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Frost Heave in Soils

The Influence of Particles on Solidification

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Abstract

Frost heave in soils is a complex solidification phenomenon, which involves the motion of water through the soil to the freezing front. The rate of heave depends on the rate of extraction of heat from the soil and on the ease of motion of water through the soil. It also depends on the details of the interaction between the ice-water interface and soil particles, which leads to the formation of ice lenses in the soil. In the present paper, we combine the concepts used by Jackson and Chalmers to discuss the formation of ice lenses with the concepts derived by Uhlmann, Jackson and Chalmers from a study of the interaction between particles and a solid-liquid interface. This permits quantitative predictions of heave rate for various soils which agree well with available experimental data.

I. Introduction

The freezing of a soil can, under certain conditions, result in a heaving of the soil surface. This complex phenomenon, appropriately termed "frost heave", is the principal subject of discussion at the present session of the Conference. Since the frost heave process represents an interesting solidification problem, we have attempted to apply the knowledge gained in other solidification studies to the situation in soils. Some results of this approach have appeared previously (Jackson and Chalmers, 1958; Uhlmann, Chalmers and Jackson, 1964; Jackson, Uhlmann and Chalmers, 1966; Jackson and Uhlmann, 1966). In the present paper we propose to effect a detailed integration of this work, placing specific emphasis on the frost heave phenomenon, and discussing at length the method adopted in our theoretical calculations.

Let us recognize at the outset that there are two alternative ways in which soil water can transform to ice. It can freeze *in situ*, around the soil particles, or it can form regions of ice almost devoid of particles. Such regions have been termed ice lenses; and their formation is recognized as a necessary condition for the occurrence of frost heave.

Jackson and Chalmers developed a theory (Jackson and Chalmers, 1956; Jackson and Chalmers, 1958) in which the critical factor in the frost heave process was identified

as the difficulty of an ice-water interface in propagating through the small channels between soil particles.

In order to propagate through such a channel of radius R_c , the interface must bend to a radius equal to or smaller than R_c (assuming zero contact angle between the ice-water and ice-particle surfaces). Propagation through the channel will not be possible if the undercooling at the interface is less than the value given by the Gibbs-Thompson relation, namely,

$$\Delta T = \frac{2\sigma T_E}{LR_c} \quad (1)$$

Here σ is the specific surface free energy of the ice-water interface, T_E is the equilibrium temperature for a flat interface, and L is the latent heat of fusion.

When the undercooling at the interface is insufficient to permit propagation of ice through the channels, freezing can continue only if water moves upward through the channels to the surface of the ice. This leads to growth of the ice to form a lens, the necessary water coming from the water table or from the pores of nearby soil.

The transport of water to the interface region is driven by a pressure gradient between the interface region and the ground water table (or nearby soil) which results from the undercooling present at the interface. This pressure gradient increases with increasing undercooling.

While the Jackson-Chalmers approach is useful for understanding many of the phenomena associated with frost heave, it does not treat the interaction between soil particles and the ice-water interface. This interaction was recently studied by Uhlmann, Chalmers and Jackson (Uhlmann, Chalmers and Jackson, 1964) for the case of isolated particles at a solid-liquid interface. The study was carried out with organic matrix materials (for example, salol and orthoterphenyl) as well as with the ice-water system.

In that investigation, particles were observed, at sufficiently low growth rates, to be rejected at and pushed along with the interface as it advances into the liquid. Not only isolated particles, but also quite sizable pile-ups were seen to be pushed by an interface.

As the growth was increased, a speed was reached for each type of particle at which the particles were no longer pushed by the interface, but were trapped in the solid. This speed was designated as the "critical velocity".

The critical velocity for a given type of particle was found to be larger for smaller particles in the range of particle sizes above a hundred microns in diameter, but to be independent of particle size for micron-size particles. The critical velocity was likewise found to depend upon the shape and chemical composition of the particles.

With organic matrix materials, particles bearing charges varying in magnitude by a factor of 10 and of both signs were all trapped at the same growth rate. These results indicate that electrical effects may be disregarded as the limiting factor in the interface behavior of soil particles. Such effects might nevertheless play an important role in frost heave by modifying the size of the effective channel for liquid transfer to the interface.

