FUNDAMENTAL STUDIES ON THE INTERNAL PRESSURE BENT TUBE FOR HOLDING NET FORM–I

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Introduction

Until now the opening and spreading of the fishing net and the holding of the net form have been solved as functions on the relative forces between the floats and the sinkers in fluid and the resistance-forces of the net weight, the rope, etc.

When a fire hose or a home vinyl hose is under water pressure, its flattened parts swell up firmly to form a fine round hose, and then if a small force shold be applied from the outside, its form will not be altered. When a bicycle tube is filled with compressed air, it becomes a circle and we can ride easily, but if it loses its air it limps again. Evidently these well known phenomena, are concerned with internal pressure.

The purpose of this study is to get basic data on the internal pressure bent tube (we shall use the term 'bent tube' for short hereafter) in order to apply them to the opening, spreading and holding of the fishing net form.

To obtain a fully pressurized bent tube, in the first place internal pressure is brought into the bent tube; little by little limp parts swell up to get a fine curved bent tube. Then we must think of a means to hold the bent tube form.

Therefore we set up the problem points of this study as follows.

I) Outward normal force of the bent tube

II) Relation between the internal pressure and the flection of the bent tube

III) The buckling of the bent tube

From a practical point of view I), II) are concerned with the holding-force of the net form, III) is concerned with the opening-force and spreading-force of the net.

In this paper we describe theoretical consideration and experiments of I) and II). III) will be discussed in a subsequent paper.

In future, if we use the bent tube together with a roundhaul net and a haul sein net, it will quicken fishing work. It will go a step forward if we can set up an automatic device for internal compression, then fishing will be more functional than today.

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Theory

I. Outward normal force of the bent tube

When pressure p acts on the whole internal surface of a Bourdon tube, its elementary ring $P_1Q_1Q_2P_2$ recieves the outward normal force ΔF. Sunatani[1] obtained this ΔF according to Green's theorem:

$$\Delta F = p\, d\varphi \oint (R + x) \cos (x, n)\, dm = p\, A\, d\varphi\,, \tag{1}$$

A being the cross-sectional area of the hollow part of the tube, R the radius of curvature of the tube, φ the included angle of the tube end faces, n the normal unit vector, dm the elemental length of the internal circumference of the cross-section.

We should obtain the holding-force of the bent tube, namely, the holding-force of the net form, by geometrically obtaining the same ΔF and slightly inquiring about the reasons of occurrence of the outward normal force of the bent tube.

Fig. 1-A shows a part of the section of the bent tube. The total outward normal force ΔF of the elemental ring $P_1Q_1Q_2P_2$ cut by two cross-sections P_1Q_1 and P_2Q_2 could be obtained as follows. When cross-section Q_2R is cut parallel to cross-section Q_1P_1, through point Q_2, the outward and inward normal forces of the part $P_1Q_1Q_2R$ are equal, therefore we may take into account only the triangular part RQ_2P_2. Fig 1-B shows the same enlarged part RQ_2P_2. T_1,S and T_2

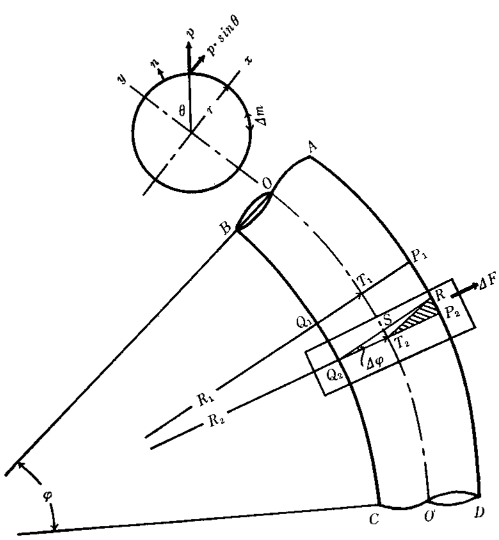

Fig. 1-A. Figure of tube $P_1Q_1Q_2P_2$ receiving the normal force ΔF caused by internal pressure p in the tube

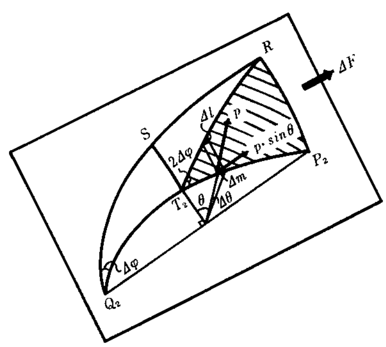

Fig. 1-B. Enlarged figure of the part RQ_2P_2 of the Fig. 1-A

become the intersecting points on the y-axis of the sectional circumference of the tube at the cross-sections Q_1P_1, Q_2R and Q_2P_2 of the tube when the x-axis is in direction of the tube cross-section and the y-axis is vertical to it. The resultant outward normal force ΔF, caused by the internal pressure p acting on the segment RP_2T_2, are calculated as follows.

