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Formation of the Density Inversions in the Intrusive Layer of the Coastal Mixing Region*

Hideo MIYAKE**

Abstract

In the coastal mixing region where two characteristic waters interacted in the subsurface layer, observations were made with a thermistor thermometer and Nansen bottles. The horizontal intrusions or interleavings were formed in the region of interaction between the salty warm water and the less salty cool water, where inversions occurred not only in vertical profiles of the temperature and the salinity, but also in those of the density.

Applying analyses of the static stability in the water column, the density inversions in the interleaving layer appear to be rised due to the difference in the eddy diffusivities of heat and salt. Namely, the negative density gradient may be generated in the isopycnal intrusion of salty warm water by the different eddy-diffusion processes.

The instability is calculated theoretically using an infinite model, in which a rectangular-pulse-shape of salty warm water is assumed as an intrusive layer. According to the result, although one of the negative density gradients generated initially at three different depths is growing till it reaches to the maximum value after about half a day, as a whole they decrease gradually to the neutral stability. The calculated maximum value agrees well with the observed one and it is found that the magnitude of instability depends both on the scale of temperature and salinity inversions and on the difference in the eddy diffusivities of heat and salt.

Introduction

In the northeast region along the Pacific coast of Japan, density inversions in vertical profiles were found in results of standard Nansen cast. Based on the geographical distribution and the statistical analyses, Kuroda[1] explained that the inversions might arise from the horizontal and vertical mixing processes between the waters of different properties. In the southern region off New England in the United States, Voorhis et al.[2] and Horne[3] examined the interleaving process and the possible mechanism between the shelf water and the slope water.

These interleaving or intrusive processes are recently considered to be one of the most active mechanisms in mixing between the different water masses. Therefore, the intrusive process is very important for the studies of hydrographic conditions in the mixing region where the coastal water and the offshore oceanic water intensely interact. And also it may be interesting to know what mechanism

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* *Contribution No. 108 from the Research Institute of North Pacific Fisheries, Faculty of Fisheries, Hokkaido University.*

** *Research Institute of North Pacific Fisheries, Faculty of Fisheries, Hokkaido University.*
(北海道大学水産学部北洋水産研究施設)

excels physically in this process. The explanation of this process may also give the available suggestions for the mixing process occurring widely in the ocean. In this paper density inversions in the intrusive layer of the coastal mixing region are described and the instabilities due to the difference in eddy diffusivities are discussed as one of the possible mechanisms.

Field observation

In Funka Bay and near the coastal area along the eastern side of Oshima peninsula in Hokkaido, several water masses are formed during the year. Ohtani[4] and Ohtani et al.[5] described the hydrographic conditions in this area and clarified the properties of these water masses, such as the Tsugaru Warm Water and the Oyashio Water and so on.

The temperature and salinity data were collected on 15 July and on 10 August 1976 in the region along the eastern side of Oshima peninsula on the RV Ushio Maru (98 GT) of the Faculty of Fisheries, Hokkaido University. The location of stations which were situated one mile apart from each other is shown in Fig. 1. In this season the Tsugaru Warm Water flows intrusively in and near Funka Bay interacting and mixing with the resident coastal water. And since the properties of these two water masses are quite different in both the temperature and the salinity, many types of structures arise in vertical sections of the temperature and the salinity[6].

Continuous records of vertical temperature profile were obtained with the thermistor thermometer, and waters in 10 meters were sampled in every other station for the salinity measurements with the Auto-Lab salinometer. Accuracies of them were about ±0.1°C and ±0.003‰, respectively.