The rejection of particles by a solid-liquid interface was attributed to a very short

range repulsion between the particles and the growing solid. Such repulsion occurs when the energy per unit area of a particle—ice boundary is larger than the sum of the energies of particle-water and water-ice boundaries—that is, when $\Delta\sigma_0 = \sigma_{sp} - (\sigma_{sL} + \sigma_{LP})$ is positive, where σ_{sp} , σ_{sL} and σ_{LP} are, respectively the solid-particle, solid-liquid, and liquid-particle specific surface free energies. In such circumstances, the system will attempt to maintain a layer of water between particle and solid, and as long as there is sufficient time for such a layer to be maintained by the transport of additional water behind the particle, pushing will take place.

Applying these concepts to the soil situation, we anticipate that when particles can be accommodated in the ice only at the cost of extra energy, they will be rejected by the interface when the freezing rate is sufficiently low that the region between particle and ice can be fed by the transport of water from the soil or ground water table. In such circumstances, an ice lens will form. At rates of growth higher than some critical value, the transport process becomes too difficult and freezing *in situ* takes place.

We have, then, two different limiting conditions for the formation of ice lenses, either one of which could be decisive in a particular case. These may be called the propagation-into-channel and the particle-trapping limits, respectively.

Before considering in detail these limits, and developing a formalism which permits quantitative prediction of frost heave rates, it seems appropriate to consider briefly the heat flow and fluid transport problems appropriate to the soil situation.

II. Heat Flow

The amount of undercooling at the interface is determined by the flux of heat through the soil and by the freezing of water in the soil. The heat conducted away through the frozen soil comes from three sources: the heat conducted to the ice lens from the soil below, latent heat from soil water freezing *in situ*, and latent heat from frost heave. The heat balance at the freezing front can then be expressed:

$$H_F = H_U + f_w \rho_I v_f L + \rho_I v L. \quad (2)$$

Here

$$H_F = k_F \left(\frac{dT}{dx} \right)_{x=l^-}; \quad H_U = k_U \left(\frac{dT}{dx} \right)_{x=l^+},$$

where k_F and k_U are the thermal conductivities of frozen and unfrozen soil, and the temperature gradients are evaluated just above and just below the freezing front; f_w is the volume fraction of soil which is water; ρ_I the density of ice; v_f the rate of advance of the freezing front; and v the rate of heave.

It will be shown below that there is a maximum rate, v_c , at which a given soil can heave. If $H_F - H_U$ is greater than $\rho_I v_c L$, then the frost line will advance through the soil. The heave rate will be v_c independent of thermal conditions, and v_f , the rate of advance of the frost line, will depend on heat flow as given by eq. (2). If, however, $H_F - H_U$ is less than $\rho_I v_c L$, the interface will not be able to advance into the soil, the frost line will remain stationary relative to the unfrozen soil, v_f will be zero, and the rate of heave, v , will depend on heat flow.

III. Fluid Transport in Soils

Frost heave may occur either by drawing water from the ground water table, or by depleting the unfrozen soil of its water. In either case, the flow of water to the ice lens occurs because of a tension developed in the water by the freezing process. The water content of the soil depends on this tension; the flow of water through the soil is therefore described by the diffusion equation:

$$D\nabla^2 P = \frac{\partial P}{\partial t}, \quad (3)$$

where P is the pressure in the soil, $D=KC$ cm²/sec; K is the permeability of the soil and C is the change in pressure per unit change in water content of the soil. While recognizing that the value of D depends upon the water content of the soil, we shall approximate the conditions during frost heave by solutions obtained for the case of constant D . Consider the steady state in a coordinate system fixed on the freezing front. For unidirectional freezing the diffusion equation becomes:

$$D \frac{d^2 P}{dx^2} + V_f \frac{dP}{dx} = 0. \quad (4)$$

This has the solution:

$$P = A + B \exp(-V_f x/D), \quad (5)$$

where x is measured from the freezing front.

