MACROECONOMICS WITH FRICTIONS

by

Noritaka Kudoh

April 20, 2000

A dissertation submitted to the

Faculty of the Graduate School of State

University of New York at Buffalo

in partial fulfillment of the requirements for the

degree of

Doctor of Philosophy

Department of Economics

Table of Contents

Acknowledgments

The journey through my graduate work would have been a lot more difficult without the help I received from many people. Here I wish to thank those people.

Everything started when I met my undergraduate advisor, Professor Hiroaki Hayakawa. His passion toward economics has changed the path I walk through. I wish to thank him for his continuous encouragement and friendship over the years. I also wish to thank Professor Koichi Futagami for organizing workshops at Ritsumeikan University for me and other students wishing to go to graduate schools. The workshops helped me learn dynamic macroeconomics.

I would like to thank the department for giving me a chance to become an economist, and for giving me a friendly and comfortable environment over the years. I am truly grateful to my dissertation committee members, Professors Peter Morgan, Joydeep Bhattacharya, and Hao Li, for their supervision, and Professor Shouyong Shi of Queen's University for his painstaking work as the outside reader. In particular, I wish to thank Professor Morgan for chairing my dissertation committee. Without his help, I would have had to give up pursuing monetary theory using search theory (Chapter 2). I also wish to thank Professor Bhattacharya for guiding me by co-authoring a paper with me (Chapter 3) and by encouraging me to do conference presentations. It was he who introduced me to macroeconomics with asymmetric information (Chapter 4).

Finally, I wish to thank my parents and my sister for their encouragement.

Abstract

The dissertation consists of three essays on macroeconomic theory. The first essay investigates monetary theory with special attention given to bilateral strategic bargaining. It uses a version of the search-theoretic model of money developed by Kiyotaki and Wright (1991) to study the implication of agents' strategic behavior on the purchasing power of money. The dependence of agents' preferences on the quality of match as well as the quantity consumed is considered in order to study the impact of heterogeneous outside options on the bargaining process and hence on the purchasing power of money. The model naturally gives rise to price dispersion due to endogenously dispersed outside option values. The purchasing power of money thus depends not only on what it will buy in the future, but also on who matches with whom. Strategic bargaining and Nash solutions do not coincide even in a steady state. Strategic bargaining results in a higher market volatility than does Nash bargaining because the values of outside options are match-specific and Nash bargaining does not use all of the information provided by the market.

The second essay reconsiders the link between tight money policies and inflation in the spirit of Sargent and Wallace's (1981) influential paper "Some Unpleasant Monetarist Arithmetic." In contrast to the previous results, this essay shows that a tight money policy engineered by open market operations can be inflationary even when the real interest rate is less than the growth rate of the economy. The key to the result is the introduction of a neoclassical concave production function, which generates the “interest rate effect”: tight money raises the real interest rate. The high interest rate reduces bond seigniorage if the economy is on the “good side” of the bond-seigniorage Laffer curve. This revenue short fall has to be made up by increasing the inflation rate. In contrast to the conventional wisdom, it is the bond-seigniorage Laffer curve, rather than the total seigniorage Laffer curve, that is the key determinant of whether an economy exhibits unpleasant monetarist arithmetic.

The third essay deals with unemployment. It reconsiders the potential role that government intervention in the labor market can have in improving the level of economic activity. In the early literature, economists such as Keynes (1936) are optimistic about the effects of public policies on employment, even though their arguments receive little support from the modern theoretical literature. However, real-world observations give a different view. Governments, especially in developing countries, routinely provide large-scale public employment programs. Using an overlapping generations model with production and asymmetric information problems in the labor market, the chapter looks for an explanation for why such governments might want to provide jobs. In such an economy, the government can attract part of the labor force at a wage rate that is lower than the private wage rate. There arise two long-run equilibria. If the low-activity steady-state is regarded as describing of a developing country, then the model suggests that the provision of public employment programs in developing countries may improve long-run economic activity. Also, it may help these economies get out of development traps. The model also provides insights as to why, in developed economies, public employment programs may be harmful and why unemployment insurance programs have better results.

Preface

When I was an undergraduate student, I was thrilled to learn debates over Keynesian and Classical macroeconomics. Because of the way undergraduate textbooks introduce Keynes, I was once an enthusiastic supporter of the Keynesian economics. As I studied economics further at the graduate level, I learned the concept of *micro foundations* of macroeconomics. In each of the essays in the dissertation, my approach is to start with an agent's utility maximization problem and then analyze a general equilibrium. In this sense, none of my dissertation chapters is truly Keynesian. Any stickiness or rigidity in price adjustment processes is imposed. At the same time, the economies modeled here are not "frictionless". In Chapter 2, I study an economy with search frictions. In Chapter 3, a legal restriction theory of money is used to study an environment in which money is dominated by other assets in rate of return. In Chapter 4, asymmetric information is assumed to model unemployment. The presence of "frictions" is essential because otherwise neither money or unemployment can be understood. In fact, in the Walrasian economy, in which all markets clear instantly, there is no role for money and full-employment will be achieved. Thus, the challenge here is to model frictions in a manner that is not *ad hoc*. For this purpose, in two of the dissertation chapters I use game theoretic equilibrium concepts. This way, a phenomenon that has been regarded as a disequilibrium by textbook macroeconomics can be studied as an *equilibrium.*

The organization of the dissertation is as follows. Chapter 1 is a general introduction to the dissertation and provides the motivations of each chapter. Chapter 2 is entitled "Matching, Bargaining and Dispersed Values of Fiat Currency." It studies the purchasing power of a unit of fiat currency using a random matching model. The project started as a term paper for Professor Peter Morgan's course "Economics of Matching," which I took in Fall 1998. The chapter was completed under Professor Morgan's guidance in Fall 1999. Chapter 3 is "Tight Money Policies and Inflation Revisited." It is a revised version of a paper jointly written with Professor Joydeep Bhattacharya in Summer 1998. Chapter 4 is "Government Employment Programs, Unemployment, and Capital Accumulation." It was written under Professor Bhattacharya's guidance in Summer 1999.

Chapter 1
Introduction

1.1 Monetary Theory

1.1.1 Kiyotaki-Wright Model

Traditionally monetary economics uses models that impose the presence of valued fiat currency into an otherwise non-monetary Walrasian environment in which all trading is centralized, frictionless and instantaneous. Since there is no natural role for fiat currency in such models, its presence is forced upon the model by an artificial assumption.

The money-in-the-utility-function (MIUF) model, for example, assumes that agents derive utility from the real cash balances they hold. The cash-in-advance (CIA) model assumes that agents need to use some real balances to obtain consumption goods. These two approaches impose valued fiat money and the Walrasian price. Although these are the two most widely used tools in monetary economics, they are often criticized for the *ad hoc* assumptions of modeling money, since money is *assumed* to be valuable. The overlapping generations (OG) model treats money as an asset: if there is no restriction on agents' asset holdings, the agents hold the assets that yield the highest return. The advantage of the approach is that the purchasing power of money is determined by what it will buy in the future. The disadvantage is that it fails to explain the "rate-of-return-dominance" problem: if there is another asset which yields a higher return than money, then no one holds money.

In other words, in an OG environment, the value of money may be zero. This is due to the absence of a transaction role for money in the model.

The challenge is to model explicitly the transaction role of money. To do so, a model of decentralized trading is needed. This is where the search theoretic approach comes in. In a search environment, an agent must first find his trading partner. A transaction is made only if he finds an agent and the transaction yields enough utility to both of the parties.

In their path-breaking contribution, Kiyotaki and Wright (1989) examined bilateral trading markets in which agents choose to hold and exchange a fiat currency because of its value in mitigating the double coincidence of wants difficulties that arise with decentralized trading. The first generation of these models, such as Kiyotaki and Wright (1989, 1991, 1993), fixes the terms of trade, or 'price,' and shows that an object called 'money' arises endogenously. The second generation consists of models such as Trejos and Wright (1995) and Shi (1995) in which the determination of the terms of trade is explicitly modeled by incorporating a bargaining problem into the search model. The third generation endogenizes the distribution of money holdings; Camera and Corbae (1999) is an example.

1.1.2 Purchasing Power of Money

To remove a fixed-price assumption of the Kiyotaki-Wright model, bargaining theory was introduced into search models of money in one form or another. Trejos and Wright (1995) and Shi (1995) introduced a version of Rubinstein's (1982) sequential bargaining game. The Rubinstein bargaining game is a powerful tool for endogenizing the terms of trade in a random matching model. The basic idea is that a bargaining party who makes an offer (the “proposer”) will offer a terms of trade that is just enough for the opponent (the “respondent”) to accept. That is, the proposer’s offer is designed so that

$$\begin{aligned} & \text{The value to the respondent of accepting the offer now} \\ = \quad & \text{Present discounted value to the respondent of rejecting the offer.} \end{aligned} \tag{1.1}$$

If the purchasing power of a unit of fiat currency is determined this way, then we must expect to see two determinants of the value of money. One is that agents discount future; that is, waiting is costly. As (1.1) suggests, rejecting an offer gets costly as the respondent’s discount rate gets large. The other force that motivates bargaining parties to reach an agreement is the fear that their bargaining partner might walk away in favor of a new bargaining partner. The right-hand-side of (1.1) contains the value of outside opportunities. Thus, the bargaining party who has better outside opportunities will have a greater bargaining power. Trejos and Wright (1995) and Coles and Wright (1998) mainly focus on the former motive for reaching an agreement. In contrast, we focus on the latter in Chapter 2.

In the previous literature, because of the way price is endogenized the latter force plays only a small role in determining the purchasing power of money. To see this point, suppose that we wish to endogenize the purchasing power of money in Kiyotaki and Wright (1991). First, since analyzing a model with divisible units of currency is quite complex, goods are instead assumed to be divisible and agents’ utilities are increasing in the amount of a good consumed. This way, the purchasing power of money is measured in units of goods consumed. The agents’ utilities thus depend upon both the quality of match made and the quantity consumed and consequently there arises the substantial complication that agents' *reservation strategies depend both on the match quality and the quantity consumed.* In order to avoid the complication, Trejos and Wright (1995) and Shi (1995) assume that the dependence of utility on the match quality is captured by a utility function that depends only on quantity consumed when the match quality is higher than an exogenous threshold level; that is,

$$U(q,z) = \begin{cases} U(q) \text{ for } z \leq x \\ 0 \text{ for } z > x, \end{cases} \tag{1.2}$$

where U is the utility function, q denotes the quantity consumed, $z \in [0,1]$ denotes the random variable for match quality, and $x \in [0,1]$ is the exogenously specified threshold match quality. This formulation severs an interaction between the agents' reservation strategy and the price of goods, since the reservation strategy is exogenously given by only the match quality.

As is clear from (1.2), the agent is indifferent over all acceptable match qualities $z \in [0,1]$. The implication for the bargaining component of the model is that the values of outside options are the same for all acceptable matches because, even if a bargaining party finds a new partner, the new partner is in utility terms *identical* to the old partner. Therefore, while the presence of outside options will alter the bargaining outcome, the homogeneity of these options across all matches will cause the outcome to be the same for all matches. In effect, the Law of One Price will prevail.

1.1.3 Introducing Heterogeneous Matching

In Chapter 2, we re-introduce the dependence of each agent's utility on the quality of each match. In particular, we specify the utility function as

$$U(q,z) = \begin{cases} U(q,z) \text{ with } U_1 > 0, U_2 < 0, U_{11} < 0 \text{ for } z \leq x \\ 0 \text{ for } z > x. \end{cases} \tag{1.3}$$

The present model also uses a constant match quality reservation strategy. However, goods are differentiated and bilateral encounters between agents occur randomly, so that each match quality is idiosyncratic. This results in bargaining partners having outside option values that depend upon the quality of their particular match. To see this point, suppose that a low quality match is formed. Since the current match quality is low, the probability that the buyer will find a higher quality match in the next period is relatively high. In other words, the value of the buyer's outside option is relatively high when a low quality match forms and will be used as a weapon to extract more production from the sellers.

More specifically, if a low quality match forms then, other things equal, the buyer's utility is directly reduced. Since the reservation strategy is constant, the buyer will walk away. Instead, there is another margin; a loss of buyer utility by a low quality match can be compensated by the seller providing to the buyer a high quantity of the good. The seller's incentive to do so is a consequence of the fact that when a low quality match is formed the probability that the buyer will discover next period a higher quality match with another seller is relatively high. This is used by the buyer as a threat in the bargaining process and, as a result, the model presented in Chapter 2 predicts price dispersion. That is, the purchasing power of one unit of fiat currency will vary from one match to another. This is a major contribution of the work done in Chapter 2. If fiat currency is valued because transactions are time-consuming and there is a double-coincidence-of-wants problem due to product-differentiation, then one should not expect the Law of One Price. The Law of One Price obtained in earlier models is due directly to the assumption of indifference over acceptable match qualities.

A second contribution of Chapter 2 is that it unifies the standard search theoretic model of money and the bargaining game developed by Rubinstein and Wolinsky (1985) and Wolinsky (1987). In particular, the bargaining game constructed here has two note-worthy attributes. One is that it is a dynamic bargaining model in the sense of Coles and Wright (1998). The other is that through the presence of outside options, it has very clear interaction between individual bargaining games and the *market*. If a bargaining game without outside options is assumed (as in most of the other models), then the only factor that determines the terms-of-trade is the bargaining parties' characteristics. Particularly, the aggregate market does *not* influence such individual bargaining processes. The bargaining game developed here itself is a contribution of the chapter. It can be applied insightfully to many other issues.

1.2 Monetary Policy

1.2.1 Fiscal Theory of Inflation

Inflation is a tax. Because of that, monetary and fiscal policies are connected by a single government budget constraint and the government cannot arbitrarily choose a mix of monetary and fiscal policies for a desired policy goal. This is the heart of the Sargent and Wallace's (1981) famous "unpleasant monetarist arithmetic." Using the interaction between monetary and fiscal policies, Sargent and Wallace (1981) produced conditions under which monetizing the government deficit does not always result in a higher inflation rate. A policy implication of their proposition is that in designing a long-run monetary-fiscal policy, it is misleading to have the quantity theory of money in mind. This point has captured the attention of many economists and has evolved into the "fiscal theory" of the price level, which emphasizes the role of the government's intertemporal budget constraint in determining the price level. Chapter 3 reconsiders the conditions under which a tight money policy through open market operations is inflationary in the long run. It asks the question in a more general environment in order to understand the necessary conditions for the unpleasant monetarist arithmetic (UMA).

Consider the following steady state government budget constraint

$$\begin{aligned} g &= m\left(1 - \frac{1}{\pi}\right) + b\,(1-\rho) \\ &= \text{Currency seigniorage + Bond seigniorage,} \end{aligned} \tag{1.4}$$

where g denotes the government deficits, m denotes the real money stock, π is the gross inflation rate, b denotes the stock of bonds, and ρ denotes the real interest rate and the growth rate of the economy is normalized to unity. Following the conventional view of the "fiscal theory" of the price level, equation (1.4) will be considered to be the (long-run) "inflation-determination condition." Since the size of the deficit is determined by the fiscal authority, it is assumed to be constant. The choice variable for the central bank is the bond-to-money ratio, b/m. Thus, the steady state rate of inflation is determined by the budget constraint once the real interest rate is known. This is clear once (1.4) is rewritten as

$$\frac{1}{\pi} = 1 + \frac{b}{m}(1 - \rho) - \frac{g}{m}.$$

1.2.2 Budget Arithmetic and Inflation: Sargent-Wallace Revisited

It is now clear that the government's budget constraint (1.4) is actually the long-run inflation determination condition. We now ask whether a tight money policy *always* leads to a lower inflation.[1] There are two cases to consider.

Sargent and Wallace (1981) restrict their analysis to a scenario where the real interest rate exceeds the growth rate of the economy (normalized to unity here), so that bond seigniorage is negative. Then, a permanent open market sale of bonds creates a revenue shortfall. If the economy is in a position in which the government could raise seigniorage by printing money faster (*i.e.*, the economy is on the "good side" of the total seigniorage Laffer curve), then a tight money policy engineered by permanent open market operations increases the steady state rate of inflation. This is the essence of the Sargent and Wallace's (1981) unpleasant monetarist arithmetic.

Consider a case with the real interest rate being less than the growth rate of the economy. This case is explored by Bhattacharya, Guzman, and Smith (1998). In such an economy, bond seigniorage is positive and a further increase in bonds increases bond seigniorage. In this case, the government could raise enough revenue from bonds. Bhat-

[1] The word "always" is important here because we are *not* trying to establish that a tight money policy *always* leads to inflation.

tacharya, Guzman and Smith (1998) point out that it is possible that an increase in bonds reduces currency seigniorage more than the increase in bond seigniorage. The intuition is as follows. As the bond-money ratio increases, capital gets crowded out of the portfolio of private agents. This reduction in deposits causes a reduction in the volume of reserves held by banks. Consequently, the inflation tax base falls. On the good side of the Laffer curve, the central bank is forced to raise the steady state inflation rate. This "tax base effect" is the channel through which the UMA proposition holds even when the rate of return is less than the growth rate of the economy.

1.2.3 Introducing Capital Accumulation: Miller-Sargent Confirmed

Chapter 3 introduces a concave neoclassical production function of the Diamond (1965) variety. This adds a new feature into the analysis. In particular, there arises a new channel through which a tight money policy engineered by a permanent open market operation increases the steady state inflation rate.

Suppose the real interest rate is less than the growth rate of the economy. Once again, in such an economy, bond seigniorage is positive and a further increase in bonds increases bond seigniorage. Thus, the government could raise enough revenue from bonds. The "tax base effect" is still present here. What is new here is the "interest rate effect". An increase in the bond-money ratio in this economy has the real effect of reducing the steady state capital stock because the tight money policy crowds capital out of the portfolio of private agents. This raises the marginal product of capital. If the primary deficit is small enough, the increase in the rate of return on capital raises the real interest rate. The increase in the real interest rate reduces bond seigniorage because of the government has to repay its debt with a higher interest. Therefore, with this “interest rate” channel, a tight money policy may reduce bond seigniorage. In such a case, the government is forced to raise currency seigniorage and therefore the UMA proposition holds.

This “interest rate effect” arises under two necessary conditions. One is that the Modigliani-Miller theorem for the government open market operations fails. That is, it is necessary that a tight money policy reduces the capital stock. Without this link, monetary policies cannot affect the real interest rate. Given that open market operations have real effects, the next qualifier is that a higher interest rate reduces bond seigniorage. This condition is met on the right-hand-side (or, “the good side”) of the bond-seigniorage Laffer curve. Now there arise two effects. One is that a tight money policy increases bond seigniorage. The other is that, through the “interest rate” channel, the tight money policy reduces bond seigniorage. If the real interest rate is not too high, the latter effect outweighs the former and the UMA proposition obtains even if the real interest rate is less than the growth rate.

The main contribution of the work done in Chapter 3 is to recognize a new channel through which a tight money policy could raise the steady state inflation. Also, the analysis suggests that it is the “bond seigniorage” Laffer curve, rather than the conventional total seigniorage Laffer curve, that the central banks have to look at in achieving a *sustainable* rate of inflation.

1.3 Public Policy

1.3.1 Public Sector Employment: Too Big to Ignore

In most of the standard growth models, public sector employment is largely ignored. Barro (1991), for instance, introduces government expenditure into an endogenous growth model to study the impact of public goods production on the growth rate of an economy. Barro (1991) assumes that one unit of government expenditure is costlessly transformed into one unit of public good. Since it is not true in the real world, it is natural to ask how important it is to understand the impact of public sector employment upon labor market.