Results of temperature section with the vertical record by the thermistor

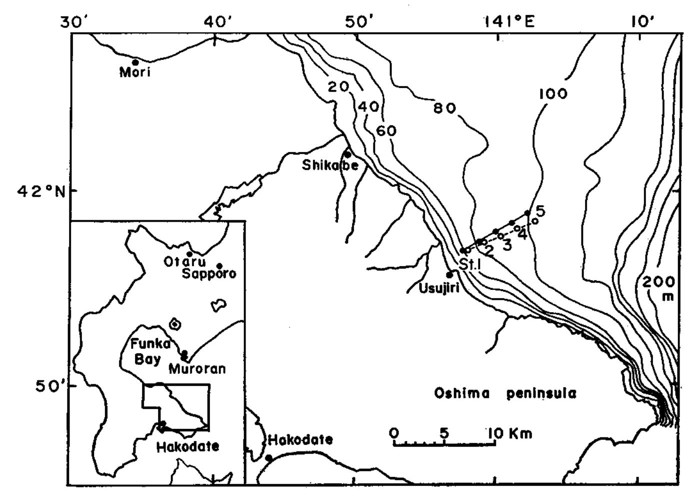

Fig. 1. Bathymetry and location of stations. Solid and open circles indicate the stations on 15 July and on 10 August 1976, respectively.

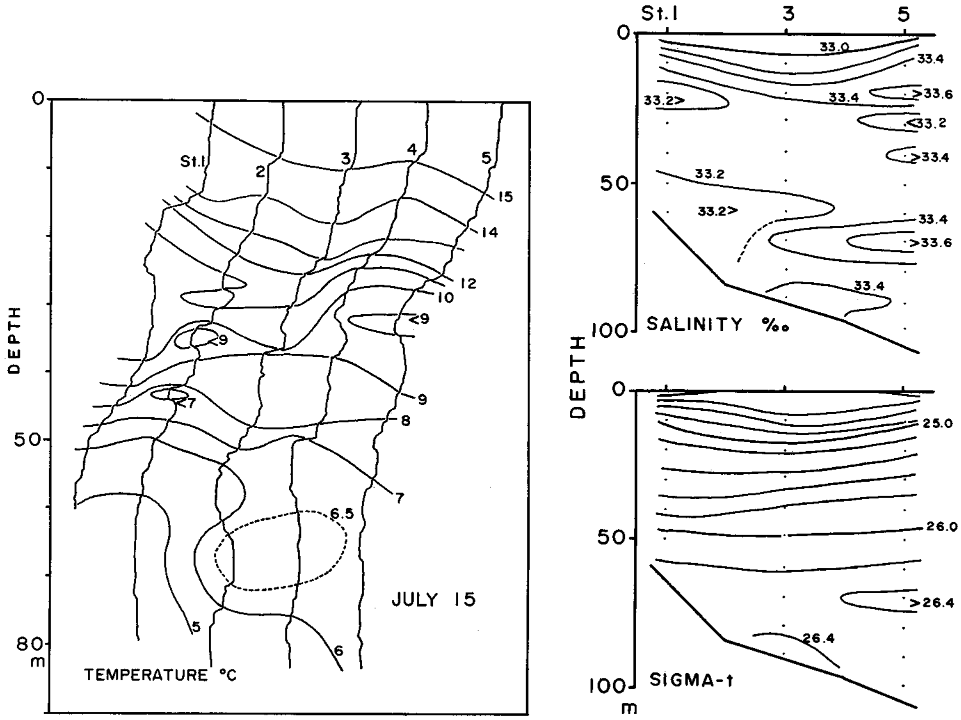

Fig. 2. Temperature, salinity and sigma-t sections on 15 July.

thermometer, and of the salinity and sigma-t section are shown in Figs. 2 and 3. As clearly from the temperature section on 15 July in Fig. 2, warmer water exists in the layer of about 70 m depth of Sts. 3 and 4, and this causes inversions of the temperature particularly at St. 3. Moreover, the small scale intrusions of warmer and cooler water are recognized in the subsurface layer. Observing the section from the salinity distribution, on the other hand, the salty waters intrude into the less saline waters in several layers; from 10 to 20 m at Sts. 1, 3 and 5, and about 70 m at Sts. 3 and 5. In particular, the intrusion of salty water in the depth of about 70 m at St. 5 induces the large salinity gradient of 0.30 to 0.32‰ per vertical 10 meters. And the layer of this salty water coincides with that of relatively high temperature. From 60 to 80 m depth at St. 5, the temperature does not certainly vary vertically, hence the decrease of salinity arises in the instability of 0.25 σ_t between the depths of 69 and 79 m as indicated in the sigma-t section in Fig. 2.

