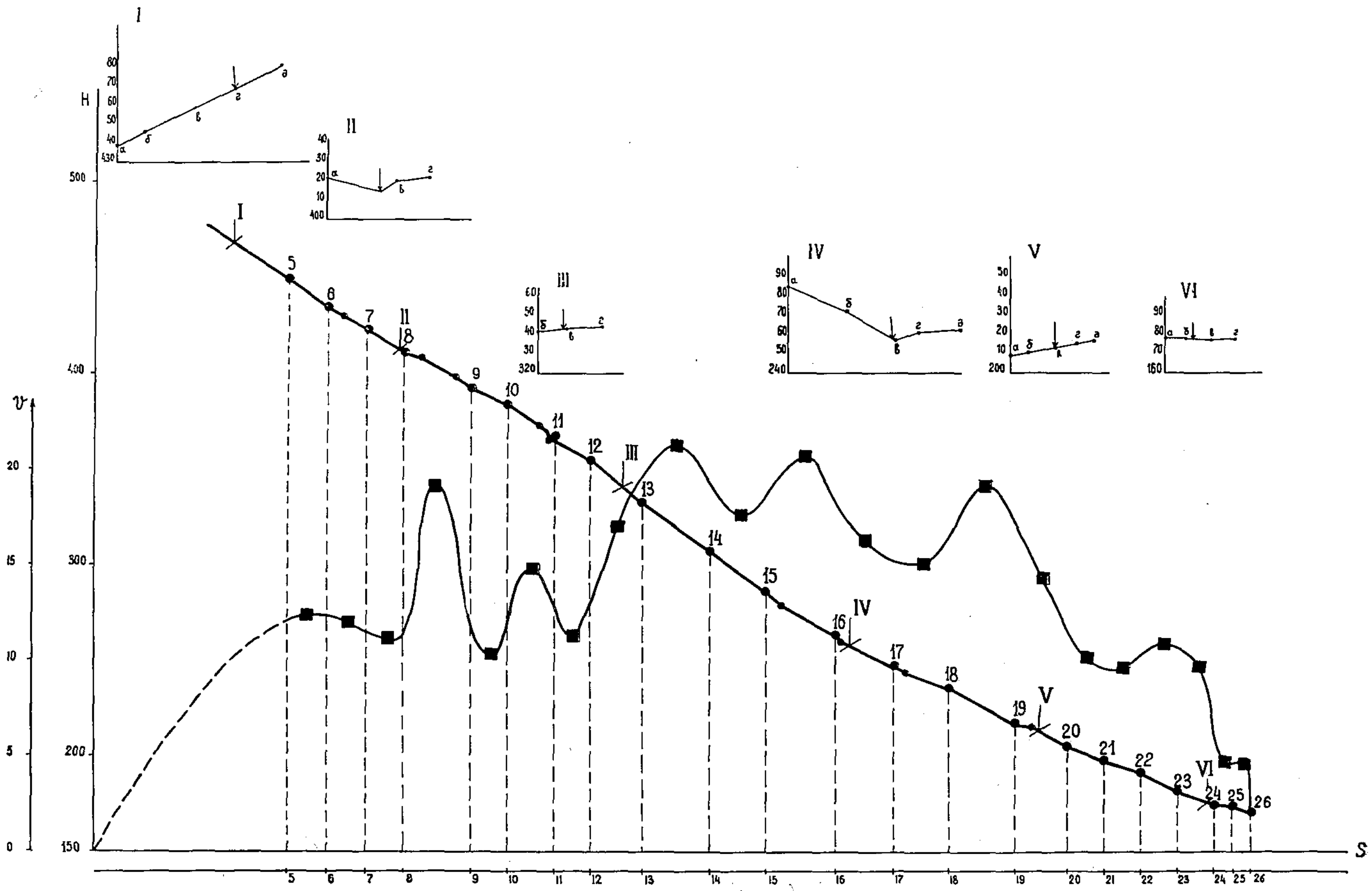
The mean square errors of determination of spatial coordinates of the forefront of the avalanche $M_y=\pm1.5$ m ; $M_x=\pm0.8$ m ; $M_z=\pm0.6$ m. The mean square error $M_{\Delta s}$ in the determination of the forefront of the avalanche determined by two stereopairs is equal to ±2.5 m. The mean square error in the determination of the value of movement velocity at the forefront of the avalanche corresponding to the least time interval Δt between the exposures is ±1.2 m/sec.

The accuracy of the stereoscopic bearings on the photograph of the avalanche body was assumed in the calculations to be equal to ±0.01 mm. The processing of materials of the stereosurvey of avalanches was carried out by analytical methods with the application of a stereocomparator and equations of the surface stereophotogrammetry. The calculations were carried out by special programming using a "Promin" type computer.

Stereophotographs of 5 avalanches were processed; the survey was made in the Khibiny region during the winter of 1964–1965. All 5 avalanches are characterised by their large volumes in the order of 10 000 m^3 and composed of old snow with a density of about 0.2 g/cm^3. Two avalanches were chutes and the others slips. The avalanches were artificially provoked by mortar shell explosions fired into the slopes.