Applying the boundary conditions $P = -P_0$ at $x=0$ and $P=0$ at $X=l$ where l is the distance to the ground water table, we have:

$$P = -P_0 \frac{\exp\left[\frac{V_f}{D}(l-x)\right] - 1}{\exp\left[\frac{V_f}{D}l\right] - 1}. \quad (6)$$

The flux of water to the interface may be written

$$J = K \left(\frac{dP}{dx} \right)_{x=0}.$$

For the general case this is:

$$J = -\frac{P_0 V_f}{C} \frac{1}{1 - \exp\left[-\frac{V_f}{D}l\right]}. \quad (7)$$

These expressions may be simplified for the two limiting cases:

Case I: Ground water table far away from freezing front ($l \gg \frac{D}{v_f}$). Then

$$P = -P_0 \exp\left[-\frac{V_f}{D}x\right],$$

and

$$J = \frac{P_0 V_f}{C}. \quad (8)$$

Here all the water is drawn from the soil. The flow of water to the interface does

not depend upon the distance to the water table, but upon the capacity of the soil to hold water.

Case II: Ground water table close to freezing front ($l \ll \frac{D}{v_f}$). Then

$$P = -P_0 \frac{l-x}{l},$$

and

$$J = \frac{KP_0}{l}. \tag{9}$$

Here the pressure gradient is linear. The flow of water comes from the water table and depends on the permeability of the soil and the distance to the water table.

IV. Theory

For frost heave to occur, two conditions must be satisfied. The ice must not grow into the channels between the soil particles, and a water layer must be maintained between the particle and the ice lens. For an isolated particle at a solid-liquid interface, only the second of these conditions must be met.

a) Isolated particle at an interface

For this case, let us consider the region of "contact" between particle and interface (Fig. 1). From the conservation of mass:

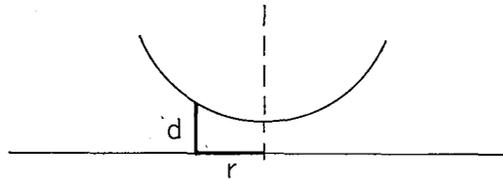


Fig. 1. Diagram for analysis of diffusion process

$$\frac{d_1 D 2\pi r_1}{kT V_0} \left(\frac{d\mu}{dr} \right)_1 - \frac{d_2 D 2\pi r_2}{kT V_0} \left(\frac{d\mu}{dr} \right)_2 = 2\pi r dr \frac{v}{V_0}, \tag{10}$$

or

$$\frac{1}{r} \frac{d}{dr} \left(r d \frac{d\mu}{dr} \right) = \frac{v k T}{D}. \tag{11}$$

Here d is the separation between particle and solid; D the diffusion coefficient for matrix liquid in the region between particle and solid; k Boltzman's constant; T the temperature; V_0 the atomic volume; v the growth rate; and μ the chemical potential of liquid in the region of contact.

Assuming the variation in surface free energy with particle-solid separation (Fig. 2) can be described by a power law of the form

$$\Delta\sigma = \Delta\sigma_0 \left(\frac{d_0}{d} \right)^n. \tag{12}$$

The chemical potential may be expressed:

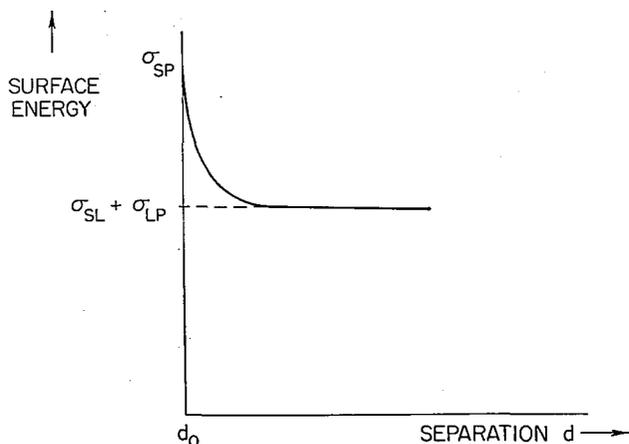


Fig. 2. Surface free energy vs. particle-solid separation (schematic)

$$\mu = -\frac{V_0}{d} \left(\frac{d_0}{d}\right)^n \Delta\sigma_0. \quad (13)$$

Here d_0 is the separation between atom centers across a particle-solid interface, and n is a positive number, of the order 6 or 12.

