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<td>Citation</td>
<td>JOURNAL OF THE RESEARCH INSTITUTE FOR CATALYSIS HOKKAIDO UNIVERSITY=北海道大學觸媒研究所紀要, 3(2): 45-51</td>
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<td>Issue Date</td>
<td>1954-03</td>
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<td>Doc URL</td>
<td><a href="http://hdl.handle.net/2115/24629">http://hdl.handle.net/2115/24629</a></td>
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ON THE DISCONTINUOUS CHANGE WITH ELECTRODE POTENTIAL OF CHARGE REQUIRED TO INCREASE UNIT AREA OF HYDROGEN ELECTRODE OF MERCURY.

By

Koshiro Miyahara* and Tetsuo Takaishi**.

(1). Introduction.

T. Honda has observed a discontinuous change of electric charge required to build up new surface of hydrogen electrode of mercury kept at a definite potential as shown in Fig. 1. Hydrogen electrode process on mercury proceeds according to Horiuti et al. through the electrochemical mechanism,

\[ 2H^+(b) \xrightleftharpoons{+2\varepsilon} 2H^+(a) \xrightarrow{\varepsilon} H^+_2(a) + \varepsilon \rightarrow H_2, \]  

i. e. through the sequence, adsorption of hydrogen ion \( H^+(b) \) in solution to form adsorbed ion \( H^+(a) \), the combination of the latter with metal electron \( \varepsilon \) on the electrode resulting in adsorbed hydrogen-molecule-ion \( H^+_2(a) \) and its neutralization to complete hydrogen molecule; the latter process determines the rate leaving the former two in partial or preliminary equilibrium.

The \( H^+(a) \) and \( H^+_2(a) \) being two possible intermediates adsorbed on the electrode surface in partial equilibrium with each other and with \( H^+(b) \), it might be suggested with Honda that the observed discontinuity has to do with the abrupt change of coverage along with the variation of the electrode potential, from \( H^+(a) \) rich to \( H^+_2(a) \) rich one, the latter being half-neutralized state of \( 2H^+(a) \).

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It was observed by Horiuti and Mituya\textsuperscript{3)\textsuperscript{)} however that the curve of logarithm of the cathodic current $i$ versus electrode potential $\eta$ has a break at $\eta = -0.035$ volt as shown in Fig. 2, Tafel's constant $\frac{RT}{F} \ln i = \eta (F: \text{Faraday})$ being 1.3 or 0.3 above or below the point respectively.

According to the theoretical result arrived at by Horiuti, Keii and Hirota\textsuperscript{2)}, $\tau$ is a proper function $a + \text{unity}$ at higher $\eta$, where $H^+_a(a)$ population is small enough to leave the interaction among $H^+_a(a)$ negligible, whereas simply $a$ at lower $\eta$ where the population is large enough to cause appreciable interaction which practically prevents its further increase with decreasing $\eta$.

It follows that the population of $H^+_a(a)$ was already appreciable around the electrode potential of the HONDA's discontinuity, much lower than that of the break and hence such transition as of the $H^+_a(a)$ rich to $H^+_a(a)$ rich coverage responsible for the observed discontinuity is very impossible. The propriety of the picture was questioned in the present paper by studying whether the discontinuity could afterall

![Figure 1](image1.jpg)

\textbf{Figure 1.}

The number $E$ of elementary charges required to build up new surface of hydrogen electrode of mercury kept at definite electrode potential $\eta$. (T. HONDA.)

![Figure 2](image2.jpg)

\textbf{Figure 2.}

The dependence of the cathodic current of hydrogen electrode of mercury on electrode potential $\eta$. (J. Horiuti & A. Mituya.)

3) J. Horiuti & A. Mituya: \textit{ibid}, 2, No. 1, 79. (1951)
to the theoretical description of the discontinuity, of their consistency and adequancy.

(2). Existence of Discontinuity.

The problem of the project was investigated by approximating (I) the interaction energy according to BRAGG and WILLIAMS and assuming (II)\(H^+(a)\) and \(H_2^+(a)\) of being exclusive constituents of the positive side of the HELMHOLTZ's double layer without diffuse potential at the electrode surface.

