On the Relationship between Partial Soil Freezing and Surface Forces

Raymond N. YONG

Department of Civil Engineering and Applied Mechanics
McGill University, Montreal, P. Q., Canada

Abstract

The existence of unfrozen water in frozen soils has been documented in previous studies (*e.g.* Kolaian and Low, 1963; Vershinin *et al.*, 1960; Yong, 1965). Of primary interest is the mobility of this unfrozen water in response to gradients arising from thermal and electrical sources in the demonstration of frost heaving.

This paper examines the relationship between unfrozen water (in frozen soil) resulting in the overall phenomenon of partial soil freezing, and the consequent mechanical performance of the frozen soil under external constraints. The parameters considered center around the energy relationship of soil for water in the initial unfrozen state and the resultant partially frozen state. The energy relationships may be described in general as surface force effects, and in particular as adsorption and capillary effects. Specifying interactions at the mineral-water interface as adsorption effects, and between water and gas phases as capillary effects, the phenomenon of unforzen water may be related directly to both adsorption and capillary effects. Included in these effects is the structural parameter which dictates or modifies the resultant complex interaction between the adsorption and capillary forces.

Relationships are drawn describing unfrozen water content as a function of moisture potential and persisting subfreezing temperature. In considering the strength of frozen soils, it is seen that actual ice content (as opposed to total initial water content) can be used to relate strength with temperature.

The frost heaving problem is examined analytically on the basis of ice-water relationships and existent multi-curvatures at the ice front. Comparisons are made between measured and predicted frost heaving pressures.

I. Introduction

The water relations of soil are for many applications the most important physical characteristics of soils. Water is the variable which has the greatest influence on properties such as strength, plasticity, compressibility, and in the case of soil freezing, on the amount of water that is able to be frozen at the persisting subfreezing temperature.

The existence of unfrozen water has been well documented in previous studies (*e.g.* Kolaian and Low, 1963; Vershinin *et al.*, 1960; Yong, 1965). One of the most important considerations is the mobility of this unfrozen water in frozen soils in response to gradients arising from thermal and electrical forces in the demonstration of frost heaving, frozen soil strength, and stability. The energy relationships may be described as surface force effects which include adsorption and capillary effects. In general, measurement of these energy relationships may be pursued on the basis of suction measurements describing therein capillary or adsorptive suction characteristics. From the thermodynamic

sense, these may be expressed in terms of moisture potentials and may be broken up into four divisions:

(1) Osmotic potential which arises from difference in the solute concentration between the soil water and free water,

(2) Matric potential which is due to the adsorptive and surface tension forces tending to hold water to soil,

(3) Gravitational potential which arises due to the difference in position or head between the soil water and free water,

and

(4) Potential due to external gas pressure.

In the considerations made in this study, it is necessary to specify only the potentials in terms of adsorption and capillary potentials. In terms of mechanical equivalence, these can be broken down in terms of the measured suction which includes matric and osmotic suctions. The osmotic potential will be large for active clays and is relatively small for silts. Since suction measurements adequately describe the general relationship between the forces holding water to soils and the particular state of the soil, it is not unreasonable to expect that these same forces could have some influence on the degree of freezing of the pore-water, and thus ultimately dictate the engineering properties of the semi-frozen soil.

The intent of this paper, therefore, is to examine (a) soil-freezing relationships in terms of the forces holding water to soil (expressed as the moisture potential or the suction capacity of the soil), (b) frozen soil strength relative to actual frozen water content (*i.e.* actual ice content) and (c) the frost heaving problem.