Let us put l parallel to the center axis on the tube wall, and $\Delta\theta$, Δl and Δm sufficiently small, the outward normal force ΔF of this small part shows that,

$$\Delta f = p \sin(\theta + \Delta\theta)\,(l + \Delta l)\,(\Delta m)\,,$$

here we can put

$$\Delta m = r\,d\theta\,, \quad l + \Delta l = 2\,r\,\Delta\varphi \sin(\theta + \Delta\theta)\,,$$

thus we have

$$\begin{aligned}\Delta f &= p \sin(\theta + \Delta\theta)\left\{2\,r\,\Delta\varphi \sin(\theta + \Delta\theta)\right\} r\,\Delta\theta \\ &= 2\,r^2\,p\,\Delta\varphi \sin^2(\theta + \Delta\theta)\,\Delta\theta\,,\end{aligned}$$

there the resultant outward normal force ΔF is obtained by integrating clockwize from zero to $\pi/2$ with respect to angle θ, provided that y-axis is angle zero, we have

$$\Delta F = 2\,r^2\,p\,\Delta\varphi \int_0^{\pi/2} \sin^2\theta\,d\theta = \frac{1}{2}\,\pi\,r^2\,p\,,$$

here πr^2 is the cross-sectional area, let us put $\pi r^2 = A$, we have

$$\Delta F = \frac{1}{2}\,A\,p\,d\varphi\,.$$

Since we get the same one as RP_2T_2 symmetrical to P_2Q_2 axis, we double the above equation and finally we have the next equation:

$$\Delta F = A\,p\,\Delta\varphi\,. \tag{2}$$

Generally when the included angle of the tube end faces is φ, we have from the above equation

$$F = A\,p\varphi, \tag{3}$$

and as for the closed tube, $\varphi = 2\pi$, we have

$$F = 2\,\pi\,\mathrm{A}\,p\,. \tag{4}$$

Next let us consider the case of the tube in Fig. 2 where the tube is in-curved in some parts and cross-sections varied parts. From (3) at each part the resultant outward normal force is,

	at part	$ABGH$	$A\,p\,a\,,$
	at part	$BCFG$	$-A\,p\,b\,,$
and	at part	$CDEF$	$A\,p\,c\,,$

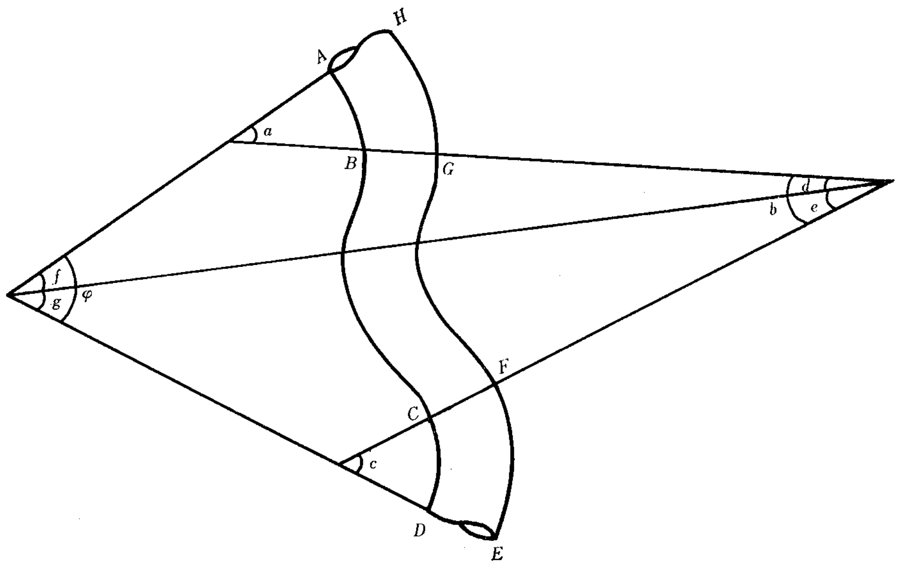

Fig. 2. Explanatory figure of the relation between the tube form and the included angle of the end faces

therefore collectively,

$$A\,p\,(a+c-b)\,,$$

here let us put $b=d+e$ and $\varphi=f+g$, we have

$$a+c=e+d+f+g=b\,,$$

hence this can be written $a+c-b=\varphi$, therefore we have

$$A\,p\,(a+c-b)=A\,p\,\varphi\,.$$

Therefore we know that, if the tube has critical parts, that is, if the included angle of the tube end face is given, the outward normal force of the whole tube is given by equation (3).