With this expression for the chemical potential, eq. (11) becomes:

$$\frac{1}{r} \frac{d}{dr} \left(\frac{r(n+1)V_0\Delta\sigma_0 d_0^n}{d^{n+1}} \frac{dd}{dr} \right) = \frac{vkT}{D}. \quad (14)$$

This has the solution:

$$\frac{1}{d^n} = a - b \ln r - \frac{1}{4} \frac{n}{n+1} \frac{vkTr^2}{DV_0\Delta\sigma_0 d_0^n}. \quad (15)$$

The separation cannot vanish at $r=0$; therefore, $b=0$. Designating the separation at

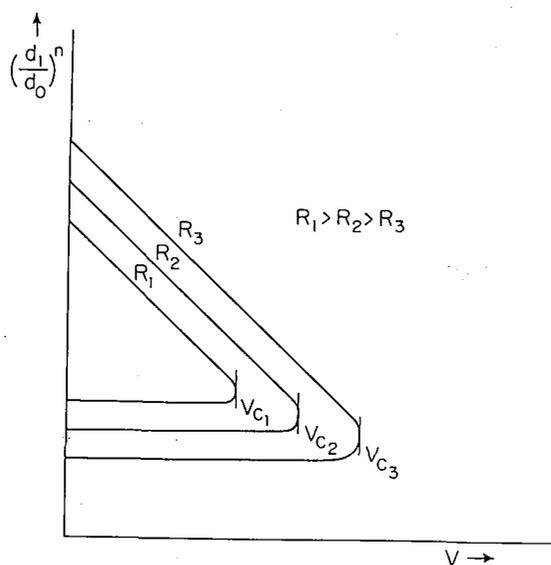


Fig. 3. Particle-solid separation vs. growth rate

$r=0$ as d_1 and the radius r_0 , at which the separation becomes infinite (beyond which interaction between particle and solid is not required to drive the material transport process) as the "effective contact radius", where

$$r_0^2 = 4 \frac{n+1}{n} \frac{DV_0 \Delta \sigma_0}{vkT} \left(\frac{d_0}{d_1} \right)^n \quad (16)$$

we have:

$$\frac{1}{d^n} = \frac{1}{d_1^n} \frac{r_0^2 - r^2}{r_0^2} \quad (17)$$

This d , we may recall, represents the separation between particle and solid which is required to drive the material transport process behind the particle, for the assumed variation in surface free energy with separation. Corresponding to this separation expression is a separation curvature $\partial^2 d / \partial r^2$, which evaluated at $r=0$ becomes

$$\frac{1}{\rho_s} = \frac{2d_1}{nr_0^2} \quad (18)$$

The curvature of the particle irregularity* is $1/R$; hence, the net curvature of the interface is $(1/R) - 1/\rho_s$. The equilibrium condition is expected to maintain when this net interface curvature compensates for the local change in chemical potential due to the presence of the particle. Curvature $1/\rho$ corresponds by the Gibbs-Thompson relation to a change in melting temperature $(a_0/\rho) T_E$, or a change in free energy $L(a_0/\rho)$. This relation defines a_0 , which generally is of the order of a molecular diameter. We may then express the equilibrium condition as

$$L_{AT} a_0 \left(\frac{1}{R} - \frac{1}{\rho_s} \right) - V_0 \Delta \sigma_0 \frac{d_0^n}{d_1^{n+1}} = 0 \quad (19)$$

Here L_{AT} is the latent heat of fusion per atom.

Writing the condition for a unit volume, and expressing ρ_s (from eq. (18)) and $\Delta \sigma_0$ (from eq. (16)) in terms of r_0 , we have:

$$L a_0 \left[\frac{1}{R} - \frac{2d_1}{nr_0^2} \right] - \frac{1}{4} \frac{n}{n+1} \frac{vkT}{V_0 D d_1} r_0^2 = 0 \quad (20)$$

Here L is the latent heat per unit volume.

The growth rate as a function of separation for different values of R is shown schematically in Fig. 3, using the relation of eq. (20). To determine the critical velocity, which corresponds to the largest growth rate obtainable for a given R , let us maximize v with respect to r_0^2 . Since r_0^2 depends on the particle-solid separation, the maximum corresponds to an instability in the pushing configuration.

The upper branch in each case corresponds to the interface remaining fairly flat behind the particle for low growth rates, and curving more to the particle periphery as the growth rate is increased. On this branch, the pushing configuration is stable against fluctuations in particle-interface separation; the chemical potential of the system decreases as the separation decreases, and the particle rides closer to the interface for

* The average R was determined from electron microscope studies of the particles to be approximately 300 Å. For particles without irregularities, this R should be taken as R_0 , the particle radius.

higher growth rates, and farther away for lower ones.

