Magnitude of Avalanche*

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Abstract

For the convenience of a statistical approach to the studies of avalanche, a definition of magnitudes of avalanche is proposed here by use of a simplified model of snow slide: Mass Magnitude (M.M.), the measure of mass of avalanched snow, Potential Magnitude (P.M.), that of total work done by an avalanched snow mass consuming its potential energy, and Destructive Magnitude (D.M.), that of the maximum destructive power by a snow slide of given mass on a given slope.

In the case of an airborne type of surface avalanche, however, further consideration on the effect of blasting wind generated by the avalanche (avalanche wind) is necessary.

By the proposed definition of magnitudes of avalanche, it was shown that the value ranges up to 6 for M.M. and up to 9 for P.M. are sufficiently large to cover almost all of the practical cases from the statistics of avalanches in Japan hitherto.

I. Introduction

In the description of avalanches, mass and volume of avalanched snow, running distance, vertical distance of slide and width of avalanche and topographical features of the avalanche site are generally described in individual cases. If uniformarized parametric descriptions of an avalanche are given in such a way that the characteristics of an avalanche can be well-described, it would be very useful for the surveying and research of avalanches, especially in a statistical approach.

For practical purposes, the parameters of the magnitude should be such that they can be practically obtained through field work. From the standpoint of research on disaster by avalanches "scale of avalanche" (amount of burying) and "intensity of avalanche", (power of destruction) are two major problems.

The motion of snow avalanches was considered in this paper through a simplified model of snow slide, and three magnitudes of avalanche were proposed.

II. Consideration on the Motion of Snow Avalanche through a Simplified Model

A most simplified model of snow slide was set up to consider the fundamental mechanism of the motion of a snow avalanche.

We assume here a snow block, mass m, on a slope AB with a declination θ, which commences to slide down the slope after release from restriction (Fig. 1). The driving

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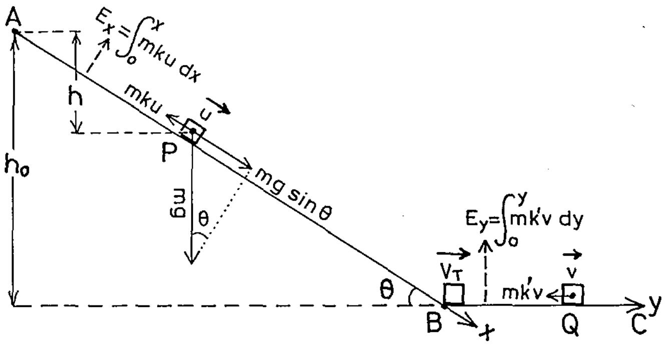

Fig. 1. A simplified model of snow slide

force of this *motion* is *the component* of gravity parallel to the slope downward, $mg\sin\theta$, with frictional resistance working against the motion. Generally, the frictional force against a motion can be expressed as a function of velocity of the motion, $f(v)$. As the frictional force increases by increase of the velocity, the motion of the snow block will be converted into a uniform motion with the terminal velocity V_T, when the frictional force balances with the driving force,

$$f(v) = mg\sin\theta\,.$$

The snow block converts the decrease of its potential energy (P.E.), through sliding down the slope, both into kinetic energy (K.E.) of its motion and into thermal and acoustical energy dissipation caused by friction or shock.

The motion of the snow block is halted when:

1) A strong obstacle (of which mechanical strength, minimum energy necessary to destroy it, is in excess of the kinetic energy of the sliding snow block) is in the path of the snow block.
2) The frictional resistance to the motion of the snow block increases and exceeds the driving force by a change of slope conditions.

Under such considerations, the model of snow slide was selected as follows (Fig. 1):

1) AB is a uniform slope of declination θ.
2) Horizontal plane BC levels off directly from the slope AB.
3) Snow block of mass m commences to slide down at A on the slope, and stops at C on the horizontal plane.