II. Outward normal force of each part of the bent tube

In the preceding section we saw that the outward normal force produced by internal pressure acting on the intrnal wall of the bent tube is $F=Ap\varphi$. Then how to express the outward normal force dimensions on each part of the tube ?

Generally each part of the curvature of the tube being different, the outward normal force dimensions are also different whether the curvature is large or small. The equation (3) doesn't show the outward normal force dimensions but the outward normal force of the whole tube. In regard to the holding of the net form, to say nothing of the outward normal force of the whole tube, we must consider the external force acting locally on the tube parts.

If we consider that each tube part forms a part of a continuous matter of the real circle with the differential curvature and express the outward normal force dimensions of the bent tube by which $2\pi Ap$ (total outward normal force of the closed bent tube) is divided by the circumference of the real circle, we could compare the external force acting on the tube parts with the holding force of the tube.

At the cross-sections P_1Q_1 and P_2Q_2 in Fig. 1-A, let us put R_1 and R_2, several radii of curvature, the several outward normal force dimensions $f(R)$ could be expressed respectively in linear densities

$$f(R_1) = A\,p/R_1\,,$$
$$f(R_2) = A\,p/R_2\,,$$

generally,

$$f(R) = A\,p/R\,. \tag{5}$$

III. Relation between the internal pressure and the flection of the bent tube

When the external force acts on the bent tube, how far does the flection of the tube, namely, how far the deformation of the net form is a factor of conclusion to choose what sort of bent tube (e.g. matter, thickness, size, length and internal pressure of the tube) could be holding the net form under outside conditions (wave, net-weight etc.).

Here, we can solve the bent tube flection by the equation of the curved beam flection. On the curved beam flection, let us consider the adequate x-y coordinates being the origin at the center axis of the beam. The flections to x and y directions of the center axis spontaneous point $F(x,y)$, δx and δy are expressed as follows:

$$\begin{aligned}\delta x &= \int (y_1 - y)\left[\frac{N}{AE} + \frac{M}{AER} + \frac{M}{AEkR}\right] d\theta + \int \left(\frac{N}{AE} + \frac{M}{AER}\right) dx\,,\\ \delta y &= -\int (x_1 - x)\left[\frac{N}{AE} + \frac{M}{AER} + \frac{M}{AEkR}\right] d\theta + \int \left(\frac{N}{AE} + \frac{M}{AER}\right) dy\,,\end{aligned} \tag{6}$$

where A is the cross-sectional area, E the modulus of longitudinal elasticity, k the form factor, M the moment, R the radius of curvature, N the vertical force to cross-section and W the load.

Here, if the diameter is less than the radius of curvature and the curved variation is small, the equation (6) could be written as follows[2]:

$$\begin{aligned}\delta x &= \int (y_1 - y)\,\frac{M}{AEkR}\,d\theta + \int \frac{N}{AE}\,dx\,,\\ \delta y &= -\int (x_1 - x)\,\frac{M}{AEkR}\,d\theta + \int \frac{N}{AE}\,dy\,.\end{aligned} \tag{7}$$

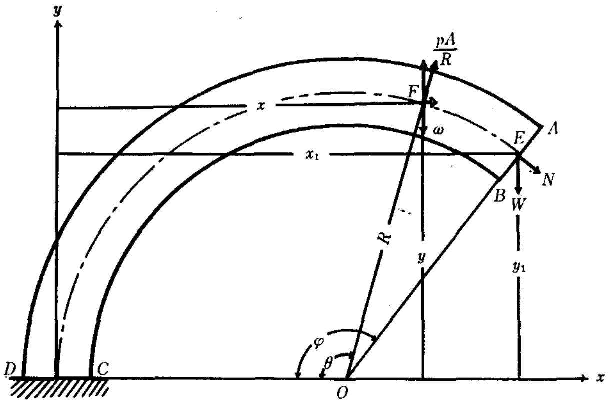

Fig. 3. Figure of the standing bent tube that the load W and the internal pressure p acting on the tube wall

Fig. 3 is an illustration of one end fixed bent tube of which the origin of x-y coordinates is put on the center axis of the fixed end. When the load W acts on the free end face AB, the vertical force N acts on the tube wall. The outward normal force Ap/R of the internal pressure and the self-weight ω acts on the whole center axis. The flection to the y-axis direction δy of the point E (x_1,y_1) on the free center axis end is calculated by the equation (7). That is, since the moment M of the spontaneous point $F(x,y)$ on the center axis is summed up, the monent of the load W, of the self-weight ω, of the outward normal force Ap/R and of the internal pressure p acting on the free end face AB; if we obtain separately each moment and finally sum them up, we could obtain the moment M of the equation (7), whose positive moment is running clockwize.