The lower branch corresponds to the interface curved around the particle irregularity. On this branch, the pushing configuration is unstable against fluctuations in particle-interface separation; the chemical potential of the system decreases as the separation decreases, and the particle-interface separation is approximately independent of growth rate.

The critical velocity, then, is the largest growth rate for which there is a stable pushing configuration. Carrying out the indicated maximization, one obtains:

$$V_c = \frac{1}{2} (n+1) \frac{La_0 V_0 D}{kTR^2}. \quad (21)$$

b) *The particle-interface interaction in a soil*

In deriving eq. (20), it was assumed that the interface far from the particle was planar. When the interface propagates between the soil particles, however, it must bend to a radius of curvature equal to the channel radius, R_c . At the maximum heave rate, therefore, the interface between the particles will have a radius slightly larger than R_c . In our calculations we will assume it is R_c . The curved interface in the channels establishes the reference undercooling at the interface, and therefore a term La_0/R_c should be added to eq. (20).

When a soil heaves, the water to form the ice lenses must be drawn from the surrounding soil or from the ground water table, as described by eqs. (18) and (19). This can be taken into account by adding a term $-FvA/r_0^2$ to eq. (20), and where from eqs. (18) and (19):

$$F = \frac{l}{K}, \quad (22 a)$$

when the ground water table is close to the freezing front; and

$$F = \frac{C}{v_t}, \quad (22 b)$$

when the water table is far away from the freezing front.

The frozen soil and any other surcharge above the freezing front must be lifted by the freezing process as heave occurs. This adds a term $-P_L A/r_0^2$ to eq. (20), where P_L is the total surcharge.

The factor A/r_0^2 which appears in these last two terms arises because the pressure which must be generated to produce water flow and the pressure from the surcharge are exerted on the whole cross section of the soil. These forces must be balanced by a repulsive force across the liquid layer separating the ice lens from the soil particles, that is, only in the region of contact of radius r_0 . The factor A/r_0^2 is defined as the ratio of the area of interface per particle to the contact area particle. For most soils, A can be approximated by R_0^2 , where R_0 is the radius of the average soil particle.

For an ice lens in a soil, then, the chemical potential balance becomes:

$$La_0 \left[\frac{1}{R} - \frac{2d_1}{nr_0^2} \right] - \frac{1}{4} \frac{n}{n+1} \frac{vkT}{V_0 D d_1} r_0^2 + \frac{La_0}{R_c} - \frac{FvA}{r_0^2} - \frac{P_L A}{r_0^2} = 0. \quad (23)$$

The highest rate of heave corresponds to the highest rate at which the freezing

front can advance without incorporating the particles or propagating between them. The highest rate may be termed the critical velocity, and represents an instability in the ice lens configuration.

To determine the critical velocity, we proceed as before. Defining $\frac{1}{R} = \frac{1}{R} + \frac{1}{R_c}$, and maximizing v with respect to r_0^2 , we obtain:

$$r_0^2 = \frac{La_0}{R} \frac{d_1 V_0 D 2(n+1)}{kTnV}. \tag{24}$$

Substituting this expression into eq. (23) and solving for v , we have:

$$V_c = \frac{1}{2F} \left(P_L + \frac{2La_0 d_1}{nA} \right) [-1 + (1+r)^{1/2}]. \tag{25}$$

where

$$r = \left(\frac{La_0}{R} \right)^2 \frac{4d_1 V_0 D(n+1) F}{nkTA \left(P_L + \frac{2La_0 d_1}{nA} \right)^2}. \tag{26}$$

Two limiting cases may be identified:

(a) **Case I:** $r \ll 1$ (limitation from trapping of particles):

$$V_c = \left(\frac{La_0}{R} \right)^2 \frac{d_1 V_0 D(n+1)}{nkTA \left(P_L + \frac{2La_0 d_1}{nA} \right)}, \tag{27}$$

(b) **Case II:** $r \gg 1$ (limitation from propagation into channels):

$$V_c = \left(\frac{La_0}{R} \right) \left[\frac{d_1 V_0 D(n+1)}{nkTAF} \right]^{1/2}. \tag{28}$$

As we shall see, Case II will be appropriate for soils which heave appreciably (small particle and channel size, large F); while Case I is the appropriate description for soils which generally do not have significantly (large particle and channel size, small F).