1) *Motion of a snow block m on a slope.* If A is taken as the original point, AB is the x-axis and u is the velocity of the snow block along the x-axis. Assuming that the frictional resistance to the motion of the snow block is proportional to m and u, the equations of the motion of the snow block may be given as follows, under the initial conditions of $x=0$ and $u=0$ when $t=0$,

$$\frac{du}{dt} = g\sin\theta - ku\,, \tag{1}$$

$$u = \frac{dx}{dt} = \frac{g}{k} + \sin\theta(1-e^{-kt})\,, \tag{2}$$

$$x = \frac{g}{k}\sin\theta\left(t+\frac{1}{k}e^{-kt}-\frac{1}{k}\right). \tag{3}$$

And the terminal velocity of the snow block for the slope is,

$$V_T = (u)_{\frac{du}{dt}=0} = \frac{g}{k}\sin\theta\,. \tag{4}$$

If we take 90% of the theoretical terminal velocity V_T as the practical terminal velocity V_T^*, the length of the slope necessary to obtain the practical terminal velocity $V_T^*(=0.9V_T)$ of the snow block is given by eqs. (2) and (3), thus

$$x_0 = 1.4\frac{g}{k^2}\sin\theta\,. \tag{5}$$

And a snow block sliding down a long uniform slope θ maintains a uniform motion of velocity V_T^* or approximately, regardless of the running distance beyond x_0 (Fig. 2 a). The snow block will produce a work converting a part of its K.E. into destructive

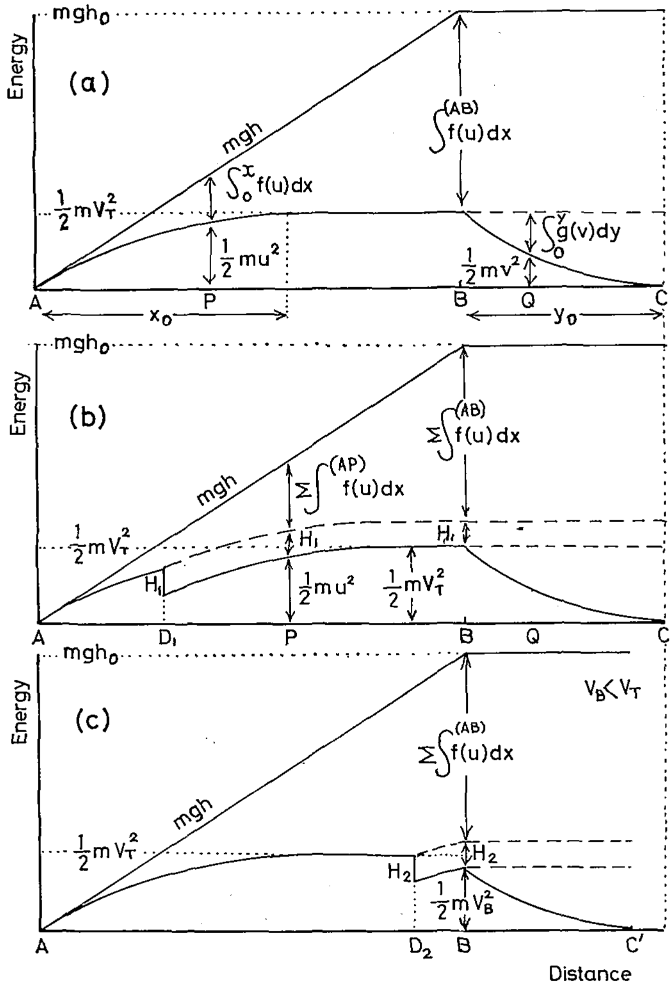

Fig. 2. Energy conditions of a snow block in the simplified model of snow slide. (a) no obstacle in the path, (b) with destruction of an obstacle at the upper part of the slope, (c) with destruction of an obstacle at the lower part of the slope

energy when it encounters an obstacle in its path. The limit of the destructive power of the snow block is $\frac{1}{2}mV_T^2$.

Let H be the mechanical strength of the obstacle, then u is the velocity of the snow mass when it encounters the obstacle, and

if $mu^2/2>H$, the snow block destroys the obstacle and proceeds.
$mu^2/2=H$, the snow block destroys the obstacle and stops there temporarily.
$mu^2/2<H$, the snow block is blocked by the obstacle without destroying the obstacle.