II. Experimentation

Most of the experimental techniques and procedures have been detailed elsewhere (*e.g.* Yong, 1965). Only a brief review will be given here. The materials used for the

Table 1. Soil properties and Characteristics

Consistency	Plastic kaolinite clay	Medium clay	Silt
LL	74	67	25.1
PL	31	28.8	17.0
Specific gravity	2.75	2.73	2.68
Grain size (by weight)			
mm % finer 0.15	100	100	100
0.10	100	100	93
0.05	100	91	83
0.01	100	72	22
0.005	100	66	2
0.002	100	30	0
0.0008	80	0	0
0.0005	67	0	0

experiments consisted of a plastic kaolinite clay identified by its trade name as "Bell" clay, a medium feldspar-quartz clay, and an inorganic silt. The physical properties of these soils are given in Table 1 in the appendix.

Suction measurements were conducted using the pressure membrane technique, while a modified double ring shear test was used to examine the strength of the frozen soils. Axial confining pressures ranging from 0 to 240 psi were used as axial pre-stressing prior to application of radial shear. In the case of the determination of the quantity of water remaining frozen, the approximate calorimetric method was used.

In the study of frost heaving pressures, a simple uniaxial system was used. A thin ice lens was allowed to develop at the frost front, before the application of an equilibrating pressure. Restriction of further growth of ice lens was taken as the criterion for equilibrating pressure.

III. Results and Discussion

Unfrozen water content

The principle of applied pressure used in the measurement of total potential of the

