A Numerical Experiment of the Bering Sea Circulation.

I. A non-divergent barotropic model

Masaaki Suzuki* and Jiro Fukuoka**

Abstract

A numerical experiment was carried out to investigate the continental shelf waves of the Bering Sea by using a non-divergent barotropic model. The continental shelf wave with about 1000 km of wave length and 6 m/sec of phase speed is generated near Fox Island and progresses along the continental shelf break, of which the speed depends on the width of the continental shelf.¹²

Introduction

The Bering Sea consists of the northernmost part of the Pacific Ocean. It is formed characteristically by a deep oceanic basin in the west and an extreme wide continental shelf in the east. The shelf break runs southeast-northwest from Fox Island, Alaska, to near Cape Navarin, USSR. It seems that the interesting flow patterns in the Bering Sea depend upon the characteristic bottom topography, coastal lines and weak planetary β effects. We have a strong interest in the phenomena having lower frequencies than the Coriolis frequency.

The mathematical model

The model in this study includes the effects of an irregular coast and bottom topography (Fig. 1). As to the formulation, we introduce the β-plane approxima-
tion \( f = f_0 + \beta y \) and assume a homogeneous water and hydrostatic balance in the vertical direction. By neglecting the bottom friction, the vertically integrated vorticity equation is as follows:

\[
\frac{\partial Z}{\partial t} + \left[ \frac{\partial}{\partial x} \left( - \frac{1}{h} \frac{\partial \Psi}{\partial y} Z \right) + \frac{\partial}{\partial y} \left( \frac{1}{h} \frac{\partial \Psi}{\partial x} Z \right) \right] \\
+ f \left[ \frac{\partial}{\partial x} \left( - \frac{1}{h} \frac{\partial \Psi}{\partial y} \right) + \frac{\partial}{\partial y} \left( \frac{1}{h} \frac{\partial \Psi}{\partial x} \right) \right] + \frac{\beta}{h} \frac{\partial \Psi}{\partial x} \\
= A_H \left[ \frac{\partial^2 Z}{\partial x^2} + \frac{\partial^2 Z}{\partial y^2} \right]
\]

(1)

The Cartesian coordinate system is used for the sake of simplicity. The formation and scheme are shown according to Endo’s method.

In the equation (1)

\[
Z = \frac{\partial}{\partial x} \left( \frac{1}{h} \frac{\partial \Psi}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{1}{h} \frac{\partial \Psi}{\partial y} \right)
\]

(2)

indicates relative vorticity and \( \Psi \) is the stream function of volume transport as follows:

\[
h u = - \frac{\partial \Psi}{\partial y}, \quad h v = \frac{\partial \Psi}{\partial x}
\]

(3)

and we define other symbols as follows:

- \( x \): distance directed eastward
- \( y \): distance directed northward
- \( t \): time
- \( h \): depth
- \( f \): Coriolis parameter \( f = f_0 + \beta y : f_0 = 11.6 \times 10^{-5} \) sec\(^{-1}\)
- \( \beta \): \( \frac{df}{dy} \) \( 0.916 \times 10^{-13} \) sec\(^{-1}\) cm\(^{-1}\)
- \( u \): eastward vertically averaged velocity
- \( v \): northward vertically averaged velocity
- \( A_H \): eddy viscosity \( (10^8 \) cm\(^2\) sec\(^{-1}\))

The volume transport of inflow and outflow in the Bering Sea has been estimated by Ohtani (Table 1). As to the lateral boundary condition, we introduce the viscous boundary condition. With such a condition, the time dependent, non-

<table>
<thead>
<tr>
<th>Strait</th>
<th>Outflow or inflow</th>
<th>Volume transport</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kamchatka Strait</td>
<td>outflow</td>
<td>10 sv.</td>
</tr>
<tr>
<td>Nea Strait</td>
<td>inflow</td>
<td>6 sv.</td>
</tr>
<tr>
<td>Amuchitka Strait</td>
<td>inflow</td>
<td>5 sv.</td>
</tr>
<tr>
<td>Bering Strait</td>
<td>outflow</td>
<td>1 sv.</td>
</tr>
</tbody>
</table>

---

*Table 1. The volume transport of inflow and outflow*
linear vorticity equation (1) is integrated by using “the Successive Over Relaxation method.”