(1) The moment caused by the load W:

$$M = W(x_1 - x),$$

here, $x_1 - x = R(\cos\theta - \cos\varphi)$, hence

$$M = W R(\cos\theta - \cos\varphi). \tag{8}$$

(2) The moment caused by the self-weight ω:

We know that the tube mass Z from the coordinates $F(x,y)$ to the coordinates $E(x_1,y_1)$. If the center axis curve is the function $y=f(x)$ and the linear density is $\rho(x)$, is read as follows;

$$Z = \int_x^{x_1} \rho(x)\sqrt{1 + \{f'(x)\}^2}\,dx$$

and the coordinates $G(X,Y)$ of the center of gravity gives

$$X = \frac{1}{Z}\int_x^{x_1} \rho(x)\sqrt{1 + \{f'(x)\}^2}\,x\,dx,$$

$$Y = \frac{1}{Z}\int_{x}^{x_1} \rho(x)\sqrt{1+\{f'(x)\}^2}\, y\, dx\,.$$

Now this case, since we can put as follows:

$$\rho(x) = \omega\,,$$
$$f(x) = \sqrt{R^2-(x-R)^2}\,,$$
$$x = R(1-\cos\theta)\,,$$
$$y = R\sin\theta\,,$$
$$\{f'(x)\}^2 = \operatorname{cosec}\theta\,,$$
$$dx = R\sin\theta\, d\theta\,,$$

thus

$$Z = \int_{x}^{x_1} \rho(x)\sqrt{1+\{f'(x)\}^2}\, dx = \int_0^{\varphi} \omega \operatorname{cosec}\theta\, R\sin\theta\, d\theta = \omega R(\varphi - \theta)\,,$$

$$X = \frac{1}{Z}\int_{x}^{x_1} \rho(x)\sqrt{1+\{f'(x)\}^2}\, x\, dx$$

$$= \frac{1}{\omega R(\varphi-\theta)}\int_0^{\varphi} \omega \operatorname{cosec}\theta\, R(1-\cos\theta)\, R\sin\theta\, d\theta$$

$$= \frac{R}{(\varphi-\theta)}(\varphi-\theta-\sin\varphi+\sin\theta)\,,$$

therefore the moment of the self-weight ω becomes

$$M = Z(X-x) = \omega R^2(\varphi-\sin\theta-\theta+\sin\theta) - \omega R^2(\varphi-\theta)(1-\cos\theta)\,. \quad (9)$$

(3) The sum of the moment caused by the normal force Ap/R and the moment caused by pressure acting on the tube free end face is zero as Sunatani[1)] obtained. Therefore the moment M finally is the sum of the moment caused by load W and the moment caused by self-weight ω. Namely, in addition to (8) and (9), we can obtain the following:

$$M = WR(\cos\theta-\cos\varphi) + \omega R^2(\varphi-\sin\varphi-\theta+\sin\theta) - \omega R^2(\varphi-\theta)(1-\cos\theta)\,. \quad (10)$$

Next N, in (7), is the vertical components of the internal pressure Ap and the load W acting on the tube free end face to the cross-section:

$$N = Ap + W\cos\varphi\,. \quad (11)$$

If we put (10) and (11) into (7) and integrate each term from zero with respect to angle θ, we can obtain the flection to y-axis direction of the coordinates $E(x_1, y_1)$.

Consequently those are;

$$\int_0^y \frac{N}{AE}\,dy = \frac{R}{AE}\int_0^{\varphi}(A\,p + W\cos\varphi)\cos\theta\,d\theta$$

$$= \frac{R}{AE}(A\,p + W\cos\varphi)\sin\varphi, \tag{12}$$

$$-\int_0^r (x_1 - x)\frac{M}{AEkR}\,d\theta = \frac{R}{AEkR}(\cos\theta - \cos\varphi)$$

$$\times\left\{\int_0^{\varphi}(W R - W R\cos\varphi - R^2\omega\varphi\cos\theta + \omega R^2\theta\cos\theta)\,d\theta\right\}$$