From the equilibrium relations in the electrochemical mechanism (1) it follows that,

\[
\begin{align*}
p_{A(a)} &= p_{A(b)} e^{|\zeta|} \\
p_{R(a)} &= (p_{A(b)})^{e^{|\zeta|}} p_0 e^{kT},
\end{align*}
\]

(2)

where \(\zeta = V - V_0\) is the electrode potential referred to that of the reversible hydrogen electrode and \(p^A\) or \(p^B\) the BOLTZMANN factor of the chemical potential of \(A\) or \(B\) respectively, which stands respectively for \(H^+\) or \(H_2^+\), notations \((a)\) or \((b)\) designating the adsorbed state or that in solution respectively and suffix 0 quantity appropriate the reversible hydrogen electrode. The \(p^\delta\) is generally expressed in terms of the BOLTZMANN factor \(q^\delta\) of the reversible work required to bring up \(\delta\), which represents \(A\) or \(B\), from its reference state onto a prerrimarily evacuated adsorption site \(\sigma\) for \(\delta\) and the probability \(\theta_{\sigma(\delta)}\) and \(\theta_{\sigma(0)}\) of \(\sigma\) being occupied by \(\delta\) or altogether empty, as\(^t\),

\[
\frac{q^\delta}{p^\delta} = \frac{\theta_{\sigma(\delta)}}{\theta_{\sigma(0)}}.
\]

We have from (2) and (3) the adsorption isotherm,

\[
\frac{q^A_{\sigma(a)}}{p^{A(b)}} = \frac{\theta_A}{1 - \theta} \exp \left( \frac{w_{AB} - \theta_B}{kT} \right) \text{ for } H^+ \tag{4}
\]

and

\[
\frac{q^B_{\sigma(a)}}{p^{A(b)} p_0} \exp \left( - \frac{e\zeta}{kT} \right) = \frac{\theta_B}{1 - \theta} \exp \left( \frac{w_{AB} \theta_A + w_{BB} \theta_B}{kT} \right) \text{ for } H_2^+, \tag{5}
\]

where \(\theta_A = \theta_{\sigma(A)}, \theta_B = \theta_{\sigma(B)},\)

\[
\theta = \theta_A + \theta_B \tag{6}
\]

4) J. HORIUTI: ibid., 1, No. 1, 13, (1948) §5.
and hence $1 - \theta = \theta_{\sigma(o)}$.

The $q^{(a)}_{\sigma}$ (or $q^{(a)}_\sigma$) is the Boltzmann factor of the portion of the reversible work for $A$ (or $B$) not due to the mutual interaction, $w_{AB}\theta_B$ (or $w_{AB}\theta_A$) the part, in accordance with approximation (1), due to the interaction with $B$ (or $A$) in the surroundings except the electrostatic part already allowed for in terms of the Helmholtz's double layer and $w_{BB}\theta_B$ the part of the reversible work for $B$ due to surrounding $B$'s: $w_{AB}$ or $w_{BB}$ is hence the value of $w_{AB}\theta_B$ or $w_{BB}\theta_B$ at $\theta_B = 1$ respectively; $w_{AA}$ similarly defined vanishes, the relevant interaction being purely electrostatic.

The $p^{(a)}_\sigma$, $p^{(a)}_\sigma$, $q^{(a)}_\sigma$ and $q^{(a)}_\sigma$ may be taken all constant determined by the experimental condition.

Differentiating the logarithms of (4) and (5) by $E$, the number of the elementary charges required to build up new surface of hydrogen electrode of mercury per unit area, we obtain,

$$0 = \frac{1}{\theta_A} \frac{\partial \eta}{\partial E} + \frac{1}{1 - \theta} \frac{\partial \eta}{\partial E} + \frac{w_{AB} \theta_B}{kT} \frac{\partial \eta}{\partial E}$$

and

$$-\frac{e}{kT} \frac{\partial \eta}{\partial E} = \frac{1}{\theta_B} \frac{\partial \eta}{\partial E} + \frac{1}{1 - \theta} \frac{\partial \eta}{\partial E} + \frac{w_{AB} \theta_A}{kT} \frac{\partial \eta}{\partial E} + \frac{w_{BB} \theta_B}{kT} \frac{\partial \eta}{\partial E}.$$  (8)