According to Heller and Tait (1983), the share of public sector employment in total employment averaged 44% in 23 developing countries and 24% in 14 industrialized countries (Gelb *et. al.*, 1991, p.1186). Extreme examples are India (72%), Ghana (74%), Tanzania (78%), and Zambia (81%) (Gelb *et. al.*, 1991, p.1186). About India, Bhagwati (1993, pp.63-64) reports that

> ...the overwhelming presence of the public sector in India must be spelled out to see why the matter of its functioning is of great importance to Indian productivity and economic performance. Thus, the 244 economic enterprises of the central government alone, excluding the railways and the utilities, employed as many as 2.3 million workers in 1990. In manufacturing, if the small 'unorganized' sector is excluded, their employment was 40 per cent of that provided by the private sector firms. In fact, the public sector enterprises in manufacturing, mining, construction, transport and communications, banking and insurance, when state-level enterprises are counted in, provided nearly 70 per cent of the 26 million jobs in the large-scale 'organized' sector in 1989.

1.3.2 Some Facts

An authoritative source on government employment across the world is Schiavo-Campo *et. al.* (1997). Their findings can be summarized as follows.

- Worldwide, total government civilian employment is about 4.7% of population. It is relatively large for the OECD economies and relatively low in Africa and Asia.

- Globally, the central government wage bill absorbs about 5.4% of GDP.

- The size of government employment and the central government wage as a multiple of GDP per capita are negatively correlated.

- Both central government employment and the relative public sector wage bill have decreased in the last 20 years. The relative size of the government has shrunk by about 1/3 when measured by employment and by about 1/4 when measured by the wage bill.

Table 1 shows the size of the government around the world in the early 90's. OECD economies have larger governments than the rest of the world. However, these data include relative large numbers of teachers and health services personnel.

Table 1.[3] **Government Employment**, *early 1990's*

(as percent of population)

	No. of Countries	General Government	Government Central	Government Local	Teaching & Health
Africa	20	2.0	0.9	0.3	0.8
Asia	11	2.6	0.9	0.7	1.0
Eastern Europe & former USSR	17	6.9	1.0	0.8	5.1
Latin America & Caribbean	9	3.0	1.2	0.7	1.1
Middle East & North Africa	8	3.9	1.4	0.9	1.6
OECD	21	7.7	1.8	2.5	3.4
Overall	86	4.7	1.2	1.1	2.4

5 This table is reproduced from Schiavo-Campo *et. al.* (1997).

Table 2 shows the fiscal weight of the central government wage bill. It is clear that the government wage bill constitutes a significant part of most economies. The last column of the Table 2 is of interest. It shows the public-to-private wage bill ratio. In many regions the ratio is less than one, suggesting that perhaps the government sector wage rate is less than the private sector wage rate.

Table 2.[7]Central Government Wages, 1990's

	No. of Countries	Central Government Wages as % of GDP	Average Central Government Wages as Multiple of per capita GDP	Ratio of Public to Private Sector Wages
Africa	21	6.7	5.7	1.0
Asia	14	4.7	3.0	0.8
Eastern Europe & former USSR	21	3.7	1.3	0.7
Latin America & Caribbean	12	4.9	2.5	0.9
Middle East & North Africa	8	9.8	3.4	1.3
OECD	16	4.5	1.6	0.9
Overall	92	5.4	3.0	0.8

1.3.3 Needed: A Theory of Government Employment

An analytical framework is needed in order to assess government employment. In contrast to other well-established fields of macroeconomics, studies of public sector employment suffer from the lack of such a framework. What will happen to an economy if the government reduces the size of publicly owned enterprises? Does government intervention in the labor market really reduce unemployment? Which income maintenance program is better; public employment or unemployment insurance? How should the wage rate in the public sector be determined? Does public sector employment crowd out private sector employ-

9 This table is reproduced from Schiavo-Campo *et. al.* (1997).

ment? Should the government provide jobs even if publicly provided jobs are not very productive? What is the optimal size of public sector employment? These and many other important issues have not been paid much attention in academia, although these public policy issues are often discussed in the real world.[10]

Schiavo-Campo *et. al.* (1997), too, admit the need for an analytical framework on which policy analyses can be based. Although public employment programs have been studied in the labor economics literature at a micro level, there is very little work investigating their macroeconomic impact. The (only) existing macroeconomic literature dates back to the study of "pump-priming" by Kahn (1931), Clark (1935), and Keynes (1936). Their advocacy of such employment programs is based on an "multiplier" view of government investment. The principal limitations of this line of inquiry are the following. First, in the old-style Keynesian models, unemployment is purely a consequence of an *ad hoc* assumption of rigid wages. As such, these models are ill-suited to study the effects of employment generation policies. Second, these models are not dynamic and the equilibrium relationships in them are not derived from primitives.

The 90's has witnessed remarkable progress in the theory of economic growth. We not only understand the determinants of long-run economic activity, but also the determinants of the growth rate of an economy. Based on recent developments of growth theory, Chapter 4 proposes an analytical framework that can be used to assess government employment.

[10] See Chapter 11 of *World Development Report* (1995) on "Public Policy and Labor Standards."

1.3.4 Defining Government Employment: A Problem

Before we start modeling government employment, the term has to be defined. There are serious measurement and definition problems. Schiavo-Campo *et. al.* (1997a, p.4) put it nicely:

> In the first place, statistics of any reliability simply don't exist in many countries. When reasonable data are available, employment comparisons are complicated by the fact that some countries include teachers and/or health workers in the civil service, others don't; some countries include contractual and seasonal (sometimes even daily) workers in government employment, others don't; local government employment may or may not include employees paid out of the central government budget but are not listed among central personnel; paramilitary personnel (*gendarmes*, *carabinieri*, etc.) may be included in civilian personnel because of their public order function, or in the armed forces because of their military status; employees of legislative bodies are sometimes included in government personnel, etc.

For the purpose of *macroeconomic policy evaluations*, it is sufficient to consider only two types of government employment: government employment as a by-product of public goods production, and government as an employer of last resort.

In the former case, the objective of the government is improvement of social welfare through providing (productive, or welfare-improving) public goods. Since labor inputs are required to produce a desired level of public goods, the government needs to hire workers. An important task is to ask the optimal level of public goods in the spirit of Barro (1991). Because of the presence of the public sector labor market, the interaction between the government wage policy and the optimal public goods provision would add a new insight into the conventional analysis.

In the latter case the provision of government employment is itself the primary purpose. Thus, the government does *not* have to provide public goods to the economy. In this view, government employment is important for two reasons. One is because it is a form of fiscal policy. As the text-book macroeconomic theory suggests, government spending may increase aggregate demand. In a sense, this is very "Keynesian." Another is because the government is providing a type of "income maintenance program." The question is whether such government intervention in the labor market can really improve the labor market outcome. The answer to the question depends crucially on how the labor market is modeled. If a frictionless model is assumed, for example, then government employment will result only in a partial re-allocation of the labor force from the private to the public sector. If, on the other hand, some market imperfections are assumed, then it is easy to envisage that there is a room for government interventions.

As Schiavo-Campo *et. al.* (1997) argue, it is often the case that government employment has multiple purposes. Because of that, it is quite hard to distinguish between the "productive" part and the "unproductive" part of government employment. Thus in Chapter 4 the two cases are considered separately.

1.3.5 An OG Model with Adverse Selection

The framework employed here is a standard overlapping generations model with production along the lines of Diamond (1965) with one exception: agents are heterogeneous in terms of their intrinsic productive abilities, and this is private information. Private sector firms use capital and labor to produce a single good via a standard neoclassical production technology. They use equilibrium unemployment as a sorting device, offering a menu of wages and unemployment probabilities that entice only the high ability people to seek private sector employment. Thus, the very presence of judiciously chosen unemployment rates keeps the low ability people (and some high ability people) out of the private sector. This is where the government can step in and set up a publicly-funded employment program which indiscriminately employs a fraction of those unemployed in the private sector and pays them a wage which is a fraction of the current private sector wage. The public sector wage bill is funded by a lump-sum tax on all agents. The public sector employees produce a "government good" which is either completely useless to private agents, or is a source of a positive externality.

In this setting, it is apparent that the very involvement of the government affects the information friction in the labor market in that it increases the payoff to the low and high ability people from not seeking private sector employment. Private sector firms react by altering the unemployment probabilities and wage rates they offer which in turn has important effects on capital accumulation.

Multiple long-run equilibria are easily possible here. The existing level of government involvement matters for capital accumulation in a precise and crucial sense. Indeed, below (above) a critical level, further increases in the volume of the public employment programs improves (lowers) long-run real activity for countries stuck at the low-activity steady state. If there are two steady state equilibria, then the high real-activity steady state is dynamically stable. It is possible for the low activity steady state to be stable too. In this case, countries stuck at the stable low activity steady state may be thought of as being caught in a development trap. Again, an increase in the volume of public sector employment programs can get the economy out of this trap. In other words, the paper provides a dynamic justification for the pursuance of Keynesian pump-priming policies in less developed countries. However, at the high-activity steady state, the provision of public employment is harmful. This is an explanation as to why developed economies typically do not have massive public employment schemes.

The main contribution of Chapter 4 is that it recognizes the need for a theory of government employment and gives a first systematic analysis on the issue. The analysis, however, is far from complete or satisfactory.

Chapter 2
Matching, Bargaining and Dispersed Values of Fiat Currency

2.1 Introduction

2.1.1 Overview

Traditionally monetary economics uses models that impose the presence of valued fiat currency into otherwise non-monetary Walrasian environments in which all trading is centralized, frictionless and instantaneous. Since there is no natural role for fiat currency in such models, its presence is forced upon the model by an artificial constraint, such as a cash-in-advance requirement. In their path-breaking contribution, Kiyotaki and Wright (1989) examined instead bilateral trading markets in which agents choose to hold and exchange a fiat currency because of its value in mitigating the double coincidence of wants difficulties that arise with decentralized trading. The first generation of the search theoretic models of money, such as Kiyotaki and Wright (1989, 1991, 1993), fixes the terms of trade, or 'price,' and shows that an object called 'money' arises endogenously. The second generation consists of models such as Trejos and Wright (1995) and Shi (1995), in which the determination of the terms of trade is explicitly modeled by incorporating a bargaining problem into the search model. The third generation endogenizes the distribution of money holdings; Camera and Corbae (1999) is an example.

In this chapter, we re-introduce the dependence of each agent's utility on the quality of each match, measured by a distance (defined later) between his most preferred good and the commodity he consumes.[11] Since goods are differentiated and agents randomly meet each other bilaterally, the qualities of matches differ and, consequently, the purchasing power of money reflects not only what money will buy in the future, but also the quality of a current match. Different match qualities result in bargains in which the terms-of-trade vary. In other words, the model predicts dispersed prices.

The model's analysis shows how earlier use of bargaining fails to capture fully the market's feedback into agents' bargaining. Bargaining theory was introduced into search models of money in one form or another to remove an *ad-hoc* fixed-price assumption. Trejos and Wright (1995) and Shi (1995) introduced a version of Rubinstein's (1982) sequential bargaining game and showed that a Nash bargaining representation can be used to approximate a strategic bargaining solution. Subsequent studies take either a Nash solution or Rubinstein (1982) and Rubinstein-Wolinsky (1985) bargaining game for granted. Indeed, in all earlier monetary models the strategic and Nash bargaining solutions coincide at a steady-state equilibrium. This turns out not to be true in general, a point already made by Wolinsky (1987).

Outside bargaining opportunities matter in monetary theory. If the price determination mechanism is restricted to being either a Nash or a Rubinstein (1982) alternating offer bargaining game, then the information used to determine the purchasing power of money is restricted to the characteristics of the bargaining parties. Nothing else can influence the

[11] I would like to thank Peter Morgan for bringing my attention to this direction of research. This is also suggested by Shi (1995).

bargaining outcome. The question to be asked is whether the value of a unit of fiat currency to the bargaining parties is determined *only* by the bargaining parties. Is there any feedback from the *market*? The present model shows that the market environment will influence, through the availability of outside opportunities, the value of fiat currency to each bargainer and, also, that the magnitude of this influence varies across matches.

In the previous literature, outside options play only a small role in determining the purchasing power of money, because of the way price is endogenized. To see this point, suppose that we wish to endogenize the purchasing power of money in Kiyotaki and Wright (1991). First, since analyzing a model with divisible units of currency is quite complex, goods are instead assumed to be divisible and agents' utilities are increasing in the amount of a good consumed. This means that the agents' utilities depend upon both the quality of the match made and the quantity consumed. As a result, agents' reservation strategies depend both on the match quality and the quantity consumed. To avoid this troublesome complication, Trejos and Wright (1995) and Shi (1995) assume a *constant reservation strategy*. In effect, agents' utilities depend upon only the quantity consumed so as long as the match quality exceeds an exogenous threshold level. As a result, the match quality will be *homogeneous across all matches*. The implication for the bargaining component of the model is that the values of outside options are the same for all matches. Therefore, the presence of outside options makes little difference in models with homogeneous match quality.

The present model also uses a constant reservation strategy. However, goods are differentiated and bilateral encounters between agents occur randomly, so that each match quality is idiosyncratic. Consequently, there are well-defined heterogeneous outside options in the economy.[12] To see this point, suppose that a low quality match is formed. Since the current match quality is low, the probability that the buyer will find a higher quality match in the next period is *relatively* high. In other words, the value of the buyer's outside option is relatively high when a low quality match forms. The exact value of this outside option depends upon the idiosyncratic quality of the current match.

2.1.2 Who Matches with whom Matters

As Binmore *et. al.* (1986) point out, there are two basic forces that induce bargaining parties to reach an agreement in a strategic bargaining problem. One is that agents discount future; that is, waiting is costly. The other force that motivates bargaining parties to reach an agreement is the fear that their bargaining partner might walk away in favor of a new bargaining partner. Trejos and Wright (1995) and Coles and Wright (1998) mainly focus on the former motive for reaching an agreement. In this paper, we focus on the latter.

In the Kiyotaki-Wright model (1991), if a low quality match is realized, then the agents simply walk away and look for a new partner. In this chapter, on the other hand, if a low quality match forms, then other things equal the buyer's utility is directly reduced. Since the reservation strategy is constant, the buyer cannot walk away. Instead, there is another margin: a loss of utility by a low quality match can be compensated by a high quantity of the good. If a low quality match is formed, then the probability that the buyer will discover next period a higher quality match with another seller is relatively high. This

[12] In Wolinsky (1987), there are heterogenous outside options, but they are completely exogenous. The presence of exogenous outside options in a general equilibrium is *ad-hoc*. In this paper, however, the dispersion in outside options is endogenous.

fact is used by the buyer as a threat in the bargaining process and, as a result, the model presented in this chapter predicts price dispersion.

The qualitative properties of the distribution of prices crucially depend on the complementarity in the agents' preferences between match quality and quantity consumed. If the complementarity is weak enough, then a low quality match will be compensated for by a low price (or, high quantity). Conversely, if the complementarity is strong, then the quantity rises with the match quality.

Because of the endogenized dispersion in the values of outside options, the use of a reduced-form bargaining solution does not give a good approximation of the underlying strategic bargaining problem. We show that a Nash solution predicts on average a higher terms-of-trade than does strategic bargaining. Also, markets in which terms-of-trade are determined by strategic bargaining will be more volatile than markets which use Nash bargaining. Those results are directly due to Nash bargaining not making use of all information provided to bargaining parties by the market.

The rest of the chapter is organized as follows. Section 2.2 describes the environment of the economy. Section 2.3 states search strategies of agents. Section 2.4 describes the bargaining game. Section 2.5 presents the market equilibrium. Section 2.6 concludes.

2.2 Environment

2.2.1 Modeling Fiat Money

The model is a hybrid of Wolinsky (1987) and Coles and Wright (1998). As is standard in search theoretic models of money, it is assumed that the economy is populated by a continuum of infinitely lived agents. The population's measure is normalized to unity. In this economy money is the only asset. We assume fiat money. That is, money does not directly generate utility. In addition, we assume that all other goods in the economy are perishable.

A fraction M of the total population is endowed with one unit of fiat money. The complementary fraction $1-M$ of the population is endowed with a production opportunity. No agent may hold more than one unit of money. This assumption is unrealistic, but it is important in that it keeps the distribution of money holdings constant over time. Though exploring the distribution of asset holdings is important, it is not examined in this paper. With this assumption, agents who have a unit of fiat money are necessarily buyers and agents who have no currency are necessarily sellers.

2.2.2 Preferences and Production Technologies

There is a continuum of differentiated, perishable goods. Each commodity is identified by a point around a circle with a circumference of length 2. Each agent is identified by a point on the same circle. It is assumed that agents are uniformly distributed around the circle. Agent i is defined as an agent whose ideal commodity is indexed by i and whose production good is indexed by i'. If he obtains a consumption good indexed by j', then his utility from consuming it depends on the *match quality* $z \equiv Z(i, j')$, the length of the arc between commodities i and j'. Obviously, $z \in [0, 1]$. The instantaneous utility from consuming q units of the good is $U(q, z)$, where

$$U(q,z) = \begin{cases} U(q,z) \text{ for } z \leq x \\ 0 \text{ for } z > x. \end{cases}$$

It is assumed that $U_1 > 0, U_2 < 0, U_{11} < 0$. In addition, we shall assume that $\lim_{q \to 0} \partial U / \partial q = \infty$ and $\lim_{q \to \infty} \partial U / \partial q = 0$. As is mentioned in the introduction, the dependence of utility on both q and z is a significant generalization of the first generations of search models of money such as Kiyotaki and Wright (1991), which assume $U(z)$, and the second generation of models such as Coles and Wright (1998), which assume $U(q)$.[13]

For simplicity, we assume that agents' preferences and technologies are sufficiently distant to preclude any agent consuming her own production. That is, $Z(i, i') > x$. This is how transactions are motivated in the search theoretic model of money. However, this restriction is not essential.[14]

Production is modeled as follows. The utility cost to agent i of producing q units of the good is $c^i(q) \equiv c(q)$ for any i, with $c' > 0$, $c'' > 0$, and $\lim_{q \to 0} c'(q) = 0$. Thus the cost of producing any good is completely *independent* of the type of good. In this economy, everyone is *completely specialized in production*, but is willing to consume a

---

13 The present model, however, is not the first attempt to include those two elements in utility. This idea goes back at least to Hayakawa and Venieris (1977), who extended the classical consumer theory to include "life-style" as well as "intensity."

14 See Kiyotaki and Wright (1993) and Burdett et al. (1995), for example.

variety of goods. She will be better off if she obtains one unit of fiat money by producing less. However, *ex-ante*, she does not know who will pay a higher price for her good.[15]

2.2.3 Matching Process and Reservation Strategy

The matching process employed in this paper is that of Mortensen (1982) and in particular, Wolinsky (1987). The Poisson arrival rate at which a buyer meets a seller is α so that during a small length of time Δ the probability that a buyer meets a seller is $\alpha\Delta$. Upon meeting, a match is formed if and only if the seller has a good that is acceptable to the buyer.

The reservation strategy of the agents is as follows. By construction of the utility function, there is an exogenously specified threshold value of z above which no one agrees to trade *at any price*. In other words, the reservation strategy is *price-independent*.