Energy conditions of a snow block which produces destruction of the obstacle at the upper part or lower part of the slope, under the condition of $\frac{1}{2}mu^2>H$, are shown in the Figs. 2 b and c.

2) *Motion of a snow block m on the ensuing horizontal plane from the slope.* Assume that a snow block of mass m is propelled into a horizontal motion from B after attaining a practical terminal velocity V_T^* on the slope (Figs. 1 and 2). Assuming that there was no energy dissipation of the snow block at B, the initial velocity of the snow block in horizontal motion would be V_T^*. Taking B as the original point, BC as the y-axis, v as the velocity of the snow block parallel to the y-axis, and $mk'v$ the frictional resistance to the motion of the snow block, the equations of motion of the snow block may be given as follows, under the initial conditions of $v=V_T^*$ and $y=0$ when $t=0$,

$$\frac{dv}{dt}=-k'v\,, \tag{6}$$

$$v=\frac{dy}{dt}=V_T^*\,e^{-k't}\,, \tag{7}$$

$$y=\frac{1}{k}V_T^*(1-e^{-k't})\,. \tag{8}$$

The energy condition of the horizontal motion is given as,

$$mgh_0=\frac{1}{2}mv^2+\int^{AB}mku\,dx+\int_0^y mk'v\,dy\,, \tag{9}$$

where h_0 is the vertical height of the slope AB. The 2nd and 3rd terms on the right side of eq. (9) indicate the energy dissipation due to friction during the motion on the slope and the horizontal plane, respectively. Immediately after the snow block changes to the horizontal motion at B, both the left side and the 2nd term of the right side of eq. (9) become constant, and the 3rd term begins to increase monotonically. Therefore, the 1st term on the right side, K.E. of the snow block, decreases monotonically. And the snow block ceases to move at C, where,

$$mgh_0=\int^{AB}mku\,dx+\int^{BC}mk'v\,dy\,.$$

The horizontal running distance of the snow block before repose is given from eqs. (7) and (8), as

$$BC=y_0=\frac{V_T^*}{k'}\,, \tag{10}$$

where y_0 is the maximum running distance of the snow block on the horizontal plane BC, without obstacles in its path.

3) *Some considerations on the frictional resistance to the motion of snow avalanche.* Assuming that the resistance to the motion of a moving body can be expressed as a function of its velocity u, then the function $f(u)$ can be expanding into a power series of u, as follows,

$$f(u) = k_0 + k_1 u + k_2 u^2 + k_3 u^3 + \cdots , \qquad (11)$$

where, $k_0, k_1, k_2, k_3, \cdots$ are constants.

k_0 is termed "solid friction" which appears predominantly in the case of friction between two solid bodies, and is independent of the relative velocity between them. $k_1 u$ is termed "fluid friction" and appears predominantly for a body moving slowly in fluid. $k_2 u^2$ is a predominant frictional factor appearing only when the velocity is remarkably high, and it can be negligibly small for a low velocity range. The higher power terms than u^2 are generally small enough to be neglected for general cases (Hagiwara, 1945).

The discussion in this paper will be carried out under the assumption of a "fluid friction" condition in which the frictional resistance to the motion is proportional to the velocity of the body, in other words, all terms on the right side of eq. (11) except the 2nd term are assumed to be negligibly small in comparison to $k_1 u$. In actual cases, the mechanism of the frictional resistance to the motion of an avalanche must be extremely complicated, as it is composed of frictions between the avalanching snow, the slope and the surrounding air, including friction of the internal motion of the avalanching snow. But no definite characteristics of frictional resistance to the motion of the avalanche have been given hitherto.

i) If the total frictional resistance to the motion of avalanche is expressed only by "solid resistance" k, regardless of the velocity of the sliding snow, the equation of motion is given as,

$$\frac{\mathrm{d}u}{\mathrm{d}t} = g \sin \theta - k_0 . \qquad (12)$$

This means a uniform acceleration motion, and the discussion made above cannot be applied to this case. However, as the mechanism of motion of avalanche can be considered to be composed of solid-solid, solid-fluid and fluid-fluid motions, the case i) may not be reasonable as an actual condition.

ii) The discussion of this paper was carried out under the assumption of "fluid friction" conditions, where the resistance to the motion is proportional to the velocity.

iii) If the resistance to the motion of avalanche is expressed by a function of higher terms of the velocity, *i.e.*, $u^2, u^3, \cdots$, and if it is an increasing function of the velocity, there should exist a terminal velocity for a snow block sliding down a slope. And an analogous discussion for the above will be made for this case, also.

