COMMENT ON MILNER'S PAPER ON
"THE POSSIBLE MECHANISMS OF COMPLEX REACTIONS"

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It will be shown in this note that the method proposed by Milner for generating "complete sets of reaction mechanisms" was no more than a bad change of the theory of the steady rate of reaction developed by Horiuti et al. with an unnecessary assumption of a single overall reaction and some confusions in the dependence among possible mechanisms. Despic et al. have, furthermore, proposed a method to list up possible steps and mechanisms for a single overall reaction on the basis of Milner's method, which was erroneously treated by the formers as "a formal method to obtain a list of all reaction paths of a single overall reaction, each of which cannot be obtained by superposition of any other members of the list."

Milner has started his discussion by solving the equations,

\[ \sum a_{is} \nu_s = 0, \quad (i = 1, 2, \ldots, I; s = 1, 2, \ldots, S) \]  

with respect to the stoichiometric number, \( \nu_s \), of step \( s \) with reference to a special value of a coefficient,

\[ B_r = \sum b_{mr} \nu_s, \quad (2) \]

of \( m \)-th reactant or product in the stoichiometric equation of a single overall reaction, where \( a_{is} \) or \( b_{mr} \) is the coefficient of \( i \)-th intermediate or the \( m \)-th reactant or product in the stoichiometric equation of step \( s \) respectively. According to this method, a value inconveniently differing from integer may be obtained for \( \nu_s \), depending upon the stoichiometric equations assumed for step \( s \) and the overall reaction, and, furthermore, such a restriction as Eq. (2) is unnecessary to solve Eqs. (1) as follows.

Eq. (1) is a special form of the steady state condition of a reaction system, which states that the amounts of respective intermediates are kept constant during the progress of reaction, and are generically given as

\[ \sum a_{is} n_s = 0 \quad (s = 1, 2, \ldots, S \text{ and } i = 1, 2, \ldots, I') \]  

or \( an = 0 \) in its matrix representation, where \( n_s \) is the number of acts of step \( s \) in...
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a specified time of reaction. On the basis of a specified set of $S$ steps and $I'$ intermediates, we can solve Eqs. (3) with respect to $n_\ell$'s and obtain $P (= S - I)$ independent sets of $n_\ell^{(p)}$-values ($p=1,2,\ldots,P$), where $I$ is the rank of the largest nonsingular matrix of $\text{a} \equiv [a_{\lambda\nu}]$, called by HORIUTI and NAKAMURA\textsuperscript{2}) as the number of independent intermediates\textsuperscript{*)}. The sets of $n_\ell^{(p)}$-values thus obtained are not unique because $n_\ell^{(p)}$'s may have a common factor, and HORIUTI and NAKAMURA\textsuperscript{2}) have defined the $p$-th independent route of reaction by a set, $\text{n}^{(p)} = [v^{(p)}, v^{(p)}, \ldots, v^{(p)}]$, of the so-called stoichiometric numbers, $v^{(p)}$'s, which are the simplest integers given by the $n_\ell^{(p)}$-values aside from their common factor. The set, $\text{B}^{(p)}$, of coefficients, $B_\lambda^{(p)}$'s, of the $m$-th species in the stoichiometric equation of route $p$ is now determined by an equation, similar to Eqs. (2),

$$\text{B}^{(p)} \equiv [\text{B}^{(p)}] = [b_{\lambda\nu}]{\text{n}}^{(p)},$$

where $b_{\lambda\nu}$ and $B_\nu^{(p)}$ are negative or positive integers according to $m$-th species being reactant or product respectively. The coefficients, $B_\lambda^{(p)}$'s, and hence the sorts of overall reactions able to occur in the specified reaction system are thus necessarily determined by Eqs. (4) on the basis of $\text{n}^{(p)}$, as illustrated by HORIUTI\textsuperscript{2}) by RICE-HERZFELD-mechanism for the pyrolysis of ethane.

Let us consider an another example, which is the system of catalytic oxidation\textsuperscript{4}) of $\text{C}_2\text{H}_4$ by $\text{O}_2$ assumed to proceed by the following steps proposed by TWIGG\textsuperscript{5}).

\begin{equation}
\begin{array}{ll}
\text{s} = 1: & \text{O}_2 \leftrightarrow 2\text{O (a)}, \\
2: & \text{C}_2\text{H}_4 + \text{O (a)} \leftrightarrow \text{C}_2\text{H}_4\text{O (a)}, \\
3: & \text{C}_2\text{H}_4\text{O (a)} \leftrightarrow \text{C}_2\text{H}_4, \\
4: & \text{C}_2\text{H}_4 + 2\text{O (a)} \leftrightarrow 2\text{C}_2\text{H}_4\text{O (a)}, \\
5: & \text{CH}_2\text{O (a)} \leftrightarrow \text{CO (a)} + 2\text{H (a)}, \\
6: & \text{CO (a)} + \text{O (a)} \leftrightarrow \text{CO}_2, \\
7: & 2\text{H (a)} + \text{O (a)} \leftrightarrow \text{H}_2\text{O}.
\end{array}
\end{equation}

Denoting the intermediates, $\text{O (a)}$, $\text{C}_2\text{H}_4\text{O (a)}$, $\text{CH}_2\text{O (a)}$, $\text{CO (a)}$ and $\text{H (a)}$ by $i = 1, 2, 3, 4$ and 5 respectively, we have the steady state condition,

$$\text{an} \equiv \begin{bmatrix} 2 & -1 & 0 & -2 & 0 & -1 & -1 \\
0 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & -1 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & 0 & 2 & 0 & -2 \end{bmatrix} \begin{bmatrix} n_1 \\
n_2 \\
n_3 \\
n_4 \\
n_5 \\
n_6 \\
n_7 \end{bmatrix} = 0.$$\textsuperscript{*)}

