ON THE STATISTICAL DISTRIBUTION OF MEASUREMENTS MADE AT EQUAL INTERVAL OF TIME OF SHIP MOTION AT SEA

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Introduction

Generally speaking, the ship's motion induced by the action of ocean waves can be treated as a stationary stochastic process. Especially we know that the method of cross-spectral density functions[1] for analysis of ship response to waves is a practical way to study the sea-keeping quality in full scale ship experiment at sea.[2]

In the treatment at these data, the author has used digital computations in a spectram analysis. According to the author's observations[3], the values were taken at one second interval from the recording charts; the number of observations used in this analysis is from 300 to 1,000, and the sum of the total time, ($M\Delta t$), 300 to 1,000 seconds.

In this paper, the author discusses the statistical distribution of the above data and especially the two dimenstional distribution, with respect to the simplified method of calculation for correlograms.

Regarding the statistical distributions, it is well known that the most common and easily determined statistical parameters are the mean and variance (or mean square). The Gaussian (or Normal) and the Rayleigh probability distributions can be completely described by their means and variances.

The theory of statistical distributions of maxima of random functions has been discussed by D.E. Carghtright and M.S. Longuet-Higgins.[4][5] According to their theory, the maxima of the distribution can be used to measure the degree of agreement between many theoretial relations and actual observations.

However, in this paper, the author discusses the following two problems from a different point of view: (1) The statistical distributions of ship's motion and wave height; and (2) Two dimensional distributions for time series.

Collection of Experimental data

Data for analysis were obtained on May 7, 1965, at Station No. 1 (42°11'N., 145°45'E.) of Cruise No. 13 (east of Cape Erimo, south of Hokkaido) aboard Training ship Oshoro Maru of Hokkaido University.[6]

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The wave heights were measured by means of a wave pole equipped with a specially designed radio telemetering apparatus[7]; the angles of roll and pitch were measured by use of vertical gyro. Measurements of motion were taken for every run in which the ship's course was 30 degrees to the estimated wave direction, the schematic diagram for relation of ship's head and wind direction in experiment is shown in Figure 2–d. The sea state of the time of the experiment was 4.

Typical brief series of values from the records are shown in Figure 1. The values used in the analysis are defined by the following constants; (1) $\Delta t=1$ sec. and (2) M=300, where Δt is the time interval between adjacent values; M, the number of samples used; and $M\Delta t$, the total length of time for all observations.

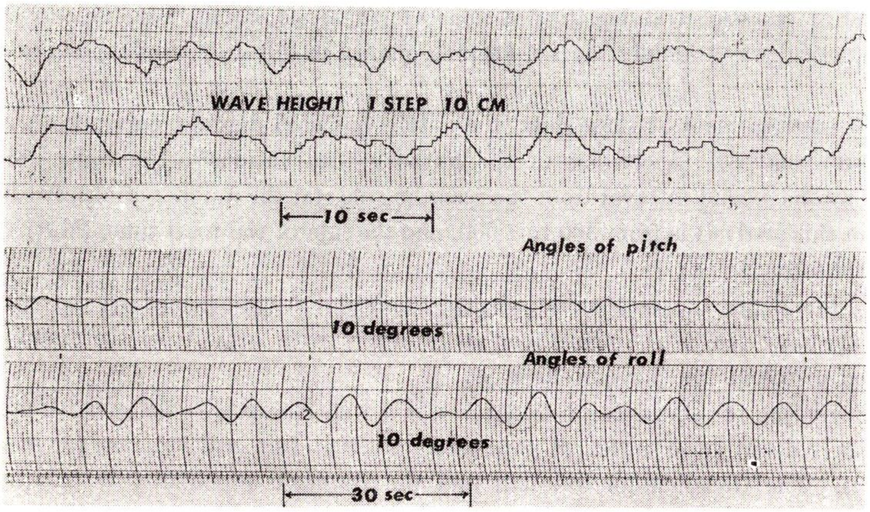

Fig. 1 Typical brief series of records used for analysis

The Probability Density Functions for Ship's Motion and Wave Height

The statistical distributions for wave height and angles of roll and pitch were estimated from the observations described above and the corresponding histograms are shown in Figure 2. The means and variances for these distributions were computed according to Gaussian probability papers and these values are given in Figure 3 and Table 1.

From the results it is concluded that the distributions are Gaussian in form; then the probability density functions can be represented by the following equation:

$$p(\lambda)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-m)^2}{\sigma^2}} \qquad (1)$$

where; m=mean value, and σ^2=variance.
Values for the variance and the means are given in Table 1.

