MATHEMATICAL ANALYSIS OF THE FILTERING-RATE OF PLANKTON NET

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Locating a cod type net, provided with extremely small sized mesh like a plankton net, in the running water, it is assumed that, owing to the so-called "Submerged Orifice" of the mesh, comparing to the current speed outside the net the one inside decreases; then the static pressure inside the net increases and the water current, having speed in proportion to the square root of the difference between the outside pressure and the inside one, passes through the mesh of the net.

As shown in Fig. 1, it is also assumed that the plankton net is made of only net excluding any other unit and that the sectional form of the net which is perpendicular to the current is circular.

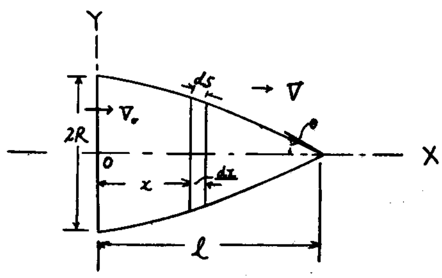

Fig. 1. Schematic figure of the plankton net

Now, the axis of reference and others are defined as follows:

X axis is on the plane containing the center of the section and is parallel to the current,

Y axis is perpendicular to X axis and is located at the entrance of the net,

V is the current speed,

V_0 is the current speed at the entrance of the net,

V_x is the current speed at the section which is x distant from the origin,

2R is the diameter of the entrance of the net,

l is the length of the net measured along the center line.

Generally the current speed has various values at any point on X axis. An expression $y=f(x)$ may be expressive of the form of the net, so that if the current speed at any section be distributed equally, the amount of the water flowing into x-section is

$$\pi f(x)^2 V_x \qquad \cdots\cdots\cdots\cdots(1)$$

Similarly, the amount of the water flowing out of x+dx section is

$$(V_x-dV_x)\{f(x)-df(x)\}^2\pi \qquad \cdots\cdots\cdots\cdots(2)$$

Then, it is assumed that the amount of the difference beween (1) and (2) flows out through the mesh of ds width.

As the current speed passing through "Orifice" is proportional to the square root of pressure difference, the amount of water passed through the "Orifice" is expressed by $c\alpha 2f(x)\pi ds\sqrt{2g(p_x-p)}$; by using Bernoulli's theorem $V^2-V_x^2=2g(p_x-p)$, then one gets

$$c\alpha 2f(x)\pi ds\sqrt{V^2-V_x^2} \qquad \cdots\cdots\cdots\cdots(3)$$

Here, it is assumed that the pressure outside the net is constant. In (3), c is Vena Contracta, α is the ratio of the area of the mesh to the total area of the net.

Accordingly, one gets

$$f(x)dV_x+2V_xf'(x)dx=c\alpha\, 2ds\sqrt{V^2-V_x^2}$$
$$=2c\alpha\sqrt{1+f'(x)^2}\sqrt{V^2-V_x^2}\, dx$$

Substituting $V\sin\varphi$ for V_x in the above expression, one obtains

$$f(x)V\cos\varphi\, d\varphi+2V\sin\varphi f'(x)dx=2c\alpha\sqrt{1+f'(x)^2}\, V\cos\varphi\, dx$$

$$\therefore \frac{d\varphi}{dx}=\frac{2\{c\alpha\sqrt{1+f'(x)^2}-\tan\varphi f'(x)\}}{f(x)} \quad \cdots\cdots(4)$$

$$\therefore \varphi=\int\frac{2\{c\alpha\sqrt{1+f'(x)^2}-\tan\varphi f'(x)\}}{f(x)}dx+\text{const.} \quad \cdots\cdots(5)$$

Constant of integration is

$$V_x=V_o=V\sin\varphi_o \qquad \text{at } x=o$$

and V_o must be determined by the following expression,

$$\pi R^2V_o=\int_0^l 2\pi y\, ds\, c\alpha\sqrt{V^2-V_x^2}$$
$$=\int_0^l 2\pi f(x)\sqrt{1+f'(x)^2}\, c\alpha\sqrt{V^2-V_x^2}\, dx$$

$$\therefore \sin\varphi_o R^2=2\int_0^l c\alpha\cos\varphi\, f(x)\sqrt{1+f'(x)^2}\, dx \quad \cdots\cdots(6)$$

Solve both (5) and (6), and it will be possible to decide the distribution of the current speed inside the net on various points on X axis and current speed at the entrance of the net V_o.