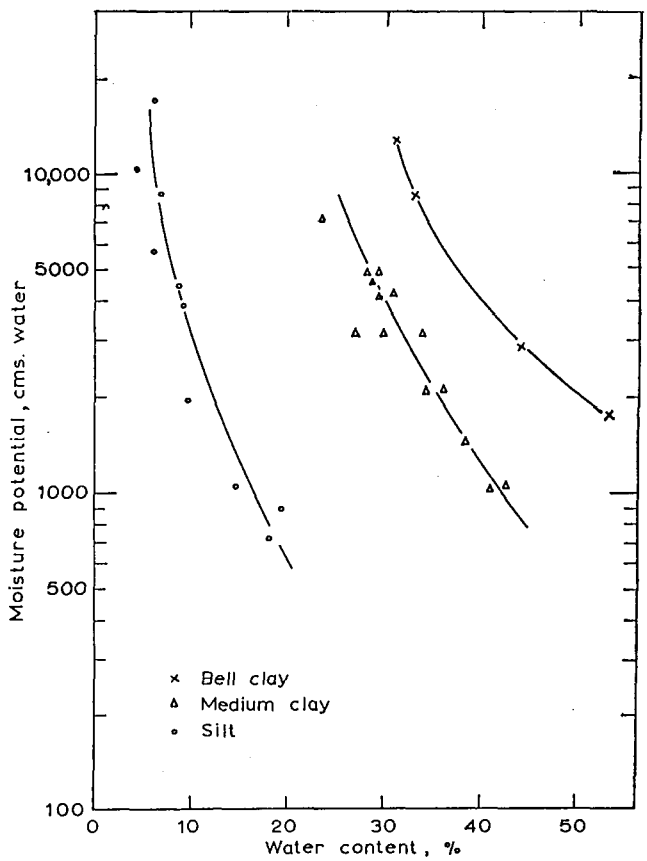


Fig. 1. Relationships between moisture potential and water content

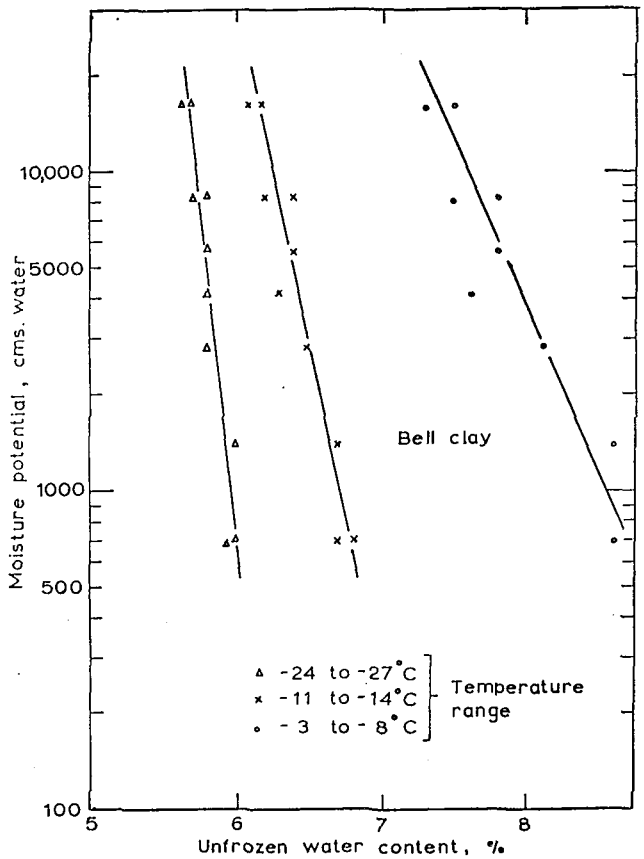


Fig. 2. Influence of moisture potential on quantity of water remaining unfrozen (unfrozen water content) for Bell clay

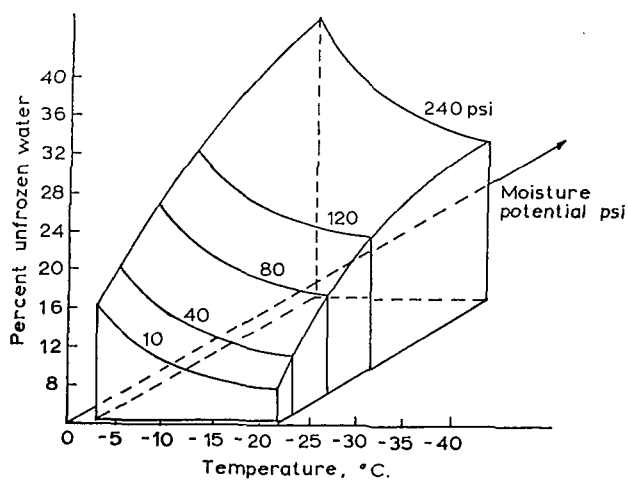


Fig. 3. Potential-unfrozen water content surface

soil water prescribes that the work required to remove water against suction forces of the soil, and osmotic forces of the soil water, defines the total potential of soil water. The relationship between water content and moisture potential for the three soils used in the study may be seen in Fig. 1. The total moisture potential at any one moisture content reflects the activity of the soil. Thus, the Bell clay with the highest plasticity index (indicative of the activity of the soil) has the higher moisture potential for the same water content, *i.e.* compared to the medium clay and the silt.

The amount of water remaining unfrozen at any one persisting subfreezing temperature can be related to the tension in the soil water arising from the geometrical arrangement of particles, forces arising from curved air-water interfaces, and interaction of the surface forces. Thus, with measurements of total moisture potential which reflect the degree of interaction arising from surface and capillary forces, and also of soil structure, an evaluation of quantity of water remaining unfrozen (designated as unfrozen water content) may be made. Such a relationship for example is shown in Fig. 2 for the Bell clay. The common base used to establish the relationship depends on initial water content. Since little is known of the effect of subfreezing temperatures on moisture potential relationships, the assumption is made that during process of freezing, initial tension in the pore water must dictate ultimate freezing of the pore water. The assumption is viable in that the critical temperature of the pore water which determines freezing of the pore water will depend on the energy status of the water within the pores. In a separate study on depressed temperature effects (1 to 20°C) on the diffuse ion-layers of sodium montmorillonite, it has been shown for example that the Gouy–Chapman theory may be used to predict osmotic swelling of the pure clay (Yong *et al.*, 1963). While this does not provide positive evidence, support may be drawn for acceptance of the assumption used.

If one expresses the quantity of water remaining unfrozen as a percentage of the original moulding water content, by interpolation from Fig. 2, the potential-unfrozen water surface shown in Fig. 3 can be established. Any part of the surface represents the percent of original water remaining unfrozen as a function of subfreezing temperature and moisture potential for the Bell clay. Similar surfaces may be drawn for the other soils tested.