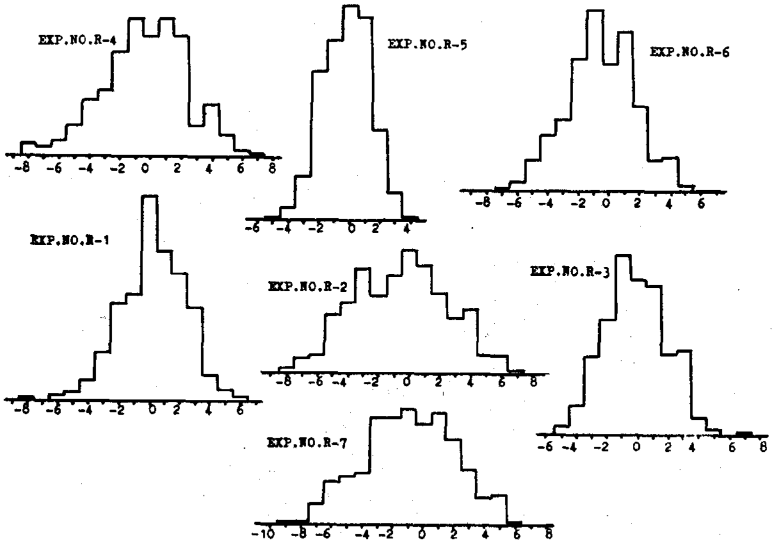

Fig. 2-a Angle of roll

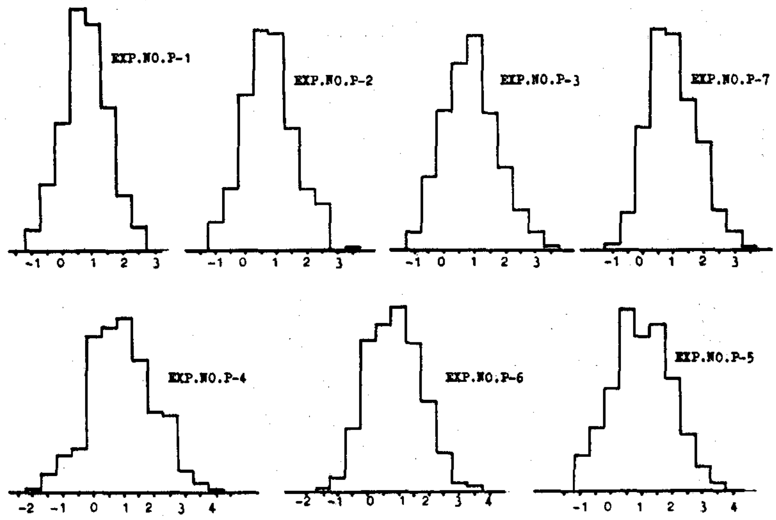

Fig. 2-b Angle of pitch

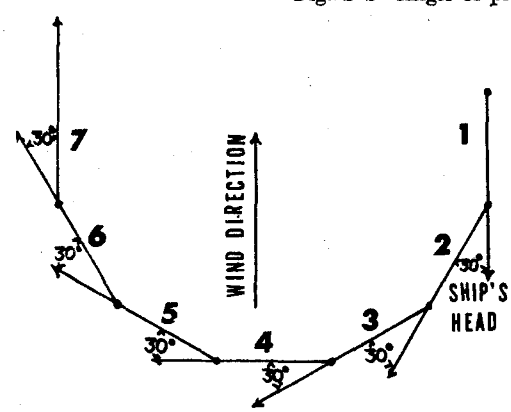

Fig. 2-d Diagram for relative angles between wind direction and ship's heads

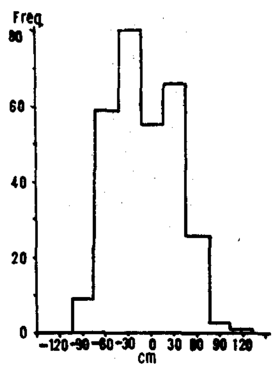

Fig. 2-c Wave heights

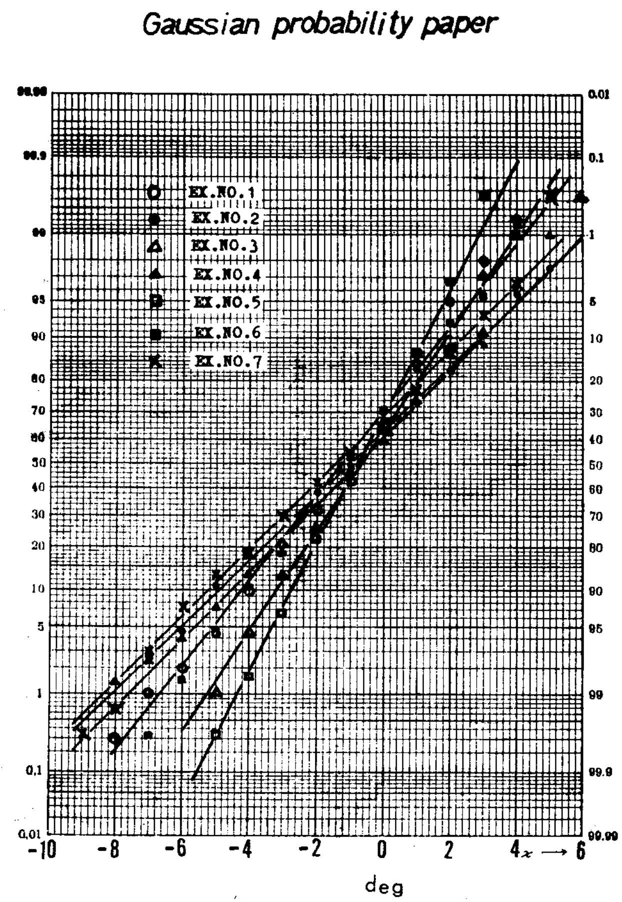

Fig. 3-b Angle oj roll

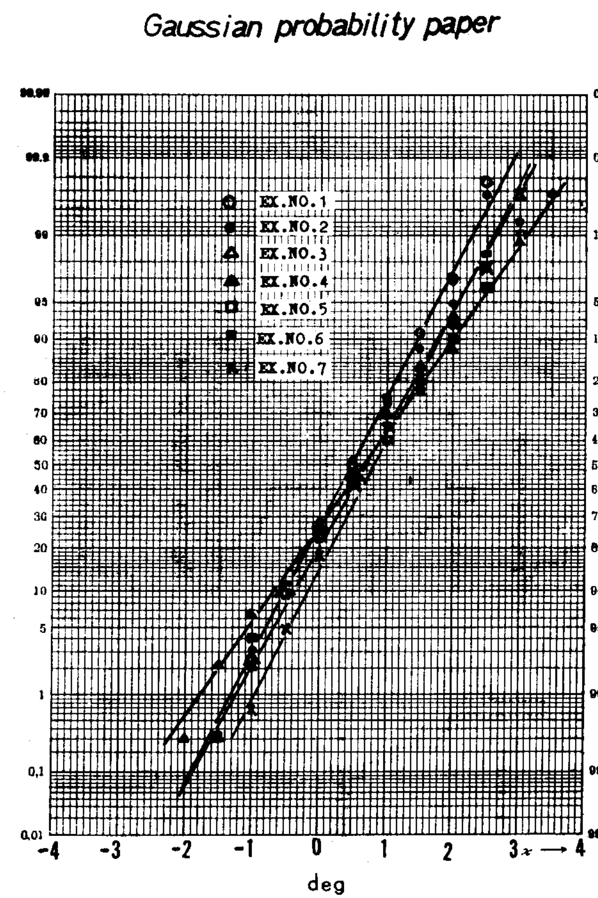

Fig. 3-a Angle of pitch

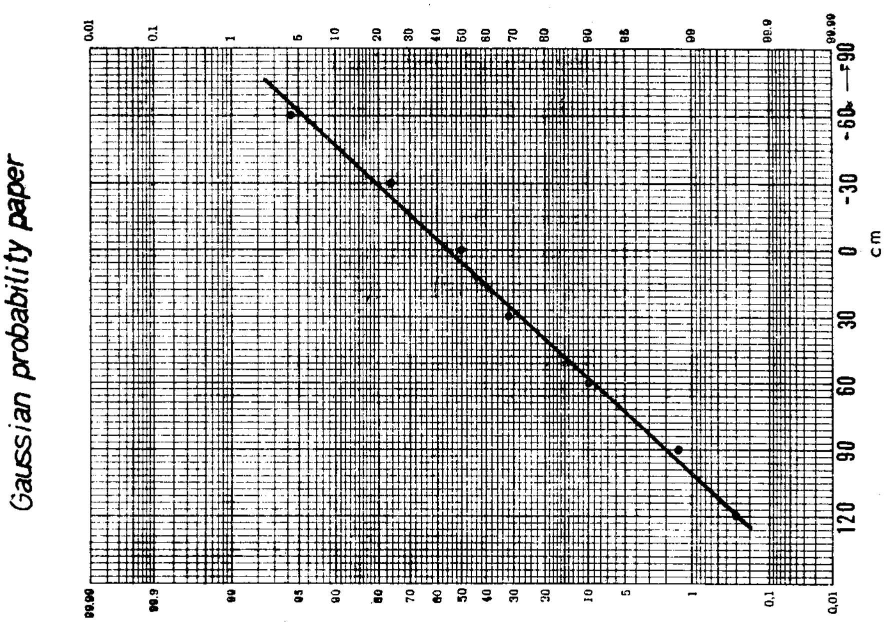

Fig. 3-c Wave heights

Table 1 Variance of ship motions and wave htights

Ex. No.	Angles of roll*				Angles of pitch**				Wave height
	Means	S	Var.	Co-var.	Means	S	Var.	Co-var.	Var.
1	−0.250	2.125	4.818	4.752	0.480	0.760	0.5776	0.5330	
2	−1.000	3.015	9.090	9.400	0.500	0.775	0.6063	0.6575	
3	−0.608	2.035	4.141	4.216	0.580	0.890	0.7921	0.7886	
4	−0.850	2.800	7.840	7.613	0.640	1.045	1.0920	1.1063	0.1764
5	−0.720	1.515	2.295	2.334	0.750	0.975	0.9506	0.9478	
6	−1.150	2.250	5.063	5.019	0.680	0.933	0.8696	0.8204	
7	−1.300	2.950	8.703	8.853	0.750	0.810	0.6561	0.6512	

* Angles of roll; Ex. No. 3 Roll
** Angles of pitch; Ex. No. 5 Pitch

The Probability Density Functions for two Dimensional Distributions

The procedures for estimating two dimensional probability density functions can be illustrated as follows:

Use the data, $x(i\Delta t)$, i=1, 2, 3,...., M, from the records. As described in Figure 4-a, plot values of $x(i\Delta t)$ along the abcissa and $x((i+\tau)\Delta t)$ along the ordinate, M-τ points can then be determined from the Figure . The points will be scattered around lines $x(i\Delta t)=x(i+\tau)\Delta t)$, and $x(i\Delta t)=-((i+\tau)\Delta t)$. (equations (B) and (A), respectively)

For these lines, the distributions were calculated for the each meshes as described in Figure 4-a, the corresponding histograms are shown in Figure 4-b and

