Heaving Force of Frozen Soils*

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Abstract

When the surface of frozen ground heaves in winter, large froces act upon structural foundations and buildings on the ground. The author measured the forces both in the field and in the cold room.

In the field research a rigid iron disc was placed as a receiver of the force caused by the frost heaving on the ground surface. This was fixed through a load-cell to a rigid frame rendered immovable throughout the winter. The area of the disc was restrained from heaving, while the surroundings were allowed to heave freely. The force exerted on the disc increased continuously, as long as the heave of the surroundings progressed, while it decreased when the heave stopped. The force changed almost parallel to the heaving rate, and it was shown that the decrease was due to the stress relaxation in the frozen soil beneath the fixed disc.

In the cold room two types of soil cylinders were set in a test box for frost heaving. One soil sample was set for free heaving, while the other was fixed to a rigid frame and restrained from heaving.

The maximum heaving pressure developed at the ice-water interface in the freezing soil was computed from these two experiments.

I. Introduction

When the ground surface is exposed to the cold weather (the opportunity of encountering such conditions has been increasing in recent years with the progress of snow removing work even in snowy districts), various foundations and buildings suffer mechanical damages by frost action. It has become necessary in practice to investigate the cause of such large forces contributing to this type of damage. On the other hand, in the study of the mechanism of frost heaving theorizations have been recently directed to the cause of heaving pressure developed in soil freezing (Everett, 1961; Penner, 1966; Yong, 1966).

In this paper the author has attempted to measure the heaving force of frozen soil both in the field and in the cold room.

In the vicinity of a region of highly frost-susceptible soil it is well known that the amount of heave is reduced when the load on the ground becomes heavier. Therefore, in field research it becomes a subject of how large a load should exist on the ground to render the heave nil.

In the cold room the pressure induced on the surface of freezing soil when it was restrained from heaving was measured for the purpose of comparison with the field

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research. The obtained results showed a definite relation to Everett's formula (Everett, 1961) which predicts maximum heaving pressure by assuming the effective pore radius within the frozen soil.

Also, it is a characteristic feature that the heaving force is accompanied with the relaxation phenomena because of the visco-elasticity of the frozen soil. This property is due to the mechanical behavior of ice within the frozen soil and the mobile layer between the ice and the soil particles.

II. Experimentation

Field research

The equipment used to measure heaving force is shown in Fig. 1 a (Kinosita, 1963). This was made on the campus of the Kitami Technical College located in the northern regions of Hokkaido, where the maximal freezing depth in the ground is 100 cm deep in winter. Two meters below the ground surface a concrete block (30 cm thick) with several iron rods (4 cm in diameter) embedded upright in the block was laid. The apparatus for measuring heaving force was attached to the top of two iron rods (spaced 40 cm apart). Next, an iron disc (12 cm in diameter; 2 cm thick) was placed on the ground surface at the centre between the two rods. Also, another iron disc (30 cm in diameter) was placed on the ground surface at the centre between two rods (spaced 90 cm apart). Load-cells were placed on the discs, and the both top touched the bottom surface of two iron channels set in layers which were fastened to the iron rods with bolts. These arrangements were made before winter, and the iron channels, the load-cell and the disc were rendered immovable throughout the whole winter. Thus, the area of the disc was restrained from heaving, while the surroundings heaved freely. The photographs of Figs. 2 a and b show the research field before winter and in the depth of winter respectively. The heaving force exerted upon the disc was electrically measured by the load-cell.

Around the research field the nature of soil was clay soil at the layer between the ground surface and at a 150 cm depth, with sand underneath the layer. The clay soil known as Kitami clay contained 36% clay, 28% silt and 36% sand; this is one of the highly frost-susceptible soils in Hokkaido. The sand under the clay soil was not frost-susceptible, so that a soil moisture transfer took place only within the clay soil: thus a closed system was developed.

The rising distance of the ground surface which heaved freely, the air temperature and the soil temperatures at several levels below the ground surface were also measured throughout the winter.

Cold room research (1)

Two cylinders of methacrylic acid resin packed with Kitami clay soil were set together in a test box for frost heaving, as shown in Fig. 1 b. Here, the cylinder on the right side was set on the base of a rigid frame, and the cover plate was fixed through a load-cell to the frame, so that the soil was restrained from heaving completely. The force induced on freezing of soil was measured by means of the load-cell.

The other cylinder on the left side was made to heave freely. The rising distance