Frozen soil strength

Evaluation of frozen soil strength was performed using a modified double ring shear device. Both the medium clay and inorganic silt soils were tested to determine the physical characteristics defining frozen soil strength. Axial prestressing up to 240 psi was used with different samples subjected to varying prestressing pressures—*i.e.* 0, 60, 120, 180 and 240 psi.

If one plots ultimate shear strength against initial water content, one obtains a fairly decipherable relationship for the silt soil, but a widely scattered plot for the clay samples—as shown in Fig. 4. The evidence shown points out the fact that while the water content parameter may provide an indication of frozen soil strength at any one freezing temperature, the fundamental parameter of unfrozen water content, which is more significant for clay soils, is perhaps more important. Since unfrozen water con-

tents for the frozen silt are small in comparison to the clay samples (Yong, 1965), correction for actual ice content, *i.e.* initial water content minus unfrozen water content, is not sufficiently large to distort the relationship shown in Fig. 4. Thus in Fig. 5 the relation between shear strength and actual ice content bears similar resemblance to

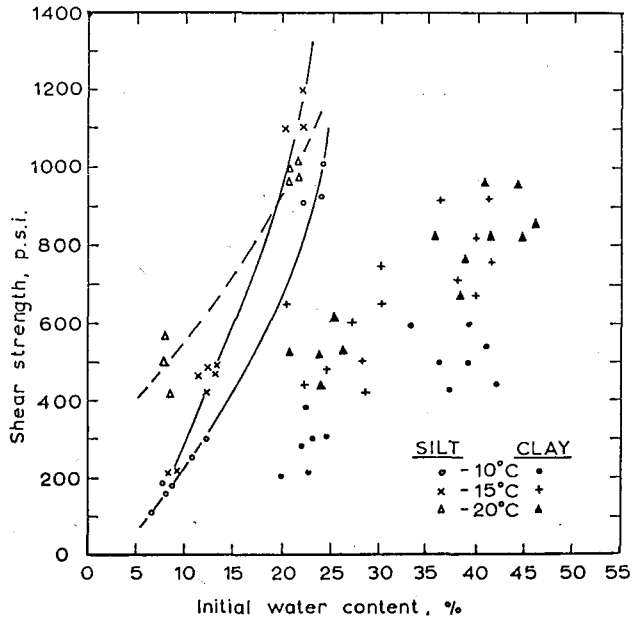


Fig. 4. Shearing strength as a function of initial water content

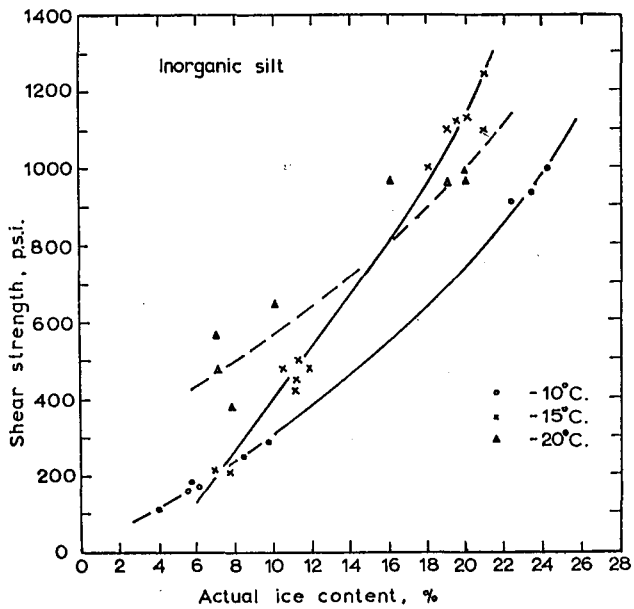


Fig. 5. Shearing strength of silt relative to actual ice content

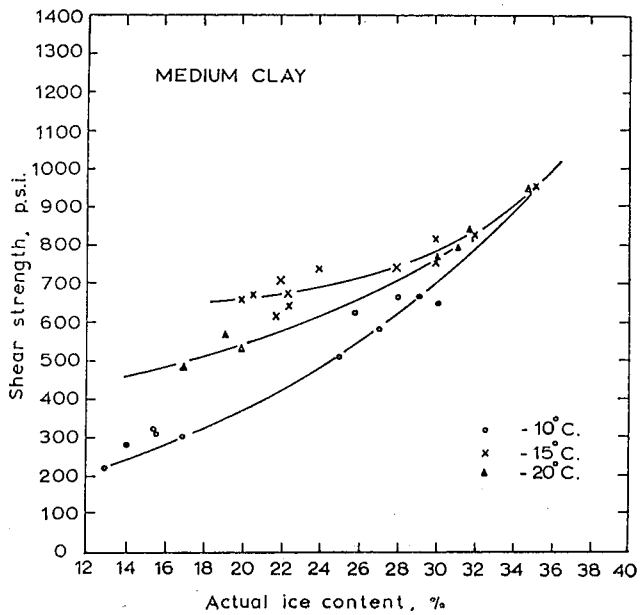


Fig. 6. Shearing strength of caly relative to actual ice content

that shown in Fig. 4.