We have, moreover, two relations according to (4) and (5) at the discontinuity ($\eta = -0.35$ volt),

$$\frac{\theta'_A}{1 - \theta'} \exp \left( \frac{w_{AB}\theta'_B}{kT} \right) = \frac{\theta'_A}{1 - \theta'} \exp \left( \frac{w_{AB}\theta'_B}{kT} \right) \left( = \frac{q^{(a)}_\sigma}{p^{(a)}_\sigma} \right)$$  (9)

and

$$\frac{\theta'_B}{1 - \theta'} \exp \left( \frac{w_{BB}\theta'_B + w_{AB}\theta'_A}{kT} \right) = \frac{\theta'_B}{1 - \theta'} \exp \left( \frac{w_{BB}\theta'_B + w_{AB}\theta'_A}{kT} \right) \left( = \frac{q^{(a)}_\sigma}{p^{(a)}_\sigma} \exp \left( \frac{\gamma}{kT} \right) \right),$$  (10)

where the single and double primes denote the quantities appropriate to higher and lower values of $E$ at the discontinuity respectively.

The $E$ is on the other hand the sum of the number of elementary charges in the adsorbed phase and that required for the reaction $2H^+(a) + e \rightarrow H_2(a)$, i.e.,

$$E = N_s (\theta_A + 2\theta_B)$$  (11)

where $N_s$ is the number of $\sigma$'s per unit area of the surface of the electrode determined at $8.8 \times 10^{14}$ cm$^{-2}$ on the basis of radius of Hg
atom at the closest packing. It is read from Fig. 1 as,

\[
\begin{align*}
\frac{E'}{N_s} &= a' = \theta'_a + 2\theta'_b = 0.0250 \\
\frac{E''}{N_s} &= a'' = \theta''_a + 2\theta''_b = 0.0174
\end{align*}
\]

(12)

and \( \frac{\partial \eta}{\partial E} = 1.48 \times 10^{14} \) volt·cm, the latter values gives 23.8 \( \mu F/cm^2 \) for the capacity, a magnitude of current acceptance.

Six unknowns, i.e., \( w_{AB}/kT, w_{BB}/kT \) and the individual fractions of coverage at \( \gamma = \gamma_c \), may now be solved from six equations (7), (8), (9), (10) and (12) as follows.

Eliminating \( \theta'_A, \theta''_A, \theta' \) and \( \theta'' \) from (6), (9) and (12) we have,

\[
\frac{w_{AB}}{kT} = \frac{1}{(1+\theta'_B-\theta'_B)} \ln \left( \frac{(1+\theta'_B-\theta''_B)}{(1+\theta''_B-\theta''_B)} \right)
\]

(13)

on one hand and eliminating similarly \( \frac{\partial \theta_A}{\partial E} \) and \( \frac{\partial \theta_B}{\partial E} \) from (7), (8) and two relations derived from (6) and (11), i.e.,

\[
\frac{\partial \theta_A}{\partial E} + 2 \frac{\partial \theta_B}{\partial E} = \frac{1}{N_s},
\]

\[
\frac{w_{BB}}{kT} = \left( \frac{w_{AB}}{kT} \right)^{\delta} \left( \frac{w_{AB}}{kT} \right) \left( \frac{w_{AB}}{kT} \right) + \frac{w'}{w''} \]

(14)

on the other hand, where,

\[
w' = \frac{(1+\theta'_B-\theta'_B)}{(1+\theta''_B)} \left( \frac{a'-2\theta'_B}{1-\theta'_B} \right),
\]

\[
v' = -w'N_s \frac{e}{kT} \left( \frac{\partial \eta}{\partial E} \right)_{\gamma = \gamma_c} \frac{2a'-2\theta'_B}{1-\theta'_B},
\]

\[
w'' = \frac{1}{1-\theta'_B} \left[ (a'-2)N_s \frac{e}{kT} \left( \frac{\partial \eta}{\partial E} \right)_{\gamma = \gamma_c} - \frac{1}{\theta'_B} \right]
\]

\( u', v' \) and \( w'' \) being similarly expressed as \( u' \) etc. respectively, with \( a' \) and \( \theta'_B \) replaced by \( a'' \) and \( \theta''_B \). According to (14) we have further,

\[
\frac{w_{AB}}{kT} = \frac{2}{\Delta u} + \sqrt{\left( \frac{\Delta v}{2\Delta u} \right)^2 - \left( \frac{\Delta w}{2\Delta u} \right)^2},
\]

