Size Selectivity Curves of Sode Hooks for Masu Salmon Derived from the Stochastic Model of Hooking Mechanism

Susumu SHIMIZU¹, Yasuzumi FUJIMORI¹, Teisuke MIURA¹ and Katsuaki NASHIMOTO²

Abstract

Size selectivity curves derived from a stochastic model of hooking mechanism were compared with the curves obtained by the method after Ishida. The stochastic model expresses the capture probability that a hook catches a fish when the fish bites the hook. A movement limit was added to the model to represent the condition that the hook could not enter the fish’s mouth if the hook was too large. The moving coefficient and hooking coefficient in the model were estimated from the results of multiple linear regression analysis, because each coefficient had shown to be related to hook width and the total length of fish in pole-and-line fishing experiments. A selectivity curve after Ishida was also estimated from the same data of the experiments as a polynomial curve with Solver of Microsoft Excel. Selectivity curves, which were plotted against total length for different hook sizes, were unimodal with a gentle long right slope and had different maximum values in the case of the stochastic model. The selectivity curves obtained by the method after Ishida were similarly unimodal and had equal maximum values. The selectivity curve by the method after Ishida fit the results of the angling experiments better than that from the stochastic model. But the maximum value of a selectivity curve might decrease as hook size increased.

Key words: size selectivity, selectivity curve, hooking mechanism, stochastic model, pole-and-line fishing, capture probability

Introduction

Size selectivity of angling gear for hook size has been inductively estimated from the results of fishing trials. It has been reported that the size selectivity curves of angling gear are unimodal and have wider selection ranges than those of gill nets (Pope et al., 1975; Koike and Kanda, 1978; Yamaguchi, 1979). But some selectivity curves of angling gear are one-sided, such as the logistic curves used for trawls according to McCracken (1963), Sætersdal (1963), Myhre (1969) and Ralston (1990). Fishing trials show several partial properties of the size selectivity of angling gear. It can not be determined only from fishing trials what type of curve (e.g., unimodal or logistic) suits the intrinsic size selectivity of angling gear, so size selectivity of angling gear is not well understood.

Theoretical models of the fishing mechanisms for a trawl by Fujiishi (1973) and

¹) Laboratory of System Design for Fisheries, Department of Marine Production System Science, Faculty of Fisheries, Hokkaido University

²) Laboratory of Production Engineering for Fisheries, Department of Marine Production System Science, Faculty of Fisheries, Hokkaido University

(北海道大学水産学部生産システム設計学講座)

(北海道大学水産学部生産工学講座)
gillnets by Sechin (1969) and Nashimoto (1979) include the cause and effect of size selection and show the typical size selections that occur for each net. Therefore they have contributed to the forming of the fundamental size selectivities for both nets. According to Shimizu et al. (1996), a stochastic model of the hooking mechanism, which included the effects of fish size and hook size, was made and fit well to the results of pole-and-line fishing experiments with “sode” hooks to masu salmon (*Oncorhynchus masou*). The moving coefficient in the stochastic model increased when the mean total length of fish decreased or the hook width increased. And the hooking coefficient increased when the mean total length of fish decreased or the hook width increased. According to Shimizu (1998) and Shimizu (1999), since these relations between fish and hook sizes and each coefficient were applied to our stochastic model, and the movement limit of a hook was added in the stochastic model, the selectivity curves of sode hooks for masu salmon were derived from the model. These selectivity curves for different hook sizes that were plotted against total length were unimodal with a gentle long right slope. Each curve had a lower maximum value and a flatter peak as hook size increased. But it was not explained well how to develop the stochastic model included a movement limit. Also, the characteristics of the obtained selectivity curves were not discussed.

In this paper, the development of our model including the movement limit is explained. The selectivity curves by the method after Ishida (1962) were also estimated from the results of the same experiments to compare them with the curves obtained from our stochastic model. The characteristics of the selectivity curve obtained from the stochastic model of hooking mechanism were discussed with regard to the shape of a selectivity curve and the parameter identification.

### Methods

**Capture Probability**

In the stochastic model of hooking mechanism, the capture probability $P_{\text{capture}}$ is the probability that hooking occurs in the mouth cavity or esophagus of a fish. This is the probability that an angler catches a fish when the fish bites the hook if the hook does not come out of the mouth during the landing. Shimizu et al. (1996) obtained the capture probability as described below. A model of the hook movement in the mouth cavity is shown in Fig. 1. Let $q$ be a distance from the snout to the position of a hook bend. The stop probability $S(q)$ represents the probability that a hook bitten by a fish stops moving inward in section $[0, q]$ of the mouth cavity. The probability density function $s(q)$ of $S(q)$ is expressed as follows.

$$ s(q) = ae^{-aq}, $$

where $a$ is the moving coefficient. Although $a$ was called the hard entering coefficient, $a$ is renamed the moving coefficient because “hard entering” appears to be an unsuitable term. The moving coefficient has been related to fish size and hook size.