The correction made for conversion of initial water content to actual ice content for the clay samples provides a lessening in the scatter of points seen in Fig. 4. A more reasonable relationship for the frozen clay strength may then be obtained—as shown in Fig. 6. In both Figs. 5 and 6, an anomaly is evidenced. The shear strength behaviour of the frozen soils at −15°C does not seem to conform to the trends shown for −10 and −20°C. No reason can be given at the present time for this behaviour. It is perhaps interesting to note that the strengths tend to similar values for all temperatures as the ice content is increased. This indication is not unrealistic.

Frost heaving

The frost heaving problem has been examined by many investigators in terms of principles, mechanisms, and pressures generated (*e.g.* Miller *et al.*, 1960; Takagi, 1965; Penner, 1959; Hoekstra *et al.*, 1965). A comprehensive review of the literature given by Takagi (1965) shows varied conclusions. It is not the intent of this section of the study to engage in a discussion on the results of different theories, but to present evidence of a viable theory for prediction of frost heaving pressures based on pore and particle radii, substantiated by corresponding measured pressures for the silt samples studied. Assuming that thermodynamic relations can be written for a curved ice-water interface in a pore, from Maxwell's thermodynamic relation:

$$\left(\frac{\partial T}{\partial P}\right)_V = \left(\frac{\partial V}{\partial S}\right)_T, \tag{1}$$

using the usual notation

$$\left(\frac{\partial V}{\partial S}\right)_T = -\frac{V_i T}{L}.$$

For small incremental changes

$$\frac{\Delta T}{\Delta P} = -\frac{V_i T}{L}, \tag{2}$$

considering ΔP as $2\sigma/r$ where

σ is ice-water interfacial energy,
r radius of curved surface,
V_i volume of ice $= 1/\rho_i$ for unit volume consideration,
ρ_i density of ice,
L latent heat of fusion,
ΔT freezing point change at curved surface.

Equation (2) may be written as

$$\Delta T = -\frac{2\sigma T}{\rho_i L r}, \tag{3}$$

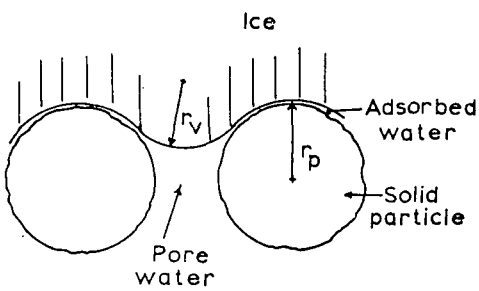
which is the same form used by Sill and Skapski (1956).