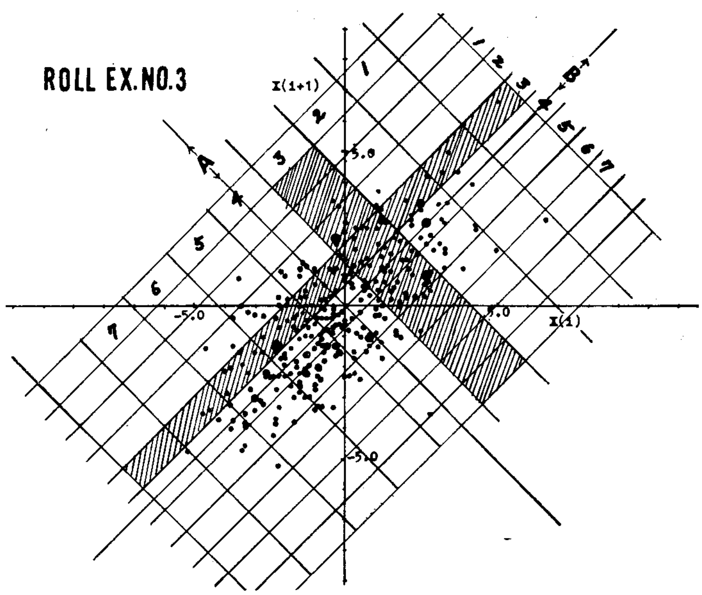

Fig. 4-a Two dimensional time series distributions for angle of roll

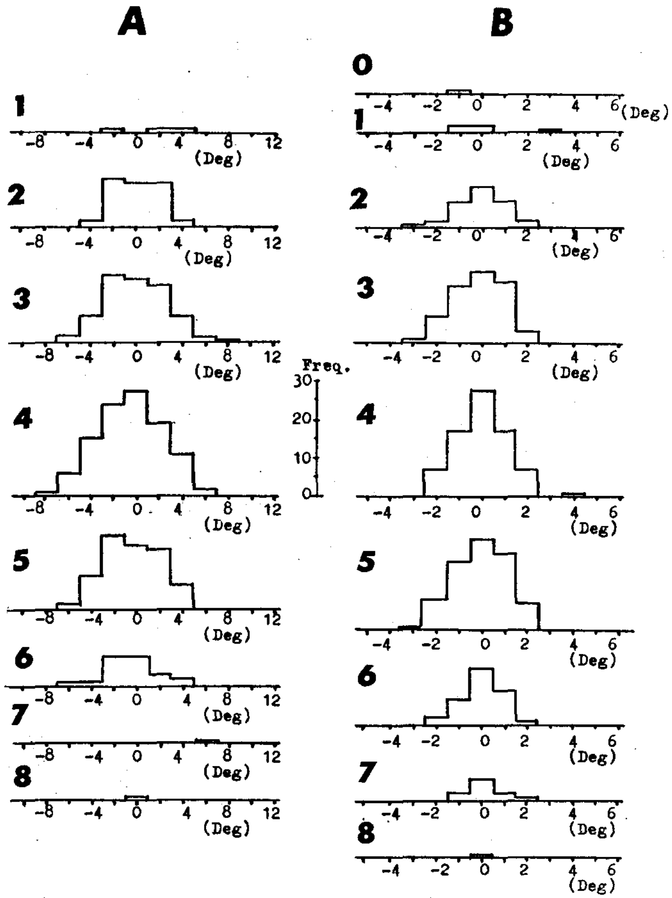

Fig. 4-b Histogram for meshes in Fig. 4-a

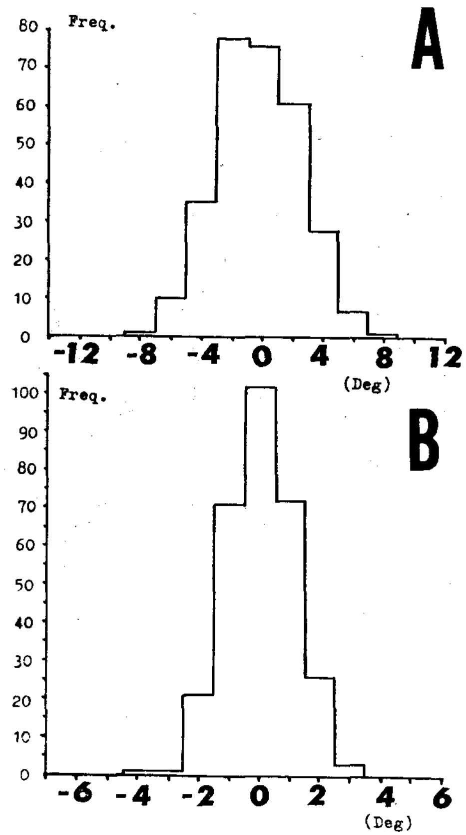

Fig. 4-c Histogram for beltlike area in Fig. 4-a

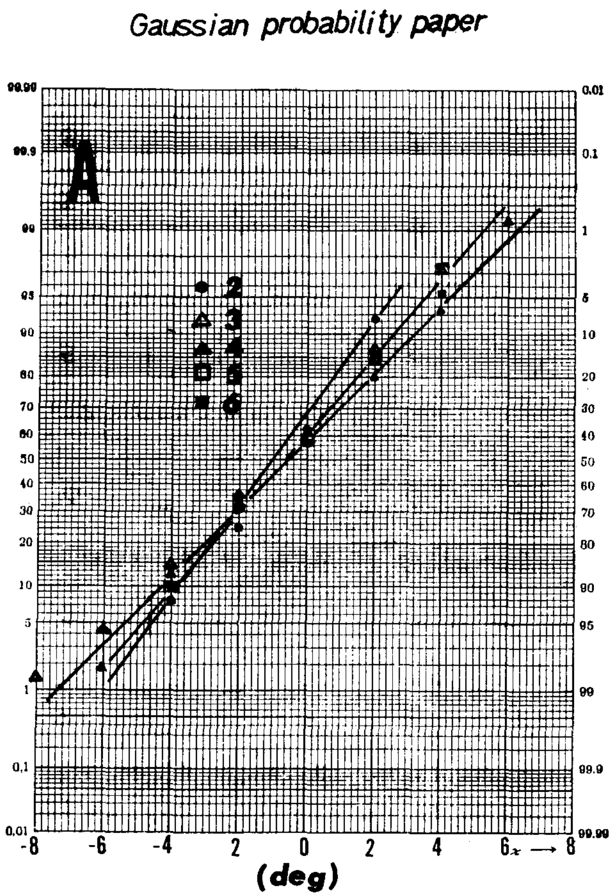

Fig. 5-a Angle of roll

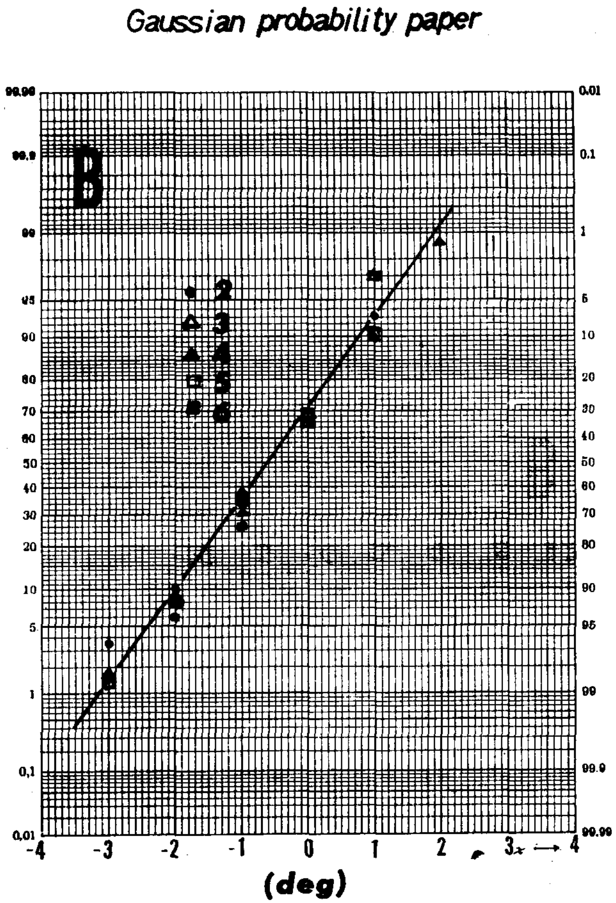

Fig. 5-b Angle of roll

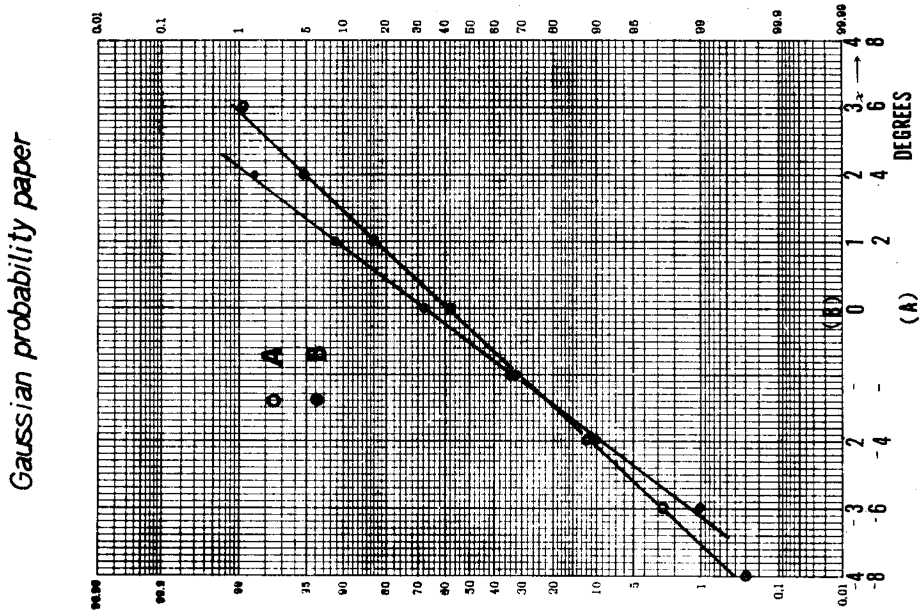

Fig. 5-c Angle of roll

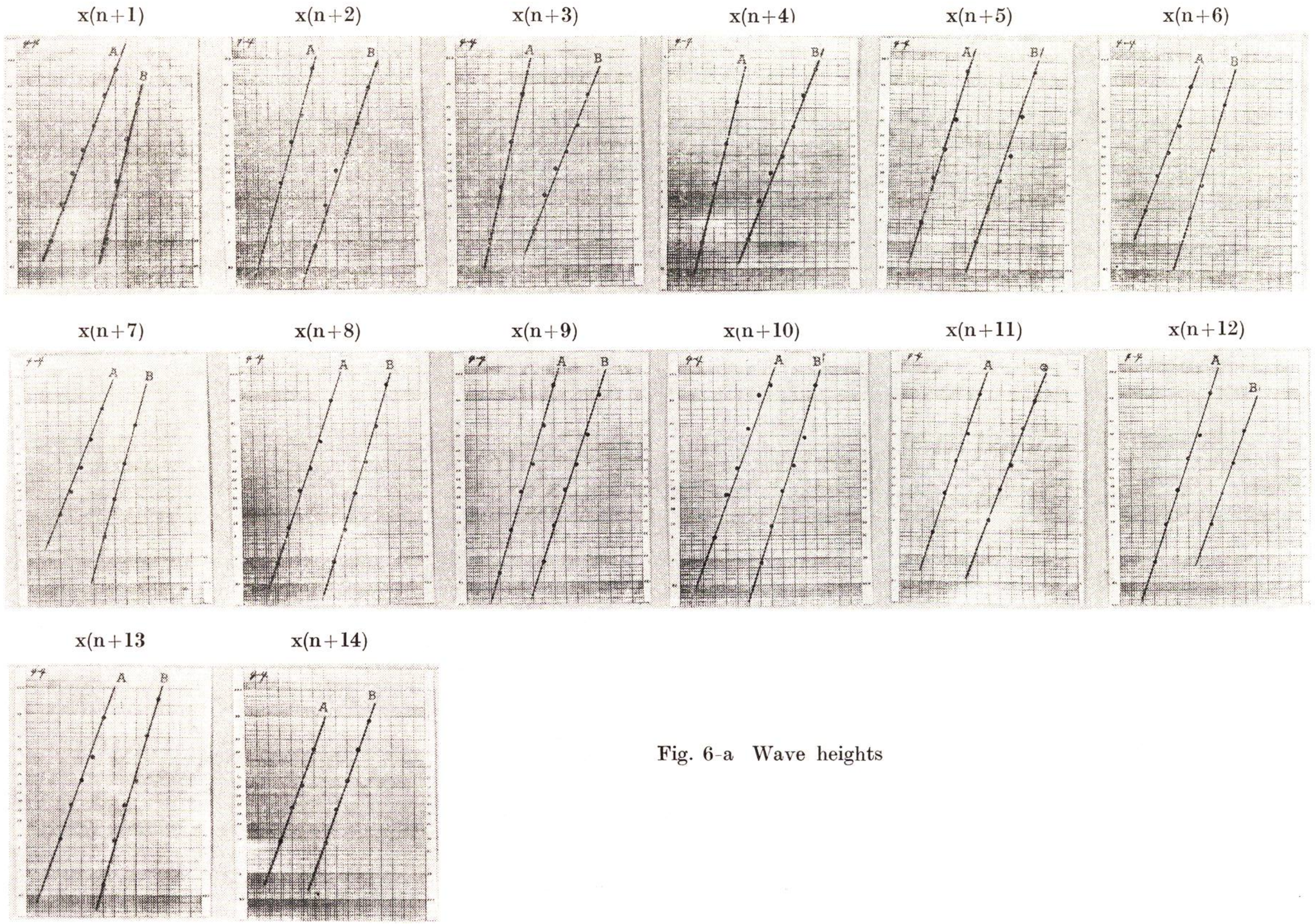