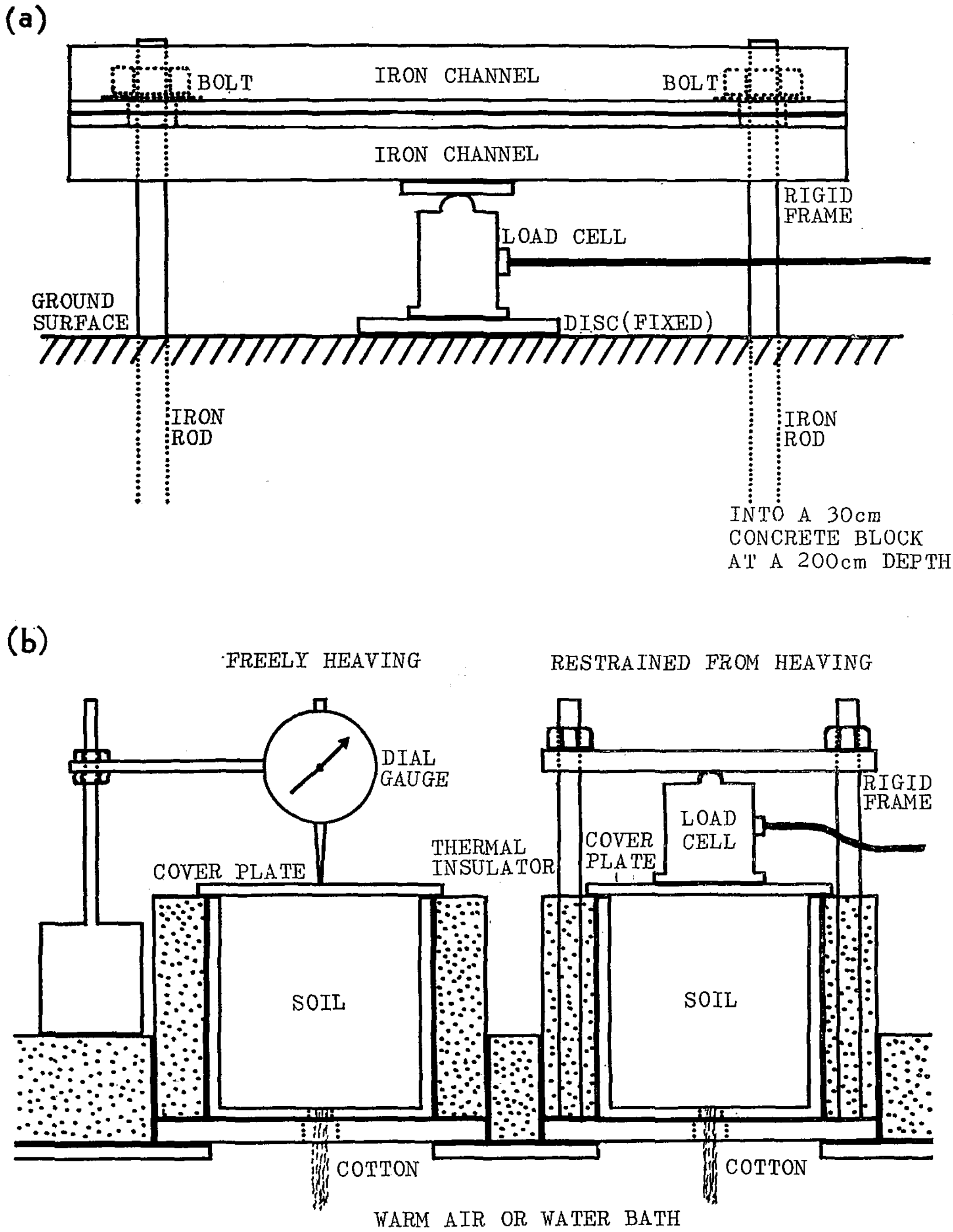

Fig. 1. Experimental apparatus

a: Field research, apparatus for measuring heaving force exerted on the fixed disc (the surroundings heave freely),

b: Cold room research. Left, for measuring the heave amount of the freely-heaving soil cylinder; Right, for measuring the heaving force exerted on the soil cylinder restrained from heaving