For comparison with the experimental results of Beskow (1947), who determined the relation between heave rate and the sum of surcharge and suction pressure, it will be useful to rewrite eq. (26) in terms of the suction pressure $P_s (=Fv)$. In these terms, our chemical potential balance then becomes:

$$La_0 \left[\frac{1}{R} - \frac{2d_1}{nr_0^2} \right] - \frac{1}{4} \frac{n}{n+1} \frac{vkT}{V_0 D d_1} r_0^2 + \frac{La_0}{R_c} - (P_s + P_L) \frac{A}{r_0^2} = 0, \tag{29}$$

and the maximum heave rate is found to vary inversely as the first power of the pressure, in the range where such pressure is significant, as:

$$V_c = \left(\frac{La_0}{R} \right)^2 \frac{d_1 V_0 D(n+1)}{nkT \left[\frac{2La_0 d_1}{n} + (P_s + P_L) A \right]}, \tag{30}$$

or:

$$P = P_s + P_L = \frac{n+1}{n} \left(\frac{La_0}{R} \right)^2 \frac{d_1 V_0 D}{AkT} \left[\frac{1}{V_c} - \frac{2\bar{R}^2 kT}{(n+1) La_0 V_0 D} \right]. \tag{31}$$

V. Comparison with Experiment and Discussion

The above results were derived for the case of a well defined, rather idealized soil. In extending them to enhance a more complex soil situation, as in comparing them with experimental data (U. S. Army Corps of Engineers, 1958; Beskow, 1947), certain ambiguities arise with regard to the parameters used to characterize the soil. Notable examples are the values to be assigned for R_0 and R_c . Further, the water content versus suction pressure relation as well as permeability data are often unavailable for specific soils of interest. Also, the shape of soil particles on a submicroscopic scale is generally not known.

While recognizing some of the difficulties involved, let us utilize a number of approximations which permit a relatively simple and generally satisfactory prediction of the maximum heave rate. In particular, we shall ignore the state of compaction of the soil, save as it affects the permeability, and will take R_0 equal to the radius of the 50% particle size and R_c equal to 1/10 the radius of the 5% particle size on the gradation curve for the specific soil. Further, we shall take R as about 300 Å, the size of irregularities observed on particles in the previous study (Uhlmann, Chalmers and Jackson, 1964). Since most soil particles are expected to have irregular peripheries on the scale of a few hundred Å, this should be a reasonable assumption in most cases.

For most physical situations, $2La_0d_1/nA$ may be neglected in comparison with P_L . For cases where the permeability and water content versus suction pressure data are not available for a particular soil, they may be approximated from data on other soils of the same general type, and F may then be estimated for the given soil situation. Values of the other parameters may be taken:

$$\begin{aligned} L &= 3.4 \times 10^9 \text{ erg/cm}^3; & a_0 &= 1.8 \times 10^{-8} \text{ cm}; & d_1 &= 10^{-7} \text{ cm}; \\ V_0 &= 3 \times 10^{-23} \text{ cm}^3/\text{atom}; & D &= 0.8 \times 10^{-5} \text{ cm}^2/\text{sec}; & n &= 6; \text{ and} \\ kT &= 3.8 \times 10^{-14} \text{ erg/atom.} \end{aligned}$$

With all these values, one may easily obtain an estimate of the maximum heave rate for a soil, which should be valid to order-of-magnitude accuracy. This procedure was carried out for a number of soils frozen under laboratory conditions by the Corps of Engineers (U. S. Army Corps of Engineers, 1958). To correspond with their experimental arrangement, l was taken as 10 cm and P_L as 5×10^3 dyne/cm².

The results of the calculations are presented in Table 1. As indicated there, the calculated heave rates agree quite well with the experimental observations—better, in fact, than had been anticipated, considering the approximations used in the calculations. The limiting interfacial process for silty soils which heave to a significant extent, as well as for clays, is propagation of the ice through channels between particles; the limit for sandy soils which generally do not heave appreciably corresponds to particle entrapment.