Fig. 6-a Wave heights

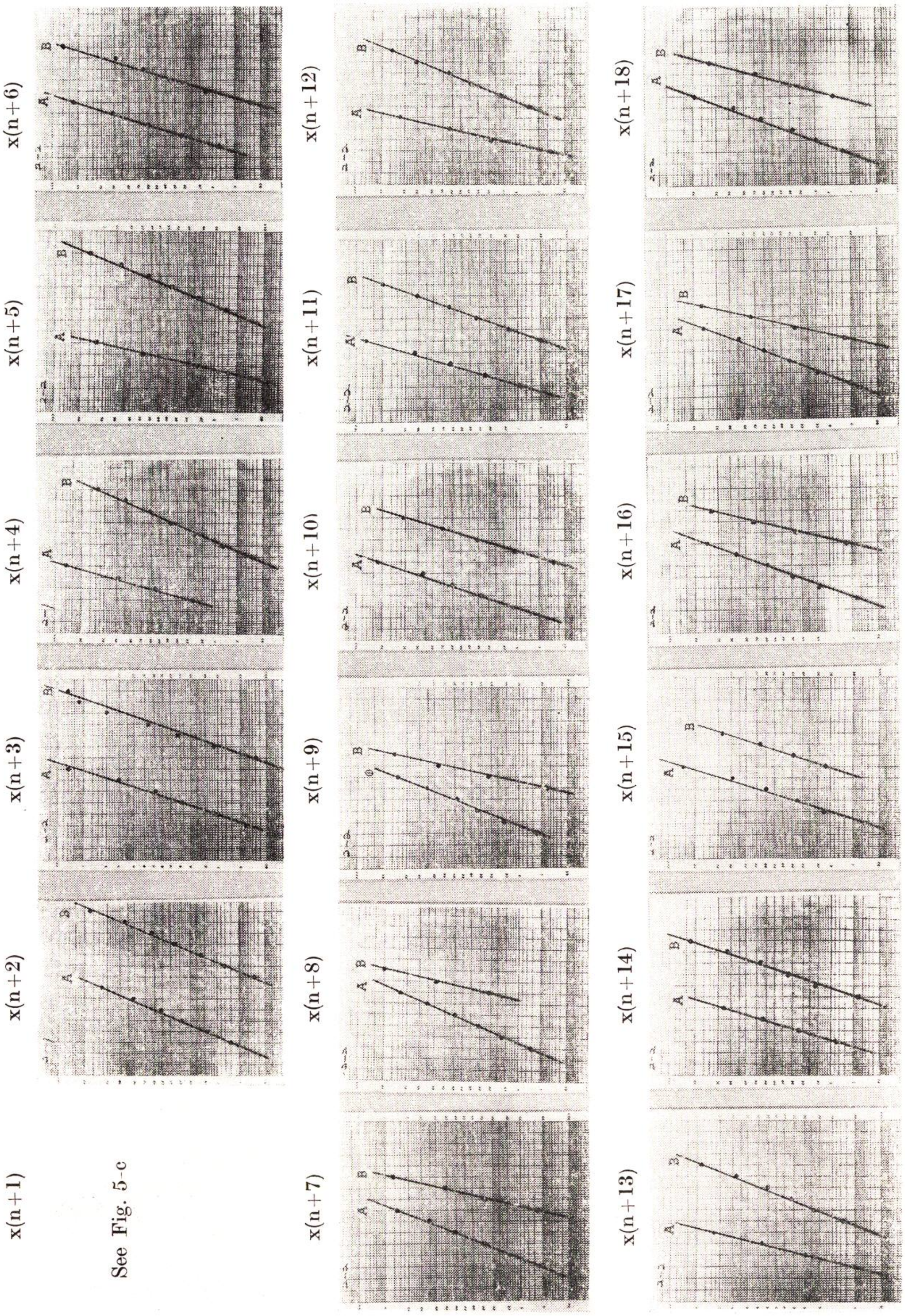

Fig. 6-b Angle of roll

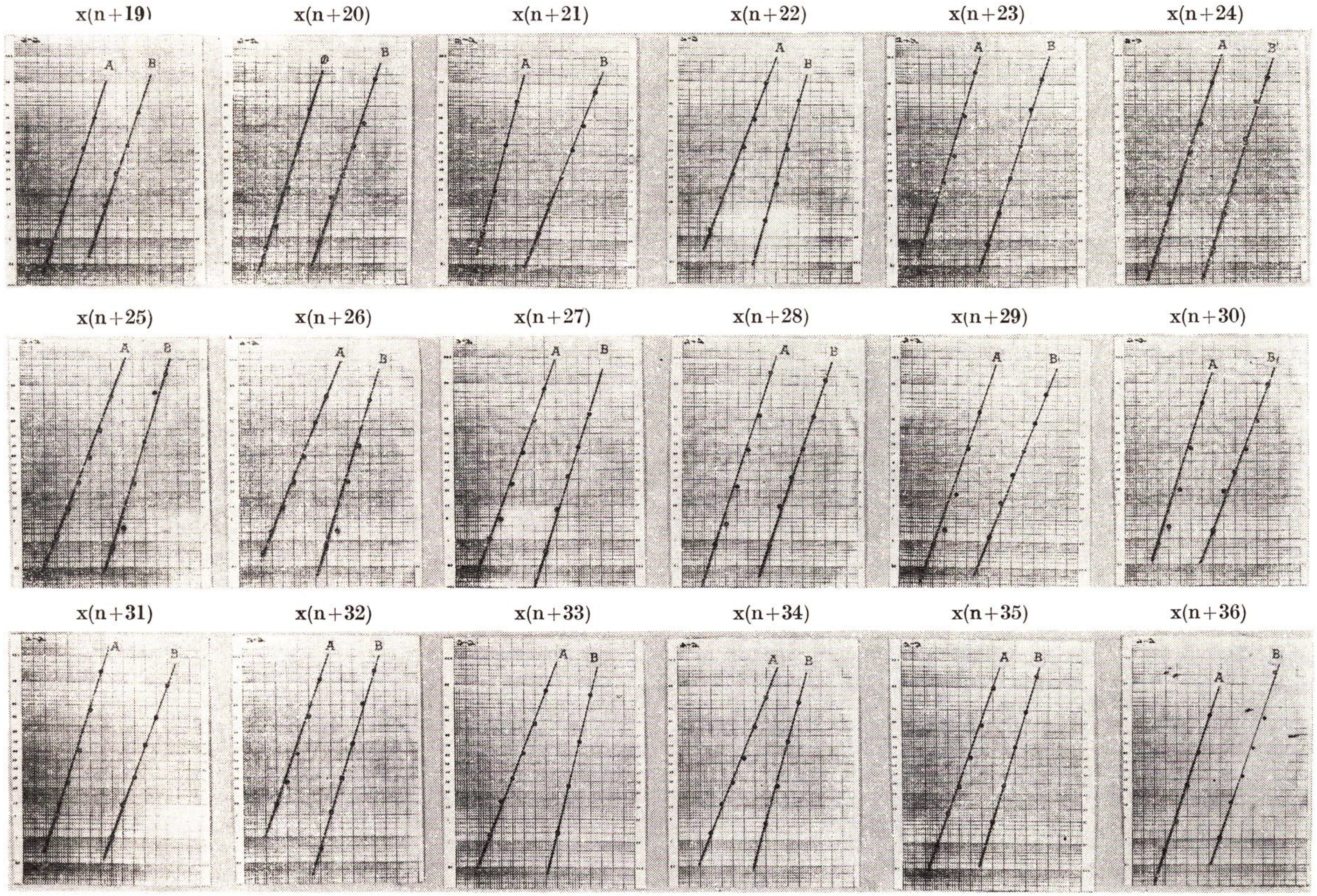

Fig. 6-b Angle of roll

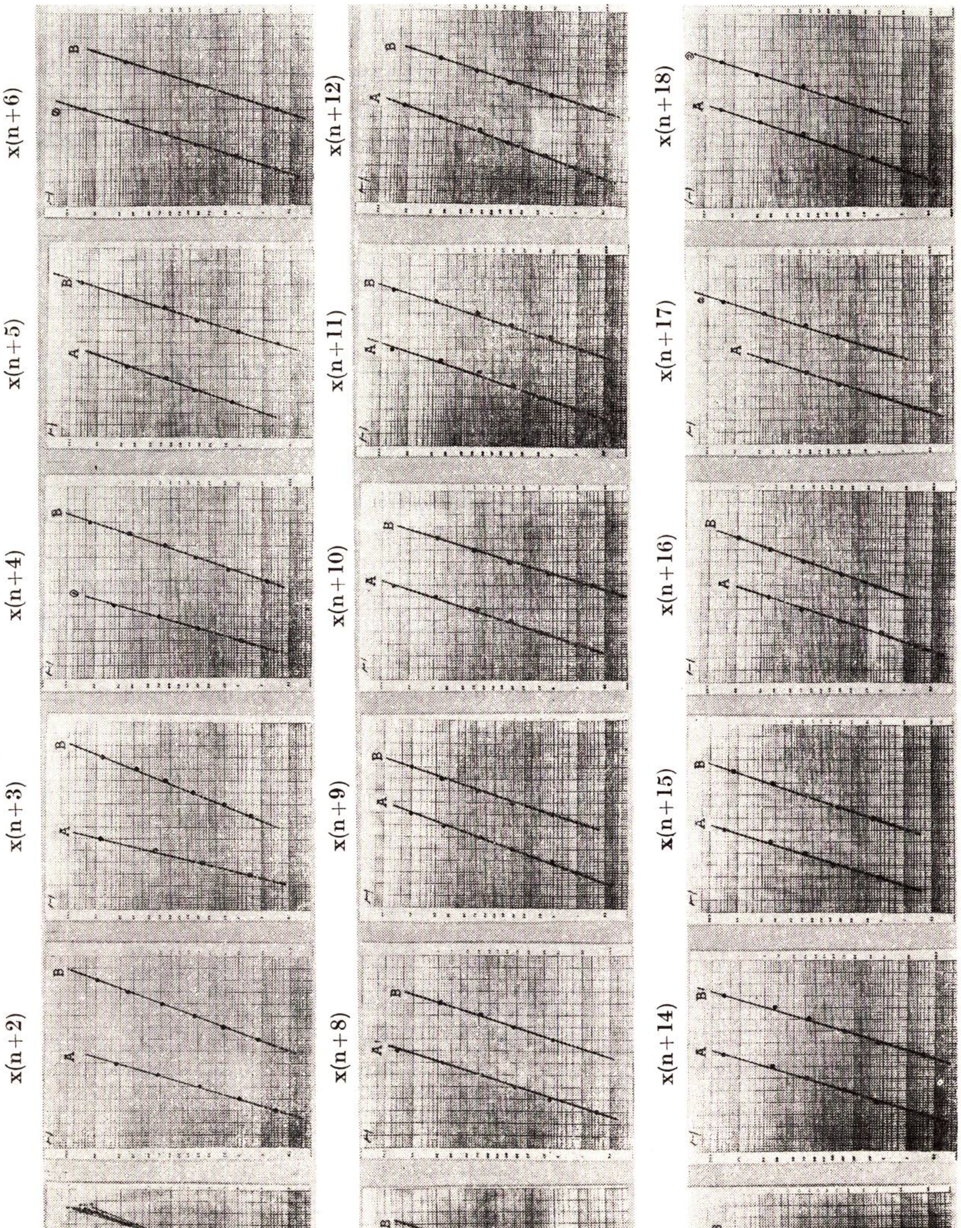

Fig. 6-c Angle of pitch

4-c. Also, the distributions are shown by Gaussian probability papers in Figure 5. However, from practical points of view, the method can be simplified by using distribution for each lines as shown by the beltlike, shaded area of Figure 4-a.