For the configuration shown in Fig. 7, since the free energy of the ice must equal that of the water at equilibrium, the reversed curvature of the ice front between pore space and particle must be accounted for. Since minimum potential energy configuration for the ice front must be a flat plane, both the ice radii in the pore r_v and around the particle r_p must be considered. In this case, if one assumed cubicle packing,

$$r_v = r_p\left(\frac{n}{1-n}\right)^{1/3},$$

$$\Delta T = -\frac{2\sigma T}{\rho_i r_v L}. \qquad \text{In the pore}$$

$$\Delta T = +\frac{2\sigma T}{\rho_i r_p L}, \qquad \text{for the particle.}$$



$$\Delta T = \frac{2\sigma T}{\rho_i L}\left(\frac{1}{r_p} + \frac{1}{r_v}\right)$$

$$\Delta P = \frac{L\Delta T}{T(v_w - v_i)}$$

$$r_v = r_p\left(\frac{n}{1-n}\right)^{1/3}$$

Fig. 7. Schematic representation of local regime at freezing front showing ice front curvature

Hence in the local region encompassing a particle and a void space, the net ΔT is given as

$$\Delta T = -\frac{2\sigma T}{\rho_i L}\left[\frac{1}{r_p} + \frac{1}{r_v}\right], \tag{4}$$

which is similar in form to that given by Miller *et al.* (1960). Since ΔP, the pressure generated between the ice and particle is given by the Clapeyron relation (see, *e.g.*, Penner, 1959),

$$\Delta P = \frac{L\Delta T}{T(V_w - V_i)}. \tag{5}$$

Substituting the appropriate values for r_p, r_v and in eq. (4) will provide the value for ΔT in eq. (5).

In the case of the silt used in this study, r_p and r_v were comparatively con-

stant throughout the soil. Since ΔP would act on the soild particles for a physical demonstration of frost heaving pressure, projection for a tolal frost heaving pressure must be made considering the effective particle surface area contacted. This can be approximated by modifying the total cross-sectional area by $(1-n)^{2/3}$, where n=porosity.

In the data obtained by McKyes (1966), using $\sigma=20$ ergs/cm^2, exact correspondence was obtained between predicted and measured values. The method was tested for results given by Hoekstra *et al.* (1965) and Penner (1959) using the 10% passing size of r_p and r_v and was found to be valid. Figure 8 shows the correlation. If frost heaving pressures can be transmitted to the surface without loss due to compression of the frozen soil above (as is the case in laboratory experimentation), the principle of multiple curvature shows valid theoretical application. The existence of adsorbed water on soil par-

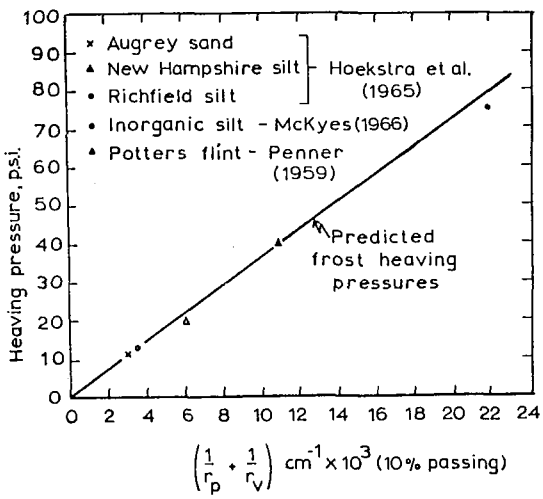


Fig. 8. Comparison between measured and predicted frost heaving pressure using actual particle radius