$$= \frac{1}{AEkR}\Big(\frac{3}{4}W R^2\sin 2\varphi - W R^2\varphi\cos^2\varphi - \frac{1}{2}W R^2\varphi$$

$$-\frac{3}{4}\omega R^3 - \frac{1}{4}\omega R^3\varphi^2 - \frac{1}{2}\omega R^3\varphi\sin 2\varphi + \omega R^3\sin^2\varphi$$

$$-\frac{5}{4}\omega R^3\cos^2\varphi + 2\,\omega R^3\cos\varphi\Big), \tag{13}$$

there

$$\delta y = \frac{R}{AE}(A\,p + W\cos\varphi)\sin\varphi + \frac{1}{AEkR}\Big(\frac{3}{4}W R^2\sin 2\varphi$$

$$-W R^2\varphi\cos^2\varphi - \frac{1}{2}W R^2\varphi - \frac{3}{4}\omega R^3 - \frac{1}{4}\omega R^3\varphi^2$$

$$-\frac{1}{2}\omega R^3\varphi\sin 2\varphi + W R^3\sin^2\varphi - \frac{5}{4}\omega R^3\cos^2\varphi + 2\,\omega R^3\cos\varphi\Big). \tag{14}$$

Then the dimensionless from factor is determined by the tube form[3] (see Fig. 4),

$$k = \frac{1}{r_2^2 - r_1^2}\left[r_2^2\left\{\frac{1}{4}\left(\frac{r_2^2}{R}\right)^2 + \frac{1}{8}\left(\frac{r_2^2}{R}\right)^4 + \cdots\cdots\right\}\right.$$

$$\left.-r_1^2\left\{\frac{1}{4}\left(\frac{r_1^2}{R}\right)^2 + \frac{1}{8}\left(\frac{r_1^2}{R}\right)^4 + \cdots\cdots\right\}\right], \tag{15}$$

where r_1 is the internal radius, r_2 the external radius and the of curvature[3].

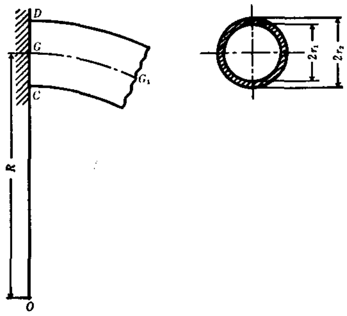

Fig. 4. Explanatory figure for the form factor k

Let us put that $\delta y(p)$ is a flection caused by only the internal pressure p and pick up the term involved p from the equation (14), we have

$$\delta y\,(p) = \frac{p\,R}{E}\sin\varphi\,. \tag{16}$$

The equation (16) shows that the flection at the tube end is caused by the internal pressure p under condition with the one end fixed bent tube on the water-surface, ignoring the tube self-weight and the water resistance. The cross-sectional area A of the tube is not included in the equation (16), namely, as a result of the bent tube flection caused by the internal pressure p was unconcerned with cross-sectional area A.

On the flection of the closed bent tube, we must think of the ring of the closed bent tube. In the present paper, we shall not try to solve the flection of the ring, but we can anticipate, that the flection of the closed tube caused by internal pressure should be zero, in other words the intrnal pressure p should not be concerned in the flection of the closed tube. This thing may be easy to anticipate by reason of the total vector of the circumference with respect to the linear density Ap/R of the normal force caused by the intrnal pressure when it is zero, that is,

$$\frac{A\,p}{R}\oint dS = 0\,, \tag{17}$$

therefore, in the case of the closed tube, $2\pi Ap$, which previously we theoretically obtained at the expression (4), shows the normal force acting on the tube wall caused by the internal force. At least let us call it the holding force of the bent tube.

Next if we put in (14) the constants A,p,k,ω,W and φ, we shall measure δy by experiment, in opposition we could obtain the modulus of longitudinal elasticity E. From the expression (7), we have

$$E = -\frac{1}{A\,\delta y}\left\{\frac{1}{kR}\int (x_1 - x)\, M\, d\theta - \int N\, dy\right\}. \tag{18}$$

The form factor k in (15) and the modulus of longitudinal elasticity E being the characteristic values of the bent tube, we can choose the bent tube having k and E to answer our purpose.