In August the intrusive or interleaving layers are clearly confirmed due to the complicated structure in the vertical section as shown in Fig. 3. Warmer and cooler waters intrude and penetrate into the middle depth as indicated by the isothermal lines of 8, 9 and 10°C. Specifically, intense interactions are recognized in the layer from 30 to 60 m at St. 3, where the warm salty waters alternate with the less saline cool waters. Comparing both the sections of the temperature and the salinity, the isohaline of 33.6‰ and the isothermal lines of 8 or

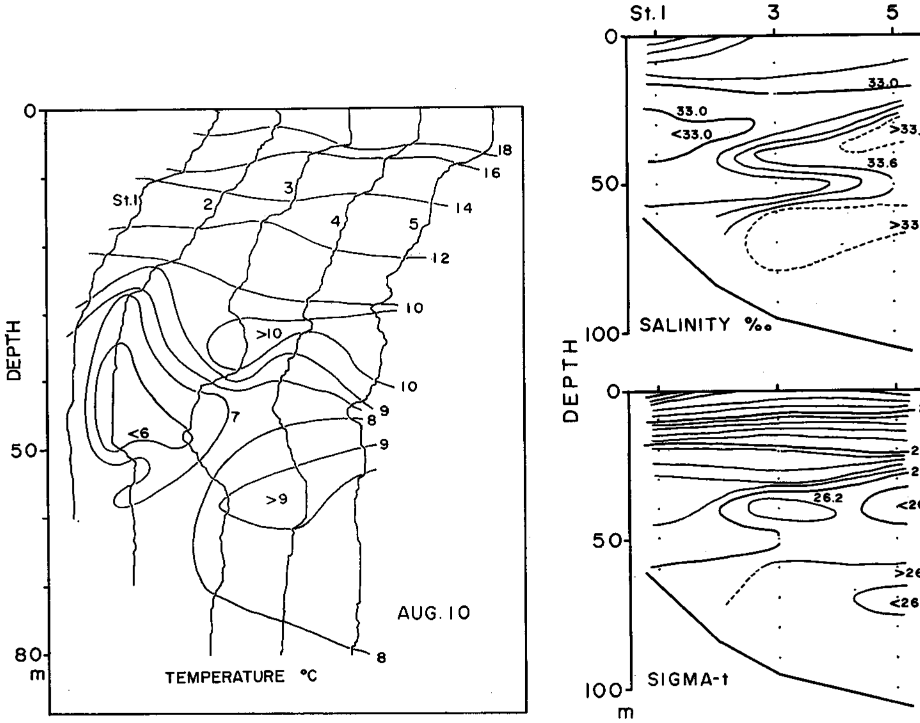

Fig. 3. Temperature, salinity and sigma-t sections on 10 August.

10°C overlap closely with each other. Since the density decrease due to the change of the salinity is not compensated sufficiently by that of temperature, density inversions appear in several depths, namely they are 0.27, 0.22 and 0.09 σ_t between the depths of 40 to 50 m at St. 3, 30 to 40 m and 60 to 70 m at St. 5, respectively.

Interpretation of the phenomena

These negative density gradients are formed between the upper saline water and the lower less saline one, and generally the salty water shows relatively high temperature. Profiles of the temperature and the salinity at the depths of 40 and 70 m at St. 5, and 40 and 60 m at St. 3 in Fig. 4 are typical examples. The warm and salty water is characterized as the Tsugaru Warm Water[4)], while the coastal water is cool and less saline.

It seems that such an interleaving of salty warm water or less salty cool water takes place along an isopycnal surface, for the negative density gradient will soon disappear by the overturnal convection even if the density inversion exists in the early stage. Thus, it may be reasonable to assume that the instability occurs during the process of mixing.