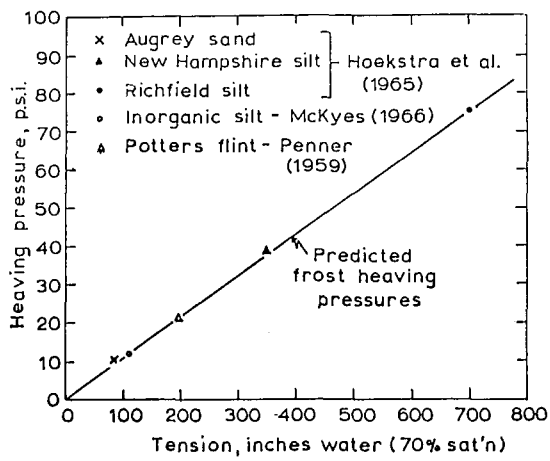


Fig. 9. Comparison between measured and predicted frost heaving pressure using radius of curvature computed from moisture potential

ticles is seen to be a requirement, without which the ice-water interfacial relationship cannot be tenable at the ice-water-particle region. In silts, since adsorbed water (controlled by surface forces) is sufficiently small, r_p can be reasonably represented by the particle radius.

A better determination of r_p and r_v may be obtained by using the capillary potential at some representative and effective saturation. In silts and sands, the total potential is due to the capillary potential, and thus an effective r_p denoted by r'_p can be computed, using $\sigma=72$ ergs/cm^2 for air-water interfacial energy. r'_v, the effective pore radius, can thus be computed. From initial controlled experiments, the representative saturation was established as 70%. Thus, a relationship between frost heaving pressure and soil moisture potential may be established (see Fig. 9), where the link is provided through the computed effective radii of curvatures r'_p and r'_v followed by the use of eqs. (4) and (5). The role of surface forces in the determination of frost heaving now becomes evident.

In clays, however, larger surface force effects will not allow an easy approximation of r'_p and r'_v based on capillary potential. The problem then remaining is to determine not only an effective r'_p, but also a corresponding r'_v.

IV. Conclusions

The role of the surface forces is seen to influence many of the soil freezing problems. Unfrozen water in frozen soils is seen to depend on the total moisture potential of the soil. In turn, this will influence frozen strength directly, and will indirectly influence the magnitude of frost heaving pressures.

Acknowledgment

The author wishes to acknowledge the Grant-in-Aid of Research provided by the Defence Research Board of Canada, and E. McKyes for assistance in the latter part of the study concerned with frost heaving pressures.

References

1) HOEKSTRA, P., CHAMBERLAIN, E. and FRATE, T. 1965 Frost heaving pressures. *Highway Res. Board*, **101**, 28–38.
2) KOLAIAN, J. H. and LOW, P. F. 1963 Calorimetric determination of unfrozen water in Montmorillonite pastes. *Soil Sciences*, **95**, No. 6, 376–384.
3) MCKYES, E. 1966 Theoretical and Experimental Determination of Frost Heaving Pressures in a Partially Frozen Silt, Thesis for McGill University Faculty of Engineering, 44 pp.
4) MILLER, R. D., BAKER, J. H and KOLAIAN, J. H. 1960 Particle size, overdurden pressure, pore water pressure and freezing temperature of ice lenses in soil. *Trans. 7 th Int. Cong. Soil Science*, **1**, 122–128.
5) PENNER, E. 1959 The mechanism of frost heaving in soils. *Highway Res. Board Bull.*, **225**, 1–13.
6) TAKAGI, S. 1965 Principles of frost heaving. *CRREL Res. Rept.*, **140**, 24 pp.
7) SILL, R. C. and SKAPSKI, A. S. 1965 Method for determination of surface tension of solids

from their melting points in thin wedges. *J. Chem. Phys.*, **24**, 644–651.

8) VERSHININ, P. V., DERIAGIN, B. V. and KIRILENKO, N. V. 1960 The non-freezing water in soil. *Translation* No. **3**, *ACAFEL*, 12 pp.

9) YONG, R. N., TAYLOR, L. O. and WARKENTIN, B. P. 1963 Swelling pressures of sodium Montmorillonite at depressed temperatures. *Clays and Clay Minerals*, **11**, 268–281.

10) YONG, R. N. 1965 Soil Suction effects on partial soil freezing. *Highway Res. Board*, **68**, 31–42.