Now, let K be the ratio of the current speed at the entrance of the net to the one outside the net, one gets

$$K=\frac{V_o}{V}=\sin\varphi_o \quad \cdots\cdots(7)$$

But it is not easy to solve (5) and (6).

As the plankton net generally used is of a rectangular conical type, one gets

$$y=\tan\theta(l-x)$$
$$R=l\tan\theta$$

Substituting these expressions for the references in (5) and (6), it follows that

$$\frac{d\varphi}{dx}=\frac{2(c\alpha\sec\theta-\tan\varphi\tan\theta)}{\tan\theta(l-x)}=\frac{2(\frac{c\alpha}{\sin\theta}-\tan\varphi)}{l-x}$$

$$\sin\varphi_o=\frac{2c\alpha}{l\sin\theta}\int_0^l\cos\varphi(1-\frac{x}{l})dx$$

and substituting μ and t for $\frac{c\alpha}{\sin\theta}$ and $\frac{x}{l}$ respectively in the above expressions, one gets

$$\frac{d\varphi}{dx}=\frac{2(\mu-\tan\varphi)}{l-x} \quad \cdots\cdots(8)$$

$$\sin\varphi_0 = 2\mu\int_0^1 \cos\varphi(1-t)dt \qquad \cdots\cdots\cdots\cdots\cdots\cdots(9)$$

Integrate (8), and it follows that

$$\frac{1}{\left(1-\dfrac{x}{l}\right)^{2(1+\mu^2)}} = \left|\frac{\cos(\varphi_0+\delta)}{\cos(\varphi+\delta)}\right| l^{\mu(\varphi-\varphi_0)} \qquad \cdots\cdots\cdots\cdots\cdots\cdots(10)$$

Where δ is nominated from the next expression, $\tan\delta = \frac{1}{\mu}$.

In order to get the value of φ_0 which satisfy (9) and (10), give various values to φ_0 of (10) for the special value of μ, find the relation between φ and $\frac{x}{l}$, put it into the right term of (9) and calculate definite integrals. And thus, among these definite integrals, it becomes possible to find the satisfactory value which coincides with the left term of the expression (9), ($\sin\varphi_0$).

After a number of such calculations, one should get the following result, "The current speed inside the plankton net is constant and its value is $\tan\varphi_0 = \mu$".

In conclusion, the next formula is obtained concerning the filtering-rate,

$$\sin\varphi_0 = \frac{\mu}{\sqrt{1+\mu^2}} \qquad \cdots\cdots\cdots\cdots\cdots\cdots(11)$$

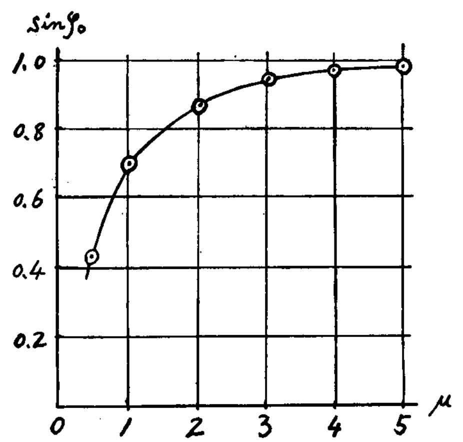

Fig. 2. The relation between $\sin\varphi_0$ and μ, which is calculated by the formula (11)

Fig. (2) gives the relation between the filtering-rate of the plankton net and μ.

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Resume

1. Towing a conical type net in water, having extremely small sized mesh like the plankton net, the relative current speed inside the net may be smaller than the one outside the net.

2. Towing the rectangular conical type plankton net in water, the current speed inside the net is constant. Let the filtering-rate be the ratio of the current speed inside the net to the one outside; it can be expressed by the formula (11).

References

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