Given the exogenous reservation strategy just described, a buyer is matched with a seller during Δ with probability $\alpha\Delta x$. The Poisson arrival rate at which a seller meets a buyer is β, making $\beta\Delta$ the probability that a seller meets a buyer in a small period of length Δ. In order for these arrival rates to be consistent with the aggregate number of matches made at each moment,

$$\alpha\Delta x M = \beta\Delta x\,(1 - M) \tag{2.1}$$

must hold in equilibrium. That is, the numbers of buyers and sellers who find bargaining partners must be equal.

[15] Recently, Peters (1991) and Montgomery (1991) amongst others have shown that sellers may have an incentive to "post" prices *ex-ante* in order to attract buyers. In this model, however, the question of whether the sellers have an incentive to post their prices is not trivial, since the surplus of a match varies across matchs.

It is assumed that no two sellers or two buyers ever meet. This assumption precludes barter trading and is introduced to simplify the analysis.[16] Though the existence of valued fiat money is assured by the assumption of no barter trading, the *purchasing power* of money is fully endogenized through bargaining.

2.2.4 Timing of Events

Time is continuous, but for convenience we consider a short length of time Δ as a period. Each period consists of two stages. The first is the bargaining stage and the second is the search stage. At the beginning of each period, an agent who is matched enters the bargaining stage. If a bargaining breakdown occurs, then the agent enters the search stage and searches for an alternative trade opportunity. If an agent has no partner at the beginning of a period, then the agent enters directly into the search stage. We seek a Perfect Equilibrium and so describe the search stage in the next section, followed by a description of the bargaining stage.

2.3 Search Problem

This section describes the agents' search problems. Let $V_b(t)$ denote the value function for buyers at time t and $V_s(t)$ denote the value function for sellers. The constant rate of time preference is r. The dynamic programming equation for the representative buyer without a

---

[16] Inclusion of barter trading alone can generate two monetary equilibria (see Shi, 1995). The price dispersion arising from inclusion of barter trading, however, is not a scope of this study.

bargaining partner is

$$V_b(t) = \frac{1}{1+r\Delta}\{\alpha\Delta\int_0^x \left[\begin{array}{c}\pi_s U(q_b(z,t+\Delta,\Delta),z)\\ +\pi_b U(q_s(z,t+\Delta,\Delta),z)+V_s(t+\Delta)\end{array}\right]dz$$
$$+(1-\alpha\Delta x)V_b(t+\Delta)+o(\Delta)\}. \quad (2.2)$$

The right-hand-side of (2.2) can be interpreted as follows. With probability $\alpha\Delta$, the buyer meets a seller. Given a meeting, with probability x the seller will have a good distant by not more than x from the buyer's most preferred good, in which case the buyer consumes the good and becomes a seller. The expected utility from consumption is $\pi_s U(q_b(z,t+\Delta,\Delta),z)+\pi_b U(q_s(z,t+\Delta,\Delta),z)$ since in the bargaining stage the seller makes an offer to the buyer with probability π_s, in which case the terms of trade is q_b, and the buyer makes an offer with probability π_b, in which case the terms of trade is q_s. For brevity, we shall define

$$B_\pi(z,t+\Delta) \equiv \pi_s U(q_b(z,t+\Delta,\Delta),z)+\pi_b U(q_s(z,t+\Delta,\Delta),z). \quad (2.3)$$

With probability $1-\alpha\Delta x$ the buyer is matched with no one and will continue to search for a trade opportunity. (2.2) uses the *constant reservation strategy*. For this to be rational, it is necessary that

$$B_\pi(z,t+\Delta)+V_s(t+\Delta) \geq V_b(t+\Delta) \quad (2.4)$$

holds for any $z \leq x$ and for any t.

Given x, the dynamic programming equation for the representative seller without a bargaining partner is

$$V_s(t) = \frac{1}{1+r\Delta}\{\beta\Delta\int_0^x \left[\begin{array}{c}-\pi_s c(q_b(z,t+\Delta,\Delta))\\ -\pi_b c(q_s(z,t+\Delta,\Delta))+V_b(t+\Delta)\end{array}\right]dz$$
$$+(1-\beta\Delta x)V_s(t+\Delta)+o(\Delta)\}. \quad (2.5)$$

The right hand side of (2.5) can be interpreted as follows. With probability $\beta\Delta$ the seller meets a buyer. Given a meeting, with probability x the seller will have a good distant by not more than x from the buyer's most preferred good, in which case the seller will exchange the good for one unit of fiat currency and become a buyer. The expected utility cost of production is $\pi_s c\left(q_b\left(z, t+\Delta, \Delta\right)\right) + \pi_b c\left(q_s\left(z, t+\Delta, \Delta\right)\right)$. For parsimony, we shall define

$$C_\pi\left(z, t+\Delta\right) \equiv \pi_s c\left(q_b\left(z, t+\Delta, \Delta\right)\right) + \pi_b c\left(q_s\left(z, t+\Delta, \Delta\right)\right). \tag{2.6}$$

With probability $1-\beta\Delta x$, the seller is matched with no one and continues to search. Rationality requires that

$$-C_\pi\left(z, t+\Delta\right) + V_b\left(t+\Delta\right) \geq V_s\left(t+\Delta\right). \tag{2.7}$$

Manipulate (2.2) and (2.5) and let $\Delta \to 0$ to get

$$-\dot{V}_b\left(t\right) + rV_b\left(t\right) = \alpha\int_0^x \left[B_\pi\left(z,t\right) + V_s\left(t\right) - V_b\left(t\right)\right] dz \tag{2.8}$$

and

$$-\dot{V}_s\left(t\right) + rV_s\left(t\right) = \beta\int_0^x \left[-C_\pi\left(z,t\right) + V_b\left(t\right) - V_s\left(t\right)\right] dz, \tag{2.9}$$

where $\dot{V}_b \equiv dV_b/dt$ and $\dot{V}_s \equiv dV_s/dt$. It is easy to show that

$$V_b\left(t\right) = \frac{\alpha\int_0^x B_\pi\left(z,t\right)dz - \frac{\alpha x}{\beta x + r}\beta\int_0^x C_\pi\left(z,t\right)dz + \frac{\alpha x}{\beta x+r}\dot{V}_s + \dot{V}_b}{\alpha x + r - \frac{\alpha x \beta x}{\beta x + r}}, \tag{2.10}$$

$$V_s\left(t\right) = \frac{-\beta\int_0^x C_\pi\left(z,t\right)dz + \frac{\beta x \alpha}{\alpha x + r}\int_0^x B_\pi\left(z,t\right)dz + \frac{\beta x}{\alpha x+r}\dot{V}_b + \dot{V}_s}{\beta x + r - \frac{\beta x \alpha x}{\alpha x + r}}. \tag{2.11}$$

(2.4) and (2.7) imply that

$$C_\pi\left(z, t+\Delta\right) \leq V_b\left(t\right) - V_s\left(t\right) \leq B_\pi\left(z, t+\Delta\right) \tag{2.12}$$

holds for any $z \leq x$ and for any t.

Lemma 1 *V_b and V_s are uniformly bounded.*

Proof. (2.12) implies $B_\pi(z, t+\Delta) \geq C_\pi(z, t+\Delta)$. Let $\widehat{q}$ solve $U(\widehat{q}, 0) = c(\widehat{q})$. Then $q(t+\Delta, \Delta) \leq \widehat{q}$ holds for all t and Δ since $U(0, z) = c(0) = 0$, $U' > 0$, $U'' < 0$, $c' > 0$, and $c'' > 0$. In other words, the terms-of-trade is bounded above by $\widehat{q}$. Therefore, the maximum possible (discounted) lifetime utility an agent can get is $U(\widehat{q}, 0)/r$. Therefore, V_b and V_s are bounded above by $U(\widehat{q}, 0)/r$. Also, the value functions are bounded below by 0, the utility of never trading. Therefore, V_b and V_s are uniformly bounded. ■

2.4 Bargaining Problem

2.4.1 Bargaining Game with Heterogeneous Outside Options

This section introduces a strategic sequential bargaining game with heterogeneous outside options. In any period in which a buyer and a seller are matched, one of them is selected randomly to make an offer: with probability π_b the buyer makes an offer, and with probability $\pi_s \equiv 1 - \pi_b$ the seller makes an offer. After an offer is made, the respondent may either accept it or search for a new bargaining partner. Following Rubinstein and Wolinsky (1985), we assume a bargaining game with *exogenous breakdowns,* in which the arrival rate of a new bargaining partner is constant.[17] If a new bargaining partner arrives during a

[17] A model in which the arrival rate of a new match is optimally chosen by the bargaining parties, as in Wolinsky (1987), is called a bargaining model with *endogenous breakdowns*.

bargaining process, then a current bargaining partner will be discarded in favor of the new partner if and only if the new match quality is at least as good as the current one.

We assume no delay in the bargaining process. In particular, we adopt Coles and Wright's (1998) Immediate Trade Equilibrium (ITE).

Definition1 *In an Immediate Trade Equilibrium (ITE), all sellers accept any offer* $q \leq q_s(z, t, \Delta)$ *and all buyers accept any offer* $q \geq q_b(z, t, \Delta)$.

In an ITE, there is actually no bargaining breakdown. However, it is the underlying market structure that induces the bargaining parties always to reach agreement. Thus, the agreed-upon terms-of-trade depends on the levels of credible threats brought by the bargaining parties.

If a seller is to make an offer, then she chooses $q_b(\widetilde{z}, t, \Delta)$ such that a buyer is indifferent between accepting and rejecting the offer, given the value functions V_b and V_s, the realization of the current match quality $\widetilde{z}$, and the length of the bargaining period Δ. The seller also takes account of the value of the terms-of-trade in the case of a new match, since it will be determined by bargaining between the seller and her new bargaining partner. Therefore, in an ITE,

$$\begin{aligned} U(q_b(\widetilde{z}, t, \Delta), \widetilde{z}) + V_s(t) &= \frac{1}{1 + r\Delta}\{\alpha\Delta \int_0^{\widetilde{z}} (B_\pi(z, t + \Delta) + V_s(t + \Delta))\, dz \\ &+ (1 - \alpha\Delta\widetilde{z})(1 - \beta\Delta x) \\ &\times \begin{bmatrix} \pi_s(U(q_b(\widetilde{z}, t + \Delta, \Delta), \widetilde{z}) + V_s(t + \Delta)) \\ +\pi_b(U(q_s(\widetilde{z}, t + \Delta, \Delta), \widetilde{z}) + V_s(t + \Delta)) \end{bmatrix} \\ &+ (1 - \alpha\Delta\widetilde{z})\beta\Delta x V_b(t + \Delta) + o(\Delta)\}. \end{aligned} \tag{2.13}$$

The right-hand-side of (2.13) is the expected discounted value of rejecting $q_b(\widetilde{z}, t, \Delta)$. The buyer meets a new potential partner with arrival rate $\alpha\Delta$. In this case, he is matched with the new seller if and only if the new seller has a good that is distant by less than $\widetilde{z}$ from the buyer's most preferred good; that is, if and only if $z \leq \widetilde{z}$. In this case, the buyer will receive the expected payoff $\int_0^{\widetilde{z}} (B_\pi(z, t+\Delta) + V_s(t+\Delta))\, dz$. With probability $(1-\alpha\Delta\widetilde{z})(1-\beta\Delta x)$ neither the buyer nor the seller finds a new partner and they continue to be matched in the next period. In the next bargaining stage the buyer makes a new offer with probability π_b and the seller makes a new offer with probability π_s. With probability $(1-\alpha\Delta\widetilde{z})\,\beta\Delta x$ the buyer does not find a new bargaining partner and the seller finds one in the interval $(t, t+\Delta)$. The seller then walks away and the buyer is left with the value of being unattached, $V_b(t+\Delta)$.

Similarly, when a buyer is to make an offer, he chooses $q_s(\widetilde{z}, t, \Delta)$ such that a seller is indifferent between accepting the offer and rejecting it, given the value functions V_b and V_s, the realization of the current match quality $\widetilde{z}$, and the length of the bargaining period Δ. Thus,

$$\begin{aligned} -c\left(q_s\left(\widetilde{z}, t, \Delta\right)\right) + V_b(t) &= \frac{1}{1+r\Delta}\{\beta\Delta \int_0^x \left(-C_\pi(z, t+\Delta) + V_b(t+\Delta)\right) dz \\ &\quad + (1-\alpha\Delta\widetilde{z})(1-\beta\Delta x) \\ &\quad \times \begin{bmatrix} \pi_s\left(-c\left(q_b\left(\widetilde{z}, t+\Delta, \Delta\right)\right) + V_b(t+\Delta)\right) \\ +\pi_b\left(-c\left(q_s\left(\widetilde{z}, t+\Delta, \Delta\right)\right) + V_b(t+\Delta)\right) \end{bmatrix} \\ &\quad + \alpha\Delta\widetilde{z}(1-\beta\Delta x)\, V_s(t+\Delta) + o(\Delta)\}. \end{aligned} \tag{2.14}$$

The right-hand-side of (2.14) is the expected discounted value of rejecting $q_s(\widetilde{z}, t)$. The seller meets a new potential partner with probability $\beta\Delta$. In this case, the seller is matched with the new buyer if and only if the new match quality is at least as good as the threshold x; that is, if and only if $z \leq x$. In this case, the seller will receive the expected payoff $\int_0^x \left(-C_\pi\left(z, t+\Delta\right) + V_b\left(t+\Delta\right)\right) dz$. With probability $\left(1-\alpha\Delta\widetilde{z}\right)\left(1-\beta\Delta x\right)$, neither the buyer nor the seller find new partners and they continue to be matched in the next period. In the next bargaining stage, the buyer makes a new offer with probability π_b, and the seller makes a new offer with probability π_s. With probability $\alpha\Delta\widetilde{z}\left(1-\beta\Delta x\right)$ the seller does not find a new bargaining partner and the buyer finds one in the interval $\left(t, t+\Delta\right)$. The buyer walks away and the seller is left with the value of being unattached, $V_s\left(t+\Delta\right)$.

Note that, as is clear from (2.13) and (2.14), $q_b\left(z, t, \Delta\right)$ and $q_s\left(z, t, \Delta\right)$ are functions not only of t and z, but also of Δ. We are interested in the limiting behavior of the terms-of-trade as $\Delta \to 0$.

Proposition 1 $\lim_{\Delta\to 0} q_b\left(\widetilde{z}, t, \Delta\right) = \lim_{\Delta\to 0} q_s\left(\widetilde{z}, t, \Delta\right) \equiv q\left(\widetilde{z}, t, \Delta\right)$ *for every* $\widetilde{z} \in \left[0, 1\right]$.

Proof. Multiply (2.13) by $\left(1+r\Delta\right)$, rearrange terms, and divide both sides by Δ to obtain

$$
\begin{aligned}
& \frac{U\left(q_b\left(\widetilde{z}, t, \Delta\right), \widetilde{z}\right) - \left[\pi_s U\left(q_b\left(\widetilde{z}, t+\Delta, \Delta\right), \widetilde{z}\right) + \pi_b U\left(q_s\left(\widetilde{z}, t+\Delta, \Delta\right), \widetilde{z}\right)\right]}{\Delta} \\
= \ & \alpha \int_0^{\widetilde{z}} B_\pi\left(z, t+\Delta\right) dz - r\left[U\left(q_b\left(\widetilde{z}, t, \Delta\right), \widetilde{z}\right) + V_s\left(t\right)\right] - \left(\beta x - \alpha\widetilde{z}\beta\Delta x\right) V_s\left(t+\Delta\right) \\
& - \left(\alpha\widetilde{z} + \beta x - \alpha\widetilde{z}\beta\Delta x\right)\left[\pi_s U\left(q_b\left(\widetilde{z}, t+\Delta, \Delta\right), \widetilde{z}\right) + \pi_b U\left(q_s\left(\widetilde{z}, t+\Delta, \Delta\right), \widetilde{z}\right)\right] \\
& + \left(1-\alpha\Delta\widetilde{z}\right)\beta x V_b\left(t+\Delta\right) + \frac{V_s\left(t+\Delta\right) - V_s\left(t\right)}{\Delta} + \frac{o\left(\Delta\right)}{\Delta}. \qquad (2.15)
\end{aligned}
$$

Similarly, multiply (2.14) by $(1+r\Delta)$, rearrange terms, and divide both sides by Δ to obtain

$$\begin{aligned}
&\frac{-c\left(q_s\left(\widetilde{z},t,\Delta\right)\right)+\left[\pi_s c\left(q_b\left(\widetilde{z},t+\Delta,\Delta\right)\right)+\pi_b c\left(q_s\left(\widetilde{z},t+\Delta,\Delta\right)\right)\right]}{\Delta} \\
= \;& \beta\int_0^x\left(-C_\pi\left(z,t+\Delta\right)\right)dz-r\left[-c\left(q_s\left(\widetilde{z},t,\Delta\right)\right)+V_b\left(t\right)\right] \\
&+\left(-\alpha\widetilde{z}+\alpha\widetilde{z}\beta\Delta x\right)V_b\left(t+\Delta\right) \\
&+\left(-\alpha\widetilde{z}-\beta x+\alpha\widetilde{z}\beta\Delta x\right)\left[\pi_s\left(-c\left(q_b\left(\widetilde{z},t+\Delta,\Delta\right)\right)\right)+\pi_b\left(-c\left(q_s\left(\widetilde{z},t+\Delta,\Delta\right)\right)\right)\right] \\
&+\alpha\widetilde{z}\left(1-\beta\Delta x\right)V_s\left(t+\Delta\right)+\frac{V_b\left(t+\Delta\right)-V_b\left(t\right)}{\Delta}+\frac{o\left(\Delta\right)}{\Delta}. \qquad (2.16)
\end{aligned}$$