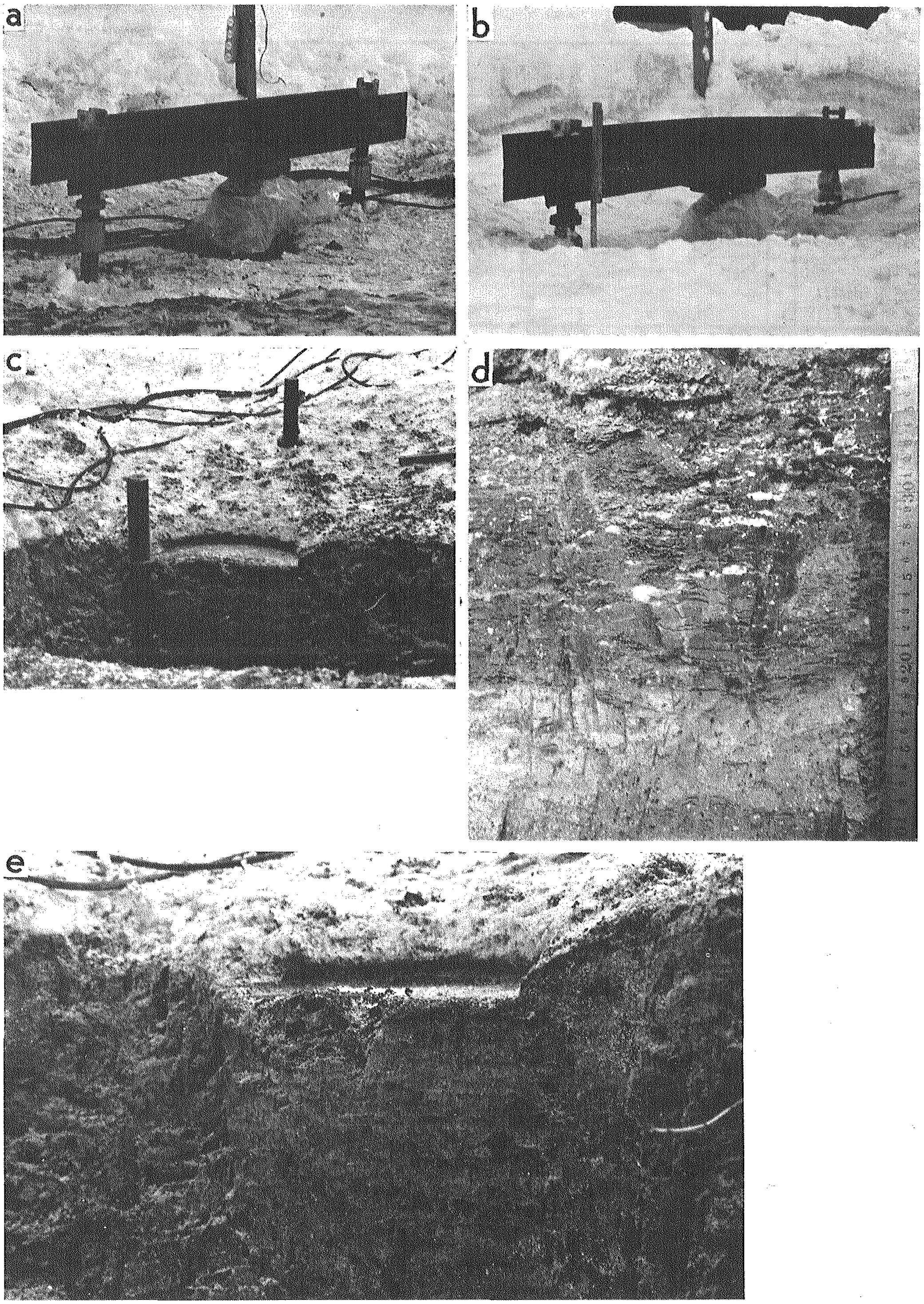

Fig. 2. Photographs of field research

a: Early winter, b: In the depth of winter, c: Vertically cut surface through the fixed disc, d: Vertically cut surface of freely-heaving ground, e: Enlarged one of c of the cover plate was measured by a dial gauge.

The entire surrounding area of both cylinders were enveloped with thermal insulator (polyurethane foam; the dotted portion of Fig. 1 b). The bottoms were exposed to warm air controlled at constant temperature above 0°C. Therefore, the soil samples were cooled from the top downwards.

Sometimes the lower box was filled with water, and bundles of cotton were suspended through the small holes in the centre of the bottom of the cylinders, and immersed in water (open system).

Cold room research (2)

As mentioned later, one of the most characteristic features on the change of the heaving force in the field researches is the decrease at the stop of the heaving advance of the freely-heaving surface. It was assumed that the decrease was due to the stress relaxation of the frozen soil. For the purpose of examining such phenomena the following experiments were conducted in the cold room. After the soil packed with a case (15×15×10 cm) was frozen, a small iron block (1.6 cm in diameter) placed in the centre of the soil surface was pushed into the soil at a constant speed by using an unconfined compression apparatus (Kinosita, 1963). The resistive force necessary for the push con-

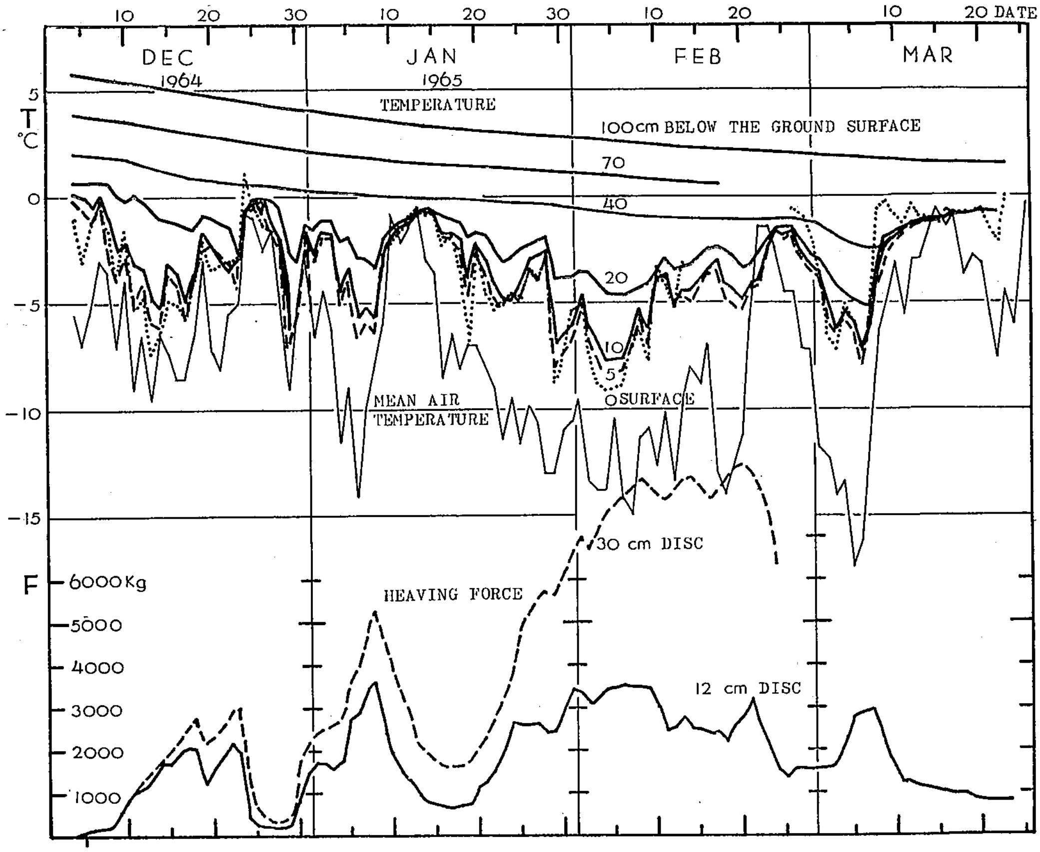

Fig. 3. Results of observation in the field during the winter of 1964–1965

tinued to increase, but when the push was discontinued, the force began to decrease. The change of the force was measured by means of the load-cell on which the sample was placed.