To check this assumption, the total difference of the density in each water column of 10 meters is separated into the density change by the temperature and the salinity. The vertical static stability in shallow water is written as follows[7].

$$E=\frac{d\sigma_t}{dz}\times 10^{-3}=\left(\frac{\partial\sigma_t}{\partial T}\frac{dT}{dz}+\frac{\partial\sigma_t}{\partial S}\frac{dS}{dz}\right)\times 10^{-3} \tag{1}$$

Now take 10 meters as dz, differences of the sigma-t, the temperature and the salinity in the same vertical scale are $\Delta\sigma_t$, ΔT and ΔS, respectively. Then the equation (1) is simplified as follows,

$$\Delta\sigma_t=\alpha\Delta T+\beta\Delta S \tag{2}$$

where $\alpha=\partial\sigma_t/\partial T$ and $\beta=\partial\sigma_t/\partial S$ are parameters of the contribution on the density by heat and salt. ΔT and ΔS are taken possitivly in the direction of increasing density with depth, and in the reverse case it is defined as "inversion". The expression (2) means that the density changes with both changes of the temperature and the salinity. Each term in the equation can be calculated from the temperature-salinity-sigma-t table. An example of such the unstable density structure is shown in Fig. 5 and Table 1. From these it is likely that the density inversion is caused by the salinity inversion without the compensation of temperature. If it is assumed that the instability of 0.26 σ_t is compensated only by the temperature change, the temperature at the depth of 69 m has to increase from 6.3 to 8.2°C as shown in Table 1. When the Tsugaru Warm Water which is inherently warm and saline intrudes into the coastal water of cool and less saline, the Water may lose its heat more quickly than its salt due to the difference in the eddy diffusivities of heat and salt. Thus the assumption taken above may be reasonable.

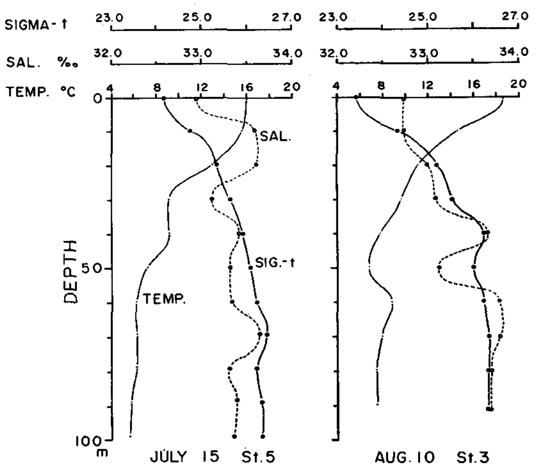

Fig. 4. Temperature, salinity and sigma-t profiles of density inversion at St. 5 on 15 July and St. 3 on 10 August.

If the temperature at 69 m depth is assumed as a broken line in Fig. 5, the water column from 69 to 79 m will be statically stable. In other words, this means that the salty warm water intrudes isopycnally into the less saline cool water. Under this assumption, the intruded layer probably has the neutral stability in the initial stage. But, since the layer may possibly diffuse its heat more rapidly than its salt, the instability will appear in the layer during the mixing process. The same way may be suitable to the negative density gradient from 40 to 50 m at St. 3 on 10 August. The isopycnal intrusion of less saline cool water which interleaves into the warm salty one may obtain its heat more rapidly than its salt, and then this layer becomes unstable due to the decrease of density