Lemma 1 implies that (2.15) and (2.16) are finite for any Δ. This implies that

$$\lim_{\Delta\to0}\frac{1}{\Delta}\left[U\left(q_b\left(\widetilde{z},t,\Delta\right),\widetilde{z}\right)-\left[\begin{array}{c}\pi_s U\left(q_b\left(\widetilde{z},t+\Delta,\Delta\right),\widetilde{z}\right)\\ +\pi_b U\left(q_s\left(\widetilde{z},t+\Delta,\Delta\right),\widetilde{z}\right)\end{array}\right]\right] \text{ and } \qquad (2.17)$$

$$\lim_{\Delta\to0}\frac{1}{\Delta}\left[-c\left(q_s\left(\widetilde{z},t,\Delta\right)\right)+\left[\begin{array}{c}\pi_s c\left(q_b\left(\widetilde{z},t+\Delta,\Delta\right)\right)\\ +\pi_b c\left(q_s\left(\widetilde{z},t+\Delta,\Delta\right)\right)\end{array}\right]\right] \qquad (2.18)$$

are bounded. (2.17) becomes

$$\begin{aligned}
&\lim_{\Delta\to0}\frac{1}{\Delta}\left[\begin{array}{c}\pi_b\left(U\left(q_b\left(\widetilde{z},t,\Delta\right),\widetilde{z}\right)-U\left(q_s\left(\widetilde{z},t,\Delta\right),\widetilde{z}\right)\right)\\ -\pi_s\left(U\left(q_b\left(\widetilde{z},t+\Delta,\Delta\right),\widetilde{z}\right)-U\left(q_b\left(\widetilde{z},t,\Delta\right),\widetilde{z}\right)\right)\\ -\pi_b\left(U\left(q_s\left(\widetilde{z},t+\Delta,\Delta\right),\widetilde{z}\right)-U\left(q_s\left(\widetilde{z},t,\Delta\right),\widetilde{z}\right)\right)\end{array}\right] \\
= \;& \lim_{\Delta\to0}\left[\begin{array}{c}\pi_b\frac{U(q_b(\widetilde{z},t,\Delta),\widetilde{z})-U(q_s(\widetilde{z},t,\Delta),\widetilde{z})}{\Delta}\\ -\pi_s\frac{U(q_b(\widetilde{z},t+\Delta,\Delta),\widetilde{z})-U(q_b(\widetilde{z},t,\Delta),\widetilde{z})}{q_b(\widetilde{z},t+\Delta,\Delta)-q_b(\widetilde{z},t,\Delta)}\frac{q_b(\widetilde{z},t+\Delta,\Delta)-q_b(\widetilde{z},t,\Delta)}{\Delta}\\ -\pi_b\frac{U(q_s(\widetilde{z},t+\Delta,\Delta),\widetilde{z})-U(q_s(\widetilde{z},t,\Delta),\widetilde{z})}{q_s(\widetilde{z},t+\Delta,\Delta)-q_s(\widetilde{z},t,\Delta)}\frac{q_s(\widetilde{z},t+\Delta,\Delta)-q_s(\widetilde{z},t,\Delta)}{\Delta}\end{array}\right] \\
= \;& \pi_b\lim_{\Delta\to0}\frac{U\left(q_b\left(\widetilde{z},t,\Delta\right),\widetilde{z}\right)-U\left(q_s\left(\widetilde{z},t,\Delta\right),\widetilde{z}\right)}{\Delta} \\
&-U'\left(q\left(\widetilde{z},t\right),\widetilde{z}\right)\left(\pi_s\,\dot{q_b}+\pi_b\,\dot{q_s}\right), \qquad (2.19)
\end{aligned}$$

where $U'\left(q\left(\widetilde{z},t\right),\widetilde{z}\right)\equiv\partial U/\partial q$. Similarly, (2.18) can be written as

$$\lim_{\Delta\to0}\frac{1}{\Delta}\left[\begin{array}{c}-\pi_s\left(c\left(q_s\left(\widetilde{z},t,\Delta\right)\right)-c\left(q_b\left(\widetilde{z},t,\Delta\right)\right)\right)\\ +\pi_s\left(c\left(q_b\left(\widetilde{z},t+\Delta,\Delta\right)\right)-c\left(q_b\left(\widetilde{z},t,\Delta\right)\right)\right)\\ +\pi_b\left(c\left(q_s\left(\widetilde{z},t+\Delta,\Delta\right)\right)-c\left(q_s\left(\widetilde{z},t,\Delta\right)\right)\right)\end{array}\right]$$

$$
\begin{aligned}
&= \lim_{\Delta\to 0}\left[\begin{array}{c} -\pi_s \frac{c(q_s(\widetilde{z},t,\Delta))-c(q_b(\widetilde{z},t,\Delta))}{\Delta} \\ +\pi_s \frac{c(q_b(\widetilde{z},t+\Delta,\Delta))-c(q_b(\widetilde{z},t,\Delta))}{q_b(\widetilde{z},t+\Delta,\Delta)-q_b(\widetilde{z},t,\Delta)} \frac{q_b(\widetilde{z},t+\Delta,\Delta)-q_b(\widetilde{z},t,\Delta)}{\Delta} \\ +\pi_b \frac{c(q_s(\widetilde{z},t+\Delta,\Delta))-c(q_s(\widetilde{z},t,\Delta))}{q_s(\widetilde{z},t+\Delta,\Delta)-q_s(\widetilde{z},t,\Delta)} \frac{q_s(\widetilde{z},t+\Delta,\Delta)-q_s(\widetilde{z},t,\Delta)}{\Delta} \end{array}\right] \\
&= -\pi_s \lim_{\Delta\to 0} \frac{c\left(q_s\left(\widetilde{z},t,\Delta\right)\right)-c\left(q_b\left(\widetilde{z},t,\Delta\right)\right)}{\Delta} \\
&\quad +c'\left(q\left(\widetilde{z},t\right)\right)\left(\pi_s\,\dot{q}_b+\pi_b\,\dot{q}_s\right), \qquad (2.20)
\end{aligned}
$$

where $c'\left(q\left(\widetilde{z},t\right)\right) \equiv \partial c/\partial q$. Since (2.17) and (2.18) are bounded, both (2.19) and (2.20) imply that $\lim_{\Delta\to 0} q_b\left(\widetilde{z},t,\Delta\right) = \lim_{\Delta\to 0} q_s\left(\widetilde{z},t,\Delta\right)$. ■

Proposition 1 asserts that the buyer and the seller offer the same terms-of-trade as the length of the bargaining period approaches to zero. The intuition is that as the length of the bargaining period goes to zero, the first mover advantage in the bargaining stage disappears.

2.4.2 Price Dispersion

Proposition 2 *In any steady state, there exists a unique equilibrium terms-of-trade for each realization of the match quality $\widetilde{z}$.*

Proof. Let $\varepsilon\left(\widetilde{z},t,\Delta\right) \equiv q_s\left(\widetilde{z},t,\Delta\right) - q_b\left(\widetilde{z},t,\Delta\right)$. We take the limits as $\Delta \to 0$ of (2.13) and (2.14) to obtain

$$
\begin{aligned}
&-\pi_b U'\left(q\left(\widetilde{z},t\right),\widetilde{z}\right)\lim_{\Delta\to 0}\frac{\varepsilon\left(\widetilde{z},t,\Delta\right)}{\Delta} \\
&= \alpha\int_0^{\widetilde{z}} B_\pi\left(z,t\right)dz - \left(\alpha\widetilde{z}+\beta x+r\right)U\left(q\left(\widetilde{z},t\right),\widetilde{z}\right)+\beta x V_b\left(t\right) \\
&\quad -\left(\beta x+r\right)V_s\left(t\right)+U'\left(q\left(\widetilde{z},t\right),\widetilde{z}\right)\left(\pi_s\,\dot{q}_b+\pi_b\,\dot{q}_s\right)+\dot{V}_s \qquad (2.21)
\end{aligned}
$$

and

$$\begin{aligned}
& -\pi_s c'\left(q\left(\widetilde{z},t\right)\right) \lim_{\Delta\to 0} \frac{\varepsilon\left(\widetilde{z},t,\Delta\right)}{\Delta} \\
= & -\beta \int_0^x C_\pi\left(z,t\right) dz + \left(\alpha\widetilde{z}+\beta x+r\right) c\left(q\left(\widetilde{z},t\right)\right) + \alpha\widetilde{z}V_s\left(t\right) \\
& -\left(\alpha\widetilde{z}+r\right) V_b\left(t\right) - c'\left(q\left(\widetilde{z},t\right)\right)\left(\pi_s \dot{q}_b + \pi_b \dot{q}_s\right) + \dot{V}_b \,. \qquad (2.22)
\end{aligned}$$

From Proposition 1, in the limit as $\Delta \to 0$, $q_b = q_s$ for any date t. This implies that $\dot{q}_b = \dot{q}_s$ for any t. Let $\dot{q}\left(z\right) \equiv \dot{q}_b\left(z\right) = \dot{q}_s\left(z\right)$. Eliminate $\lim_{\Delta\to 0} \frac{\varepsilon(\widetilde{z},t,\Delta)}{\Delta}$ from (2.21) and (2.22) to obtain

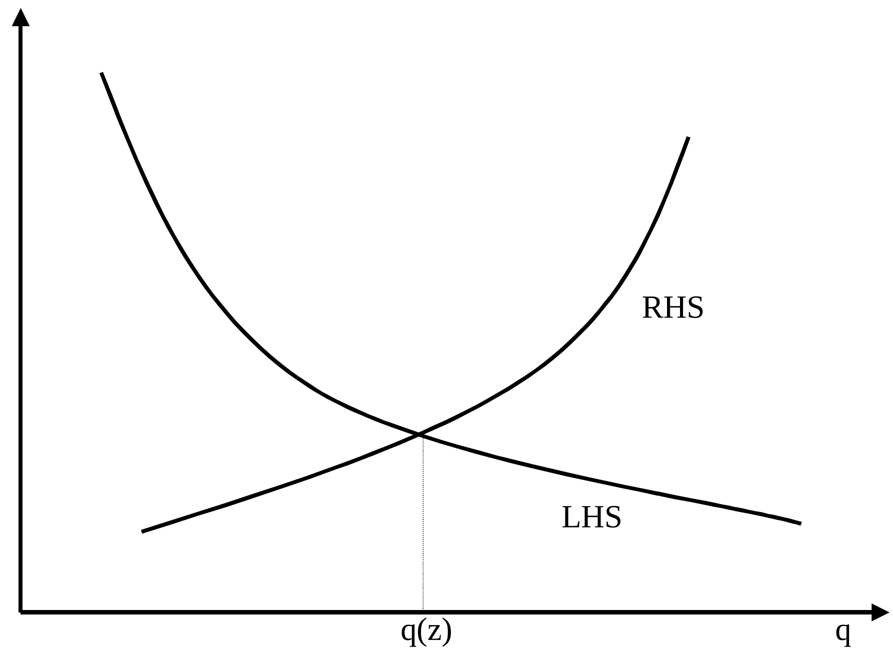
$$\frac{U'\left(q\left(\widetilde{z},t\right),\widetilde{z}\right)}{c'\left(q\left(\widetilde{z},t\right)\right)} = \frac{\pi_s}{\pi_b} \frac{\left[\begin{array}{c} -\alpha\int_0^{\widetilde{z}} B_\pi\left(z,t\right)dz + \left(\alpha\widetilde{z}+\beta x+r\right)U\left(q\left(\widetilde{z},t\right),\widetilde{z}\right) \\ +\left(\beta x+r\right)V_s\left(t\right) - \beta x V_b\left(t\right) - U'\left(q\left(\widetilde{z},t\right),\widetilde{z}\right)\dot{q}\left(\widetilde{z}\right) - \dot{V}_s \end{array}\right]}{\left[\begin{array}{c} \beta\int_0^x C_\pi\left(z,t\right)dz - \left(\alpha\widetilde{z}+\beta x+r\right)c\left(q\left(\widetilde{z},t\right)\right) \\ -\alpha\widetilde{z}V_s\left(t\right) + \left(\alpha\widetilde{z}+r\right)V_b\left(t\right) + c'\left(q\left(\widetilde{z},t\right)\right)\dot{q}\left(\widetilde{z}\right) - \dot{V}_b \end{array}\right]}. \qquad (2.23)$$

In a steady state,

$$\frac{U'\left(q\left(\widetilde{z}\right),\widetilde{z}\right)}{c'\left(q\left(\widetilde{z}\right)\right)} = \frac{\pi_s}{\pi_b} \frac{-\alpha\int_0^{\widetilde{z}} B_\pi\left(z,t\right)dz + \left(\alpha\widetilde{z}+\beta x+r\right)U\left(q\left(\widetilde{z}\right),\widetilde{z}\right) + \left(\beta x+r\right)V_s - \beta x V_b}{\beta\int_0^x C_\pi\left(z,t\right)dz - \left(\alpha\widetilde{z}+\beta x+r\right)c\left(q\left(\widetilde{z}\right)\right) - \alpha\widetilde{z}V_s + \left(\alpha\widetilde{z}+r\right)V_b}. \qquad (2.24)$$

Both sides of (2.24) are positive for any given $\widetilde{z}$. Let $H\left(q\left(\widetilde{z}\right),\widetilde{z}\right)$ be the left-hand-side of (2.24) minus the right-hand-side. From the assumptions on preferences and technologies, it is easy to show that $\lim_{q\to 0} H\left(q\left(\widetilde{z}\right),\widetilde{z}\right) = \infty$ and $H\left(q\left(\widetilde{z}\right),\widetilde{z}\right)$ becomes negative for large q. Therefore, there exists a value of q that satisfies $H\left(q\left(\widetilde{z}\right),\widetilde{z}\right) = 0$. The left-hand-side of (2.24) is decreasing in $q\left(\widetilde{z}\right)$. The right-hand-side of (2.24) is an increasing function of $q\left(\widetilde{z}\right)$. Therefore, as shown in Figure 2-1, there is a unique terms of trade $q\left(\widetilde{z}\right)$ for each realized value of the match quality $\widetilde{z}$. ■

Figure 2-1 Price Dispersion



Corollary 1 *Prices are dispersed in a steady state.*

The above results are particularly important in that the Law of One Price does not hold in the current framework. What matters here is the realization of match quality, rather than the characteristics of a particular good. Thus, because buyers' preferences are diverse, the same good can be priced differently due to different realizations of match qualities, even though there is no vertical heterogeneity in terms of the quality of goods. Note that the source of price dispersion here is very different from other recent research. Camera and Corbae (1999), for instance, study price dispersion by allowing multiple units of money holdings. In the present model, a price dispersion arises even though any agent's money holding is limited to one unit.

Notice that the price dispersion here is not purely a monetary phenomenon. Rather, it is the agents' preferences that play the central role. The important implication for monetary theory is that the purchasing power of money is derived not only from what one unit of money will buy in the future, but also the current match quality. As Binmore *et. al.* (1986) point out, there are two basic driving forces that motivate bargaining parties to reach an agreement. One is that agents discount future; prolonging a negotiation results in a lower payoff. The other is the fear that their bargaining partner might walk away in favor of a new bargaining partner. In models with no bargaining breakdowns (*e.g.*, Rubinstein (1982) and Trejos and Wright (1995)) only the first force can impact the purchasing power of money. In models which insist that match qualities are identical (*e.g.*, Rubinstein and Wolinsky (1985) and Trejos and Wright (1995)) the purchasing power of money is affected by both forces, but to the same extent in all matches. In the present model, however, match qualities are diverse and so the purchasing power of money depends upon the idiosyncratic quality achieved by each match. This results in dispersed terms-of-trade.

We now turn to describing the cross-sectional distribution of steady-state dispersed prices. The following proposition shows that there is generally not a monotonic relationship between the terms-of-trade achieved by bargaining partners and the qualities of their matches. This dependency is complex. One ingredient is the manner in which buyers' marginal utilities of consumption are affected by match qualities. It is natural to presume that a buyer's marginal utility increases with the quality of the match achieved. Provided that this effect is not too pronounced, a decrease in match quality results in a larger terms-of-trade. This is also true if buyers' marginal utilities of consumption are unaffected, or lowered, by improved match qualities. If, however, there is for buyers a large complementarity between match quality and quantity consumed, then there may be a nonmonotonic relationship between match quality achieved by a buyer and a seller and the terms at which they trade.

Proposition 3 *In a steady state, the terms-of-trade $q(\widetilde{z})$ is strictly increasing with respect to $\widetilde{z} \in [0, x]$ if buyers' preferences do no exhibit strong complementarity between quality and consumption; that is, if $\partial^2 U / \partial \widetilde{z} \partial q \geq 0$ or is not too negative.*

Proof. From (2.24), it is straightforward to compute $dq/d\widetilde{z}$ as follows.

$$\frac{dq}{d\widetilde{z}} = \frac{(\alpha \widetilde{z} + \beta x + r) \frac{\partial U}{\partial \widetilde{z}} \pi_s c'(q(\widetilde{z})) - (\alpha \widetilde{z} + r) \pi_b W_s \frac{\partial^2 U}{\partial \widetilde{z} \partial q} - \pi_b \frac{\partial U}{\partial q} \alpha (V_b - V_s - c(q))}{(\alpha \widetilde{z} + r) \pi_b \left\{ \frac{\partial W_s}{\partial q} \frac{\partial U}{\partial q} + W_s \frac{\partial^2 U}{\partial q^2} \right\} - (\beta x + r) \pi_s \left\{ \frac{\partial W_b}{\partial q} c'(q) + W_b c''(q) \right\}}, \tag{2.25}$$

where $W_b \equiv U(q(\widetilde{z}), \widetilde{z}) + V_s - T_b$ is the buyer's payoff less the value of outside options and $W_s \equiv -c(q(\widetilde{z})) + V_b - T_s$ is the seller's payoff less the value of outside options. The values of these outside options are

$$T_b = \frac{1}{\beta x + r} \left(\alpha \int_0^{\widetilde{z}} B_\pi(z, t) \, dz - \alpha \widetilde{z} U(q(\widetilde{z}), \widetilde{z}) + \beta x V_b \right) \tag{2.26}$$

$$T_s = \frac{1}{\alpha \widetilde{z} + r} \left(-\beta \int_0^x C_\pi(z, t) \, dz + \beta x c(q(\widetilde{z})) + \alpha \widetilde{z} V_s \right). \tag{2.27}$$

Since $\partial W_s / \partial q < 0$ and $\partial W_b / \partial q > 0$, the denominator of (2.25) is unambiguously negative. Since $V_b - V_s - c(q) \geq 0$ from (2.7), the numerator of (2.25) is also negative provided that either $\partial^2 U / \partial \widetilde{z} \partial q \geq 0$ or is not too negative. ■

The content of the above result is that buyers, rather than sellers, are advantaged and quantities produced are greater when low quality (*i.e.*, high $\widetilde{z}$) matches form, unless there is a large enough complementarity in buyer preferences between match quality and quantity consumed. Two effects are at work. The first of these does not depend upon how match qualities alter buyers's marginal utilities. The second effect does.

The intuition for the first effect is as follows. The outside option value for a seller is the same for every match because the seller's utility depends only upon the quantity that she produces. In contrast, the buyer's outside option value depends upon the quality of the current match because of his idiosyncratic preferences. If a high quality match is formed (*i.e.* small $\widetilde{z}$), then both parties know that there is only a small probability that the buyer will discover a match of greater quality in the search stage of the current period. Consequently the buyer's outside option's value is low when the current match quality is high. The outcome is a low term-of-trade that reflects the seller's possession of the greater bargaining threat. The converse is the high terms-of-trade that result for low current match qualities, for then the buyer is relatively likely to discover a better partner in the search stage of the current period. The seller's response is to increase her production to acquire the buyer's immediate agreement.

The intuition for the second effect is more subtle. The seller's and buyer's outside option values are both maxima, each conditioned upon the value of the other's offered terms-of-trade and each optimized with respect to his own offer. Suppose that $\partial^2 U/\partial q \partial \widetilde{z} > 0$. Then, ignoring the probabilistic effects described in the preceding paragraph, an improvement in the match quality (*i.e.* lower $\widetilde{z}$) will, for every possible terms-of-trade value, both raise the buyer's utility function and reduce its slope. The buyer's best response to the seller's proposed terms-of-trade is therefore relocated to a smaller value for his proposed terms-of-trade, thereby reinforcing the first effect. If $\partial^2 U/\partial q \partial \widetilde{z} = 0$ then, again ignoring probabilistic effects, a change to $\widetilde{z}$ causes no change to the buyer's best offer and the first effect is the only effect. If $\partial^2 U/\partial q \partial \widetilde{z} < 0$, then the intuition for the second effect is reversed, making it possible that the first effect may overwhelm the first for at least some match qualities.

2.4.3 Relation to a Nash Solution

In the majority of the search theory literature a Nash bargaining solution is used as an approximation of a strategic bargaining game in order to "endogenize" prices. However, as Wolinsky (1987, p.326) pointed out, "the two solutions coincide only in certain special cases" because a Nash solution does not make use of all the information provided to the bargainers by the market. The conditions for the equivalence pointed out by Wolinsky (1987) are (i) the search intensities are not decision variables, (ii) the match qualities are the same for all matches and (iii) in the bargaining process an agent *always* drops an old partner in favor of a new one. In this study, the match qualities are heterogenous, and a buyer drops his old partner *if and only if* the new match quality is at least as good as the current one. Thus, the question that is addressed in this subsection is whether the use of a Nash solution is justified here.