III. Magnitudes of Avalanche

Various types of parametric descriptions of avalanche may be considered to suit the purposes. A classification of avalanches by the magnitude of some fundamental quantity of the avalanche could be a useful means especially when the avalanches are processed statistically. Through previous discussions, three magnitudes of avalanche were proposed as follows:

A snow block which slides down a slope converts the decrease of its potential energy, P.E., into kinetic energy of the snow block, K.E., and energy dissipation by friction with the surroundings and work done by the snow block, D.E. The relation between these three quantities may be expressed as,

$$\Delta \text{P.E.} = \text{K.E.} + \text{D.E.}$$

Therefore,

i) The mass of avalanched snow is one of the fundamental quantities to describe an avalanche.

ii) The decrease of potential energy of the avalanched snow, ΔP.E. (=the total work done by avalanched snow), is also an important quantity to describe the magnitude of energy of an avalanche.

iii) As the destroying work by an avalanche is done through a consumption of kinetic energy of the avalanche, K.E., the destructive power of an avalanche is limited by the maximum kinetic energy the avalanche can aquire, $\frac{1}{2} m V_T^2$. The maximum speed of a snow block for a uniform slope is limited by the slope conditions, declination and frictional coefficient, regardless of the running distance. Therefore, particular attention must be paid to the limit of kinetic energy an avalanche can aquire which is also the limit of the destructive power of the avalanche.

1) *Mass Magnitude* (*M.M.*). This is the measure of mass of avalanched snow. As a practical unit, common logarithm of the mass of the avalanched snow in tons, is taken as,

$$\text{M.M.} = \log_{10} m . \quad (13)$$

Relation between mass of avalanche snow and its mass magnitude is shown in Fig. 3.

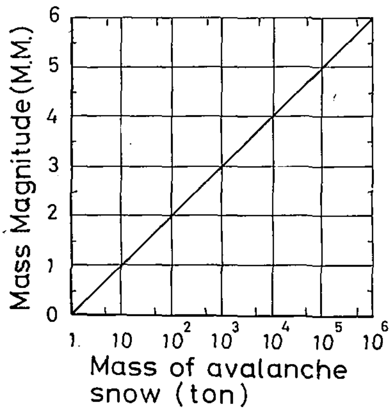

Fig. 3. Mass of avalanche snow, in tons, and Mass Magnitude (M.M.)

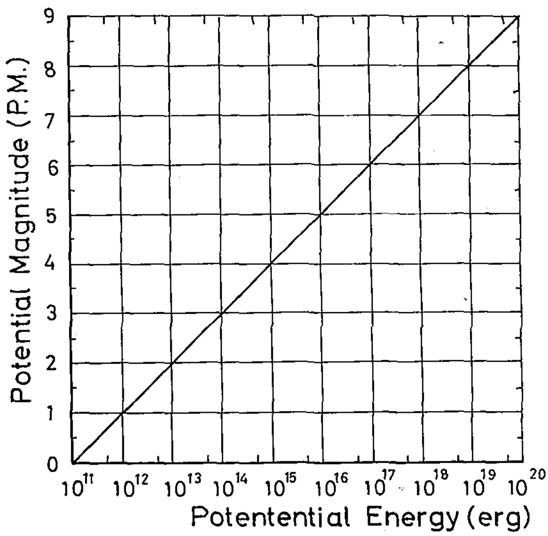

Fig. 4. Total work made by avalanche, in ergs, and Potential Magnitude (P.M.)