Once the hook stops moving inward at $q$ in the mouth cavity, the hook starts moving outward from $q$. While the hook moves outward, hooking occurs somewhere in the mouth cavity. Let $L$ be the hook point height and $x$ be the distance from the snout to the position of the point of the hook. The hooking probability...
$H(q - L - x)$ represents the probability that hooking occurs in section $[x, q - L]$ after the hook has stopped moving inward at $q$. The probability density function $h(q - L - x)$ of $H(q - L - x)$ is as follows:

$$h(q - L - x) = be^{-b(q - L - x)},$$

where $b$ is the hooking coefficient. The hooking coefficient also has been related to fish size and hook size.

Let $Q$ be the mouth cavity length. The probability $P_{me}$ that hooking occurs somewhere in the mouth cavity represents the probability that hooking occurs in section $[0, Q]$ after the hook has stopped moving inward in section $[x + L, Q + L]$. The swallowing probability $P_s$ is the probability that hooking occurs in the esophagus. It is assumed that hooking always occurs when the point of the hook reaches the esophagus farther than $Q$. In this case, the hooking probability becomes 1.

The capture probability $P_{capture}$ is expressed by the following equation:

$$P_{capture} = P_{me} + P_s = \int_0^Q \left( \int_{x + L}^{Q + L} s(q)h(q - L - x) dq \right) dx + \int_{Q + L}^{\infty} s(q) dq$$

$$= \left( \frac{be^{-al} + ae^{-a(Q + L) - bq}}{a + b} - e^{-a(Q + L)} \right) + e^{-a(Q + L)}$$

$$= \frac{1}{a + b} \left( be^{-al} + ae^{-a(Q + L) - bq} \right)$$

(3)

**Capture Probability with the Movement Limit**

To obtain the selectivity curve for a given hook size, the capture probability must also include the probability of the case where an extremely small fish bites the hook. The hook cannot reach the esophagus of a small fish, even if it enters the mouth. The stochastic model represents one-dimensional movement of a hook on the $X$ axis in the mouth cavity. Conditions in which a hook can become embedded in the mouth cavity or esophagus are determined by the movement limit $Z$, which is the maximum distance to which the hook can move inward in the mouth as shown
When the point of the hook does not reach the esophagus in the case of $Z \leq Q + L$, the fish cannot swallow the hook, but hooking can occur in the mouth cavity. When the point of the hook does not reach into the mouth in the case of $Z < L$, hooking cannot occur. Therefore, a movement limit was added to the stochastic model so that the capture probability could be obtained when $L \leq Z \leq Q + L$ where a fish was not able to swallow a hook.

Let $P_{iz}$ be the probability that hooking occurs somewhere in the mouth cavity after the hook has stopped moving inward within the movement limit. Let $P_{oz}$ be the probability that hooking occurs somewhere in the mouth cavity after the hook has stopped moving inward at the movement limit. The capture probability consists of two probabilities, $P_{iz}$ and $P_{oz}$, when a fish cannot swallow the hook. $P_{iz}$ is obtained from the product of the probability densities $s(q)$ and $h(q-L-x)$ integrated on $q$ through section $[x+L, Z]$, then integrated on $x$ through section $[0, Z-L]$ as follows:

$$P_{iz} = \int_{x+L}^{Z-L} \left( \int_{x}^{Z} s(q) h(q-L-x) dq \right) dx$$

$$= \frac{a}{a+b} e^{-aZ-b(Z-L)} + \frac{b}{a+b} e^{-aL} - e^{-az}$$

$$0 < L \leq Z \leq Q + L \tag{4}$$

On the other hand, $P_{oz}$ is derived from the product of two probability densities. One is the probability density that the hook stops moving inward at the movement limit. The other is the probability density that hooking occurs at $x$ while the hook moves outward from the movement limit. Because the probability density that the hook stops moving inward at the movement limit is equal to the probability density that the hook enters farther than $Z$ without the movement limit, the product of the probability densities is expressed as the following equation:

$$s(q) h(Z-L-x) = a e^{-aq} b e^{-b(Z-L-x)} = ab e^{-b(Z-L-x)} e^{bx} e^{-aq}$$

$$0 < Z + L \leq Z \leq Z \tag{5}$$

$P_{oz}$ is obtained from equation (5) after it was integrated on $q$ through section $[Z, \infty]$ and integrated on $x$ through section $[0, Z-L]$ as follows:

$$P_{oz} = \int_{0}^{Z-L} \left( \int_{Z}^{\infty} s(q) h(Z-L-x) dq \right) dx$$

$$= e^{-az} - e^{-aZ-b(Z-L)}$$

$$0 < L \leq Z \leq Q + L \tag{6}$$

The capture probability $P_{\text{capture}}$ in the case where a fish cannot swallow a hook is expressed as the following equation:

$$P_{\text{capture}} = P_{iz} + P_{oz}$$

$$= \frac{b}{a+b} \left( e^{-az} - e^{-aZ-b(Z-L)} \right)$$

$$0 < L \leq Z \leq Q + L \tag{7}$$

Therefore equation (3) is applied to the case of $Z > Q + L$ where a fish can swallow a hook.
Data Used in Calculations

The data used in the calculations were obtained from the pole-and-line fishing experiments described by Shimizu et al. (1996). The conditions and the results of the angling experiments are shown in Table 1. The mean total length $MTL$ of fish and hook width $W$ were adopted as the representative size of a fish and a hook. According to Shimizu et al. (1996), the moving coefficient $a$ and the hooking coefficient $b$ each were related to the mean total length $MTL$ (mm) of masu salmon and the hook width $W$ (mm) of a sode hook used in the experiments as follows:

$$a = 0.96 \frac{W^{0.30}}{MTL^{0.58}}$$

(8)

$$b = 6.8 \times 10^5 \frac{W^{4.2}}{MTL^{4.5}}$$

(9)

When these coefficients for given sizes of fish and hook were calculated from equations (8) and (9), the total length $TL$ of the given fish was substituted for the mean total length $MTL$. According to Shimizu et al. (1996), the mouth cavity length $Q$ of masu salmon was expressed as the following equation:

$$Q = 0.17 TL \text{ (mm).}$$

(10)

Hook point height $L$ could be related to hook width $W$ by linear regression analysis for the measurements of every hook size. It was assumed that the movement limit $Z$ was shorter than $Q + L$ when the experimental swallowing probability was zero, and that the movement limit $Z$ was related to the total length $TL$ and the hook width $W$. Values of $Z$ were estimated from the experimental values of swallowing probability because $Z$ could not be measured. The size selectivity of sode hooks for masu salmon was obtained from the capture probability of the stochastic model by applying the above relationships to equations (3) and (7).

Selectivity curves after Ishida was also estimated using Solver of Microsoft Excel from the same data used in the stochastic model to be compared with those derived from the stochastic model. This estimation of a selectivity curve using Solver of Microsoft Excel was introduced to fit a polynomial to the results by Fujimori and Tokai (1999).

Results

The relationship for a sode hook between hook width $W$ and hook point height $L$ is shown in Fig. 2. By linear regression analysis, hook point height $L$ was expressed as follows:

$$L = 0.66 W + 0.86 \text{ (mm)}$$

(11)

(coefficient of determination $R^2 = 0.98$)

Experimental values of the swallowing probability were zero in experiments No. 6, 7, 9, 12 and 13, as shown in Table 1. The values of $Z$ appeared to be less than $Q + L$ for these experiments, but more than $Q + L$ for the other experiments, according to the experimental values of the swallowing probabilities. The experimental values of $Z$ were estimated to get the good result of multiple linear regression.
Table 1. Conditions and results of pole-and-line fishing experiments with “sode” hooks to masu salmon, *Oncorhynchus masou* (Shimizu et al., 1996)