As a result of the stereophotogrammetric processing of the survey materials, the spatial coordinates of the avalanche body were obtained in a condition photogrammetric system of coordinates corresponding to definite moments of time.

The results of processing allowed for a presentation of the process of avalanche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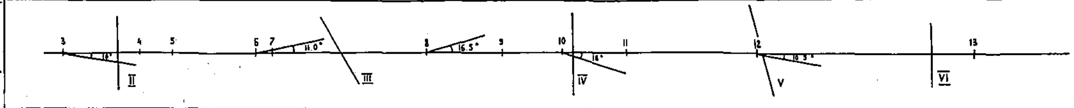
		5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
t_{sec}	From beginning of movement to 5 th stereopair 9.37 sec.	2.04	2.02	2.05	2.04	2.03	2.04	2.00	2.07	2.08	2.05.	2.06	2.06	2.04	2.04	2.02	2.08	2.07	2.03	2.07	2.05	2.04	$\Sigma t \approx 42,96+9,37$
S_m	Actually traversed path	25.0	24.0	23.0	39.0	21.0	30.0	22.5	35.0	44.0	36.0	42.5	33.5	31.0	39.0	29.0	21.0	20.0	22.0	20.0	9.4	9.5	$\Sigma S \approx 576,4$
$v^{m}/_{sec}$	Average velocity on parts of path	12.3	11.9	11.2	19.1	10.3	14.7	11.3	16.9	21.2	17.6	20.6	16.3	15.2	19.1	14.4	10.1	9.7	10.8	9.7	4.6	4.7	$v_{cp} \approx 11,01$

Axis of the traverse in plane with angles of turning and cross-sections



Tilt angles	32.5°	29°	28°	30°	47	36.5°	36.5°	34.5°	26°	24°	21.5°	
Characteristic of the slope by part the chute	1-3 smooth slope	3-5 deeper and narrower part of the chute		5-6 smooth spole	rocks	7-8 smooth ded of chute	8-10 smooth bed of chute with turning at 8th point		*	11-13 smooth slope		
Underlying surface	S n o w											

1. The form of the avalanche body and the position of characteristic point of the forefront on the plane with indication of the number of the stereopairs

2. Position of the characteristic point of the fore-front on the longitudinal profile and the number of stereopairs

3. The characteristic point of the longitudinal profile.

4. The place of intersection of the transversal profile with longitudinal

5. Magnitude of the average velocity of the forefront of the avalanche on a given part of the path in scale 1: 100

* 10-11 narrowed part of chute bed with turning at 10th point

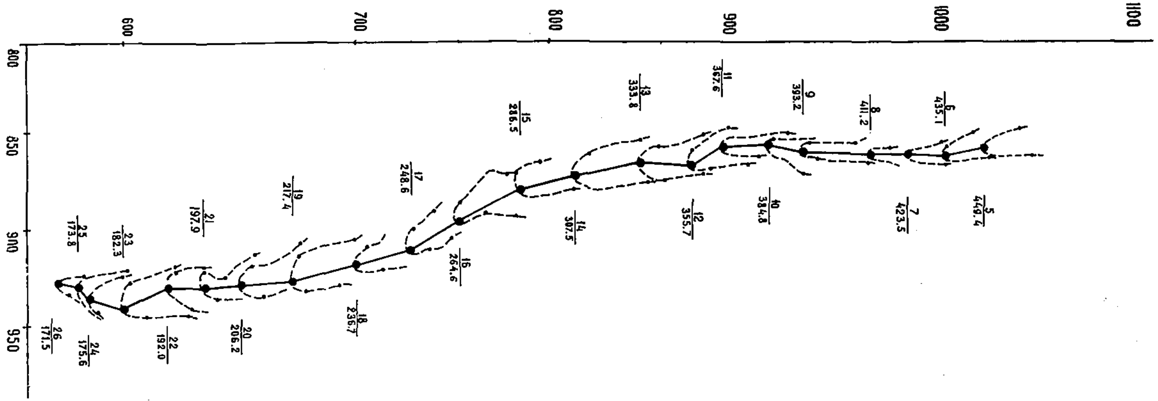


Fig. 4. The plot of the path and velocities of avalanche (chute avalanche on Yukspor Mountain, Khibiny, March 30, 1965, basis 51.32 m, angle of gradient 0°, volume 10 000 m³). density = 0.2 g/cm³

motions on a graphic plan and longitudinal profile in a scale of 1 : 1 000 (Fig. 4). The process of avalanche movement on the plane is expressed in the form of a series of contours corresponding to the form of the avalanche body at the moment of its photographing. The positions of the forefront of avalanches are marked by black circles on the longitudinal profile at the moment of photographing.

Figure 4 shows data on the geometry of the avalanche chute (tilt angles, transversal profiles, turning angles of the path), and the characteristics of the surface of the slope along the path were obtained as a result of the stereophotogrammetric reduction of stereophotographs at the moment where the avalanche stopped. These photographs clearly show the whole path of the avalanche along the slope, which allows for a selection of points for measurement at any characteristic point of the path (Fig. 5).