Consider the Nash bargaining problem

$$q^n(\widetilde{z},t) = \arg\max \left[U\left(q\left(\widetilde{z},t\right),\widetilde{z}\right) + V_s(t) - Y_b(t)\right]^{\theta}\left[-c\left(q\left(\widetilde{z},t\right)\right) + V_b(t) - Y_s(t)\right]^{1-\theta}, \tag{2.28}$$

where $Y_b(t)$ and $Y_s(t)$ denote the threat points for the buyer and the seller. The first order condition requires that

$$\frac{U'(q(\widetilde{z},t),\widetilde{z})}{c'(q(\widetilde{z},t))} = \frac{1-\theta}{\theta}\frac{U(q(\widetilde{z},t),\widetilde{z}) + V_s(t) - Y_b(t)}{-c(q(\widetilde{z},t)) + V_b(t) - Y_s(t)}. \tag{2.29}$$

Rewrite the equilibrium condition (2.24) for the strategic bargaining game as

$$\frac{U'(q(\widetilde{z},t),\widetilde{z})}{c'(q(\widetilde{z},t))} = \frac{(\beta x + r)\pi_s}{(\alpha\widetilde{z} + r)\pi_b}\frac{U(q(\widetilde{z},t),\widetilde{z}) + V_s(t) - T_b(t)}{-c(q(\widetilde{z},t)) + V_b(t) - T_s(t)}, \tag{2.30}$$

where $T_b(t)$ and $T_s(t)$ are defined in (2.26) and (2.27). (2.29) and (2.30) imply that the Nash bargaining and the strategic bargaining solutions coincide if and only if $Y_b(t) = T_b(t)$, $Y_s(t) = T_s(t)$, and

$$\theta = \frac{(\alpha\widetilde{z} + r)\pi_b}{(\alpha\widetilde{z} + r)\pi_b + (\beta x + r)\pi_s} \in (0,1).$$

The buyer's bargaining power rises with π_b, α and $\widetilde{z}$, and falls as π_s or β rises. Since $\alpha M = \beta(1-M)$, the buyer's bargaining power rises as M falls. Since $\widetilde{z}$ is a random variable, the bargaining power is also random.

Notice, however, that the exact components of $T_b(t)$ and $T_s(t)$ are not known unless the strategic bargaining game is explicitly considered as in (2.13) and (2.14). What has been done in the literature is instead to assume either $Y_b(t) = Y_s(t) = 0$, or $Y_b(t) = V_b(t)$ and $Y_s(t) = V_s(t)$; that is, the threat points are assumed to be either zero or the value functions of search. Consequently, the question of whether the Nash representation and strategic bargaining outcomes coincide reduces to asking whether $T_b = V_b$ and $T_b = V_b$ or not.

In the present model the threat points $T_b(t)$ and $T_s(t)$ do not coincide with the expected values of being without a bargaining partner, $V_b(t)$ and $V_s(t)$. In particular, (2.4) and (2.8) imply $T_b \leq V_b$, and (2.7) and (2.9) imply $T_s \leq V_s$. Therefore, the Nash and strategic bargaining solutions do not coincide *even in a steady state* and so the model's predictions are sensitive to the bargaining game form selected. This contrasts sharply with previous search-theoretic models of money with prices, such as Trejos and Wright (1995), which assume that the purchasing power of one unit of fiat money is determined either by Nash bargaining or by strategic bargaining with homogeneous outside options. The main rationale for using of a Nash solution in the previous literature is that it coincides with a strategic bargaining solution in steady states.[18] This requires that the values of outside options are either zero or identical for all matches, a condition met in previous research by *assuming* that the quality of all matches is the same in order to keep the issue of outside options trivial. This assumption causes not all market information to be used to characterize the market's price level.

Proposition 4 *In the limit as* $\alpha \to 0$ *and* $\beta\left(=\alpha\left(1-M\right)/M\right) \to 0$*, the terms-of-trade in a steady state is the unique solution to*

$$\frac{U'\left(q\left(\widetilde{z},t\right),\widetilde{z}\right)}{c'\left(q\left(\widetilde{z},t\right)\right)} = \frac{\pi_s}{\pi_b}\frac{U\left(q\left(\widetilde{z},t\right),\widetilde{z}\right)+V_s\left(t\right)}{-c\left(q\left(\widetilde{z},t\right)\right)+V_b\left(t\right)}.$$

Proposition 4 states that in the limit as all outside options disappear, the strategic and the Nash bargaining coincide in a steady state.

Notice that the Nash bargaining solution also supports a unique terms-of-trade for each realized match quality and generates price dispersion. This is because the trade surplus is match-specific. Thus the terms-of-trade is also match-specific. However, the terms-of-

[18] Coles and Wright (1998) show that a Nash solution and a bargaining outcome do not necessarily coincide outside steady states.

trade predicted by a Nash solution are typically not the same as is predicted by strategic bargaining.

2.5 Equilibrium

2.5.1 Steady State

This section describes aggregate equilibrium conditions and closes the model. Rational expectations are assumed. Thus, in equilibrium expectations are correct in that $B_\pi(z,t) = U(q(z,t),z)$ and $C_\pi(z,t) = c(q(z,t))$. In order to describe the market equilibrium, we need an aggregate measure of the price level. Since there is no aggregate uncertainty, we construct an aggregate variable by taking expectations. Define the average production (the inverse of the average price level) at date t by

$$q(t) \equiv \frac{1}{x}\int_0^x q(\widetilde{z},t)\,d\widetilde{z}.$$

We describe below the properties of the average price level in a steady state. Special attention will be given to the relationship between the strategic and Nash solutions.

Proposition 5 *Let $q^n(\widetilde{z},t)$ solve*

$$\frac{U'(q^n(\widetilde{z},t),\widetilde{z})}{c'(q^n(\widetilde{z},t))} = \frac{\pi_s}{\pi_b}\frac{U(q^n(\widetilde{z},t),\widetilde{z}) + V_s(t) - V_b(t)}{-c(q^n(\widetilde{z},t)) + V_b(t) - V_s(t)}.$$

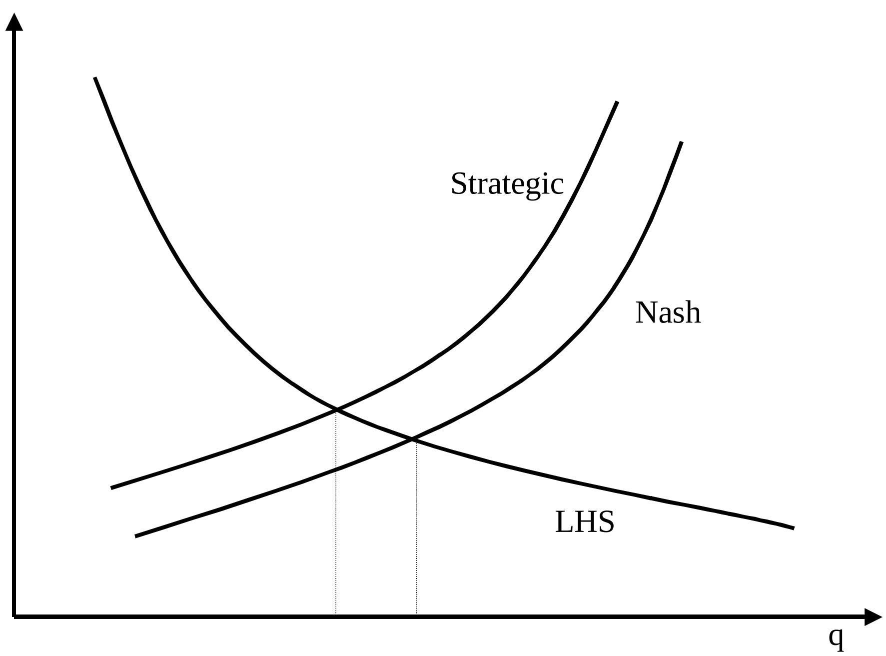
Then, $q(\widetilde{z},t) \leq q^n(\widetilde{z},t)$ for any $\widetilde{z} \leq x$ and $q(x,t) \leq q^n(x,t)$.

Proof. Substitute (2.8) and (2.9) into (2.24) to obtain

$$\frac{U'\left(q\left(\widetilde{z}\right),\widetilde{z}\right)}{c'\left(q\left(\widetilde{z}\right)\right)}=\frac{\pi_s}{\pi_b}\frac{U\left(q\left(\widetilde{z}\right),\widetilde{z}\right)+V_s-\left(V_b-\frac{\alpha}{\alpha\widetilde{z}+\beta x+r}\int_{\widetilde{z}}^{x}U\left(q\left(z,t\right),z\right)dz\right)}{-c\left(q\left(\widetilde{z}\right)\right)+V_b-V_s}. \tag{2.31}$$

$\frac{\alpha}{\alpha\widetilde{z}+\beta x+r}\int_{\widetilde{z}}^{x}U\left(q\left(z,t\right),z\right)dz$ is decreasing in $\widetilde{z}$ and is zero if $\widetilde{z}=x$. Thus, the right-hand-side of (2.31) is greater than the right-hand-side of (NS). This implies that $q\left(\widetilde{z},t\right)\leq q^n\left(\widetilde{z},t\right)$ for any $\widetilde{z}\leq x$ (see Figure 2-2). ■

Figure 2-2 Comparison to a Nash Solution



Proposition 5 asserts that a Nash solution with outside options coincides with the strategic bargaining solution when the realized match quality is at the minimum acceptable level. The value of the seller's outside options is homogeneous across all matches because the production cost is independent of to whom she produces. In contrast, the buyer's outside options are heterogenous across matches. According to (2.31), for a buyer, a higher match quality results in a lower threat. This fact is used by sellers only if bargaining is strategic and with outside options.

Corollary 2 *Market volatility is higher for* $q\left(\widetilde{z},t\right)$ *than for* $q^{n}\left(\widetilde{z},t\right)$.

Markets in which terms-of-trade are determined by strategic bargaining will be more volatile than markets which use Nash bargaining, in the sense that the variance of the terms-of-trade is higher for the strategic bargaining case.

Corollary 3 *Let* $q^{n}\left(t\right)\equiv\frac{1}{x}\int_{0}^{x}q^{n}\left(\widetilde{z},t\right)d\widetilde{z}$. *Then,* $q\left(t\right)\leq q^{n}\left(t\right)$ *for all* t.

The average price level $(1/q)$ predicted by the Nash solution described by (NS) is lower than is predicted by the strategic solution.

2.5.2 Dynamics

This subsection derives the dynamical differential equations of the model. From (2.23), for a given match quality $\widetilde{z}$,

$$\begin{aligned}\dot{q}\left(\widetilde{z}\right) &= \frac{\pi_{s}}{U'\left(q\left(\widetilde{z}\right),\widetilde{z}\right)}\left\{\begin{array}{c}-\alpha\int_{0}^{\widetilde{z}}U\left(q\left(z\right),z\right)dz+\left(\alpha\widetilde{z}+\beta x+r\right)U\left(q\left(\widetilde{z}\right),\widetilde{z}\right)\\ +\left(\beta x+r\right)V_{s}-\beta xV_{b}-\dot{V}_{s}\end{array}\right\}\\ &\quad-\frac{\pi_{b}}{c'\left(q\left(\widetilde{z},t\right)\right)}\left\{\begin{array}{c}\beta\int_{0}^{x}c\left(q\left(z\right)\right)dz-\left(\alpha\widetilde{z}+\beta x+r\right)c\left(q\left(\widetilde{z}\right)\right)\\ -\alpha\widetilde{z}V_{s}+\left(\alpha\widetilde{z}+r\right)V_{b}-\dot{V}_{b}\end{array}\right\}. \end{aligned} \tag{2.32}$$

Substitute (2.8) and (2.9) into (2.32) to obtain

$$\begin{aligned}\dot{q}\left(\widetilde{z}\right) &= \frac{\pi_{s}}{U'\left(q\left(\widetilde{z}\right),\widetilde{z}\right)}\left\{\begin{array}{c}\left(\alpha\widetilde{z}+\beta x+r\right)U\left(q\left(\widetilde{z}\right),\widetilde{z}\right)\\ -\alpha\int_{0}^{\widetilde{z}}U\left(q\left(z\right),z\right)dz-\beta\int_{0}^{x}c\left(q\left(z\right)\right)dz\end{array}\right\}\\ &\quad+\frac{\pi_{b}}{c'\left(q\left(\widetilde{z},t\right)\right)}\left\{\begin{array}{c}\left(\alpha\widetilde{z}+\beta x+r\right)c\left(q\left(\widetilde{z}\right)\right)+\alpha\left(x-\widetilde{z}\right)V\\ -\beta\int_{0}^{\widetilde{z}}c\left(q\left(z\right)\right)dz-\alpha\int_{0}^{x}U\left(q\left(z\right),z\right)dz\end{array}\right\}. \end{aligned} \tag{2.33}$$

Since the quality of a match $\widetilde{z}$ is a random variable that is Uniformly distributed between zero and x, we can transform (2.33) into the aggregate equilibrium condition by taking expectations. Define

$$\dot{q} \equiv \frac{1}{x}\int_0^x \dot{q}\left(\widetilde{z}\right) d\widetilde{z}.$$

Then the aggregation of (2.33) yields

$$\begin{aligned}\dot{q} \;=\; & \int_0^x \frac{\pi_s}{xU'\left(q\left(\widetilde{z}\right),\widetilde{z}\right)}\left\{\begin{array}{c}\left(\alpha\widetilde{z}+\beta x+r\right)U\left(q\left(\widetilde{z}\right),\widetilde{z}\right)\\ -\beta\int_0^x c\left(q\left(z\right)\right)dz-\alpha\int_0^{\widetilde{z}} U\left(q\left(z\right),z\right)dz\end{array}\right\}d\widetilde{z}\\ & +\int_0^x \frac{\pi_b}{xc'\left(q\left(\widetilde{z}\right)\right)}\left\{\begin{array}{c}\left(\alpha\widetilde{z}+\beta x+r\right)c\left(q\left(\widetilde{z}\right)\right)+\alpha\left(x-\widetilde{z}\right)V\\ -\beta\int_0^{\widetilde{z}} c\left(q\left(z\right)\right)dz-\alpha\int_0^x U\left(q\left(z\right),z\right)dz\end{array}\right\}d\widetilde{z}, \qquad (2.34)\end{aligned}$$

where from (2.8) and (2.9),

$$\dot{V} = -\alpha\int_0^x U\left(q,z\right)dz-\beta xc\left(q\right)+\left(\left(\alpha+\beta\right)x+r\right)V. \qquad (2.35)$$

(2.34) and (2.35) describe the dynamics of the economy. Unfortunately, the analysis of the system of differential equations is cumbersome. But intuition suggests that the dynamic properties of the model do not differ much from those of Trejos and Wright (1995) and Coles and Wright (1998); that is, the monetary steady state is most likely a source.[19]

2.6 Conclusion

This chapter has presented a model of money and prices in which the quality of matches between buyers and sellers is heterogenous across matches. Strategic bargaining is employed as a price determination mechanism. Extending agents' preferences to depend upon the quality of match as well as the quantity consumed allows endogenization of the val-

[19] One way to simplify the analysis is to consider a special case in which the match quality is always at $\widetilde{z} = x$. (2.33) evaluated at $\widetilde{z} = x$ and (2.35) construct such an example. From this restricted system of differential equations, it is straightforward to verify that the monetary steady state is a source.

ues of outside options in the bargaining process. The model gives rise to dispersed prices even though money is indivisible and money holding is limited to one unit. The qualitative properties of the distribution of prices depends on the complementarity between the match quality and the quantity consumed in agents' preferences. If the complementarity is weak enough, then a buyer in a low quality match will be compensated by a low price (*i.e.* high quantity supplied). Conversely, if the complementarity is strong, then the price may rise with some match qualities. The model asserts that the purchasing power of money depends not only on what it will buy in the future, but also on the type of match made. The reason can be easily explained by examining the role that outside options play in reaching agreement in strategic bargaining problems. Particularly, a higher outside option value increases the chance that an agent's bargaining partner might walk away in favor of a new bargaining partner. In the Trejos-Wright and the Coles-Wright models, outside options are either zero or homogeneous. Thus, in their models agents are motivated to reach agreement only because delay is costly, causing money to be valued only for what it will buy in the future. The framework presented in this chapter focuses on the presence of outside options with values that are endogenously dispersed, reflecting all information contained in the market. The resulting purchasing power of money not only captures what it will buy in the future, but also reflects the qualities of specific match made at each moment. This breaks the Law of One Price. The results obtained in this paper have some similarity to those in Bester (1988), who studies equilibrium price dispersion in a consumer search problem.

The price dispersion here is *not* purely a monetary phenomenon. It is the prices of *commodities* that is dispersed. This raises an important question: how should the value of money be defined? Since fiat currency does not directly generate utility, the value of money is measured only in terms of the utility of other commodity. On the other hand, the price of a good is measured by money. In a sense, the values of money and commodities are mutually determinate. To fix the unit of measure, a *numeraire* has to be chosen. If commodities or match qualities are homogeneous across all matches (as in previous models), then the value of money can easily be measured in terms of units of goods it purchases because in such an economy there is virtually only one commodity and therefore the choice of a *numeraire* is trivial. If match qualities are heterogeneous (as in the present model and in the real world), on the other hand, then there is no clear candidate for *numeraire* because the value of each commodity itself is subject to change depending upon match quality. If the value of money is simply what it will buy in future transactions, then the model presented in this chapter suggests that the value of money is quite idiosyncratic and one should not expect that a unit of currency has the same value even at a given point in time. One should expect a cross-sectional distribution in the value of money.

In the model presented in this chapter the strategic bargaining solution does not coincide with a Nash bargaining solution, even in a steady state. This is because the values of outside options are match-specific. This clearly suggests that the use of a Nash solution in matching models requires extra care. As Trejos and Wright (1993) recognizes, even though it is possible to find a connection between a strategic solution and a Nash solution, the threat points need to capture the underlying market conditions. The results obtained in this paper reinforce the claim.

Because an agent's outside option values are determined endogenously and are dispersed, the framework developed here can be extended in several useful directions. First, the interaction between agents' reservation strategies and market prices is an important issue. This is particularly important in the context of the labor market where it has long been a question as to why an unemployed person cannot persuade firms to hire him by offering a low wage. Another important extension is to consider the impact of agents' search activities on prices in a manner similar in spirit to Li (1995). Wolinsky (1987) asked a similar question by extending the Rubinstein-Wolinsky model (1985). In the Wolinsky model (1987), the values of outside options are exogenous. Since we are dealing with a general equilibrium model, however, the values of outside options must be determined by the "market". The present model achieves this requirement and so is suitable for studying market equilibria with endogenous search intensities.

Chapter 3
Tight Money Policies and Inflation Revisited

Consider Japan first. ... The economy is now beginning to pick up, but only because the government has repeatedly tried to restart growth by spending, and borrowing, on a massive scale. The strategy has been correct, given deflation and a long (albeit shallow) recession–though it ought recently to have been accompanied by aggressive monetary expansion.

(from "Debt in Japan and America," *Economist*, January 22, 2000)

3.1 Introduction

3.1.1 Issues

Inflation is a tax. Because of that, monetary and fiscal policies are connected by a single "government budget constraint" and the government cannot arbitrarily choose a mix of monetary and fiscal policies for a desired policy goal. This is the heart of Sargent and Wallace's (1981) famous "unpleasant monetarist arithmetic." Using the interaction between monetary and fiscal policies, Sargent and Wallace (1981) produced conditions under which monetizing the government deficit does not *always* result in a higher inflation. A policy implication of their proposition is that in long-run policy making, it may be misleading to have the quantity theory in mind. This point has attracted many economists and has evolved to the "fiscal theory" of price determination, which emphasizes the government's intertemporal budget constraint. This chapter reconsiders the conditions under which a tight money policy through open market operations is inflationary in the long run. It asks the question in a general environment to understand the *necessary conditions* for the unpleasant monetarist arithmetic (UMA).