Methods and Materials

As an experimental tube, we used a bicycle tube ($26\times 1\frac{3}{8}$, B/E, JIS K 6304, JIS K 6305), see Fig. 5. The tube self-weight is 1.1 g/cm, the thinkness is 1.0 mm and the stopper load of the tube end is 7.0 g. Fig. 6 shows the relation between the stress and strain of the experiment tube using the tensile testing machine (Type

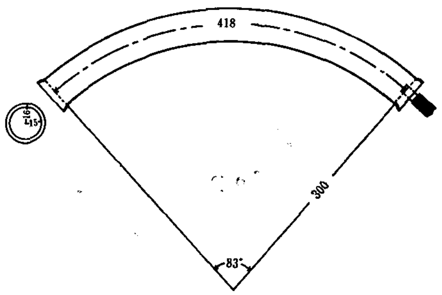

Fig. 5. Schematic figure of experimental tube

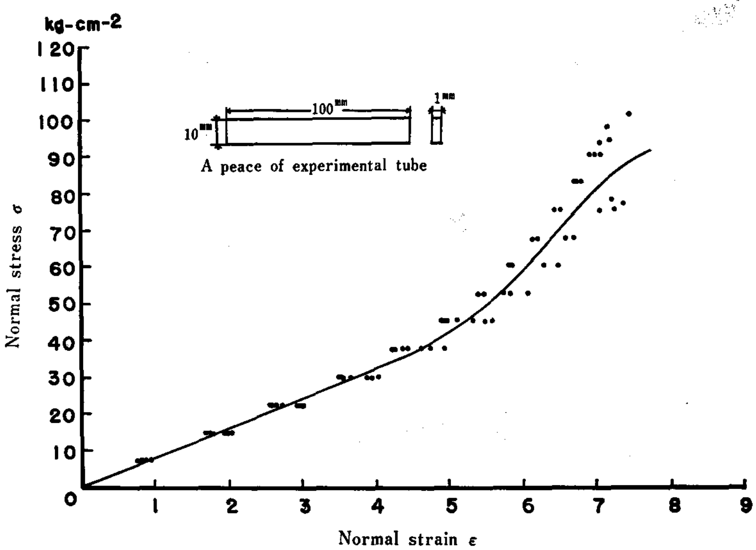

Fig. 6. Relation between stress and strain of the experimental tube

SH 100 kg). From this graph we obtained aboht 8000 g/cm² as modulus of longituidinal elasticity E_0.

The apparatus used in the experiment is shown in Fig. 7. The pressure gauge was at the maximum range of 0.2 kg/cm² (BT×100×0.2). In the experiment we first put air into the tube by compressor, switched off the compressor motor, loaded on the tube end and then photographed the tube form and the gauge index with a camera (see Fig. 8.). Varying the pressure within the tube, we repeated as

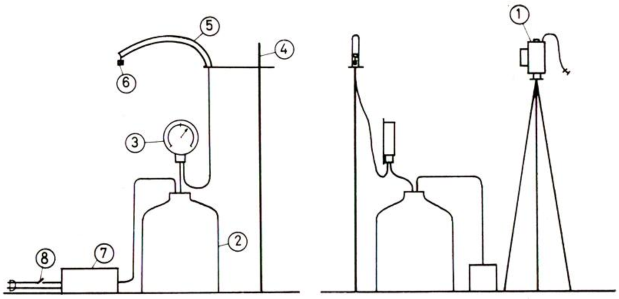

Fig. 7. Schematic figure of experimental apparatus
1: Camera 2: Air tank 3: Pressure gauge 4: Stand
5: Tube 6: Load 7: Compressor 8: Switch