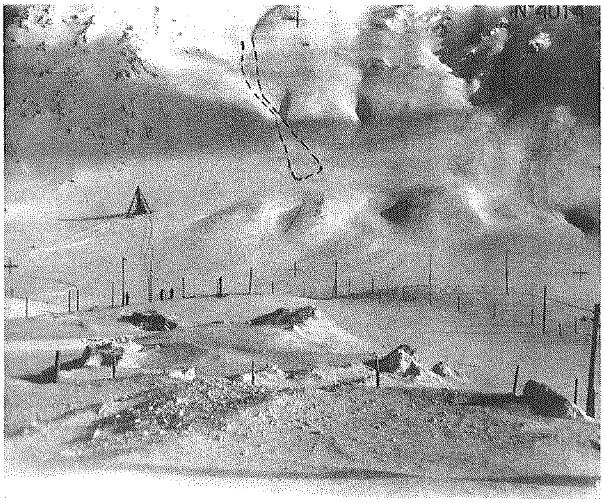


Fig. 5. Photograph of the falling avalanche

The volume of the avalanche is determined by the horizontal net method according to one of the stereopairs on which the boundaries of the avalanche body were expressed with the greatest clarity, thus allowing for a determination with reliability of its surface area and mean height. Generally speaking, by rigidly adhering to definite conditions of photographing the problem of determination of the avalanche volumes can be solved by every stereopair, and thus the change of the volume of the avalanche can be determined in the process of motion.

According to the actual distance covered by the avalanche for Δt time intervals between the moments of photographing, the corresponding velocity value was determined which in turn corresponds to the mean velocity of the avalanche at a given part of the path.

The plot showing the motion velocity of the forefront of an avalanche, depending on the distance covered, was combined with the longitudinal profile. For this purpose the mean velocity at a given part was plotted in a scale of 1 : 100 perpendicularly at the corresponding point of the horizontal axis of the profile.

As can be seen from Fig. 4, the plot of the forefront of the avalanches shows a pulsation characteristic for the wave motion. Analogical pulsation of velocity was also observed on the plots of velocities drafted for the remaining four observed avalanches indicated earlier as belonging to the chute type of avalanches and slips.

It should be noted that the plot of velocities characteristic of wave motions were previously obtained for avalanches not large in volume (about 2 000 m^3) moving with a velocity of only about 5 m/sec.

The results obtained in this paper are only the first results of the application of the method of surface stereophotogrammetric survey in the study of avalanches. Much is to be discovered still in regard to the nature of this phenomenon. It is impossible, for instance at present, to state with any conviction as to what the frequency of velocity momentum changes in the moving avalanches is or as to how the frequency depends on the physico-mechanical properties of snow, morphology of the slope, volume of avalanche etc. Additional data may be obtained by surface stereophotogrammetric survey methods when improved equipment which can specifically meet the requirements of the maximal reduction of the time interval between exposures (of an order of 0.5 sec) is introduced. Positive results can also be expected in this respect by cinema filming carried out together with stereosurvey. In such a case a way may be found by using film sequences for measuring purposes.

III. On the Calculations of Pressure Produced by Impact of the Avalanche against Obstacles

ON THE POSSIBILITY OF SUPERSONIC MOTION IN SNOW AVALANCHES

Snow avalanches are a mixture of air and crystals of ice of different density. This mixture under the influence of gravity runs down the slope and a complex turbulent movement is produced.

We consider that for the calculation of loads on the impedimenta the snow avalanche can be expressed as a certain continuous medium composed of a mixture of air and snow particles. This supposition is based on the fact that due to the smallness of size of snow particles these particles practically move with the air surrounding them. Actually the force of the aerodynamic resistance of these particles is proportional to the square linear size, while the inertia powers are proportionate to mass, consequently to the cubic of linear size. Based on this it may be inferred that with the very small size of ice crystals the force of inertia is of the smallest order compared with the aerodynamical forces and we may disregard them. Therefore, the air and snow particles velocities quickly equalise. Thus we can introduce the concept of some small volume of snow-air mass with an unchanging composition of air and snow particles.

In this case the motion of the avalanche can be described by the system of hydrodynamic equations:

$$\rho \frac{d\bar{v}}{dt} = -\operatorname{grad} p + \rho \bar{F}_e , \quad \operatorname{div}(\rho \bar{v}) + \frac{\partial \rho}{\partial t} = 0 , \qquad \rho = F(p) , \tag{6}$$

where ρ is the mean density, $\bar{v}$ is the mean velocity, $\bar{F}_e$ are the body forces. In particular this system will be closed if $F(p)$ is known. Let us consider the expression of the mean density

$$\rho = \frac{M_a + M_i}{\Omega_a + \Omega_i}, \tag{7}$$

where M_a is the air mass in a certain observed volume Ω; M_i is the mass of ice crystals in the volume Ω; Ω_a, Ω_i are the volumes filled with air and ice corresponding to volume Ω.