With respect to, for instance, the formation of $\text{HCl}$ from $\text{H}_2$ and $\text{Cl}_2$ through steps, 1: $\text{Cl} + \text{H}_2 \leftrightarrow \text{HCl} + \text{H}$ and 2: $\text{H} + \text{Cl}_2 \leftrightarrow \text{HCl} + \text{Cl}$, of NERNST chain, we have, according to Eqs. (3), that $-n_1 + n_2 = 0$ for Cl and $n_1 - n_2 = 0$ for H, which are linearly dependent with each other, and thus the independent intermediate in this case is either of two intermediates, H and Cl.
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The rank of the matrix \( a \) is 5 and \( S=7 \) as seen from scheme (5) and hence we have two independent routes of reaction, which are

\[
\mathbf{n}^{(1)} = (1, 2, 2, 0, 0, 0, 0) \quad \text{and} \quad \mathbf{n}^{(2)} = (3, 0, 0, 1, 2, 2, 2).
\]

Introducing Eqs. (7) and the matrix,

\[
[b_{mx}] = \begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix},
\]

into Eqs. (4), where \( \text{O}_2, \text{C}_2\text{H}_4, \text{C}_2\text{H}_4\text{O}, \text{CO}_2 \) and \( \text{H}_2\text{O} \) are assigned by \( m=1, 2, 3, 4 \) and 5 respectively, we have

\[
\mathbf{B}^{(1)} = (-1,-2, 2, 0, 0) \quad \text{and} \quad \mathbf{B}^{(2)} = (-3,-1,0,2,2)
\]

and thus find the two independent routes can be represented by the stoichiometric equations,

\[
2\text{C}_2\text{H}_4+\text{O}_2 = 2\text{C}_2\text{H}_4\text{O} \quad \text{and} \quad \text{C}_2\text{H}_4+3\text{O}_2 = 2\text{CO}_2+2\text{H}_2\text{O},
\]

\( i.e. \) the partial and the complete oxidations of \( \text{C}_2\text{H}_4 \) respectively.

It is now pointed out that Milner has introduced some confusions into his list of possible mechanisms by leaving the dependence among the routes of reaction out of consideration.

Let us consider the hydrogen electrode reaction, \( 2\text{H}^+ + 2e = \text{H}_2 \), which was exemplified by Milner as comprising three steps, 1: \( \text{H}^+ + e \rightarrow \text{H} \), 2: \( \text{H}^+ + \text{H} + e \rightarrow \text{H}_2 \) and 3: \( 2\text{H} \rightarrow \text{H}_2 \). We have \( S=3 \) and \( I=1 \) in this case, and hence two independent routes of reaction, which are \( \mathbf{n}^{(1)} = (1, 1, 0) \) and \( \mathbf{n}^{(2)} = (2, 0, 1) \), assigned by Milner as mechanisms A and B, respectively. Milner has listed up another one, \( \mathbf{C} = (\mathbf{n}^{(3)} = (0, 2, -1)) \), and discussed that this mechanism is possible only on some specific electrode material, which forms \( \text{H} \) atoms only through step 1 but not step 3, and necessarily behaves a phenomenon of induction period. The mechanism \( \mathbf{C} \) is, however, as pointed out by Horiuti et al., no more than one of linear combinations of A and B, \( i.e. \) 2A−B, which corresponds to the so-called ‘slow discharge mechanism’ where step 1 is the slowest and consequently \( \text{H}_2 \) is evolved predominantly through steps 2 and 3.

With respect to the other two overall reactions, \( \text{Cu}^{2+} + 2e = \text{Cu} \) and \( 4\text{OH}^- = \text{O}_2 + 2\text{H}_2\text{O} + 4e \), illustrated by Milner, we can easily see \( P=4 \) and 5 respectively on the basis of steps assumed by Milner and that only four or five mechanisms from those, A, B, C etc., listed up by Milner are linearly independent with each other and the other mechanisms are all given as linear combinations of these.
independent ones as follows:

For \( \text{Cu}^{2+} + 2e = \text{Cu} \), it follows

\[
A = \frac{1}{2}(D+E), \quad B = 2A - F, \quad C = \frac{1}{2}(D+J), \quad F = \frac{1}{2}(E+H), \quad G = \frac{1}{2}(H+J) \quad \text{and} \quad I = 2G - F
\]

as linear combinations of \( D, E, H \) and \( J \).

For \( 4\text{OH}^- = \text{O}_2 + 2\text{H}_2\text{O} + e^- \), we have

\[
B = 2(A-D) + H, \quad C = 2D - H, \quad F = E + D - G, \quad I = 2(E-D) + H, \quad J = 2(E-G) + H \quad \text{and} \quad K = 2(G-D) + H
\]

as linear combinations of \( A, D, E, C \) and \( H \).

These dependent routes of reaction are just the "combinations of simultaneous mechanisms" of Milner and omitted by him from his list of the possible mechanisms. We see thus, so far as Milner's analysis concerns, that it is sufficient for the discussion of possible mechanisms to list up independent routes of reactions, which has already been done by Horiuti et al.\(^2\)

References

2) J. Horiuti and T. Nakamura, Z. Physik. Chem. (Neue Folge) 11, 358 (1959);
   Advances in Catalysis 17, 1 (1967), J. Horiuti, This Journal 5, 1 (1967) etc.
4) K. Miyahara, This Journal, the preceding paper.