III. Results of Field Researches

The measurements have been successively conducted since the winter of 1961–1962 (Kinosita, 1963, 1966). The results obtained during the winter of 1964–1965 are graphed in Fig. 3. The upper diagrams show the daily mean of the air and soil temperatures of 0, 5, 10, 20, 40, 70 and 100 cm below the ground surface. The soil temperatures at the levels under 40 cm depth continues to go down slowly from the early winter till

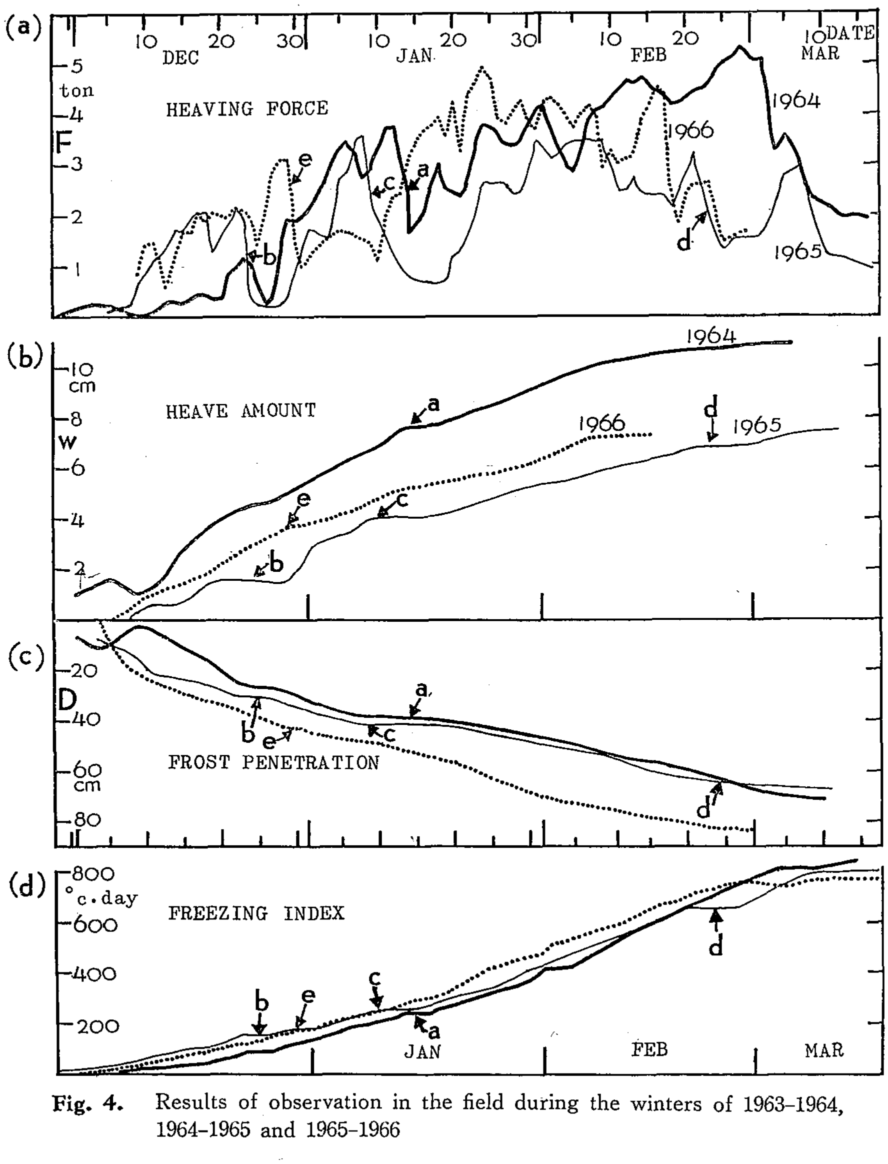

Fig. 4. Results of observation in the field during the winters of 1963–1964, 1964–1965 and 1965–1966

the spring. However, at levels above 40 cm depth they fluctuate, and the amplitude becomes larger, as the level approaches the surface. The lower diagrams of Fig. 3 show the heaving forces. The solid and the broken curves represent respectively the forces for the 12 cm diameter disc and 30 cm diameter disc. They have several cyclic changes which run inversely parallel to the smoothed curve of the soil temperatures near the ground surface. On the other hand, the heave amount of the freely-heaving surface and the frost penetration increase continuously till the end of winter. In order to examine the interrelation between them in more detail, the curves of the change of heaving force, heave amount of freely-heaving surface, frost penetration and freezing index during the winter are plotted in Figs. 4 a, b, c and d respectively for the three seasons of 1963–1964, 1964–1965 and 1965–1966. The curves of heaving force show sharp falls at the middle of January (arrow mark a) and the last part of December b in 1964, at the early part of January c, the last part of February d and the end of December e in 1965, while the curves of heave amount, frost penetration and freezing index during those periods maintain almost constant values. Also, when the heaving force shows a sharp rise or keeps a large value, the other quantities show an advance. It may be said roughly that the heaving force goes nearly in parallel to the heaving rate. These results are suggestive of the existence of a relaxation phenomena which is one of the characteristic mechanical behaviors of visco-elastic material.

The maximum values of heaving force F at each winter are presented in Table 1, together with the heave amount W, the frost penetration D and the freezing index I. These forces contributed directly to the mechanical damages of foundations and buildings.