Assuming that the air is an ideal gas and the ice crystals practically cannot be compressed we can obtain the following expression for density:

$$\rho = \frac{\frac{M_a}{M_i}+1}{\frac{M_a}{M_i}\frac{\mathrm{R}T}{p}+\frac{1}{\rho_i}}, \quad \Omega_a = \frac{M_a \mathrm{R}T}{p}, \quad \Omega_i = \frac{M_i}{\rho_i}. \tag{8}$$

Here R is the gas constant; T is the absolute temperature; p is the pressure. This expression includes the air temperature. This temperature can be determined by two extreme assumptions:

a) there is no thermal exchange between the air and the snow; in this case air motion is adiabatic;

b) the thermal exchange between the air and snow is so intensive that the temperature of air and snow is practically the same and, as the thermal capacity of snow is much more than that of air, their joint temperature will have a small difference from the initial snow temperature.

The last assumption seems more reliable. By making any of the indicated earlier assumptions we can obtain a connection between the density of the snow-air mass and pressure in it:

$$p = \frac{\mathrm{R}T\rho}{1+\frac{M_i}{M_a}\left(1-\frac{\rho}{\rho_i}\right)}. \tag{9}$$

The velocity of sound "c" is determined by the equality:

$$c = \sqrt{\frac{\mathrm{d}p}{\mathrm{d}\rho}} = \frac{\sqrt{\mathrm{R}T\left(1+\frac{M_i}{M_a}\right)}}{1+\frac{M_i}{M_a}\left(1-\frac{\rho}{\rho_i}\right)}. \tag{10}$$

The dependence of sound velocity on density ρ with $p=p_{atm}$ is presented on Fig. 6.

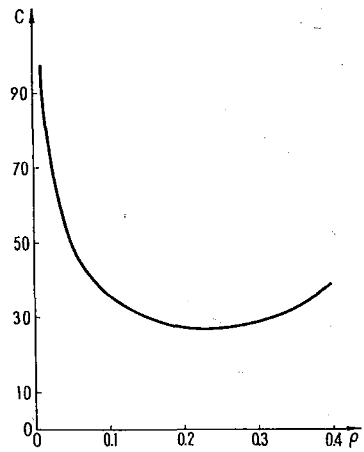


Fig. 6. Dependence of sound velocity on density with $p=p_{atm}$

The plot shows that with a density of the order 0.15–0.35 g/cm³ the sound velocity does not exceed 30 m/sec. Analogical research was carried out for other media, in particular, water saturated soil by Liakhov (1959). Thus, the sound velocity in an avalanche can be less than the avalanche motion velocity, *i.e.* the motion in avalanche can be supersonic. This presumes that the character of the flow over of obstacles will be similar to a flow over in a supersonic stream, *i.e.* accompanied by the presence of shock waves. In particular, a simple method can be suggested for calculating loads on obstacles, which have the form of planes, wedges, etc.

Let the avalanche move with a velocity v_0 (Fig. 7). In the path of the avalanche we assume an obstacle of a wedge form with an angle α to the direction of the velocity vector. This problem does not contain the characteristic linear size either in the velocity equation, or in the boundary conditions. The stream, therefore, shall be a shock wave starting from the summit of the wedge at a certain angle β with two progressive streams before the shock wave and after it. The velocity of the stream after the shock wave is parallel to the side of the wedge. The conditions of the shock wave in this case are as following:

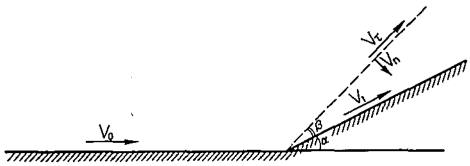


Fig. 7. The scheme of overflowing of wedge-like impediment by avalanche

$$
\begin{aligned}
&1^\circ \quad v_n = v_0 \sin(\alpha+\beta)\,,\\
&2^\circ \quad v_{n_1} = v_0 \cos(\alpha+\beta)\tan\beta\,,\\
&3^\circ \quad \rho v_n = \rho_1 v_{n_1}\,,\\
&4^\circ \quad p+\rho v_n^2 = p_1+\rho_1 v_{n_1}^2\,,\\
&5^\circ \quad \rho = \frac{1+\dfrac{M_a}{M_i}}{\dfrac{M_a}{M_i}\dfrac{RT}{p}+\dfrac{1}{\rho_i}}\,,\\
&6^\circ \quad \rho_1 = \frac{1+\dfrac{M_a}{M_i}}{\dfrac{M_a}{M_i}\dfrac{RT}{p_1}+\dfrac{1}{\rho_i}}\,.
\end{aligned}
\tag{11}
$$

1° and 2° are the expressions for normal to the shock wave velocity components with the allowance that the tangent components do not suffer discontinuity in the shock wave;
3° is the equation for continuity;
4° is the equation of momentum conservation;
5° and 6° are the equations for the state written before the shock wave and after it.