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>Hook No. (Gou)*1</th>
<th>Hook width <em>W</em> (mm)</th>
<th>Hook point height <em>L</em> (mm)</th>
<th>Mean total length <em>MTL</em> (mm)*2</th>
<th>Size ratio</th>
<th>Experimental value of probability</th>
<th>Entering limit (Z) (mm)</th>
<th>Number of trials</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>capture</td>
<td>swallowing</td>
<td>calculated</td>
</tr>
<tr>
<td>1</td>
<td>0.3</td>
<td>2.0</td>
<td>2.3</td>
<td>145.5</td>
<td>73</td>
<td>0.37</td>
<td>0.12</td>
<td>31.8</td>
</tr>
<tr>
<td>2</td>
<td>0.8</td>
<td>2.6</td>
<td>2.4</td>
<td>134.9</td>
<td>52</td>
<td>0.49</td>
<td>0.11</td>
<td>29.4</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>3.2</td>
<td>2.6</td>
<td>133.5</td>
<td>42</td>
<td>0.42</td>
<td>0.14</td>
<td>29.0</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3.7</td>
<td>3.2</td>
<td>129.9</td>
<td>35</td>
<td>0.56</td>
<td>0.14</td>
<td>28.1</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>4.7</td>
<td>4.1</td>
<td>145.1</td>
<td>31</td>
<td>0.60</td>
<td>0.03</td>
<td>31.3</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
<td>7.0</td>
<td>5.8</td>
<td>145.8</td>
<td>21</td>
<td>0.52</td>
<td>0.00</td>
<td>31.0</td>
</tr>
<tr>
<td>7</td>
<td>13</td>
<td>10.0</td>
<td>7.2</td>
<td>132.9</td>
<td>13</td>
<td>0.48</td>
<td>0.00</td>
<td>27.6</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>3.7</td>
<td>3.2</td>
<td>125.1</td>
<td>34</td>
<td>0.79</td>
<td>0.22</td>
<td>27.0</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>4.7</td>
<td>4.1</td>
<td>124.7</td>
<td>27</td>
<td>0.75</td>
<td>0.00</td>
<td>26.8</td>
</tr>
<tr>
<td>10</td>
<td>7</td>
<td>5.7</td>
<td>4.7</td>
<td>125.7</td>
<td>22</td>
<td>0.62</td>
<td>0.01</td>
<td>26.8</td>
</tr>
<tr>
<td>11</td>
<td>9</td>
<td>7.0</td>
<td>5.8</td>
<td>123.7</td>
<td>18</td>
<td>0.55</td>
<td>0.01</td>
<td>26.1</td>
</tr>
<tr>
<td>12</td>
<td>11</td>
<td>7.7</td>
<td>6.0</td>
<td>125.2</td>
<td>16</td>
<td>0.51</td>
<td>0.00</td>
<td>26.3</td>
</tr>
<tr>
<td>13</td>
<td>12</td>
<td>9.3</td>
<td>6.6</td>
<td>127.1</td>
<td>14</td>
<td>0.49</td>
<td>0.00</td>
<td>26.5</td>
</tr>
<tr>
<td>14</td>
<td>3</td>
<td>3.7</td>
<td>3.2</td>
<td>217.4</td>
<td>59</td>
<td>0.46</td>
<td>0.10</td>
<td>47.4</td>
</tr>
<tr>
<td>15</td>
<td>7</td>
<td>5.7</td>
<td>4.7</td>
<td>218.3</td>
<td>38</td>
<td>0.60</td>
<td>0.01</td>
<td>47.3</td>
</tr>
</tbody>
</table>

*1 Gou number indicates the hook size or the leader size according to the Japanese numbering system. (Larger hooks have higher Gou numbers than smaller hooks).

*2 Sample size = 100
according to the experimental values of the swallowing probability as shown in Table 1. By multiple linear regression analysis, the following equation was obtained:

\[ Z = 0.22MTL - 0.18W \ (\text{mm}) \]  
(coefficient of determination \( R^2 = 0.99 \) )

(12)

The capture probability of a sode hook for masu salmon was calculated from equations (3) or (7), which were substituted from equation (8) through equation (12), using the total length \( TL \) of a given fish size as the mean total length.

Let \( S_r \) be the size ratio, which is the ratio of total length to hook width \( (TL/W) \). Figure 3 shows the experimental values of the capture probability by marks, which are plotted against the size ratio. The bold line curves were obtained from the calculated capture probability of the stochastic model for different total lengths. The maximum value of the curve for a large fish was lower than that for a small fish, and the maximum value for the large fish was obtained at a higher size ratio than for the small fish. The other curve (fine line), which has been fit to the experimental values using Solver of Microsoft Excel, is identical to the selectivity curve obtained by the method of Ishida except the maximum value of 0.64. Let \( P_{\text{capture}}(S_r) \) be the capture probability of this selectivity curve by the method after Ishida. \( P_{\text{capture}}(S_r) \) was expressed as follows:

\[ P_{\text{capture}}(S_r) = -0.044 + 0.055S_r - 0.0013S_r^2 + 0.92 \times 10^{-5}S_r^3 \]  

(13)

Table 2 shows a comparison of the fitness between the selectivity curve by the method after Ishida and that from the stochastic model. Because the mean square of the residual by the method after Ishida (0.0071) was smaller than that from the stochastic model (0.012), the selectivity curve by the method after Ishida appeared to fit the data better.