Table 1

Date		Diameter of disc (cm)	F (kg)	Mean pressure (kg/cm²)	W (cm)	D (cm)	I (deg·day)
1 Mar.	1962	15	5 150	29		54	660
22 Jan.	1963	12	2 600	23	6.0	52	380
27 Feb.	1964	12	5 300	47	10.6	68	750
8 Jan.	1965	12	3 600	32	4.1	42	250
20 Feb.	"	30	8 700	12	7.0	64	650
24 Jan.	1966	12	4 900	43	5.7	62	410
17 Feb.	"	30	9 200	13	7.2	78	670

The heaving force for the 30 cm diameter disc changed almost in parallel to that for the 12 cm diameter disc (Fig. 3). The former was larger only by 20–30% during the first half of the winter, but the difference increased as the winter progressed and in the depth of winter the ratio of both values became equal to 2.5, the ratio of both diameters.

IV. Stress Relaxation within the Frozen Soil

The problem of the visco-elasticity of frozen soil has been studied in recent years (Vyalov, 1963). Here also, it is noted that the quick decrease of the heaving force at the termination of the heave advance of freely-heaving surface is assumed to be due to the stress relaxation which is a characteristic phenomena for a general visco-elastic material.

A schematic figure of this research is pictured in Fig. 5. It shows a vertically cut surface of the frozen ground. The MM plane is the original surface and P is the fixed disc. The LL plane represents the heaving ground surface, leaving the disc at the original level. The NN plane is the freezing front at that time. A further heave amount of the ground surface is brought about by the segregation of the ice layer at the freezing front just under that ground surface. Therefore, the cause of the displacement in this system is made at the NN plane.

In the cold room research as described in section II of this paper, a block of frozen soil was pushed at the centre of the surface by an iron rod of small diameter, in such a way that the bottom was fixed, while the centre of the surface was displaced. When the push was discontinued, the resistive force decreased, that is to say, a stress relaxation took place. In this experiment the cause of force was made at the surface as an external condition, and there were no effect of ice segregation. However, both in the field and in the cold room the developed force was exerted along the entire frozen soil

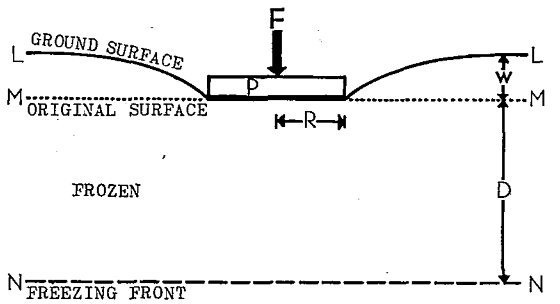

Fig. 5. Schematic diagram of a vertically cut surface of the frozen ground in the field research

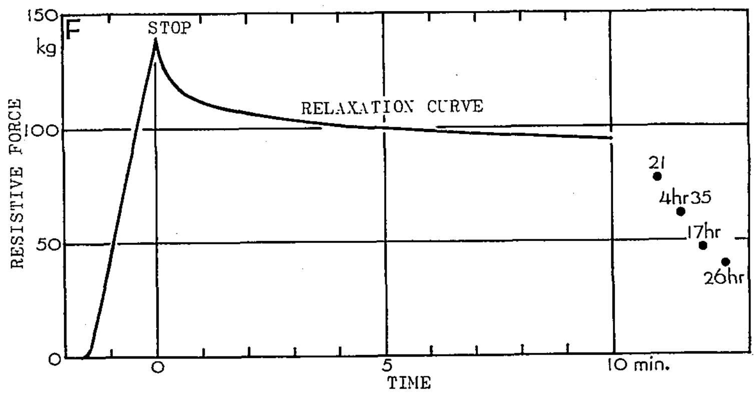

Fig. 6. Resistive force for pushing a small rod (1.6 cm in diameter) into the surface of frozen soil. Pushing speed: 1.3 mm/min, temperature: $-4°C$. After the termination of pushing, relaxation curve was recorded.

mass. In this sense the relaxation curve obtained by this experiment was comparable to the derease of heaving force in the field research.

The curve of Fig. 6 gives the relation of resistive force to time (pushing speed, 1.3 mm/min; temperature, -4°C). When the push was discontinued, the force which had already reached 140 kg began to decrease though the displacement retained the value at the time of stoppage. This is a relaxation phenomena which is characteristic of visco-elastic material. The force fell to 30% of the value at the stoppage in 26 hours.

On the experiments for -8 and -23°C the force fell to respectively 50 and 75% of the value at the stoppage in one day.

In field research a near stop of heaving advance was observed several times as seen in the arrow marks a, b, c, d and e of Fig. 4b. The stress relaxations experienced at these times gave the rates of stress decrease, 52% in a, 24% in b, 67% in c, 80% in d and 47% in e, in one day. Now, since external conditions were not completely stationary in the field, it may be said that these results agree well with the above-mentioned hypothesis for stress relaxation.

From the experimental results the decrease in force with time can be described by

$$F = F_0 (at+1)^{-n}\,, \tag{1}$$

where F_0 is the force at the time of stoppage, and a and n are constants obtained by experiments; for the temperature of -4°C a and n are respectively 35 hour^{-1} and 1/6.

The increase of the resistive force against the pushing distance may be extended to the large scale of the field research. The numerical values of heaving force for 1 mm heave of freely-heaving ground surface in the field (fixed disc of 12 cm in diameter) may be computed from the results of the cold room research (pushing rod of 1.6 cm in diameter) (Table 2).