While solving the system of these algebraic equations relative to p_1, *i.e.* pressure after the shock wave (the same pressure is assumed on the side of wedge), we obtain the following expression for p_1;

$$
p_1 = \frac{v_0^2 a^2 m RT(m+1)}{p\left(m\dfrac{RT}{p}+\dfrac{1}{\rho_i}\right)^2}\,, \tag{12}
$$

here

$$
a = \sin(\alpha+\beta)\,, \qquad m = \frac{M_a}{M_i}\,.
$$

Then after transformation, considering that $\dfrac{RT}{p}=\dfrac{1}{\rho_a}$ we obtain:

$$
p_1 = \rho_i v_0^2 a^2 \frac{800m(1+m)}{(1+800m)^2}\,. \tag{13}
$$

For the evaluation of the maximal pressure we can assume $a=1$. This formula can be used in the case of a true supersonic current, *i.e.* when $v_0>c$ where c is the velocity of sound in an avalanche.

Let us consider the examples:

I. $\rho=0.05\ \mathrm{g/cm^3}$; $v_0=50$ m/sec; then $p_1=1.20$ atm.

II. $\rho=0.33\ \mathrm{g/cm^3}$; $v_0=40$ m/sec; then $p_1=3.55$ atm.

CALCULATIONS OF PRESSURE ON AN OBSTACLE PRODUCED BY DIRECT IMPACT OF THE AVALANCHE

First, let us consider the problem of pressure of avalanches on a vertical immovable wall during the shock.

The snow avalanche here is assumed to have a velocity v_0 and density ρ_0 before impact with the obstacle, while the pressure in it is equal to the atmospheric pressure. As the equation of state eq. (9) does not include the values with dimensions independent of the dimensions, of p, ρ_0, v_0 then the dimensions of these values are also dependent, and as the formulation of the problem and the equation of motion does not include any physical values, then the problem under consideration is a similarity. Similar formulation of problems was considered in gas dynamics, for instance, in the problems with a piston (Landau and Lifshits, 1965).

The solution of the problem will be found in the form of two progressive streams divided by the shock wave (Fig. 8).

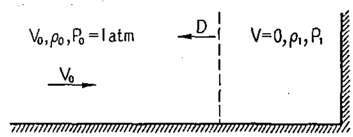


Fig. 8. The scheme of shock of avalanche against the immobile impediment

The stream adjacent to the obstacle has a velocity equal to zero with a density ρ_1 and pressure p_1. Let the velocity of the shock wave, propagating upwards along the avalanche, be equal to D. Then we have the following relations on the shock wave;

$$\begin{aligned}
&\rho_0(v_0+D)=\rho_1 D\,,\\
&p+\rho_0(v_0+D)^2=p_1+\rho_1 D^2\,,\\
&\rho_1=\frac{1+m}{\dfrac{mRT}{p_1}+\dfrac{1}{\rho_i}}\,.
\end{aligned} \tag{14}$$

After transformation we obtain the following square equation relative to density ρ_1:

$$\left(\frac{\rho_1}{\rho_i}\right)^2\left(1+\frac{mRT\rho_i}{\rho_0 v_0^2}+\frac{p}{\rho_0 v_0^2}\right) - \frac{\rho_1}{\rho_i}\left[1+m+\frac{mRT}{v_0^2}+\frac{(1+m)p}{\rho_0 v_0^2}+\frac{p}{\rho_i v_0^2}\right]+\frac{(1+m)p}{\rho_i v_0^2}=0\,. \tag{15}$$

This equation has two solutions: one solution is for the compression wave, the other is for the rarefaction wave. The coefficients of this equation do not depend on the sign of v_0. As we are interested only in the root of the equation, corresponding to the compression wave, the smaller root of this equation, corresponding to the rarefaction wave, can be disregarded.

Having found ρ_1 from this equation and inserting it into formula (9) we obtain the value of the shock pressure p_1.

Figure 9 presents the dependence of shock pressures on the density and velocity of the avalanche. The plots show that the shock pressure can get large values from 10 to 20 times of that of the atmosphere with large avalanche velocities and considerable density. These pressures are much greater than the pressures which appear during the flow over of an obstacle by an avalanche. It should be noted that the obtained pressures are smaller, because the scheme considered above disregards the strength of ice crystals and interaction between them. Evidently this interaction can be neglected in the case of sufficiently small densities of snow and at large velocities of avalanche motion.

With small velocities of the snow mass motion it is necessary to make allowance for the air motion relative to the ice crystals. In this case we cannot use the theory considered.

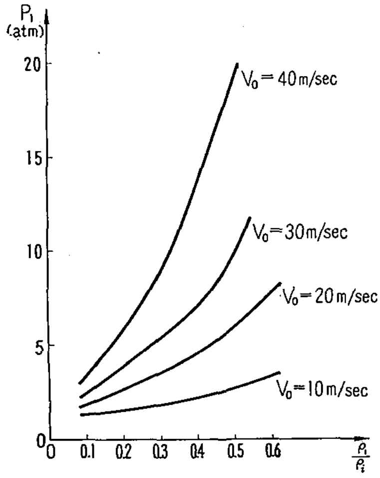


Fig. 9. Dependence of shock pressure on velocity and density of the avalanche

IV. On the Nature of the Air Waves Caused by Snow Avalanche

During the fall of some types of snow avalanches in mountains a powerful "air wave" is created, which is, as a rule, accompanied by avalanches of loose snow. The "air wave" is one of the most dangerous events associated with snow avalanches. This is explained, firstly, by its unusually destructive force and, secondly, by the fact that if the snow falls along the same paths then the path of the "air wave" might be completely unexpected. The observers have noted that the "air wave" is often in advance of an avalanche.