The distributions of these simplified method, based on Gaussian probability papers are shown in Figure 6. The alphabetical characters for lines in the Figures correspond to equations (A) and (B) noted above.

Since the two dimensional distributions for angles of roll and pitch and for wave heights in the sea are approximately Gaussian in form, then, the joint probability density functions can be given by the following equation:

$$p(X(i\Delta t),\ X(i+\tau)\Delta t) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}\exp\left[-\frac{1}{2(1-\rho^2)}\right.$$
$$\times\left\{\left(\frac{X(i\Delta t)-m_1}{\sigma_1}\right)^2 - \frac{2\rho\,(X(i\Delta t)-m_1)\,(X((i+\tau)\Delta t)-m_2)}{\sigma_1\sigma_2}\right.$$
$$\left.\left. + \left(\frac{X((i+\tau)\Delta t)-m_2}{\sigma_2}\right)^2\right\}\right] \tag{2}$$

where; m_1; mean for $x(i\Delta t)$, m_2: mean for $x((i+\tau)\Delta t)$, σ_1^2: variance for $x(i\Delta t)$, σ_2^2: variance for $x((i+\tau)\Delta t)$, $i=1, 2, 3 \ldots, M.$, ρ: correlation coefficient for $x(i\Delta t)$ and $x((i+\tau)\Delta t)$.