Table 2. Increase of force for a 1 mm push or heave

Temperature (°C)	Cold room observed (kg)	Field		Date
		Computed (kg)	Observed (kg)	
−4	70	930	450	last part of Jan. 1965
−8	170	2 300		
−23	220	2 900		

The values observed in the field were smaller than those computed from the cold room experiment. This difference was due to the speed of displacement; in the field the heaving rate was at most several mm/day, while in the cold room experiment it was much larger, at a rate of several mm/min.

Assuming the relation between resistive force $F(t)$ and heave amount $W(t)$ of freely-heaving surface to be a visco-elastic one, the following equation may hold.

$$F(t) = C\,w(t) - \int_0^t \mathrm{f}\,(t-t')\,C\,w(t')\,\mathrm{d}t'\,. \tag{2}$$

Here C is constant, and f is the so-called memory function. When $w(t)$ maintains constant value, $F(t)$ decreases as eq. (1), so that $\mathrm{f}(x)$ can be given by

$$f(x) = -an\,(ax+1)^{-n-1}\,. \tag{3}$$

From eqs. (2) and (3) resistive force $F(t)$ changes with time for the constant rate of heaving ($w(t)=bt$, where b is constant) as follows

$$F(t) = \frac{Cb}{a\,(1-n)}\left\{1-\frac{at\,(2n-1)+1}{(at+1)^n}\right\}. \tag{4}$$

It should be explained from eq. (4) that F becomes smaller as the heaving rate b becomes larger.

The above considerations were made for the external appearance of the visco-elastic phenomena of frozen soil. It becomes a subject of study as to how the relaxation phenomena is conducted within the internal structure. Owing to the Wakahama's theory (1962) the stress relaxation of ice takes place following the microscopic movements within the ice, that is, the dislocations in the basal plane of the ice crystal. Since ice is the most plastic material among the constituent elements of frozen soil, it should be expected that the stress relaxation in the frozen soil has the same rate as in the ice. However, the rate for frozen soil is smaller than for ice: the relaxation time taken till the stress decreases to 1/e of that at stoppage is one day or so in frozen soil (−4, −8°C), while it is ten minutes or so for polycrystalline ice (−10°C). As a reason for the slow rate of stress relaxation in frozen soil it is assumed that the movements of dislocations along the basal plane of ice crystal are restricted around the boundary between ice and soil particle.

On the other hand the existence of unfrozen water in frozen soils has been documented in previous studies. Such unfrozen water exists around the soil particles as thin layers and owing to its mobility water molecules are transfered through the layer to the freezing front from the soil or the ground water table, forming an ice lens. Such a mobile layer should play an important role for stress relaxation of frozen soil. The content of this unfrozen water decreases as the temperature falls (Williams, 1962; Yong, 1962), so that the thickness of the mobile layer becomes thinner. Therefore, the restriction against the movements of the dislocations within the ice might increase as the temperature falls. This agrees well with the above described expemeintal result in which the rate of stress relaxation decreases with temperature.

V. Pressure Developed at the Freezing Front

In field research it is very difficult to give a precise analysis on the stress distribution at the freezing front. Here, let us consider a semi-infinite elastic solid where its boundary is compressed by a rigid disc: the freezing front may be assumed to form one plane at that depth within the semi-infinite body although there is discontinuity across the freezing front in actuality. The distributions of stress are taken to be equal to those in an elastic body.

The normal stress σ_z (z is directed downwards) is given by

$$(\sigma_z)_{z=0} = \frac{F}{2\pi R\sqrt{R^2-r^2}}\,, \tag{5}$$

$$(\sigma_z)_{r=0} = \frac{F}{2\pi R^2} \frac{1+3\left(\frac{Z}{R}\right)^2}{\left\{1+\left(\frac{Z}{R}\right)^2\right\}^2} . \tag{6}$$

where R is the radius of the fixed disc and r is taken radially from the centre and F is the resistive force exerted on the disc.

The normal stress in the vertical direction across the freezing front just under the centre of the disc where displacement does not take place is computed from the above eq. (6). The maximum value of that normal stress amounted to 1.4 kg/cm^2 at February 27th of 1964.

The increase of the resistive force, dF, is related to the increase of the heave amount, dw, at the freely-heaving ground surface at a considerable distance from the disc by the following equation.

$$dF = \frac{2E}{1-\nu^2} R dw , \tag{7}$$

where E and ν are respectively Young's modulus and Poisson's ratio. dF is proportioned to the radius R of the fixed disc. Therefore it should be expected that the heaving force for the 30 cm diameter disc is larger by 2.5 times than that for the 12 cm diameter disc. Actually this relation became settled on February as seen in Fig. 2, while during the first half of the winter it failed; then the force for the larger disc was only slightly larger than that for the smaller one.

When the frost penetration does not reach a depth corresponding to a distance larger than the diameter of the disc, the above approximation should fail, and the force should be smaller.

VI. Cold Room Research

As above-stated in section II, both the heave amount of freely-heaving soil sample and the pressure exerted on the soil sample which was restrained from heaving were measured in the cold room. Both soil samples (Kitami clay) was packed into cylinders of methacrylic acid resin of 10 cm in diameter and 11 cm in length. The former corresponds to the heave amount of the freely-heaving ground surface distant from the fixed disc in the field research, and the latter to the pressure at the freezing front just under the centre of the disc; it is $(\sigma_z)_{r=0}$ at the freezing front.

The density of the used soil cylinder was 1.8 g/cm^3, and the real density of the soil was 2.4 g/cm^3. Since the original water content was 35%, the sample was saturated.