It should be noted that as the velocity of motion of avalanches along the slope, as a rule, does not exceed 100 m/sec, which is considerably less than the velocity of sound in the air, there can be no shock waves in the air similar to the shock waves caused by supersonic airplanes.

However, some authors think that origin of the air wave is caused by the increased air pressure before the avalanche. Such a wave can exist only during the movement of the body and stretches to a very small distance before that body. At the same time it is known that the air wave accompanied by an avalanche is also observed after the avalanche itself stops.

The assumption that the air wave, similar to explosion shock waves, is formed during the fall of the avalanche from scarps is also untrue. It is known that such a wave propagates as a spherical shock wave with its center at the site of explosion. This is in contradiction with the observed direction of air wave propagation.

Thus the points of view existing at present on the origin of the air wave are in contradiction with the observed facts.

We assume that the mechanism of formation and movement of the "air wave" is analogical to the formation and movement of a vortex circle. Similar circles are often demonstrated by means of a box filled with smoke with a round hole. At a sudden reduction of the volume of the box a vortex circle appears which can move to a distance of tens and hundreds of diameters of the circle. If we cut such a circle across by a symmetry plane, we obtain a model for the "air wave" in the form of a semicircle moving over the plane.

There are sufficiently detailed descriptions of similar vortex circles (Kochin, Kibel and Roze, 1965). In particular, if we consider the movement of the infinite vortex line near the plane (a two-dimensional problem), we arrive at the conclusion that the velocity near the plane is four times greater than the velocity of displacement of the vortex line.

During the fall of an avalanche of loose snow certain conditions may arise favourable to the formation of a powerful vortex. Actually, a snow avalanche involves the air on the slope; in loose snow avalanches an intensive mixing of the snow with the air surrounding the tongue of the avalanche promotes this phenomenon, while the air at a somewhat higher height is not involved. As a result, we obtain a velocity circulation which is different from zero, and the vortex lines encircling the tongue of the avalanche are supported by the slope which serves as the symmetry plane. Even after the avalanche slows down and stops, the vortex semicircles continue to move down the slope and produce destruction similar to those described above.

At points where the vortex line enters the slope phenomena analogical to tornado effects can be observed. Such phenomena during snow avalanches were noted by many observers. An additional increase of energy of the air current can take place due to the potential energy of the snow particles carried away by the vortex down the slope. Other mechanisms of formation of the vortex can be also shown.

With an avalanche velocity of about 100 m/sec at the moment of the vortex formation, the air pressure on the impedimenta due to the velocity pressure reaches about 500 kg/cm^2. In order to understand the different mechanisms of formation of the vortex special experiments were carried out with models of these events.

The experiments showed that in reality, as a rule, the movement of the avalanche is accompanied by a vortex movement of the surrounding media. This movement still continues after the avalanche stops, and a semicircle vortex may propagate after the avalanche stops to a distance governed by the order of the characteristic size of the avalanche.

V. On the Evaluation of Avalanche Release

GEOGRAPHICAL METHODS OF EVALUATION

In developing the mountain territories of the Soviet Union the problem of evaluation of the limits of destructive possibilities of avalanche snow and air waves at various construction sites becomes important. The greater part of the mountain regions of our country have an extremely low population, therefore, an accumulation of similar experiences of observations of avalanches collected over many centuries as recorded in the Alps is lacking here. Thus in most cases before a construction site is determined it is necessary to carry out a survey on the exact boundaries of avalanche distribution.

This work begins with the determination of the places of falling of avalanches ("avalanche seats") carried out by the application of large-scale photosurvey preliminary field work.

Such data collected at the Laboratory of Snow Avalanche Problems, Moscow State University, shows that with the aid of accurate deciphering of photographs, especially supported by further field research reveals the seat of chute avalanches in the majority of cases (Tushinsky, 1949).

It is more complicated to find avalanche danger on flat slopes, because traces of avalanches on them are less distinct than those of the chute avalanches. Finally, it is most difficult to find seldom falling and, therefore, the most dangerous avalanches because of their suddenness. In order to avoid such drastic errors the researchers, out of necessity, usually give a somewhat (indefinitely) larger number of avalanche seats within each territory.