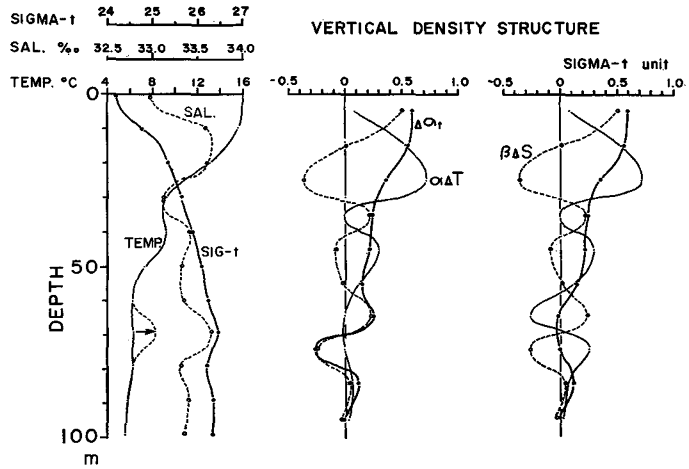

Fig. 5. Temperature, salinity and sigma-t profiles, and the density structures at St. 5 on 15 July. On the right side the density structures are shown when the temperature at 69 m depth is increased to 8.2°C. Thick, thin and broken lines indicate $\Delta\sigma_t$, $\alpha\Delta T$ and $\beta\Delta S$ in the text.

by the thermal expansion. In this case the density inversion may also be raised by the decrease in temperature at the depth of 40 m. Therefore, it appears that the instability of this water column is caused by two ways; the decrease in temperature of 40 m depth and the increase in that of 50 m depth.

Calculation of the instability

As mentioned already it seems qualitatively evident that the density inversion may be raised during the different diffusive processes. According to Voorhis et al.[2], the interleaving phenomena had the time scale of 1–3 days and they suggested the existence of vertical turbulent exchanges of heat and salt accompaning the microstructures. Thus the eddy diffusion process has to firstly be considered, for the process may be thought of as turbulent.

In order to calculate the instability of the water column, let us consider the vertical diffusion equation of one dimension in an infinite fluid, that is,

$$\frac{\partial T}{\partial t} = K_T \frac{\partial^2 T}{\partial z^2} \quad (-\infty < z < \infty,\ t > 0) \tag{3}$$

where T, t, K_T and z are temperature, time, the vertical eddy diffusivity for heat assumed a constant and a vertical coordinate, respectively. The solution of this equation with the initial conditions of $T = T_{o,l}$ (constant) in $|z| > l$ and of $T = T_{o,l} + \Delta T$ (constant) in $-l < z < l$ is obtained as follows[8],

$$T = T_{o,l} + \frac{\Delta T}{2}\left\{\phi\left(\frac{l+z}{2\sqrt{K_T t}}\right) + \phi\left(\frac{l-z}{2\sqrt{K_T t}}\right)\right\} \tag{4}$$

where $\phi(x)$ is the error function defined by the following equation.

Table 1. *Evaluation of $\alpha\Delta T$, $\beta\Delta S$ and $\Delta\sigma_t$ in the intrusive layer. Parentheses indicate the*

Depth m	Temp. °C	Sal. ‰	Sigma-t g cm$^{-3}\times 10^{-3}$	ΔT °C
0	16.0	32.95	24.19	
				0.4
10	15.6	33.59	24.77	
				2.6
20	13.0	33.61	25.34	
				3.8
30	9.2	33.13	25.64	
				0
40	9.2	33.42	25.87	
				1.9
50	7.3	33.32	26.08	
				1.0
60	6.3	33.34	26.23	
				0 (−1.9)
69	6.3 (8.2)	33.64 (33.64)	26.46 (26.21)	
				0 (1.9)
79	6.3	33.32	26.21	
				0.5
89	5.8	33.39	26.33	
				0.2
99	5.6	33.35		

$$\phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-\beta^2} d\beta$$

The order of $(l+z)$ is about 1 to 10 m. If the time scale is about one day on referring to results of the shelf/slope water front[2,9], the order of K_T will be determined as the following way. Namely, since the temperature has to change effectively in the spatial and temporal scale mentioned above, the value of $(l+z)/\sqrt{K_T t}$ has almost the order of 1. Consequently, substituting $t=10^5$ sec and $(l+z)=10^2 \sim 10^3$ cm, we have K_T as the order of 10^{-1} to 10 cm^2 sec^{-1}. When K_T is given, the temperature at different depths and times is able to be calculated. Replacing T and K_T in equation (3) with S of the salinity and K_S of the vertical eddy diffusivity for salt, the salinity profile can be easily calculated in a similar way. Thus, the vertical density profiles are derived with respect to both the temperature and salinity changes.