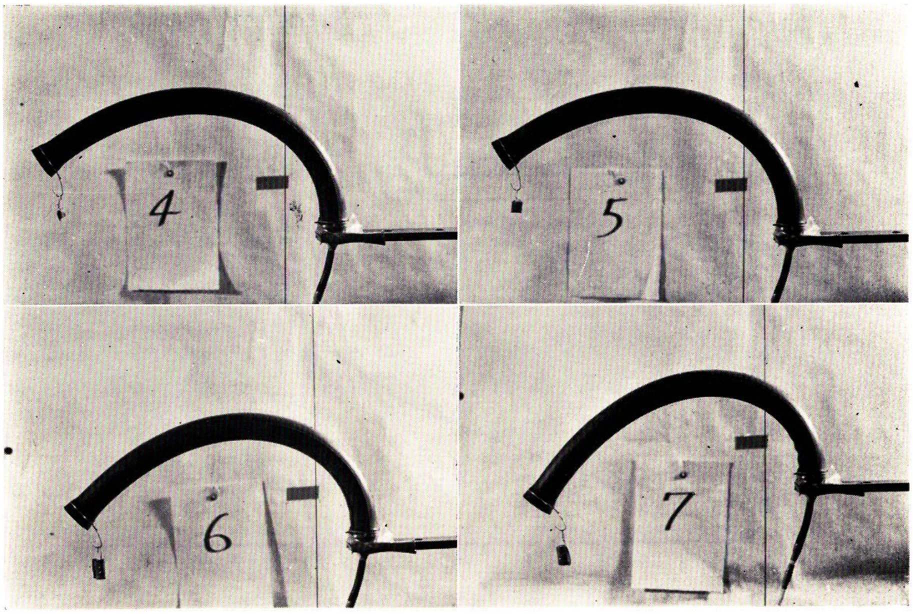

Fig. 8. Examples of tube bending under the internal pressurs p=164 g/cm²

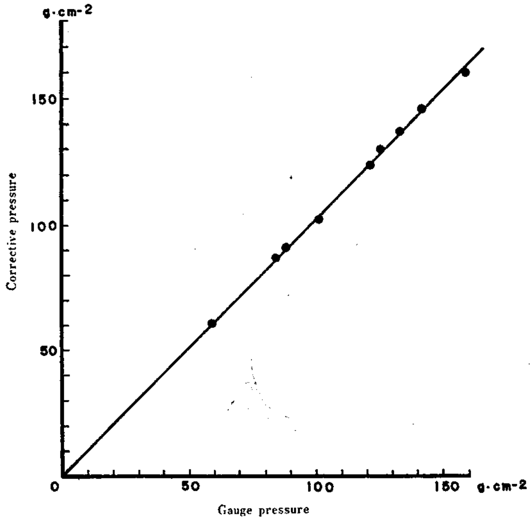

Fig. 9. Correction of gauge pressure

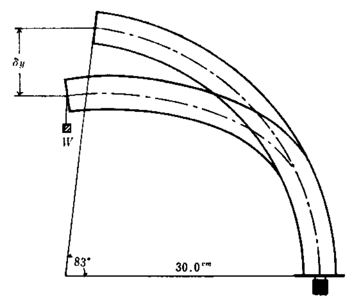

Fig. 10. Method to measure the flection

above. The correction of the gauge pressure is shown in Fig. 9. There was hardly a difference in correction on different days.

Then we magnified the negative film, and we read the flection of the tube end and then the index of the pressure gauge. For measuring the flection, we have drawn ready beforehand the tube form under conditions without suspending the load, the self-weight and the internal pressure, that is, we drew an arc of a radius of 30 cm with the center angle of 83°, and projected the above negative film on it, and we measured the traveling length of the tube end (see Fig. 10).

Next we obtained flections caused by internal pressure only. Previously we had fixed the one end of the bent tube filled by compressed air, set it on the water surface in a basin, the tube self-weight being negligible, we then obtained the flections of the tube free end under several pressures.

Results and Discussion

We measured the flections of the tube end as described above. Next, we obtained the modulus of longitudinal elasticity E by using these flections.

We put into the expression (18) the flection δy, then the internal pressure p and the load W, and calculated the modulus of the longitudinal elasticity E. Result shown on Table 1. So calculation constants were as follows, $\varphi=83°$, $R=30.0$ cm, $A=8.0$ cm^2, $\omega=1.1$ g/cm, $W=7.0$ g and $k=0.0013$ (from the equation (15)). We used the empirical formula of E and p by the method of least squares (see Fig. 11).

Table 1. Tube flections and modulus of longitudinal elasticity

No	p (g.cm^{-2})	W (g)	$-\delta y$ (cm)	$\frac{1}{kR}\int(x_1-x)M\,d\theta$ (g. cm)	$-\int N\,dy$ (g. cm)	$\frac{1}{kR}\int(x_1-x)M\,d\theta - N\,dy$ (g. cm)	$-A\,\delta y$ (cm^{-1})	E (g.cm^{-2})
1	164	7.0	19.0	274×10^3	—391×10^2	235×10^3	152	155×10
2	〃	8.4	20.5	293	〃	253	164	154
3	〃	9.4	21.4	306	〃	267	171	155
4	〃	12.3	23.4	344	〃	305	187	162
5	159	7.0	18.5	274	—379	236	148	160
6	〃	8.4	20.4	293	〃	255	163	156
7	〃	9.4	21.4	306	〃	268	171	156
8	157	7.0	19.4	274	—374	237	155	152
9	154	7.0	19.7	274	—367	238	157	151
10	〃	8.4	20.7	293	〃	256	165	154
11	〃	9.4	21.5	306	〃	269	172	156
12	〃	12.3	23.7	344	〃	307	189	162
13	133	7.0	20.7	274	—317	243	165	146
14	〃	8.4	22.7	293	〃	261	181	143
15	〃	9.4	23.2	306	〃	274	185	147
16	〃	12.3	25.7	344	〃	312	205	151
17	113	7.0	22.0	274	—269	247	176	140
18	〃	8.4	23.2	293	〃	266	185	143
19	〃	9.4	24.0	306	〃	279	192	145
20	〃	12.3	27.5	344	〃	317	220	144
21	103	7.0	22.7	274	—245	250	181	137
22	〃	8.4	24.2	293	〃	268	193	138
23	〃	9.4	25.7	306	〃	281	205	136
24	101	8.4	24.0	293	—241	269	192	140
25	99	7.0	23.0	274	—236	251	184	136
26	97	7.0	23.2	274	—231	251	185	135