After the avalanche seats are discovered a new stage follows in the evaluation of the covered territory, *i. e.* the stage of determination of limits of avalanche danger zones for each avalanche seat. In the first approximation this evaluation is carried out in the first stage with the application of the same external signs of the avalanche fall (broken or damaged forests, "avalanche debris" on the debris cones, etc.). Thus an effective geomorphological-geobotanical method in current use in the USSR is being developed. This may be seen especially in the numerous work of Prof. G. K. Tushinsky (1949). However, notwithstanding its simplicity and obviousness this method of determination of the boundaries of avalanche danger zones has a serious drawback: It tends to produce underrated results inasmuch as the used criterion, *i. e.* the traces of avalanches present only certain indefinite conditions of avalanche activity of the last one or two centuries and cannot be accepted as evidence of the maximal distance of avalanche release.

During the last 30 years in the Soviet Union several calculation methods were worked out for the determination of boundaries of avalanche danger zones. The method of S. M. Kozik (1962) is now most widely used. The practical determination of the maximal distance of avalanche release by this method is very simple and is reduced to a graphical solution of the longitudinal profile of the slope. Evidently this method, as all those similar to it, presents the movement of the avalanche in an idealised form. A series of allowances and simplifications tend to make the results rather overrated.

Thus, at present two methods are used in competition in the evaluation of avalanche danger on the territory of the USSR: Namely the geomorphological-geobotanical method and the calculations suggested by S. M. Kozik. Naturally the researchers favour the calculating method, though its inaccuracy leads to great extra costs. Therefore, the problem of the method of evaluation of the distance of avalanche release is still very acute.

A possible solution of the indicated problem is the development of a reliable determination of the maximal distance of avalanche release for a number of seats according to the signs on the landscape of the fall of avalanches and improvement of calculating methods on this basis. Earlier we mentioned the external signs of avalanche falls which reflect avalanche activity in recent years. However, the landscapes of mountain regions bear evidence of all avalanches in various states of display, including the largest of them. Among these the easiest study method is made on the deposits of the debris materials brought down by the avalanche. Finally, by obtaining the boundary of avalanche deposits as a result of detailed research, we can proceed to the boundary of distribution of avalanches for the whole period of avalanche activity with the help of a certain transit coefficient, *i.e.* to the boundary of their maximal distribution. The significance of the transit coefficient lies in the ratio between the distance of the avalanche release and the deposit of stone material by these avalanches.

Work in the described trend was conducted for four years for a group of projects called "Apatit" in the Khibiny (Kola Peninsula). The Khibiny is a low (1 200 m) mountain massif with a fall of heights of about 500–700 m and with a rather close network of avalanches (up to 4–5 avalanches per 1 linear km at the foot of the slope). Continuous avalanche activity has been repeated here for about 10–12 thousand years since the last glaciation. The simplicity of the slopes construction and avalanche seats (funnels and cories of regular form), the intensive deposit by avalanches of debris material and considerable width of flat bottoms of valleys, which allow the avalanche cones to develop freely, have made this place almost ideal for the study of avalanche deposits. The complex of the field work included the tacheometric study of the form of cones, the measurement of the size and form of the debris material on their surface and within their layer, stratigraphic, mineralogic and geochemical researches of avalanche and moraine, proluvial and gravitation-talus deposits. In different combinations this work was conducted approximately on 30 avalanche cones. Numerous observation data were collected, which allowed, among other results, the drawing of conclusions concerning the maximal distance of avalanche deposits. The boundaries are obtained due to the reduction of materials on the changeability of debris cover. The "density" of the debris cover, expressed by the number of fragments larger than 10 cm for an area 10 m^2, according to the measurements in the continuous bands along the axes of the cones, was in the lower part of the cones and showed a regular (approximately parabolically) decreasing. Extrapolation of parts of density curves, obtained by direct measurements, to zero values of density allows for the determination of the boundaries of distribution of debris carried by the avalanches. These boundaries are equal to the limits of deposition of the avalanches themselves. Accurate extrapolation based on the methods of statistics of extreme values will allow, evidently, for the obtaining of boundaries with an accuracy of $\pm$50 m of the actual position. At present the reduction allowed for obtaining boundaries with an accuracy of about $\pm$100 m of the actual position (the boundaries, calculated for the same region by the S. M. Kozik method, have the probable error of $\pm$60 m from the assumed overrated limit). The obtained boundaries are 150–200 m apart from the boundaries drawn according to the outer traces of avalanche falls, while the boundaries obtained for 80 seats by the calculating method are 600 m apart (100–1 300 m) on the average. The advantage of an application of a more labour consuming, but more objective than the calculating method, mode of determination of boundaries based on the analysis of avalanche cone structure is demonstrated by the following figures: Within the studied part of the Khibiny from a point of view of the calculating method 10.5 hectares of area are safe from avalanches, while the new method allows for the evaluation of the safe territory of about 15 hectares.

Further on empirical method of determination of the maximal release distance of avalanches is described.

MORPHOMETRIC METHOD OF EVALUATION

At present there are a number of studies which within various simplified suppositions concerning the character of movement of the snow avalanche (laws of friction, snow cover properties, etc.) can be used in the determination of the distance of release of avalanches (Kozik, 1962). However, these suggestions appear too rough, which results, as a rule, in considerable overrating of the boundary of the avalanche termination. While to meet many practical requirements the boundary of avalanche danger zones should be indicated with greater accuracy.