(15)

where \( \Delta u = u' - u'' \), \( \Delta v = v' - v'' \) and \( \Delta w = w'' - w'' \), and from (10) and (13),

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$\frac{w_{BB}}{kT} = \frac{1}{\theta_B' - \theta_B''} \left[ \ln \frac{\theta_B'' (1 + \theta_B'' - a')}{\theta_B' (1 + \theta_B'' - a'')} - \left( \frac{a' - a''}{\theta_B'' - \theta_B''} - 2 \right) \ln \frac{(1 + \theta_B' - a') (a' - 2 \theta_B)}{(1 + \theta_B'' - a'') (a' - 2 \theta_B)} \right]. \quad (16)$

Eqs. (13) and (15) incorporated with above values of $a'$, $a''$ and $(\frac{\partial \eta}{\partial E})_{\eta=\nu}$ determines now $(\theta_B', \theta_B'')$ relation, which falls within the range,

$$0 < \theta_B' < \frac{a'}{2} = 0.0125 \quad 0 < \theta_B'' < \frac{a''}{2} = 0.0087$$

required by (12) for positive sign of the root of (15) but not for negative one. The expression of $w_{AB}/kT$ with the former sign substituted in (14) gives another expression for $w_{BB}/kT$ which provides a further $(\theta_B', \theta_B'')$ relation when combined with (16). Both the relations were now solved graphically for $\theta_B'$ and $\theta_B''$ and henceforth $\theta_A'$, $\theta_A''$, $w_{AB}/kT$ and $w_{BB}/kT$ determined as below.

$$\theta_A' = 1.62 \times 10^{-2}, \theta_B' = 5 \times 10^{-4} \text{ for the higher value of } E \quad (18)$$
and

$$\theta_A'' = 5 \times 10^{-5}, \theta_B'' = 1.25 \times 10^{-2} \text{ for the lower one,} \quad (19)$$
and

$$w_{AB}/kT = 490.3 \text{ and } w_{BB}/kT = 398.6 \text{ at } 19^\circ C. \quad (20)$$

(3). Conclusion.

Conclusion is that there may exist at least formally* such discontinuity as observed by Honda and the change of appropriate $\theta_A$ and $\theta_B$ required by (12) for positive sign of the root of (15) but not for negative one.

* It has been shown [K. Miyahara and T. Takaisi: "Busseiron-Kenkyu", 40, 70 (1951) (printed in Japanese), or partly Fowler and Guggenheim: "Statistical Thermodynamics" (Cambridge Univ. Press, 1939) Chap. X.] that the condition required for the stable coexistence of two adsorbed phases is, besides the equality of the chemical potentials of individual components, that of spreading pressures $\pi'$ and $\pi''$ i.e.,

$$\pi' = \pi''$$

of the phases denoted respectively by single and double primes. The expression of $\pi$ is,

$$\frac{\pi}{kT} = -\ln (1-\theta) + \frac{1}{2} (\theta_A w_{AA} + \theta_B w_{BB} + 2 \theta_A \theta_B w_{AB}),$$

which gives $\pi'/kT = 4.4 \times 10^{-2}$ and $\pi''/kT = 2.1 \times 10^{-2}$ at $19^\circ C$ for this case. These values and that of $(\frac{\partial \pi}{\partial \eta})$ at $\pi'$ and $\pi''$, i.e., $(\frac{\partial \pi}{\partial \eta})_{\eta=\pi'} = 3.05$ and $(\frac{\partial \pi}{\partial \eta})_{\eta=\pi''} = 1.93$, indicate, according to the general behaviour of $\pi$ in the neighbourhood of the discontinuity [cf. the paper quoted above], that the consistent value of $\nu_e$ is somewhat higher than $-0.35$ volt of the observed one as shown in Fig. 3.

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Figure 3.

The behaviour of $\pi$ near the transition point $p_c; p$ is the equilibrium pressure of adsorptives in gas phase.

$\theta_n$ there shown above is compatible with the suggested picture; relevant magnitude of $w_{AB}$ and $w_{BB}$ is however extravagant for the picture reasonably to be accepted.

We are much indebted to Dr. J. Horimti, the Director of the Institute for Catalysis, for his kind and valuable direction on this work, which was submitted to the 6th Annual Meeting of the Japan Chemical Society.