$$E = 3.1\,p + 1064 \ (\text{g/cm}^2)\,. \tag{19}$$

Now, we should consider the phsical mean of the constant 1064 (g/cm^2). Calculating the modulus of longitudinal elasticty by using (18), we put A the cross-sectional area of the bent tube like the column. Here, we should consider the cross-sectional area A_t of the tube wall except by the cross-sectional area of the hollow of the tube, and we compare A with it, that is,

$$\frac{A}{A_t} = \frac{\pi\,(1.6)^2}{\pi\left\{(1.6)^2-(1.5)^2\right\}} \fallingdotseq 8.2\,. \tag{20}$$

Then if we make the constant 1064 (g/cm^2) in the expression (19) times 8.2, we have 8.7×10^3 (g cm^2). This 8.7×10^3 (g/cm^2) could fit likely for E_0=8.0×10^3 (g/cm^2) that was obtained at the tensile test of the experiment tube. Namely, when we would use the equation (7) that shows the flection of the curved beam, as the modulus of longitudinal elasticity E, by using the whole tube cross-sectional area A or the tube wall cross-sectional area A_t, we must distinguish whether E is given by the equation (19) or is given by the tensile test. Accordingly, if we

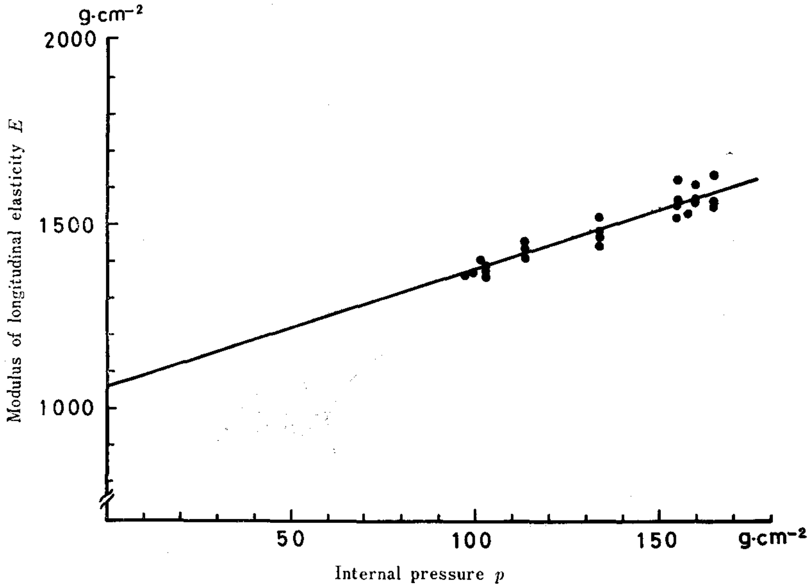

Fig. 11. Relation between the internal pressure and the modulus of longitudinal elasticity

consider that the bent tube is a column made of two different materials and the cross-sectional area A, the modulus of longitudinal elasticity E of the bent tube is shown in it next equation, but m is a constant,

$$E = m\,p + (A_t/A)\,E_0\,. \tag{21}$$

Next we compared the experimented flections with the calculated flections by formula (16). The results are shown on Table 2. The calculated value fairly well coinciding with the experimented value, we may then consider that the flection caused by internal pressure is given by formula (16). But here is one question whether the formula (16) doesn't involve the form factor k. This means, in spite of a different form factor, that if the same material, the cross-sectional area and

Table 2. Compared the experimented and calculated flections

pressure	experimented value	calculated value
164 (g/cm^2)	2.9 (cm)	3.1 (cm)
144	2.6	2.8
123	2.3	2.5
101	2.0	2.0
81	1.7	1.6

the included angle are given, the flection caused by internal pressure could be obtained. That solves the flection of the bent tube, so far as we use the formula of a curved beam. This should be taken into consideration.

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