**Result**

Calculations are made in three cases (Table 2) and the initial condition is given in Fig. 2. A convenient measure of the system equilibrium is given by the total kinetic energy. In a typical run, the kinetic energy integral decreases rapidly at first, then becomes steady in about 30 days.

<table>
<thead>
<tr>
<th>Case</th>
<th>Mesh size</th>
<th>Bottom</th>
</tr>
</thead>
<tbody>
<tr>
<td>CASE I</td>
<td>54 km</td>
<td>bottom topography considered</td>
</tr>
<tr>
<td>CASE II</td>
<td>54 km</td>
<td>flat</td>
</tr>
<tr>
<td>CASE III</td>
<td>27 km</td>
<td>bottom topography considered</td>
</tr>
</tbody>
</table>

Fig. 2. Isopleths of volume transport stream function at initial

(I) **CASE I**

In the first case, the continental shelf wave of which its wave length is about 1000 km and its phase speed is 6 m/sec is generated near Fox Island and progresses toward Cape Naverin with rising barotropic instability. At Cape Naverin, the width of the continental shelf becomes very narrow, and the kinetic energy of the eddy is decreased by viscosity. Then the eddies travel with their right shoulder against the coast with decreasing energy (Fig. 3).

Fig. 4 shows the vorticity balance at section A. (see Fig. 1) The local time change of the vorticity and bottom topography terms balance each other. This balance is the characteristic of the continental shelf wave (Topographic Rossby wave). After several days, the local time change and bottom topography terms decrease, and then other terms (non-linear, \(\beta\)-term) increase. After 100 days, isopleths of volume transport stream function become parallel to isobath.

The vorticity balance at section B (see Fig. 1) is shown in Fig. 5. The viscous
term remarkably increases. And then, at section C (see Fig. 1), the bottom topography term balances with viscous term. Namely, in the area where the width of the continental shelf is narrow, the eddy progresses but its kinetic energy is remarkably decreased by viscosity.
Fig. 4. Vorticity balance at 12th, 44th, 68th days. Solid line denotes bottom topography term, dotted line denotes local time change term.

Fig. 5. Vorticity balance at section B. Solid line with circle denotes bottom topography term, solid line with square denotes local time change term and dotted line with triangle denotes viscous term.
(II) CASE II

In order to check the effects of bottom topography (topographic $\beta$ effect), we consider the flat bottom model. Fig. 7 shows the isopleths of the volume transport stream function after 15 days. Two large eddies are generated and the righthand eddy progresses westward as Planetary Rossby wave. And the flow through Amuchitka Strait runs westward on account of planetary $\beta$ effects. These circulation patterns can not be noticeable in the Bering Sea from mere observation. This conclusion suggests that topographic $\beta$ effects are more dominant than planetary $\beta$ effects in the Bering Sea.

(III) CASE III

In order to check reproducibility, we resolved the vorticity equation with 27 km mesh size. The flow patterns are the same as in CASE I.
Suzuki & Fukuoka: A numerical experiment of the Bering Sea

Discussion

A numerical experiment was carried out by using a non-divergent barotropic model to investigate the low frequency phenomena of the Bering Sea circulation. For the model which includes bottom topography, the inflow through Amuchitka Strait runs eastward. When the flow crosses the continental shelf break, the vorticity is generated as a result of the conservation of potential vorticity \( \frac{d(f + z)}{dz}/dt = 0 \). Then the eddy travels along the shelf break. This is the reason why the continental shelf wave is generated in the present model. As the width of the continental shelf from Fox Island to Cape Navarin is very wide, this progressive wave may be called Double Kelvin wave.\(^5\)

Nevertheless the present model does not contain the wind stress where the continental shelf wave is generated. This wave should be also generated by the wind action.\(^6\) It is possible that we can easily find the phenomena accompanying the continental shelf wave.

Finally, our present model is too simple to describe the Bering Sea circulation. In order to simulate the Bering Sea circulation, we must construct a more realistic model which contains the baroclinic component, wind stress and seasonal variation of the heat flux through the sea surface.

Acknowledgement

The authors wish to express their appreciation to Mr. Jhong Hwan Yoon, Mr. Masahisa Kubota, Geophysical Institute, University of Tokyo and Dr. Nobuo Suginohara, Institute of Geophysical Fluid Dynamics, University of California, for their available advice and continuous encouragement. Thanks should be extended to Miss Chieko Yoshioka for her kindness about drafting.

Reference