The coefficient of the vertical eddy diffusivity for heat was formerly thought of as the same for salt and oxygen. These values derived from the observations in the moderate and stable conditions in the subsurface layer are ranged from 1 to 90 cm^2 sec^{-1} [7], of which a mean value is about 30 cm^2 sec^{-1}. Several investigators, however, reported recently the values of K_S different from K_T. One of them, Horne[3] obtained the values of 8 cm^2 sec^{-1} as K_T and 3 cm^2 sec^{-1} as K_S for the interleaving layer. In this work, values of K_T and K_S are assumed as 25 cm^2 sec^{-1} and 1 cm^2 sec^{-1} on referring to the results mentioned above.

On the basis of the observation at St. 5 on 15 July, each value of 6.3°C, 1.9°C, 5 m, 33.32‰ and 0.32‰ is used as $T_{o,l}$, ΔT, l, $S_{o,l}$ and ΔS in the calculation, respectively. The values of 6.3°C and 33.32‰ are almost equal to the mean values of the depths 50, 60, 79, 89 and 99 m, and the sigma-t of the water shows 26.21, which is nearly isopycnal with the intrusive layer appearing from 65 to 75 m. The equation (4) is calculated numerically for values of the time as 0.25, 0.49, 4 and 9×10^4 sec and of the depth as 0, $\pm l/2$, $\pm l$, $\pm 3l/2$, $\pm 2l$ and $\pm 3l$. Results of the *values when the temperature at 69 m depth is increased from 6.3°C to 8.2°C.*

α g cm^{-3} °C$^{-1}\times10^{-3}$	$\alpha\Delta T$ g cm$^{-3}\times10^{-3}$	ΔS ‰	β g cm^{-3}‰$^{-1}\times10^{-3}$	$\beta\Delta S$ g cm$^{-3}\times10^{-3}$	$\Delta\sigma_t$ g cm$^{-3}\times10^{-3}$
0.225	0.090	0.64	0.80	0.512	0.60
0.210	0.546	0.02	0.80	0.016	0.56
0.190	0.722	−0.48	0.80	−0.360	0.36
0.170	0	0.29	0.80	0.232	0.24
0.160	0.304	−0.10	0.80	−0.080	0.22
0.135	0.135	0.02	0.80	0.016	0.15
0.135	0	0.30	0.80	0.240	0.24
(0.135)	(−0.260)				(−0.02)
0.135	0	−0.32	0.80	−0.260	−0.26
(0.135)	(0.260)				(0)
0.135	0.068	0.07	0.80	0.056	0.12
0.135	0.027	−0.04	0.80	−0.032	−0.01

calculation are presented in Fig. 6. Depths of $z=-2l$, 0 and $2l$ in the calcuation correspond to 60, 70 and 80 m in the actual depth, respectively, because the origin of the vertical axis is taken at the center of intrusive layer.

As is evident from Fig. 6, the processes of diffusion by heat and salt are quite different. At the time of 0.49×10^4 sec, the temperature of 70 m depth decreases to about one third of ΔT, but the salinity of the intrusive layer remains as it was. After about one day (9×10^4 sec), the temperature profile becomes nearly flat in contrast to the saline one.