On the same basis, taking $A=R_0^2$, our predictions may be compared with the experimental work of Beskow (1947) on soil fractions of different particle sizes. The calculated and observed curves are shown graphically in Fig. 4. As seen there, the agreement between theory and experiment is quite satisfactory for the larger particle-

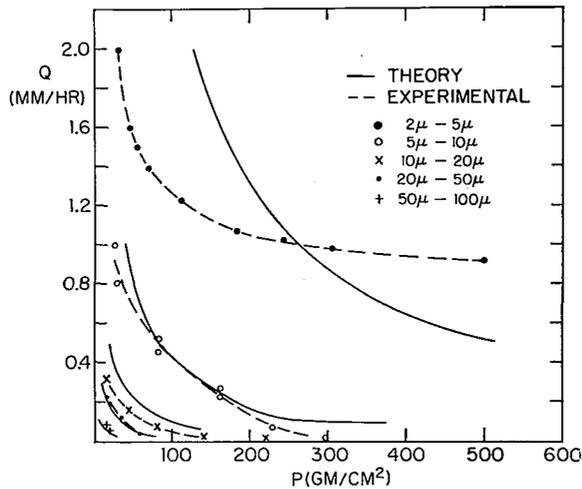


Fig. 4. Heave rate vs. pressure for soil fractions of various particle sizes. Experimental data from Beskow, 1947

Table 1. Comparison of calculated heave rates with experimental observations by U. S. Army Corps of Engineering (Reference 7)

Soil	Limitation	V_c (10^{-6} cm/sec)	Observed heave rate (10^{-6} cm/sec)
New Hampshire Silt	II	23	9
Indiana Silt	II	19	12
Yukon Silt	II	6.4	2.6
Fairbanks Silt	II	12*	5.2-13.4
Searsport Clay	II	11	5.5-10
Portsmouth Silty Sand	I	2.6	1.1

* Permeability data for this soil was not presented in Reference 7). In our calculations we used the average permeability for the other silts tested.

size soil fractions, but not so good for the 2-5 μ fraction although even in this case the calculated heave rates are within a factor of 2 of the experimental values for all measured rates.

The poorer agreement at small particle sizes can be attributed to variations in compaction of the soil. The effect of pressure in compacting a soil, with the resultant effect on heave rate, is expected to be greatest for the fractions of smallest size. Nonuniformities in the fractions, also expected to be most significant for the smallest particle sizes, should further enhance this effect.

The approximations suggested here should be best for silty soils, and less reliable for sands or clays (where the assignments for R_0 and R_c are more questionable). Where greater precision is desired, the permeability and water content versus suction pressure should be directly measured for the specific soil. Better estimates of the effective channel size may be derived from considering the packing of spheres with

a distribution of sizes or from the water content-suction pressure relations*. The effective particle size may more appropriately be taken as the root mean square particle size of the distribution, since the effective area at the interface occupied by the particle is relevant to the theoretical development.

VI. Conclusions

The view has been adopted that the decisive conditions for ice lens formation may be understood by considering the channel propagation and particle trapping aspects of the problem. Using an approximate treatment for the water transport problem in soils, expressions have been derived for the heave rate as a function of various parameters used to characterize the soils. Two limiting cases have been identified, one corresponding to particle entrapment, appropriate for coarse sandy soils, and one corresponding to channel propagation, appropriate for fine silty soils. The heave rate expressions are completely specified by the various soil parameters. Despite some uncertainty in the specification of these parameters for soils previously studied, the agreement between experiment and theory was found to be satisfactory.

A more definitive test of the theory can be provided by experiments carried out on a soil which has been separated into fractions of different particle size—or, perhaps more simply, on an artificial soil composed of readily characterized particles. In either case, a detailed knowledge of the particle peripheries, on the scale of a few hundred Å, will be required, as will the permeability and the water content as a function of suction pressure. With such fractions, frozen at a variety of freezing rates, surcharge pressures and suction pressures, frost heave data can be obtained in which the relevant parameters are well specified.

While such data will be extremely interesting when they become available, it is apparent that many of the critical parameters which govern the frost heave process have been identified, and that the maximum heave rate may conveniently be predicted with relatively little information about a particular soil.

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* We are postponing for other discussion the effect of adsorbed water in reducing effective channel size (an effect which should be most significant for clayey soils) just as we have not explicitly considered effects such as the possibility of air rejected at the water-ice interface blocking the channels. Such effects, which are potentially significant in some frost heave situations, should properly form the subject of separate discussion.

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