Understanding the conditions for the UMA is important for the following three reasons. First, the original Sargent-Wallace (1981) model has several seemingly strong assumptions. It is therefore useful to check the robustness of their claim. In particular, as Darby (1984) states, one of the conditions for the UMA –the rate of return on bonds has to exceed the growth rate of the economy– is empirically implausible. A previous attempt to produce more general (and weaker) conditions for the UMA is carried out by Bhattacharya, Guzman, and Smith (1998).

Second, when the quantity theory of money fails, making a conceptual link between monetary policy and inflation gets complicated. In particular, when the interest rate is *lower* than the growth rate of the economy, the economy is dynamically *inefficient* and so the government should be able to roll over its debt without causing inflation. This chapter provides, for a very general environment, the conditions for the UMA to hold and gives an intuition for the counterintuitive phenomenon and reasons for why the proposition holds *even when* the interest rate is *lower* than the growth rate of the economy. It is shown that what is central for the UMA is whether an economy is in a position of raising bond seigniorage by increasing the interest rate. This chapter demonstrates that UMA can be understood better by considering the "bond-seigniorage Laffer curve" rather than the conventional total seigniorage Laffer curve.

Finally, this chapter provides an alternative indicator for the long-run monetary policy making: bond seigniorage. The Sargent-Wallace proposition is truly an "unpleasant" example in that it shows that tightening money through open market operations may be inflationary. Then, what should the central bank do to achieve *sustainable* low inflation? The analysis of the government's intertemporal budget constraint in this chapter reveals that in the long run, the lowest (positive) rate of inflation is achieved when the central bank chooses the bond-to-money ratio so as to maximize bond seigniorage.

3.1.2 Budget Arithmetic

Consider the following steady state government budget constraint

$$\begin{aligned} \text{Deficits} &= \text{Real money balance}\left(1-\frac{1}{\text{Inflation rate}}\right)+\text{Bonds}(1-\text{Real interest rate}). \\ &= \text{Currency seigniorage + Bond seigniorage.} \end{aligned}$$

Here we have normalized the growth rate of the economy to unity. Following the conventional view of the "fiscal theory" of the price level, we shall consider this equation as the inflation-determination condition.[20] Since the amount of deficit is determined by the fiscal authority, it is assumed to be constant. The choice variable for the central bank is the bond-to-money ratio. Thus, the steady-state rate of inflation is determined by the budget constraint once the real interest rate is known. There are two cases to consider.

Sargent and Wallace (1981) restrict their analysis to a scenario where the real interest rate exceeds the growth rate of the economy, so that bond seigniorage is negative. Then, a permanent open market sale of bonds creates a revenue shortfall. If the economy is in a position in which the government could raise seigniorage by printing money faster (i.e., the economy is on the "good side" of the Laffer curve), then a tight money policy engineered

[20] See Cochrane (1998, 2000).

by permanent open market operations *increases* the steady state rate of inflation. This is the essence of the Sargent and Wallace's (1981) unpleasant monetarist arithmetic.

Consider a case with the rate of return being less than the growth rate of the economy. This case is explored by Bhattacharya, Guzman, and Smith (1998). In such an economy, bond seigniorage is positive and a further increase in bonds increases bond seigniorage. In this case, the government could raise enough revenue from bonds. Bhattacharya, Guzman and Smith (1998) point out that it is possible that an increase in bonds reduces currency seigniorage more than the increase in bond seigniorage. The intuition is as follows. As the bond-money ratio increases, capital gets crowded out of the portfolio of private agents. This reduction in deposits causes a reduction in the volume of reserves held by banks. Consequently, the *inflation tax base* falls. On the good side of the Laffer curve, the central bank is forced to raise the steady state inflation rate. This "tax base effect" is the channel through which the UMA proposition holds even when the rate of return is less than the growth rate of the economy.[21]

3.1.3 Introducing Neoclassical Production Function

This chapter introduces a (concave) neoclassical production of the Diamond (1965) variety. The presence of a concave production function adds a new feature into the analysis. In particular, there arises a new channel through which a tight money policy engineered by a permanent open market operation increases the steady state inflation rate.

---

[21] The "tax base effect" is pointed out by Espinosa-Vega and Russell (1998).

Suppose the real interest rate is less than the growth rate of the economy. Once again, in such an economy, bond seigniorage is positive and a further increase in bonds increases bond seigniorage. Thus, the government could raise enough revenue from bonds. The "tax base effect" is still present here. What is new here is the "interest rate effect". An increase in the bond-money ratio in this economy has a real effect: it reduces the steady state capital stock because the tight money policy crowds capital out of the portfolio of private agents.[22] This raises the marginal product of capital. If the primary deficit is small enough, then the increase in the rate of return on capital raises the real interest rate. The increase in the real interest rate reduces bond seigniorage because the government has to repay its debt with a higher interest. Therefore, with this "interest rate" channel, a tight money policy may *reduce* bond seigniorage. In such a case, the government is forced to raise currency seigniorage and therefore the UMA proposition holds.

This "interest rate effect" arises under two *necessary* conditions. One is that the Modigliani-Miller theorem for government open market operations fails. That is, it is necessary that a tight money policy reduces the capital stock. Without this link, monetary policies cannot affect the real interest rate. Given that open market operations have real effects, the next qualifier is that a higher interest rate *reduces* bond seigniorage. This condition is met on the good side of the bond-seigniorage Laffer curve. Now there arise two effects. One is that a tight money policy increases bond seigniorage. The other is that, through the "interest rate" channel, the tight money policy reduces bond seigniorage. If the

[22] See Espinosa-Vega and Russell (1998) for a similar result.

real interest rate is not too high, then the latter effect outweighs the former and the UMA proposition obtains even if the real interest rate is less than the growth rate.

The rest of the chapter is organized as follows. Section 3.2 describes the environment and the nature of trade in our model, while Section 3.3 states its equilibrium conditions. Section 3.4 contains the main unpleasant monetarist arithmetic results. Section 3.5 concludes.

3.2 The Model

3.2.1 Environment

We consider an economy consisting of an infinite sequence of two-period lived overlapping generations, an initial old generation, and an infinitely-lived government. Let $t = 1, 2, ...$ index time. At each date t, a new generation comprised of N identical members appears.

There is a single final good produced using a standard neoclassical production function $F(K_t, L_t)$ where K_t denotes the capital input and L_t denotes the labor input at t. Let $k_t \equiv \frac{K_t}{L_t}$ denote the capital-labor ratio (capital per young agent). Then, output per young agent at time t may be expressed as $f(k_t)$ where $f(k_t) \equiv F(\frac{K_t}{L_t}, 1)$ is the intensive production function. We assume that $f(0) = 0$, $f' > 0 > f''$, and that the usual Inada conditions hold. The final good can either be consumed in the period it was produced, or it can be stored to yield capital the following period. Capital depreciates 100% between periods.

Each agent is endowed with one unit of labor when young and is retired when old. In addition, the initial old agents are each endowed with $M_0 > 0$ units of fiat currency and $k_0 > 0$ units of capital.

Let c_{1t} (c_{2t}) denote the consumption of the final good by a representative young (old) agent born at t. All such agents have preferences representable by the utility function $U(c_{1t}, c_{2t})$ where U is twice-continuously differentiable, strictly increasing, and strictly concave in its arguments.

Finally, the government has a constant net-of-interest deficit of $g \geq 0$ in each period. The government finances this entirely by issuing bonds and money. Let M_t denote the per capita stock of money outstanding at the end of period t, and B_t denote the outstanding per capita supply of bonds (in nominal terms) where $B_0 = 0$.

3.2.2 Trade

Young agents supply their labor endowment inelastically in competitive labor markets, earning a wage income of ω_t at time t where

$$\omega_t \equiv \omega(k_t) = f(k_t) - k_t f'(k_t) \qquad \forall t > 1. \tag{3.1}$$

In addition, capital is traded in competitive capital markets, and earns a real return of q_{t+1} between t and $t+1$ where,

$$r_{t+1} = f'(k_{t+1}).$$

Let p_t denote the time t price level, m_t denote the holdings of real balances by a representative young agent at t, k_{t+1} denote the amount she stores and b_t denote her real bond holdings at t. We assume that all storage is subject to a reserve requirement.[23] More specifically, each agent faces the constraint

$$m_t \geq \lambda k_{t+1};\ \lambda \in (0,1)\,. \tag{3.2}$$

In addition to this reserve requirement, each young agent faces the following budget constraints at t :

$$c_{1t} + m_t + k_{t+1} + b_t \leq \omega_t \tag{3.3}$$

and

$$c_{2t} \leq r_{t+1}k_{t+1} + \rho_{t+1}b_t + \left(\frac{p_t}{p_{t+1}}\right) m_t, \tag{3.4}$$

where ρ_{t+1} is the gross real rate of return on government bonds between t and $t+1$.

The problem of a young agent at t is to maximize $U(c_{1t}\,, c_{2t})$ subject to (3.2)-(3.4). If

$$r_{t+1} = f'(k_{t+1}) > \frac{p_t}{p_{t+1}} \tag{A.1}$$

holds, then the reserve requirement binds, and (3.2) holds as an equality. We exclusively focus on equilibria that satisfy (A.1).[24] It is now possible to rewrite the young agents' problem as follows. Let $d_t \equiv k_{t+1} + m_t$ and let $\phi \equiv \frac{1}{1+\lambda}$. Then $d_t = (1+\lambda)k_{t+1}$ denotes storage plus reserves, (which we refer to as "deposits") and ϕ represents the fraction of deposits held in the form of storage, while $(1-\phi)$ can be interpreted as the fraction of deposits required to be held as reserves (real money balances).

---

[23] We assume that holders of government bonds are not subject to any such reserve requirement. Our description closely follows Wallace (1984) and Espinosa and Rusell (1998).

[24] It is important to note that fiat money is valued (held) in this setup solely because of the presence of a reserve requirement on storage. Therefore, in this environment, if money is dominated in return and *no* legal restrictions (*e.g.* reserve requirements) are present, the demand for money would be zero. Since the current purpose is to study alternative modes of deficit finance, we restrict our attention *only* on equilibria where a positive demand for money exists.

Armed with this notation, we can now rewrite the problem (**P1**) of a representative young agent at t as

$$\max U(c_{1t}, c_{2t})$$

subject to

$$c_{1t} + d_t + b_t \leq \omega_t \tag{3.5}$$

and

$$c_{2t} \leq \left[\phi r_{t+1} + (1-\phi)\left(\frac{p_t}{p_{t+1}}\right)\right] d_t + \rho_{t+1} b_t. \tag{3.6}$$

Of course, if bonds and deposits are both to be held, then

$$\rho_{t+1} = \left[\phi r_{t+1} + (1-\phi)\left(\frac{p_t}{p_{t+1}}\right)\right] \tag{3.7}$$

must hold. Equation (3.7) requires that the return on government bonds equals the appropriately weighted return on storage and currency (which is, in effect, the return on deposits). When (3.7) holds, we can simplify the young agents' problem further. Let $S_t \equiv d_t + b_t$ denote total savings by a young agent at t. Then problem **(P1)** is identical to one where an agent chooses S_t so as to maximize $U\left[\omega(k_t) - S_t, \rho_{t+1} S_t\right]$. Let

$$S(\omega(k_t), \rho_{t+1}) \equiv \arg\max \ U\left[\omega(k_t) - S_t, \rho_{t+1} S_t\right]. \tag{3.8}$$

The function $S(.)$ summarizes an agent's optimal savings behavior. We assume that current and future consumption are normal goods and that current and future consumption are gross substitutes. Then the following lemma is obvious.

Lemma 2 *(a)* $S_\omega \in (0,1)$, *(b)* $S_\rho \geq 0$, *and (c)* $S\left(0, \rho_{t+1}\right) = S\left(\omega\left(k_t\right), 0\right) = 0$.

Finally, note for future reference that when (A.1) and (3.7) hold, $f'(k) > \rho$ automatically holds. The reserve requirement may then be interpreted as driving a wedge between the return on capital and the return on bonds.

3.2.3 The Government

The government finances a net-of-interest deficit of $g \geq 0$ every period by issuing indexed one-period default-free bonds and by printing money.[25] The government's budget constraint is

$$g = \frac{M_{t+1} - M_t}{p_{t+1}} + b_{t+1} - \rho_{t+1} b_t \; ; \; t \geq 1. \tag{3.9}$$

Equation (3.9) states that the government finances its expenditures and interest obligations on outstanding government debt, from seignorage revenue earned by money creation, $\frac{M_{t+1} - M_t}{p_{t+1}}$, and from the sale of new bonds at date $t+1$, b_{t+1}.

Suppose the central bank conducts its monetary policy by choosing at time 1 a ratio

$$\mu \equiv \frac{b_t}{m_t}; \; \mu > 0; \forall t \geq 1 \tag{3.10}$$

of bonds issued to money created. It may be convenient to think of variations in μ as *permanent* open market operations. In addition, at time 1, the central bank sets the reserve requirement $(1 - \phi)$. Using $m_t \equiv \frac{M_t}{p_t}$, we can rewrite the government budget constraint as

$$g = m_{t+1} - m_t\left(\frac{p_t}{p_{t+1}}\right) + b_{t+1} - \rho_{t+1} b_t. \tag{3.11}$$

3.3 General Equilibrium

[25] Click (1998) documents that between 1971-90, in a wide cross-section of countries, currency seignorage as a percentage of GDP ranged from 0.3% to 14% and seignorage as a percentage of government spending ranged from 1% to 148%.

3.3.1 Characterization

A general equilibrium of the economy is described as follows.

Definition2 *A monetary competitive equilibrium is a set of sequences for allocations* $\{S_t\}$, $\{b_t\}$, $\{k_t\}$, $\{m_t\}$, *and prices* $\{r_t\}$, $\{\omega_t\}$, $\{p_t\}$, $\{\rho_t\}$ *such that*

(a) Factor markets clear:

$$r_{t+1} = f'(k_{t+1})$$

$$\omega_t = f(k_t) - k_t f'(k_t) \equiv \omega(k_t)$$

(b) Asset market clears:

$$m_t + k_{t+1} + b_t = S(\omega_t, \rho_{t+1})$$

(c) The government budget constraint holds:

$$g = m_{t+1} - m_t(\frac{p_t}{p_{t+1}}) + b_{t+1} - \rho_{t+1} b_t.$$

Noting that $b_t + d_t = \left[\frac{1+\mu(1-\phi)}{\phi}\right] k_{t+1}$, we can rewrite the asset market clearing condition as

$$[1 + \mu(1 - \phi)]k_{t+1} = \phi S(\omega(k_t), \rho_{t+1}). \tag{3.12}$$

The equilibrium gross return on real balances (the inverse of the gross rate of inflation) for given ϕ and g can now be determined as

$$\frac{p_t}{p_{t+1}} = \frac{(1 + \mu)k_{t+2} - \mu\phi k_{t+1} f'(k_{t+1}) - \frac{\phi}{1-\phi} g}{[1 + \mu(1 - \phi)]k_{t+1}}. \tag{3.13}$$

Substituting (3.13) in (3.7), it is possible to write (3.7) as

$$\rho_{t+1} = \frac{\phi k_{t+1} f'(k_{t+1}) + (1 - \phi)(1 + \mu)k_{t+2} - \phi g}{[1 + \mu(1 - \phi)]k_{t+1}}. \tag{3.14}$$

Equation (3.14) describes the equilibrium rate of return on government bonds as a function of the capital-labor ratio. Therefore, equations (3.12) and (3.14) jointly constitute the equilibrium conditions in this model. Together they determine the time path of the capital-labor ratio, $\{k_t\}$, given the government's prior pre-committed choices of μ, ϕ, and g.

There are a number of conditions that an equilibrium sequence $\{k_t\}$ must satisfy. First, $k_t > 0$ must hold at each date t. Second, (3.13) must yield a non-negative gross return on real balances, i.e. $\frac{p_t}{p_{t+1}} > 0$. This requirement is satisfied if

$$k_{t+2} > \frac{\mu\phi k_{t+1} f'(k_{t+1})}{(1+\mu)} + \frac{\phi}{1-\phi}\frac{g}{1+\mu}\ ;\ \forall t. \tag{3.15}$$

Third, (A.1) must be satisfied at each t, i.e. $f'(k_{t+1}) > \frac{p_t}{p_{t+1}}$ must hold. This condition is satisfied if

$$f'(k_{t+1}) > \frac{k_{t+2}}{k_{t+1}} - \frac{\phi}{1-\phi}\frac{g}{1+\mu}\frac{1}{k_{t+1}}\ ;\ \forall t. \tag{3.16}$$

3.3.2 Steady State Equilibria

We first explore the properties of steady state equilibria. To simplify notation, define $A \equiv 1 + \mu\,(1-\phi)$. Then, imposing $k_t = k$ for all t in (3.12) and (3.14) yields

$$k = \frac{\phi}{A} S(\omega(k), \rho\,(k)), \tag{3.17}$$

where

$$\rho\,(k) \equiv 1 + \frac{\phi}{A}\left[f'(k) - 1 - \frac{g}{k}\right]. \tag{3.18}$$

It is easy to check that

$$\rho'\,(k) \equiv \frac{\phi}{A}\left[f''(k) + \frac{g}{k^2}\right]. \tag{3.19}$$

Legitimate steady state equilibria must, in addition, satisfy restrictions imposed by (3.15) and (3.16). It is easy to verify that these reduce to

$$1-\frac{\phi g}{(1-\phi)(1+\mu)k}<f'(k)<\frac{(1-\phi)(1+\mu)k-\phi g}{\mu\phi(1-\phi)k},$$

which in turn imposes the condition on ρ that

$$1-\frac{\phi g}{(1-\phi)(1+\mu)k}<\rho<\frac{1+\mu}{\mu}-\frac{\phi g}{\mu(1-\phi)k}. \tag{3.20}$$

(3.20) therefore imposes restrictions that the steady state gross real rate of return on capital (and bonds) must satisfy. Notice that if $g=0$ holds, then (3.20) immediately implies that

$$1<\rho<1+\frac{1}{\mu}.$$

Therefore, in our formulation, the presence of primary deficits is essential for a consideration of the case with $\rho<1$.

3.3.3 Existence and Uniqueness

We now explore the issue of existence of steady state equilibria. To begin with, note that given (A1), $k=0$ solves (3.17). Furthermore, the left-hand-side of (3.17) is (trivially) strictly increasing in k and has a slope equal to 1. The slope of the right-hand-side is given by

$$\frac{\phi}{A}\left[S_{\omega}\omega'(k)+S_{\rho}\rho'(k)\right]\equiv\Theta\left(k\right). \tag{3.21}$$

It is clear that (3.17) has no non-trivial solutions if $\Theta(k)>1$ or $\Theta(k)<1$ for all k. The following proposition establishes sufficient conditions for the existence of a unique steady state equilibrium.

Proposition 6 *There exists a unique non-trivial steady state equilibrium if* $k_0 > 0$ *and*

(a) $\lim_{k\to 0} \Theta(k) > 1$,

(b) $\lim_{k\to\infty} f'(k) = 0$, *and*

(c) the right-hand-side of (3.17) is strictly concave.