2) *Potential Magnitude* (*P.M.*). This is the measure of the total work done by the avalanche snow consuming its potential energy. Taking m as the mass of avalanched snow in tons, and h_0 the vertical drop of the center of gravity of the avalanche snow in meters, the common logarithm of the decrease of potenital energy of the avalanche snow is given as

$$\log_{10} mgh_0 = \log_{10} g + \log_{10} mh_0 . \tag{14}$$

where, g is the acceleration of gravity.

For practical purposes, only the 2nd term on the right side of eq. (14) may be sufficient to classify the amount of the total work done by the avalanched snow. Therefore, the practical unit of the Potential Magnitude of an avalanche is given as,

$$\text{P.M.} = \log_{10} mh_0 . \tag{15}$$

Relation between potential energy of avalanche and its Potential Magnitude is showen in Fig. 4. It must be noted that an avalanche with high P.M. is not always powerful in destruction.

Destructive Magnitude (*D.M.*). This is the measure of the maximum power of destruction of an avalanche. Taking V_T^* as the practical terminal velocity of a snow block of mass m which slides down a uniform slope θ with no obstacle, the common logarithm of the maximum destructive energy of the snow block is given as,

$$\log_{10} \frac{1}{2} m V_T^{*2} . \tag{16}$$

By the eq. (4)

$$\log_{10} \frac{1}{2} m V_T^{*2} = \log_{10} \frac{1}{2} m \left(\frac{g}{k} \sin \theta\right)^2 ,$$
$$= \log_{10} \frac{1}{2} mg^2 + \log_{10} \frac{m}{k^2} \sin^2 \theta .$$

As the 1st term on the right side of this equation is constant, only the 2nd term may be sufficient to classify the distructive power of an avalanche. Therefore, the practical unit of the Destructive Magnitude of an avalanche is proposed as a function of mass of avalanching snow and the slope condition,

$$\text{D.M.} = \log_{10} \frac{m}{k^2} \sin^2 \theta , \tag{17}$$

where, k is the resistance coefficient of an avalanche to the surroundings. Further study is necessary to evaluate k.

4) *Destruction by blasting avalanche wind.* In the previous section, destructive power of an avalanche only by the force of inertia of the avalanching snow mass was considered on a simplified model. The actual mechanism of an avalanche must be far more complicated than this model: for example, it is necessary to consider the effect of duration of the force acting on the obstacle encountered to discuss the destruction even only by the inertia force of snow mass.

Especially, blasting wind generated by a dry snow avalanche seems to have an extremely strong destructive power which has an essentially different mechanism from those by a sliding snow mass (Ogasawara, 1965).

Table 1. Statistics of avalanches in Hokkaido, Japan (Only those which have applicable records for calculations of M.M. and P.M.)

Date	Location	m (ton)	M.M.	h (meter)	P.M.	Killed	Injury	Reference
'61/1/26	Teshio	3 000	3.5	150	5.7			Fukuyama (1963)
'61/4/ 5	Niikappu, Hidaka	4 500	3.7	80	5.6	21	6	Yosida *et al.* (1963)
'61/4/ 5	Nukabira, Hidaka	5 600	3.8	70	5.6	12	6	Yosida *et al.* (1963)
'64/4/ 1	Furubira	60 000	4.8	70	6.6			Huzioka *et al.* (1964)
'65/3/14	Satsunai-Gawa, Hidaka†	300 000	5.6	700	8.5	6	6	Shimizu *et al.* (1966)
'65/4/15	Yubari	300	2.5	20	3.8	1		Huzioka *et al.* (1965)

m: Mass of avalanche snow, h: Vertical fall of avalanche snow measured by its center of gravity.

† Satsunai-Gawa Avalanche, 1965, was one of the biggest avalanches on record in Japan.

5) *Examples of magnitudes of avalanche.* Some examples of avalanches in Japan which have sufficient records or description to calculate the prescribed magnitudes, Mass Magnitude (M.M.) and Potential Magnitude (P.M.), are illustrated in Table 1.

As a result, the value ranges up to 6 for M.M. and up to 9 for P.M. seems sufficiently large to cover avalanches in Japan.*

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** In Japanese with English summary.

* Mass Magnitude M.M.=7 was recorded by a glacier avalanche at Huasccran, Peru, 1962.