The experimental restults are graphed in Figs. 7 a and b. The upper diagrams of both graphs show the change of the heaving pressure p and the heave amount w with time, and the lower the frost penetration D. The experiments were carried out in a closed or open system. On both systems p and D increased with time, rather rapidly at first and then slowly, and approached maximum value in a short time. However, w increased at a constant rate in an open system, while it changed in the same way as p and D in a closed system.

This experiment suggests that if the frozen soil which can heave by w is restrained

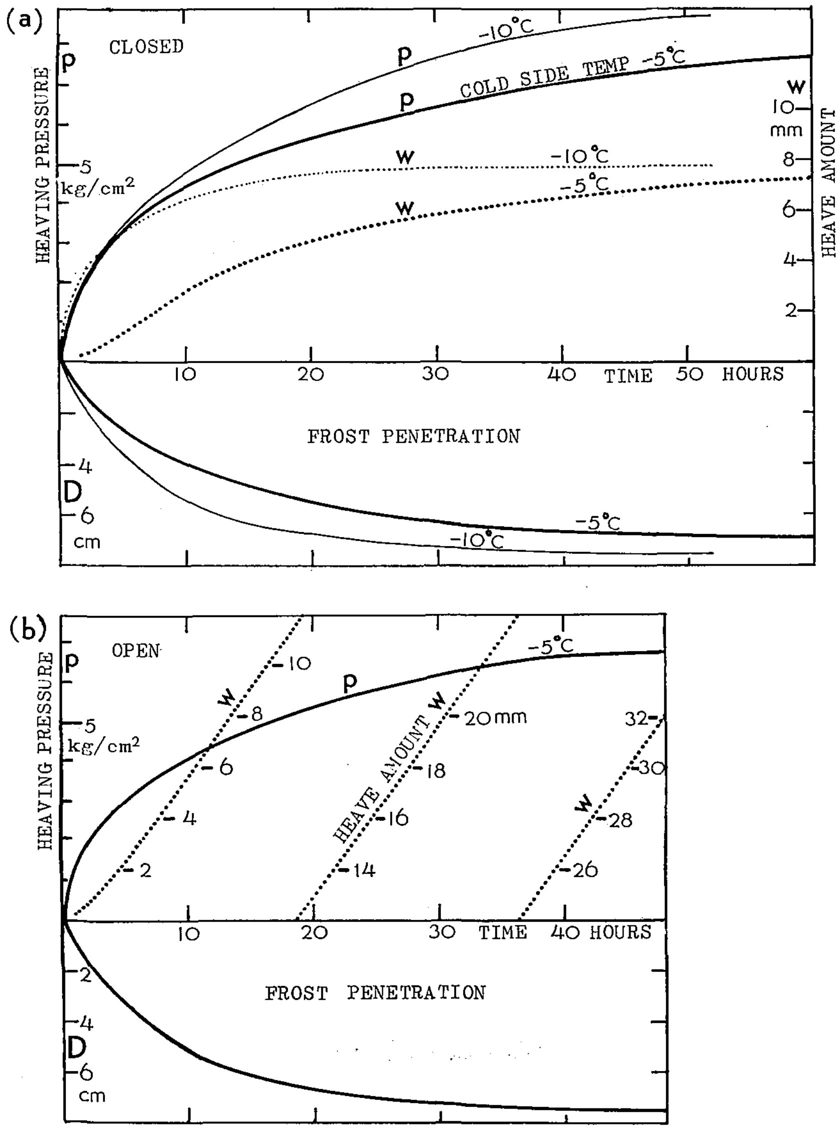

Fig. 7. Results of observations in the cold room research

p: Heaving pressure exerted on the frozen soil restrained from heaving,
W: Heave amount of freely-heaving soil

completely from heaving, it would suffer the resistive pressure *p*. Also, if *w* could be reduced by a certain amount, *p* should decrease to some degree. From this point of view *p* may be said to have maximum heaving pressure. The maximum value of *p* amounts to 7.5 kg/cm^2 for -5°C (both closed and open systems) and 9.0 kg/cm^2 for -10°C. These values are larger than those obtained by Penner (1959), and Hoekstra, Chamberlain and Frate (1965), 1–2 kg/cm^2. It is supposed that the difference is due to the size of the effective pore of the used soil sample, as will be explained in the next section.

The computed increase dp of the pressure at the freezing front in the period of the sharp rise of heaving force F shall be compared with the increase dp in this experiment for the same increase dw of heave amount of freely-heaving soil surface (Table 3).

The values in the field are smaller than those in the experiments. Also, the maximum value of the pressure at the freezing front in the field is a small value, 1.4 kg/cm^2. It is assumed that the difference is due to the rate; the heaving force becomes larger with rate owing to the visco-elasticity of frozen soil as above-described in section IV. In the cold room the heaving rate is 4–10 mm/day, while in the field it is 1 mm/day or so.

Table 3

Field					
Date	dF (kg)	dp (kg·cm^2)	dw (mm)	Elapsed time (days)	Frost penetration (cm)
19–23 Jan. 1965	1 800	0.9	4	4.3	42–45
10–16 Jan. 1966	2 600	0.9	5	6	49–54

Experiment					
Type	Temperature (°C)	dp (kg/cm^2)	dw (mm)	Elapsed time (hours)	Frost penetration (cm)
Closed	−5	5.0	4	15	4.9
		5.8	5	22	5.8
Open	−5	3.3	4	7.6	4.3
		3.8	5	9	4.9

VII. Some Considerations on Maximum Heaving Pressure

It seems that in the above described cold room research the maximum heaving pressure p in a closed system can be estimated from the heave amount w of the freely-heaving soil by using a simple method such as a calculation of thermal stress. Considering that an expansion $\frac{w}{D}$ induced by frost heaving is entirely suppressed by applying the compressive stress p to the soil, p may be expressed by

$$p=\frac{w}{D}E, \tag{8}$$

where E is Young's modulus. From the results of pushing the frozen soil (section IV) E is obtained as 440 kg/cm^2, assuming ν^2 as 0 in eq. (7). In the curve of Fig. 6a the value of w for $D=7$ cm is 7.2 mm, so that $\frac{w}{D}E$ is 45 kg/cm^2, while the observed pressure is 7.5 kg/cm^2. Of course, the expansion in this case is not an elastic one. Therefore, the difference is large. However, it may be said that this hypothesis should give a rough estimation in practice.