Proof. It is easy to see from Figure 3-1 that if the right-hand-side of (3.17) has the configuration shown in Figure 3-1, then a unique steady state exists. If condition (c) is violated, then multiple equilibria occur (Figure 3-2). ■

Figure 3-1 Unique Equilibrium

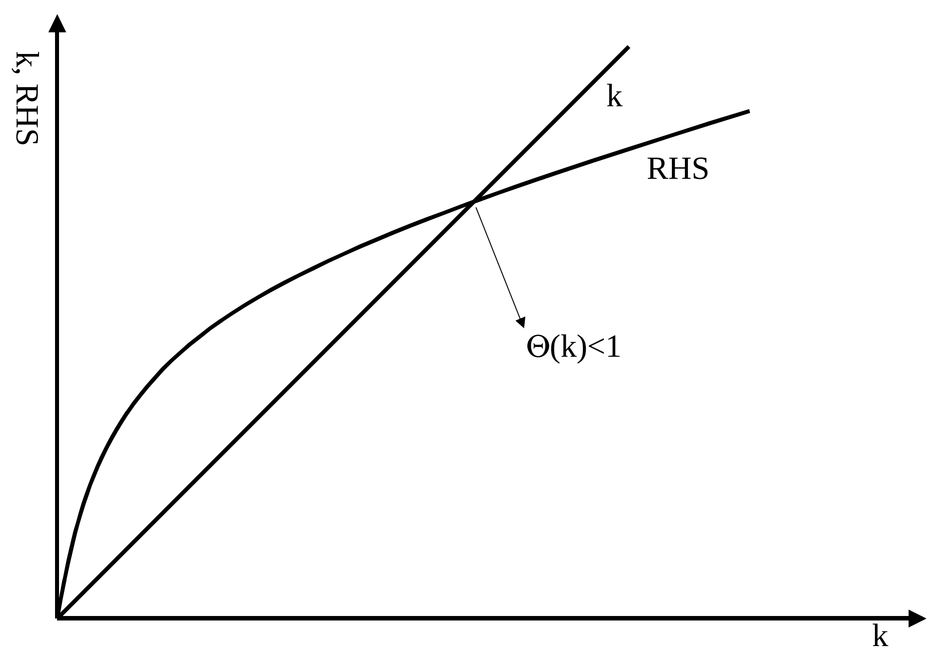
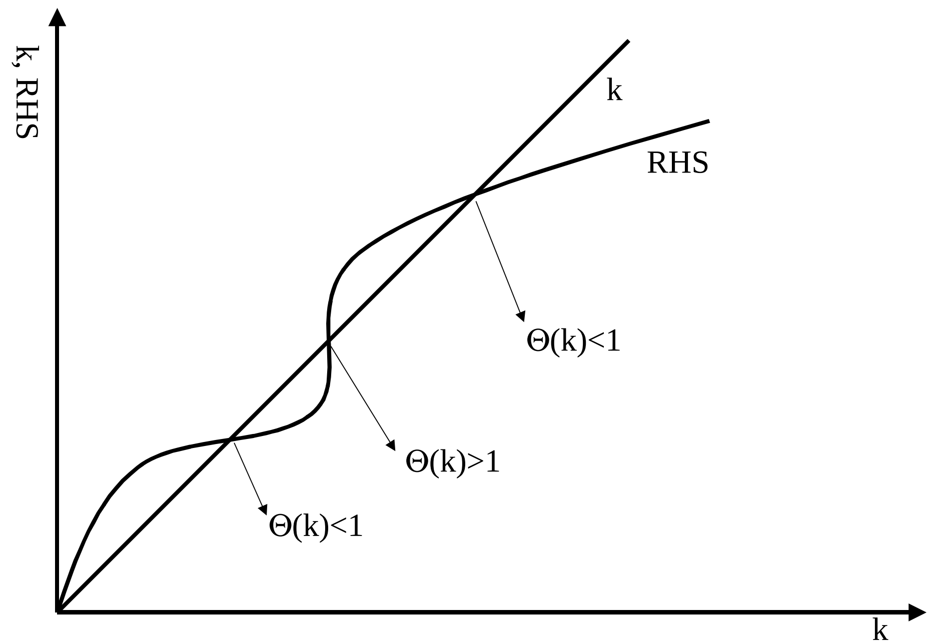


Figure 3-2 Multiple Equilibria



3.3.4 Dynamic Properties of Equilibria

Proposition 7 *Steady state equilibria characterized by* $\Theta(k) < 1$ *are saddle-point stable; those characterized by* $\Theta(k) > 1$ *are sources.*

A proof is provided in the appendix to this chapter. The main results of this chapter are based on comparative statics. A comparative statics exercise is of no use if a steady state is a source. Therefore, the rest of this chapter will focus only on equilibria that are approachable. Thus, throughout, only equilibria with $\Theta(k) < 1$ will be considered.

3.3.5 Total Seigniorage

The government's ability to make use of currency seignorage and bond sales to raise revenue obviously faces certain endogenous restrictions. In particular, there may arise a tension between the inflation tax base and the tax rate when it comes to raising a fixed amount of revenue. Denote by $R = \dfrac{p_t}{p_{t+1}}$, the inverse of the steady state rate of inflation. Then (3.11) may be written as

$$g = H(R, k, \mu) \equiv \lambda k[1 + \mu - AR(k) - \mu\phi f'(k)]. \tag{3.22}$$

The function $H(.)$ and (3.17) summarize all the information pertaining to the inflation-tax Laffer curve for this economy. The following issue then arises: how do we know if a particular steady state is on the "good side" of the Laffer curve? The following lemma addresses exactly this issue.

Lemma 3 *The slope of the total seigniorage Laffer curve is*

$$\frac{dH}{dR} = -\frac{(1-\phi)\, S\, [1 - \Theta\,(k)]}{[1 - \Theta\,(k)] - (1-\phi)\, \frac{\phi}{A} S_\rho \Gamma\,(k)}, \tag{3.23}$$

where

$$\Gamma\,(k) \equiv \mu\rho'\,(k) - \frac{\phi}{1-\phi}\frac{g}{k^2}. \tag{3.24}$$

Proof. The following three equations characterize the Laffer curve.

$$g = \frac{1-\phi}{\phi} k\,[1 + \mu - R - \rho\mu]\,, \tag{3.25}$$

$$\rho = \phi f'\,(k) + (1-\phi)\, R, \text{ and} \tag{3.26}$$

$$Ak = \phi S\,(\omega\,(k)\,, \rho)\,. \tag{3.27}$$

Totally differentiate (3.25)-(3.27) to get

$$\frac{dH}{dR} = \frac{g}{k}\frac{dk}{dR} - \frac{1-\phi}{\phi}k - \frac{1-\phi}{\phi}k\mu\frac{d\rho}{dk}\frac{dk}{dR}, \quad (3.28)$$

$$\frac{d\rho}{dk} = \phi f''(k) + (1-\phi)\frac{dR}{dk}, \text{ and} \quad (3.29)$$

$$\frac{d\rho}{dk} = \frac{A - \phi S_\omega \omega'(k)}{\phi S_\rho}. \quad (3.30)$$

(3.27) and (3.28) give

$$\begin{aligned}\frac{dH}{dR} &= \frac{g}{k}\frac{dk}{dR} - \frac{1-\phi}{\phi}k - \frac{1-\phi}{\phi}k\mu\left[\phi f''(k) + (1-\phi)\frac{dR}{dk}\right]\frac{dk}{dR} \\ &= \left[\frac{g}{k} - (1-\phi)k\mu f''(k)\right]\frac{dk}{dR} - \frac{1-\phi}{\phi}Ak. \quad (3.31)\end{aligned}$$

Equate (3.29) with (3.30) to get

$$\frac{dR}{dk} = \frac{A - \phi S_\omega \omega'(k)}{(1-\phi)\phi S_\rho} - \frac{\phi}{1-\phi}f''(k). \quad (3.32)$$

Combine (3.31) and (3.32) to get dH/dR. We present some of the steps as follows.

$$\begin{aligned}\frac{dH}{dR} &= \frac{\left[\frac{g}{k} - (1-\phi)k\mu f''(k)\right](1-\phi)\phi S_\rho}{A - \phi S_\omega\omega'(k) - \phi f''(k)\phi S_\rho} - \frac{1-\phi}{\phi}Ak \\ &= \frac{\left[\frac{g}{k^2} + f''(k) - Af''(k)\right]k(1-\phi)\phi S_\rho - \frac{1-\phi}{\phi}Ak\left[A - \phi S_\omega\omega'(k) - \phi f''(k)\phi S_\rho\right]}{A - \phi S_\omega\omega'(k) - \phi f''(k)\phi S_\rho} \\ &= \frac{-\left[-\phi S_\rho\rho'(k) + \phi S_\rho\phi f''(k) + A - \phi S_\omega\omega'(k) - \phi f''(k)\phi S_\rho\right]S(1-\phi)}{A - \phi S_\omega\omega'(k) - \phi f''(k)\phi S_\rho} \\ &= \frac{-S(1-\phi)\left[A - \phi S_\rho\rho'(k) - \phi S_\omega\omega'(k)\right]}{A - \phi S_\omega\omega'(k) - \phi f''(k)\phi S_\rho}.\end{aligned}$$

We then establish that

$$\frac{dH}{dR} = -\frac{(1-\phi)S\left[1-\Theta(k)\right]}{\left[1-\Theta(k)\right] + \frac{\phi}{A}S_\rho\left[\rho'(k) - \phi f''(k)\right]}. \quad (3.33)$$

Use (3.19) to write

$$\rho'(k) - \phi f''(k) = -(1-\phi)\left[\mu\rho'(k) - \frac{\phi}{1-\phi}\frac{g}{k^2}\right].$$

The rest is then immediate. ■

This Lemma implies that any approachable steady state is on the right-hand-side ("good side") of the Laffer curve.

3.3.6 Bond Seigniorage

It is helpful to consider the bond-seigniorage Laffer curve. The seignorage from issuing bonds is defined by

$$s_b \equiv b(1-\rho) = \mu\lambda k(1-\rho(k)). \tag{3.34}$$

The slope of the bond-seigniorage Laffer curve is therefore

$$\frac{\partial s_b}{\partial \rho} = \mu\lambda\left[\frac{1-\rho(k)}{\rho'(k)} - k\right] \equiv G(k). \tag{3.35}$$

Lemma 4 *(a) $s_b = 0$ if and only if $k = 0$, or $\rho(k) = 1$.*

(b) Let k_b solve $f''(k)k + f'(k) = 1$. Then $G(k_b) = 0$.

(c) The economy is on the right-hand-side ("good-side") of the bond-seignorage Laffer curve if and only if

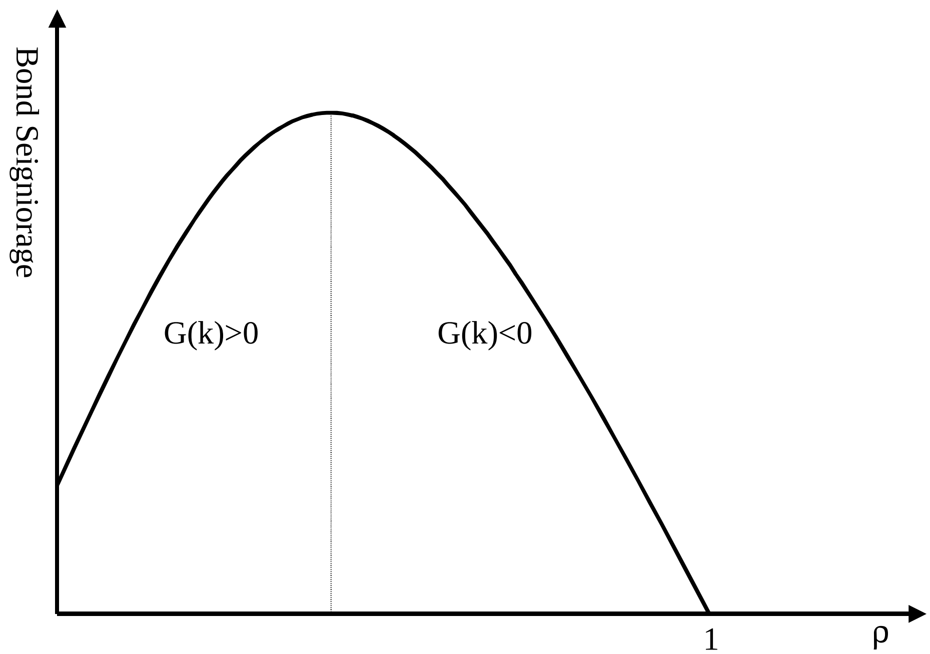
$$\frac{1-\rho(k)}{k\rho'(k)} \leq 1, \tag{3.36}$$

or equivalently,

$$k \leq k_b.$$

The bond-seigniorage Laffer curve has the configuration shown in Figure 3-3. Lemma 4 provides a testable condition regarding whether an economy is on the good side of the bond-seigniorage Laffer curve. If an economy is characterized by $f''(k)\,k + f'(k) \leq 1$, then in such an economy the government cannot raise bond seigniorage by increasing the reliance on bonds.

Figure 3-3 Bond Seigniorage Laffer Curve



If $\rho > 1$ as in Sargent Wallace (1981), then in such an economy, *bond seigniorage is actually negative*. So the government needs to raise currency seigniorage by increasing the steady state inflation rate. Therefore, it is clear that if the rate of return on bonds is greater than the economy's growth rate (which is normalized to unity here), then any further bond reliance will only result in a higher inflation.

Proposition 8 *Suppose* $1 > \rho(k)$ *. If the economy is on the right-hand-side (good side) of the bond-seigniorage Laffer curve, then it is on the good side of the total-seigniorage Laffer curve.*

Proof. From (3.23), the slope of the total-seigniorage Laffer curve is given by

$$\frac{dH}{dR}=-\frac{(1-\phi)\,S\,[1-\Theta\,(k)]}{[1-\Theta\,(k)]-(1-\phi)\,\frac{\phi}{A}S_{\rho}\Gamma\,(k)},$$

where

$$\Gamma\,(k)\equiv\mu\rho'\,(k)-\frac{\phi}{1-\phi}\frac{g}{k^{2}}.$$

If the economy is on the good side of the bond-seigniorage Laffer curve, then

$$\frac{k\rho'\,(k)}{1-\rho\,(k)}<1.$$

So

$$\mu\frac{k\rho'\,(k)}{1-\rho\,(k)}-\frac{1}{1-\rho\,(k)}\frac{\phi}{1-\phi}\frac{g}{k}<\mu-\frac{1}{1-\rho\,(k)}\frac{\phi}{1-\phi}\frac{g}{k}.$$

Consider the right-hand-side. From (3.20), it is straightforward to show that

$$\mu-\frac{1}{1-\rho\,(k)}\frac{\phi}{1-\phi}\frac{g}{k}<0.$$

Therefore,

$$\mu\frac{k\rho'\,(k)}{1-\rho\,(k)}-\frac{1}{1-\rho\,(k)}\frac{\phi}{1-\phi}\frac{g}{k}<0,$$

which can be rewritten as

$$\mu\rho'\,(k)-\frac{\phi}{1-\phi}\frac{g}{k^{2}}<0. \tag{3.37}$$

■

Lemma 5 *Suppose $\rho<1$. If the real interest rate and capital stock are negatively related, then the economy's state is on the right-hand-side of the bond-seigniorage Laffer curve.*

Proof. From (3.37), $\Gamma\,(k)<0$ if $\rho'\,(k)<0$. ■

3.4 Main Results

Following Sargent and Wallace (1981), we now investigate how changes in the bond-money ratio, μ, affect the steady state equilibrium levels of the rate of inflation. Clearly an increase in μ corresponds to a contractionary (and permanent) open market operation. In other words, tight money policies involve increases in μ. The following lemma describes the expression for the effect of an increase in the bond-money ratio on the steady state inflation rate.

Lemma 6 *In a steady state,*

$$\frac{dR}{d\mu} = (1-\rho) - \Gamma(k)\frac{dk}{d\mu}, \tag{3.38}$$

where

$$\frac{dk}{d\mu} = \frac{(1-\phi)\left[\frac{\phi}{A}S_\rho(1-\rho) - k\right]}{A\left[1-\Theta(k)\right]}. \tag{3.39}$$

Proof. The following equations characterize the steady state equilibria.

$$g = m(1-R) + b(1-\rho), \tag{3.40}$$

$$\rho(k) \equiv 1 + \frac{\phi}{1+\mu(1-\phi)}\left[f'(k) - 1 - \frac{g}{k}\right], \text{ and} \tag{3.41}$$

$$k = \frac{\phi}{A}S(\omega(k), \rho(k)). \tag{3.42}$$

Rewrite (3.40) as

$$\begin{aligned} R &= 1 + \frac{b}{m}(1-\rho) - \frac{g}{m} \\ &= 1 + \mu(1-\rho) - \frac{g}{\lambda k}. \end{aligned} \tag{3.43}$$

Totally differentiate (3.43) to obtain

$$\begin{aligned} \frac{dR}{d\mu} &= (1-\rho) - \mu \frac{d\rho}{dk}\frac{dk}{d\mu} + \frac{\phi}{1-\phi}\frac{g}{k^2}\frac{dk}{d\mu} \\ &= (1-\rho) - \left[\mu \rho'(k) - \frac{\phi}{1-\phi}\frac{g}{k^2}\right]\frac{dk}{d\mu}. \end{aligned} \tag{3.44}$$

$dk/d\mu$ can be computed from (3.42). Totally differentiate (3.42) to obtain

$$(1-\phi)\, k d\mu = -\left\{A - \phi S_\omega \omega'(k) - \phi S_\rho \rho'(k)\right\} dk + \phi S_\rho \frac{(1-\phi)(1-\rho)}{1+\mu(1-\phi)} d\mu.$$

The rest of the proof is immediate. ■

Notice that if $dk/d\mu = 0$, then $dR/d\mu = 1-\rho$. This implies that, if a permanent open market operation is irrelevant (i.e., a Modigliani-Miller theorem for government open market operations holds), then the necessary and sufficient condition for the UMA to hold is $\rho > 1$. Therefore, in order to come up with a condition for the UMA that is weaker than the Sargent-Wallace condition, it is necessary that a Modigliani-Miller theorem for government open market operations do *not* hold. Whether open market operations are irrelevant is an empirical question.[26] It is the presence of a reserve requirement that breaks the Modigliani-Miller theorem.

3.4.1 Economy with $\rho > 1$

The analysis will be complex if $dk/d\mu \neq 0$. Consider first the case with $\rho > 1$. This corresponds to the original Sargent-Wallace environment.

Corollary 4 *(Sargent and Wallace, 1981) A tight money policy engineered by a permanent open market operation raises steady state inflation rate if*

---

26 See, for example, Highfield, O'Hara, and Smith (1996).

(a) monetary authority needs to raise revenue (Fiscal Dominance),

(b) the economy is on the right-hand-side ("good side") of the "bond seigniorage" Laffer curve, and

(c) $\rho > 1$.