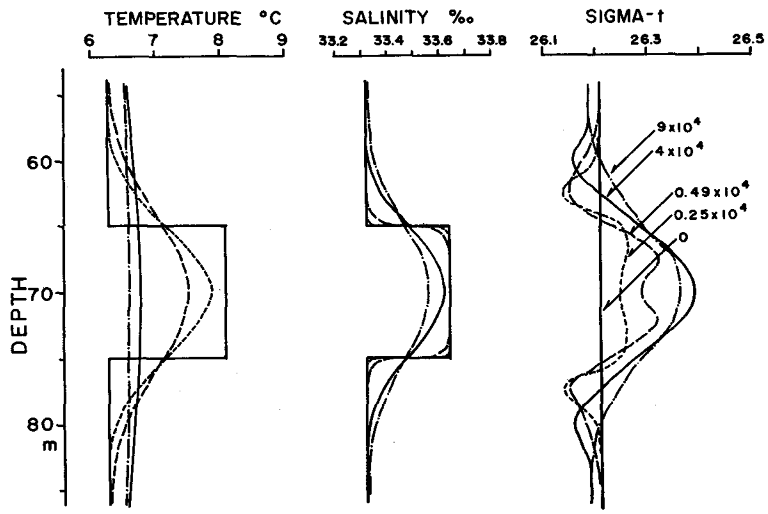

Fig. 6. Time variations of temperature, salinity and sigma-t profiles near the warm salty intrusive layer calculated from an infinite model. Numerals denote time in seconds.

The density inversions are generated initially at three different depths; below 60 m, above 70 m and below 75 m. Among them, two negative gradients which appear near the boundaries of intrusive layer are large, and they extend toward the outer side of the initial layer with time. Though the weak inversion of density which lies in the center of the intrusive layer disappears at the time of 4×10^4 sec, the density inversion from 70 to 80 m develops to its maximum value of 0.23 σ_t at that time. The value agrees well with the observed one of 0.25 σ_t at St. 5 on 15 July, and furthermore the temperature profiles at Sts. 3, 4 and 5 presented in Fig. 2 seem to suggest the vestige of the diffusion process of heat. At the time, the upper instability above 60 m depth has been already weakened, while the lower instability in the water column from 70 to 80 m begins to weaken very gradually after this time.

With respect to time, the deformation of this density inversion is remarkably small on comparing with that in the early stage, because the rate of deformation in the later stage depends nearly on the diffusion of salt whose process is very slow. Thus the large inversion of density which is detectable enough by the method of Nansen cast is kept for a relatively long time, if the overturnal convection does not happen. Even after one day, as presented in Fig. 6, the difference of sigma-t between 70 and 85 m depth shows 0.17 being the unstable condition.

Discussion

The main cause for the instability in the intrusive layer does not lie in the shape of its layer but in the difference in diffusivities of heat and salt. Therefore the rectangular-pulse-shape assumed in the calculation is only used for the convenience to clear the phenomena of intrusive instability. And it is needless to describe that in order to estimate the diffusive fluxes across the interfaces exactly, more precise profiles of the temperature and the salinity must be given.

There still remain some problems in the theoretical treatments that the vertical axis extends to infinity and the initial profiles of $T_{o,l}$ and $S_{o,l}$ are constant. Because the depth of water is finite and the assumptions of $T_{o,l}$ and $S_{o,l}$ are satisfied only within the layer from $-4l$ to $4l$. Specifically, in the upper layer above 40 m depth, both profiles of the temperature and salinity are very complicated as shown in Fig. 4. But, as evident from the time variation of profiles in Fig. 6, heat and salt are not diffused to the outer layer of $\pm 4l$. Hence, the model used in the calculation may be available under the conditions within half a day and the depth of 50 to 90 m.

The ratio of K_T/K_S is more important than each value, for the magnitude of instability is influenced by the ratio. For example, if $K_T/K_S=1$, the instability may scarcely appear in the intrusive layer. In the calculation, the vertical eddy diffusivities of K_T and K_S are given as mentioned above. If the time series of vertical profiles of the temperature and salinity are obtained, the reliability of the assumed values of K_T and K_S will be confirmed, and moreover the formation process of the intrusive instability will be clear. Recently Posmentier and Houghton[9] measured the fine instabilities in the intrusion using CTD, and it was explained qualitatively as double diffusion mixing. Their treatment may be suitable for our profiles though the relationship between the fine scale mixing and the large scale is little understood.

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