The measurements of maximum heaving pressure have been conducted recently by several researchers (Penner, 1959, 1966; Hoekstra, Chamberlain and Frate, 1965). Also, Everett (1966) predicted a formula from a thermodynamical theory: heaving pressure p

is related to the radius r of effective pore and the ice-water interfacial energy σ_{iw} by

$$p = \frac{2\sigma_{iw}}{r}. \tag{9}$$

Penner (1966) examined this formula and derived σ_{iw} as 35 erg/cm^2. Ôura (1951) calculated the value of σ_{iw} as 18.5 erg/cm^2 theoretically, and McKyes (1966) obtained the value of 20 erg/cm^2.

Using these values as σ_{iw}, the effective pore radius r amounts to 0.05–0.09 μ in this experiment for Kitami clay. It is small as compared with the value 0.5–0.6 μ obtained by Penner (1966).

As seen in the photograph of Fig. 8 taken from ice lens in frozen soil there are many columns of air bubbles (Kinosita, 1966). Also, Maeno (1966) observed the process of the growth of such air bubbles. Therefore the possibility of the sandwiching of air bubbles between ice and thin water film around the soil particle at freezing front may be assumed. In such a case σ_{iw} of eq. (9) must be rewritten to ice-air interfacial energy σ_{ia} or water-air interfacial energy σ_{wa}, where p becomes lager with the smaller effective pore radius.

It has been suggested that a thin liquid like mobile layer exists between ice and soil particles, and through this layer water molecules transfer upwards from the region of unfrozen soil, reaching the freezing front to form an ice lens. In this experiment such a layer maintains the heaving pressure as large as several kg/cm^2. Its thickness is conjectured to be in the order of 8–50 Å, that is, it is a layer of 3–20 molecules.

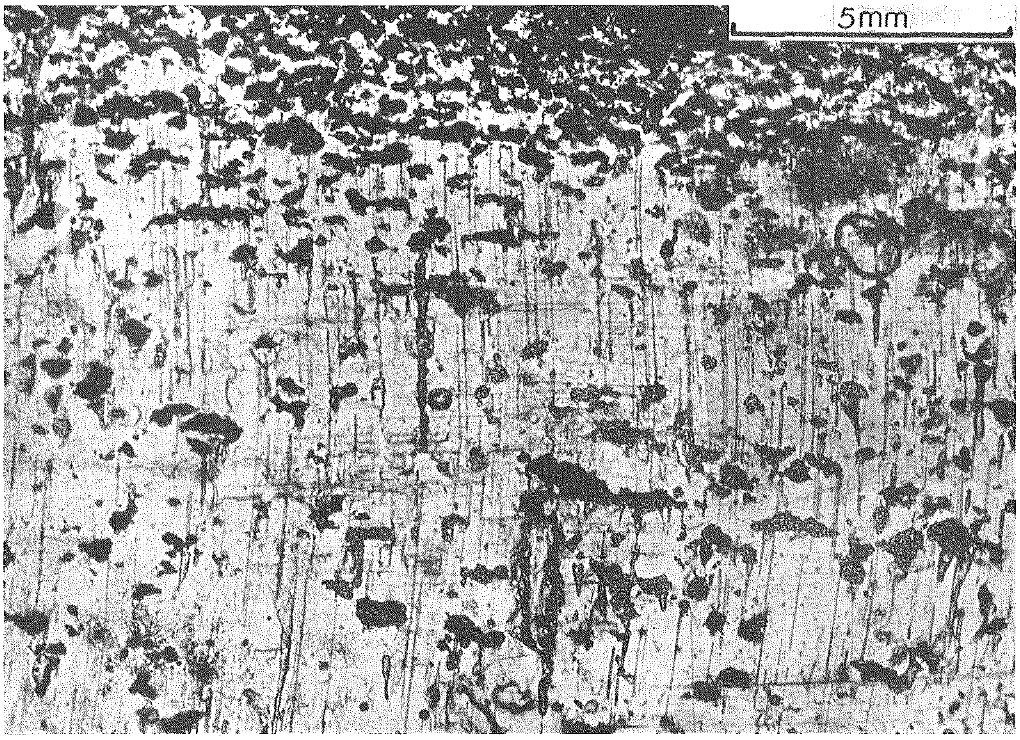

Fig. 8. Thin section of ice lens developed near the ground surface

Since the heaving rate is about several μ/sec, water molecules must move through the mobile layer at a rate of the same order. From the theory of Uhlmann and Jackson (1966) it has been suggested to be possible. Now one interesting experiment has been recently carried out by Nakamura (1966). He observed the existence of a water film sandwiched between ice and a glass plate under large nonhydrostatic pressure as large as 40 kg/cm^2. The thickness was that in the order of visible size under microscope. Maeno (1966) also observed the existence of a mobile layer between ice and air bubble in the experiment of air bubble formation in ice crystals.

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