For some time to come we can hardly expect sufficiently accurate solutions of the problem from theoretical discussions, because any theoretical solution is inevitably connected with more or less radical simplifications concerning the character of a slope and the mechanism of movement of a snow avalanche.

Now a certain empirical approach for the determination of the distance of release of snow avalanches will be given.

For simplicity's sake we shall assume that the distance of release depends only on the geometry of the longitudinal profile of the avalanche seat with all other conditions being equal, including the relative height of the slope. Let there be a sufficiently complete album of profiles of slopes and avalanche seats, for which the actual maximal distances of avalanche release are known. Let us also assume that there is a certain definite profile of slope or avalanche seat for which the maximal boundary of avalanche termination is unknown. Then we can proceed to the next step.

We consider the profiles from the album to be similar in character by superimposing the termination point of the avalanche and the assumed point of stop on the studied profile. If all the points of the profile from the album lie at higher points than that of a given profile, *i. e.* if the profile from the album is steeper, it is natural to assume that the point of avalanche termination on that profile will not exceed the assumed point of stop. If the profile is much lower than the profile from the album, then the termination point thus obtained shall be considerably overrated.

In order to formulate the indicated empirical approach we shall consider the following statement.

Let us discuss two profiles. Here and further only those parts of the slope are compared which lie higher than their points of coincidence. Let them intersect at the point of avalanche termination of a less steeper slope which with all its points lies lower than the steeper one. As before, it is natural to assume that with all things being equal, the conditions of the avalanche falling from a steeper slope does not stop at the intersection point but will pass further. Thus, if one profile lies completely above the other then the points of avalanche termination cannot coincide. Consequently if we superimpose the points of avalanche termination on two profiles, then because they cannot lie one higher than the other they should either coincide, or intersect even if only in one other point.

These reasons can be applied for the evaluation of the maximal avalanche release. In fact, the maximal avalanche releases are observed in the most favourable conditions of formation of snow cover on the slope. We can discuss different conditions of snow cover formation for a given profile and among them choose the most favourable for the maximal distance of avalanche release. These conditions of formation may with more or less frequency be repeated in different climatic conditions. In cases when they occur, a simultaneous maximal falling of avalanches may be observed. Therefore, the maximally farthest possible release does not depend on climatic conditions for regions where the avalanches fall more or less regularly.

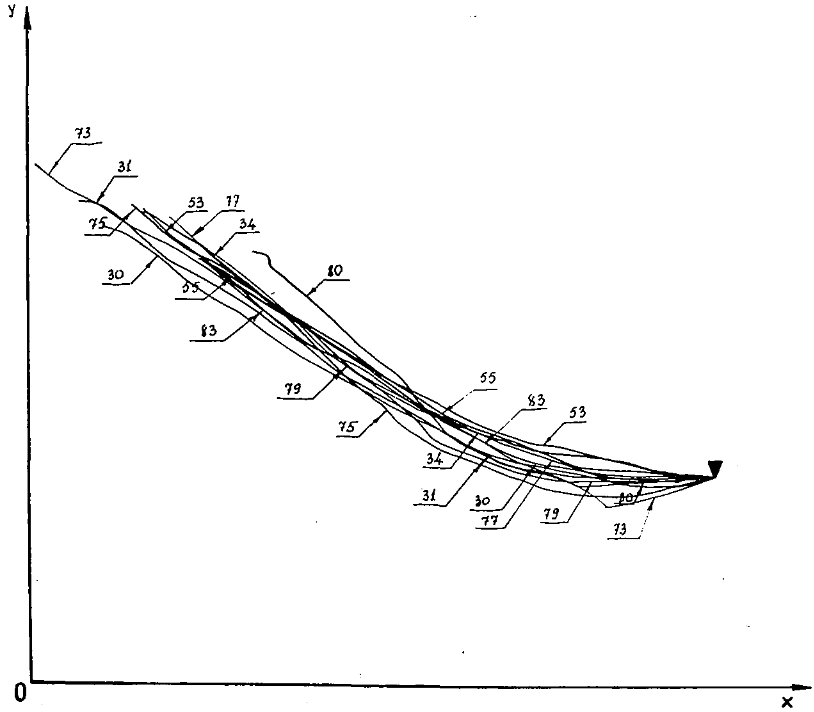


Fig. 10. Profiles of slopes in the Khibiny region
▼: Point of termination of maximal avalanches

Figure 10 represents a series of profiles of paths of avalanche falls in the Khibiny region on the Kola Peninsula. All the points of stops of maximal avalanches are superimposed, and the maximal release of avalanches is determined by the geomorphological method described above. The behaviour of curves, showing their numerous intersections, confirms the statement formulated above and, consequently, the supposition presented at the demonstration.

In future during the compilation of an avalanche album it seems important to group the avalanche profiles separately according to the height of slopes, to the character of avalanche seats in the plane and, possibly, to the roughness of the underlying surface.

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