The covariance and variance for the data are given in Tables 2 and 3.

Table 2 Means and variance

Lag	Angles of roll*				Angles of pitch**			
	A		B		A		B	
	Means	Var.	Means	Var.	Means	Var.	Means	Var.
1	−0.17	0.8464	−0.30	0.4225	−0.60	7.6176	−0.52	1.2544
2	−0.30	0.2304	−0.30	0.3025	−0.80	3.6864	−0.80	4.0000
3	−0.30	0.1225	−0.30	0.3600	−0.56	1.5625	−1.20	3.6864
4	−0.28	0.1806	−0.25	0.2500	−0.40	2.5600	−1.00	6.7600
5	−0.25	0.2756	−0.20	0.2256	−0.22	0.4489	−1.20	6.2500
6	−0.23	0.2256	−0.23	0.2256	−0.72	3.3856	−1.04	3.2400
7	−0.28	0.1936	−0.30	0.2500	−0.80	5.7600	−0.96	1.9600
8	−0.30	0.2162	−0.28	0.2304	−0.88	6.7600	−0.88	1.9600
9	−0.25	0.3025	−0.20	0.2601	−0.80	5.8564	−0.60	1.9600
10	−0.30	0.2601	−0.21	0.2401	−0.88	4.0000	−0.88	3.3856
11	−0.30	0.2401	−0.30	0.2209	−0.88	3.0976	−0.88	4.6656
12	−0.32	0.2304	−0.31	0.2401	−0.40	2.3716	−1.04	5.9536
13	−0.30	0.2916	−0.28	0.2500	−0.76	2.3104	−1.20	5.7600
14	−0.25	0.2025	−0.25	0.2025	−0.76	3.0976	−1.20	4.0000
15	−0.25	0.2209	−0.25	0.2209	−1.08	3.5344	−1.00	3.8416
16	−0.20	0.2401	−0.22	0.2401	−0.48	4.8400	−0.80	2.3104
17	−0.20	0.2500	−0.25	0.2304	−0.80	5.7600	−1.00	1.9600
18	−0.30	0.2401	−0.30	0.2401	−1.28	4.8400	−1.00	2.6896
19	−0.30	0.2500	−0.22	0.2401	−0.72	4.0000	−1.08	4.4944
20	−0.25	0.2500	−0.25	0.2401	−0.88	3.5344	−1.00	4.0000

* Angles of roll; Ex. No. 3 Roll

** Angles of pitch; Ex. No. 5 Pitch

Table 3 Covariance for ship motions

Lag	Angles of roll	Angles of pitch	Lag	Angles of roll	Angles of pitoh
	Ex. No. 3	Ex. No. 5		Ex. No. 3	Ex. No. 5
0	4.2157	0.9478	19	0.0069	0.0942
1	2.9427	0.3996	20	−1.1797	0.0545
2	0.1312	−0.2484	21	−1.6892	−0.0921
3	−2.4437	−0.4828	22	−1.2501	−0.2161
4	−3.3729	−0.2252	23	−0.1345	−0.1648
5	−2.3825	0.0109	24	1.0128	0.0535
6	−0.3000	0.0291	25	1.5641	0.2496
7	1.6748	−0.0391	26	1.2650	0.2540
8	2.6040	0.0124	27	0.2994	0.0995
9	2.1817	0.0637	28	−0.8197	−0.0350
10	0.7798	0.0927	29	−1.5262	−0.0568
11	−0.8293	0.0807	30	−1.4254	−0.0335
12	−1.8764	0.0706	31	−0.5195	−0.0401
13	−1.9260	0.0296	32	0.7289	−0.0765
14	−1.0451	−0.0063	33	1.6029	−0.0703
15	0.2957	−0.0109	34	1.5611	−0.0322
16	1.4208	0.0028	35	0.6184	0.0551
17	1.7870	0.0185	36	−0.6569	0.1448
18	1.2150	0.0552			

Discussion

There are many applications in which knowledge of the frequency distributions of motion and wave height can be used to advantage. A few examples are (1) description of the statistical properties of ship motion, (2) estimation of the maximum values of ship response to wave, and (3) the use of the method for simplified computation of correlograms. As for item (3), as long as the ship's motions are due to ocean wave action, the simplified method for calculations of correlograms can be used. The author has already used this method to solve ship's problems.[8]
If the characteristics of the ship's response to waves are considered to be both a second order process and a Gaussian stationary process[9], then use of the simplified method for calculation of correlograms will provide a fairly good fit for the data. This is an important characteristics in the studies of ship motion at sea.

Conclusion

These conclusions are based on observations of the relation of ocean waves to ship's motion at sea and from the above discussion, the following conclusions have been reached:
(1) It is considered that the distributions for the data used in this experiment are Gaussian in form ;(2) For one sample from these experiments, it is shown that the two dimensional distributions are Gaussian in form; and (3) From the statistical properties of ship motions due to wave action, the covariance and the mean can be used as parameters to represent the characteristics of random motions of a ship at sea.

Acknowledgements

The data presented in this paper were obtained as the results of the cooperative effort of the entire crew of T.S. Oshoro Maru. The author express his hearty thanks to Dr. M. Huzii and to Dr. Y. Yamanouchi for their helpful comments and advice. Thanks are also due to Mr. K. Amagai and Miss E. Ohtaki for processing for all of the needed to obtain the numerical results.

Summary

The main objective of this paper is to define the statistical properties of ship motion using records collected during a cruise of T.S. Oshoro Maru.

Data for analysis were sampled from the records of motion using the following constants: (1) The time interval between adjacent data values was 1 second, and (2) the number of observations was 300.

From these data, (1) statistical distributions for various conditions are calculated according to Gaussian (or Normal) probability theory; (2) two dimensional time-series distributions for ship's motions are estimated according to Gaussian probability papers; (3) all distributions can be represented by Gaussian probability density functions; and (4) the variance for ship motion and wave height defines the characteristics of a ship's motion at sea.

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