The curves shown in Fig. 3 were plotted against total length, as shown in Fig. 4. The upward arrow shows that a fish of about 220 mm in total length sometimes stretched the bend of a 3 Gou hook during hooking in the experiment. The selectivity curves of fine lines by the method after Ishida were calculated for the corresponding size ratio range within which the angling experiments were carried
Fig. 3. Selectivity curves plotted to the size ratio of total length to hook width (TL/W).
Marks, experimental values; Fine line, selectivity curve by the method after Ishida; Bold lines, selectivity curve from the stochastic model; Numbers, total length of fish.

Table 2. Comparison of the fitness between two selectivity curves.

<table>
<thead>
<tr>
<th>Method of Ishida</th>
<th>Sum of squares of residual</th>
<th>Number of samples</th>
<th>Number of parameters</th>
<th>Degree of freedom</th>
<th>Mean square of residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capture probability</td>
<td>0.078</td>
<td>15</td>
<td>4</td>
<td>11</td>
<td>0.0071</td>
</tr>
<tr>
<td>Capture probability</td>
<td>0.083</td>
<td>15</td>
<td>8</td>
<td>7</td>
<td>0.012</td>
</tr>
</tbody>
</table>

Discussion

The two types of selectivity curves of sode hooks for masu salmon were similar unimodal curves. Our selectivity curves derived from the stochastic model had a sharp, high peak for a small hook, and a flat, low peak for a large hook. The selectivity curves obtained by the method after Ishida were geometrically similar and had the same maximum values. The selectivity curve by the method after Ishida fit the results of the angling experiments better than that from the stochastic model. But the method of Ishida was based on the assumption after Baranov (1914) who assumed that every hook whose size was proportional to fish size had an equal efficiency. Generally, this assumption has been applied to relative efficiency.
The capture probability is not relative value. The data of our angling experiments did not have enough samples of different fish sizes to confirm the geometrical similarity of selectivity for each hook. Therefore, the maximum value of a selectivity curve might decrease as hook size increased. Nashimoto (1979) showed that the selectivity curves of a gillnet obtained by dynamic analysis also had different maximum values, because the maximum body retention by a mesh produced by elasticity of a net filament and a fish body became stronger according to the enlarging of mesh size. In the case of the angling gear, it is still unknown what mechanism causes the maximum value of the selectivity curve to vary.

A selectivity curve obtained by the method after Ishida can be estimated for the range of the size ratio used in the angling experiments, although that derived from the stochastic model can be calculated for every given fish size and hook size. By the method after Ishida, a selectivity curve can be obtained within the size range of hooks and fish examined in experiments. In the case of the stochastic model, both coefficients and every variable were related to hook size and fish size. Each simple relation was included in the stochastic model, so the selectivity curve can be calculated from the stochastic model, even if it is an extrapolation. Since the fish of about 220 mm in total length sometimes stretched the bend of the 3 Gou sode hook in the angling experiments, this hook could not catch larger fish than 220 mm. Our stochastic model did not include the effect of hook strength. Selectivity curves
from the stochastic model show overall size selectivity, when every hook is not broken.

Both coefficients and the movement limit were identified from the results of the angling experiments to calculate the selectivity curve from the stochastic model. Now there is no other way to determine the parameters of a stochastic model than by conducting angling experiments. When the parameters were identified from the angling experiments, they might include the effects of factors other than hook size and fish size, so the selectivity curves in Fig. 4 might include the effects of factors other than these sizes. But it is clear that hook size and fish size are fundamental factors. It is assumed that starvation of a fish and bait size influence the movement limit. It is assumed that starvation, bait size and the timing of pull up a hook influence the moving coefficient, and that bait size and the timing of pull up a hook influence the hooking coefficient. It will be expected that bait size will have a strong effect on the hooking mechanism, which will be analyzed based on the stochastic model in future studies.

The characteristics of the selectivity curves of the sode hooks and the simple relations included in the stochastic model will be helpful in further analysis of the hooking mechanism. In the future, we will establish an identification method of parameters in the stochastic model of the hooking mechanism to get more reliable selectivity curves of angling gears.

Acknowledgments

This work was partly supported by a Grant-in-Aid for Scientific Research from the Ministry of Education, Science, Sports and Culture of Japan (05806025).

References