Proof. From (3.38) and (3.39),

$$\frac{dR}{d\mu} = (1-\rho) - \Gamma(k)\frac{(1-\phi)\left[\frac{\phi}{A}S_\rho(1-\rho) - k\right]}{A\left[1-\Theta(k)\right]}. \tag{3.45}$$

If the economy is on the good side of the bond-seigniorage Laffer curve, then

$$\frac{k\rho'(k)}{1-\rho(k)} < 1.$$

So,

$$\mu\frac{k\rho'(k)}{1-\rho(k)} - \frac{1}{1-\rho(k)}\frac{\phi}{1-\phi}\frac{g}{k} < \mu - \frac{1}{1-\rho(k)}\frac{\phi}{1-\phi}\frac{g}{k}.$$

Consider the right-hand-side. From (3.20), it is straightforward to show that

$$\mu - \frac{1}{1-\rho(k)}\frac{\phi}{1-\phi}\frac{g}{k} < 0.$$

Therefore,

$$\mu\frac{k\rho'(k)}{1-\rho(k)} - \frac{1}{1-\rho(k)}\frac{\phi}{1-\phi}\frac{g}{k} < 0,$$

which can be rewritten as

$$\mu\rho'(k) - \frac{\phi}{1-\phi}\frac{g}{k^2} = \Gamma(k) < 0.$$

Therefore, $\Gamma(k) < 0$ holds on the right-hand-side of the bond-seigniorage Laffer curve. The rest of the proof is immediate. ■

Here the government earns negative revenue from bond-seignorage because interest obligations on outstanding bonds exceed revenue from new bond sales. Suppose the government raises μ through a permanent open market operation. This will reduce bond seigniorage further if the economy is on the on the right-hand-side (or, the good side) of the bond-seigniorage Laffer curve. In this case, the government can make up the revenue shortfall by raising the steady state inflation rate. The Laffer curve consideration is required here because of the real effect of open market operations. A tight money policy reduces capital stock and raises the real interest rate. If the economy is on the left-hand-side (or, the "bad side") of the bond-seigniorage Laffer curve, then an increase in the interest rate raises bond seigniorage. In this case, it might be the case that the bond seigniorage actually *rises* and the UMA proposition does *not* hold even when $\rho > 1$ holds.

It is important to note that raising the steady state inflation rate is not the only possible way in which the government can raise revenue to make up the shortfall. An alternative is tax revenue. In fact, in the real world, it is rather common for a government to use tax revenue to repay its deficits. In this sense, the condition (a) ("Fiscal Dominance") seems strong and it is not very realistic. However, assuming that a government can raise tax rates or tax base as much as it wishes is not realistic either. The reason is because (i) it may be hard to collect tax directly and an inflation tax might be a more efficient way to collect tax revenue in some countries, (ii) the government may not wish to raise the tax rate for political reasons[27], or (iii) tax rates in a country might be at the point where tax revenue

27 Burnside, Eichenbaum, and Rebelo (1998) adopt the assumption of the fiscal dominance. Their claim is that even though agents could believe that the government deficits will be financed by raising taxes or lower expenditure, this may not be credible.

is already maximized (i.e., the economy is at the peak of the tax revenue Laffer curve). Therefore, one should think of this model economy as being in a situation in which the fiscal authority for some reason cannot raise tax revenue and the monetary authority needs to raise revenue. In such an economy, the Sargent-Wallace proposition tells us that it is not advisable to increase the government debt. This is inflationary.

A Numerical Example

Suppose that the production function and the utility function are specified as follows:

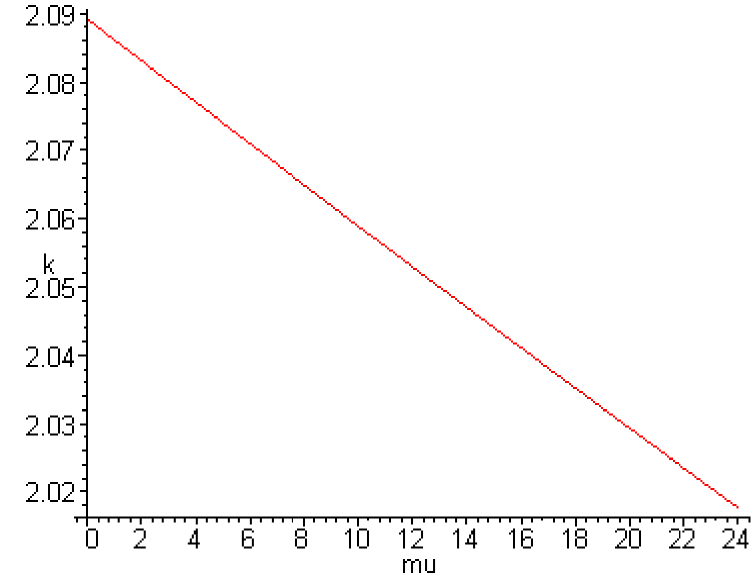
$$f(k) = ak^{\alpha} \tag{3.46}$$

and

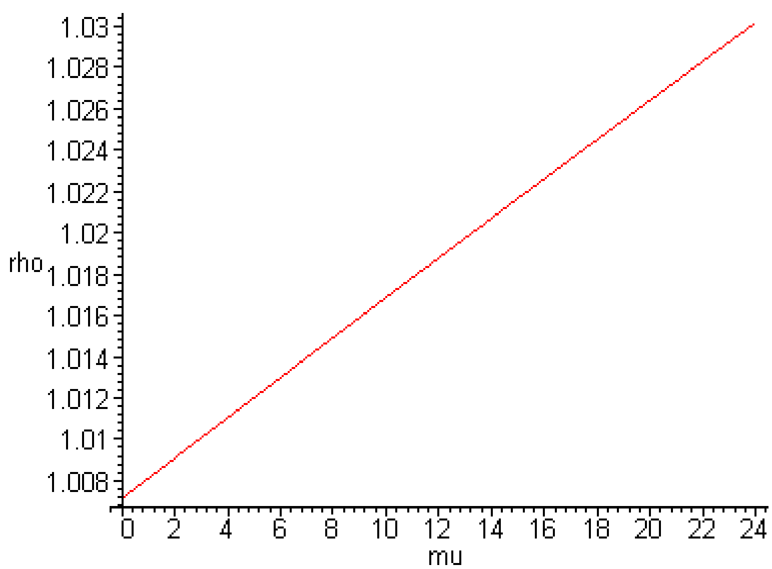
$$U(c_1, c_2) = \frac{c_1^{1-\gamma}}{1-\gamma} + \beta \frac{c_2^{1-\gamma}}{1-\gamma}. \tag{3.47}$$

Here, the parameters of the model economy are the following: $a = 5$, $\alpha = 0.33$, $\phi = 0.9$, $\beta = 0.96$, $\gamma = 0.97$, $g = 0$. Figures 3-4(a)-(d) report the consequence of choosing different values of the bond-money ratio, μ, on the capital stock, real interest rates, the inflation rate, and seignorage revenues respectively. Here the initial steady state real interest rate is greater than one. This example therefore conforms to the setting studied in Sargent and Wallace (1981) in that the initial real rate of interest exceeds the growth rate of the economy, and the economy is on the good side of the currency Laffer curve. From Figure 3-4(a) and 3-4(b), it is clear that tight-money policies reduce the steady state capital stock and raise steady state real interest rates. There are at least two things to note here. First, the effect of tight-money on both the capital stock and the real interest rate is monotonic in this model economy. Second, the figures provide us with an idea of the magnitude of the effect (in the case of the real interest rate, the change is roughly 2% points). Figure 3-4(c) illustrates a nice monotonic and convex relationship between steady state inflation and the bond-money ratio implying not only that tight-money increases the long-run inflation rate but also that the rate of increase in the inflation rate *increases* with monetary policy tightness.

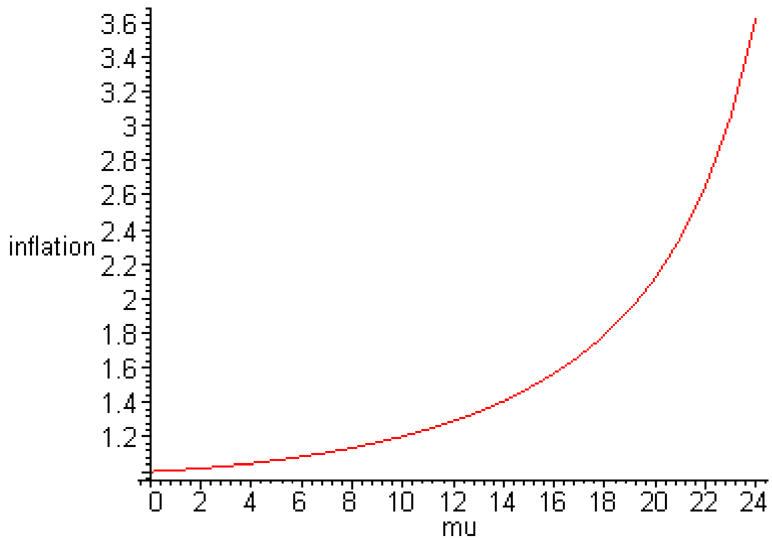
Figure 3-4 Example with $\rho < 1$.
($\alpha = 0.33$, $\phi = 0.999$, $a = 5$, $\beta = 0.96$, $\gamma = 0.97$, $g = 0$)



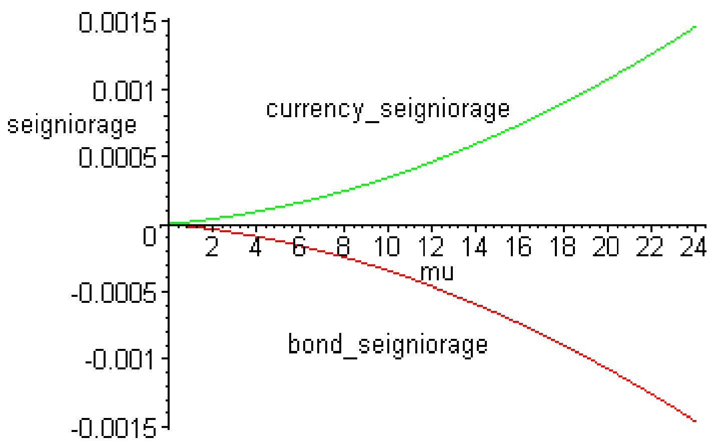
(4a) Effect of change in μ on k



(4b) Effect of change in μ on ρ



(4c) Effect of change in μ in inflation



(4d) Effects of change in μ on seigniorages

3.4.2 Economy with $\rho < 1$

Is $\rho > 1$ necessary for the UMA result? The following proposition provides a set of necessary and sufficient conditions for the UMA and shows that the UMA proposition goes through with $\rho < 1$.

Proposition 9 *A tight money policy engineered by a permanent open market operation raises the steady state inflation rate if and only if*

(a) the monetary authority needs to raise revenue (Fiscal Dominance),

(b) the economy is on the right-hand-side ("good side") of the "bond seigniorage" Laffer curve,

(c) the tight money policy reduces capital stock ($dk/d\mu < 0$), and

(d)

$$\rho > 1 - \Gamma(k)\frac{dk}{d\mu},$$

where

$$1 - \Gamma(k)\frac{dk}{d\mu} < 1.$$

Proof From (3.38), $dR/d\mu < 0$ if and only if

$$(1-\rho) - \Gamma(k)\frac{dk}{d\mu} < 0. \tag{3.48}$$

■

Intuition for the result is as follows. When $1 < \rho$, the government raises a positive revenue from bonds. The question is whether the bond seigniorage is large enough to finance the government's primary deficit, g, without increasing the reliance on currency seigniorage. Suppose that the government raises the bond-money ratio. There are two effects to consider. One is that an increase in the bond-money ratio directly increases the bond seigniorage $b(1-\rho)$. The other is that the tight money reduces the capital stock and this in turn increases the interest rate $\rho(k)$. This increase in the interest rate reduces the bond seigniorage if the economy is on the good side of the bond-seigniorage Laffer curve. Suppose it is. Then, when the latter effect outweighs the former, the bond seigniorage decreases and the government has to increase its reliance on currency seigniorage by raising the inflation rate. Loosely speaking, this central result obtained in this chapter can be regarded as a formal analysis of the example given by Miller and Sargent (1984). Miller and Sargent (1984) graphically showed the possibility that an economy is on the right-hand-side of the bond-seigniorage Laffer curve and used it as a counterexample for Darby (1984), who claimed that the interest rate being less than the growth rate is unrealistic. As Miller and Sargent (1984) suggest, what is important is that an economy cannot increase bond seigniorage by raising the bond-money ratio, rather than a simple comparison between the interest rate and the growth rate of an economy.

From the statement of Proposition 9, it is thus readily apparent that the Sargent and Wallace requirement that the real rate of return on bonds exceed the growth rate of the economy is *sufficient* but is by no means necessary to guarantee "unpleasant monetarist arithmetic". In other words, we have now established that the necessary conditions for unpleasant monetarist arithmetic to hold are weaker than those stated by Sargent and Wallace.[28]

Notice that to weaken the Sargent-Wallace condition $(\rho > 1)$ in this economy, it is *necessary* that the Modigliani-Miller theorem for government open market operations fails. That is, what is central is that a tight money policy reduces the capital stock. Without this

[28] In fact, the necessary conditions we state are even weaker than those stated in Bhattacharya, Guzman, and Smith (1998). There, it was shown that for unpleasant monetarist arithmetic to hold, it is required that there be an asset (here capital) with a rate of return that exceeds unity. Our results indicate that even if $f'(k) < 1$, unpleasant arithmetic is a possibility as long as $f'(k) > \rho$ holds (a "positive equity premium").

link, the UMA result is obtained only if the real rate of return on bonds exceeds the growth rate of an economy. Given that open market operations have real effects, the next qualifier for the UMA is that a higher interest rate *reduces* bond seigniorage. This condition is met on the good side of the bond-seigniorage Laffer curve. Now there arise two effects. One is that a tight money policy increases bond seigniorage. The other is that, through reducing the capital stock, the tight money policy reduces bond seigniorage. The final qualifier is that the latter effect exceeds the former, which is true if $\rho > 1 - \Gamma(k)\, dk/d\mu$. And this condition does *not* require that the real interest rate is greater than unity. But it is true only if $dk/d\mu < 0$. That is, the weakening of the Sargent-Wallace condition demonstrated here requires the Modigliani-Miller theorem for open market operations to break.

Bhattacharya, Guzman, and Smith (1998) uses a model similar to the one presented here. They show that the UMA proposition holds in an economy with $\rho < 1$. The difference between the two models is that they use an endowment economy, while the model presented here is a production economy with a standard (concave) neoclassical production function. The driving force of their result is the “tax base effect.” A higher bond-money ratio through open market operations crowds out deposits from the portfolio of private agents. This reduction in deposits causes a reduction in the volume of reserves held by banks. Consequently, the *inflation tax base* falls. On the good side of the Laffer curve, the central bank is forced to raise the steady state inflation rate.

The “tax base effect” reduces currency seigniorage, forcing the central bank to increase the “tax rate”. Thus, it is only necessary that the economy is on the good side of the total seigniorage Laffer curve, so that an increase in the inflation rate (or, the tax rate) increases seigniorage. On the other hand, our "interest rate effect" reduces bond seigniorage (as well as currency seigniorage). For this effect to be present, it is necessary that the economy is on the good side of the *bond-seigniorage* Laffer curve.

The following result corresponds to "pleasant monetarist arithmetic."

Corollary 5 *(Espinosa-Vega and Russell, 1998) Suppose* $\rho < 1$. *Then, a tight money policy engineered by a permanent open market operation reduces the steady state inflation rate if and only if the economy is on the left-hand-side ("bad side") of the "bond seigniorage" Laffer curve (*$\Gamma(k) \geq 0$*).*

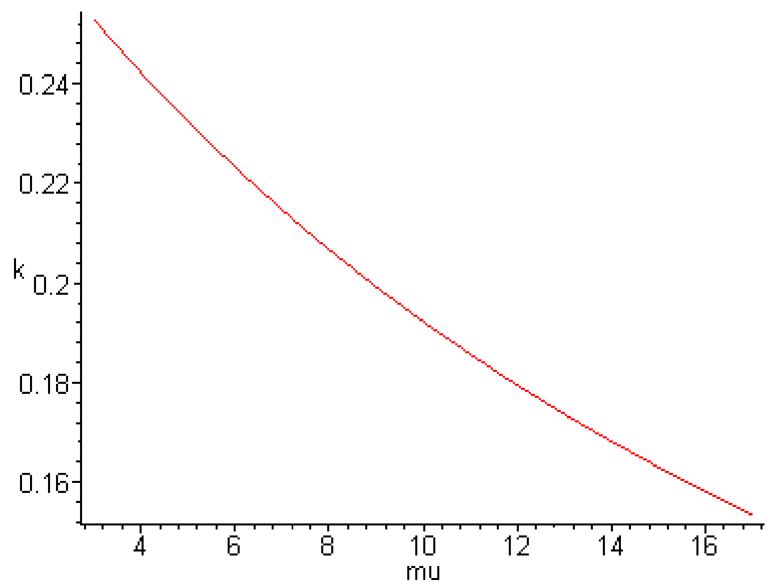
The above corollary demonstrates that a necessary condition for the UMA result under $\rho < 1$ is that the economy is on the good side of the bond seigniorage Laffer curve. Now it seems the question of whether an economy exhibits UMA or not is reduced to asking whether the economy is on the good side of the bond seigniorage Laffer curve or not.

A Numerical Example

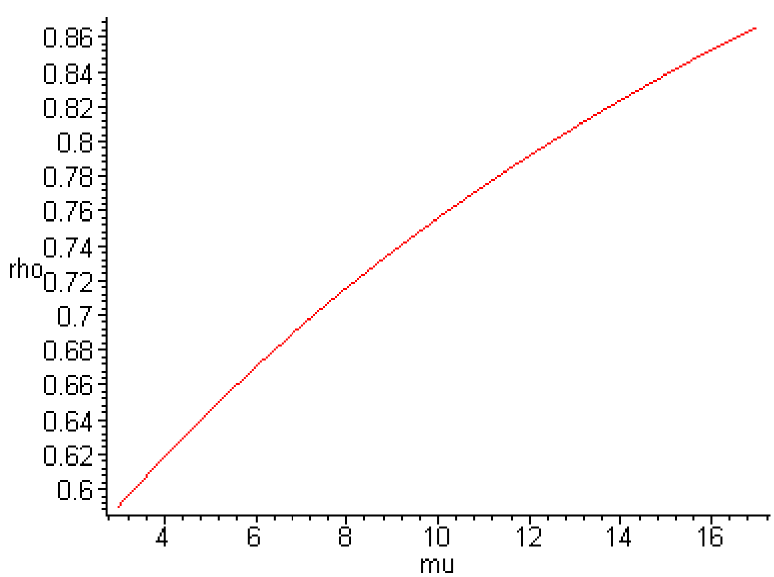
Here, the parameters of the model economy are the following: $a = 1$, $\alpha = 0.2$, $\phi = 0.9$, $\beta = 0.96$, $\gamma = 0.97$, $g = 0.2$. Here the initial steady state real interest rate is *less* than one. As before, from Figure 3-5(a) and 3-5(b), it is apparent that tight-money policies reduce the steady state capital stock and raise steady state real interest rates. Figure 3-5(c) illustrates something interesting: a non-monotonic relationship between inflation and monetary policy tightness. In fact, for the model economy, it is the case that for low values of the bond-money ratio, the steady state rate of inflation actually *falls* (the "pleasant arithmetic" of

Darby (1984)) but for sufficiently high values (here $\mu > 9$), the inflation rate increases at an increasing rate. [29] The figures for bond seignorage and currency seignorage are consistent with this observation.

Figure 3-5 Example with $\rho < 1$
($\alpha = 0.2$, $\phi = 0.96$, $a = 1$, $\beta = 0.96$, $\gamma = 0.97$, $g = 0.02$)

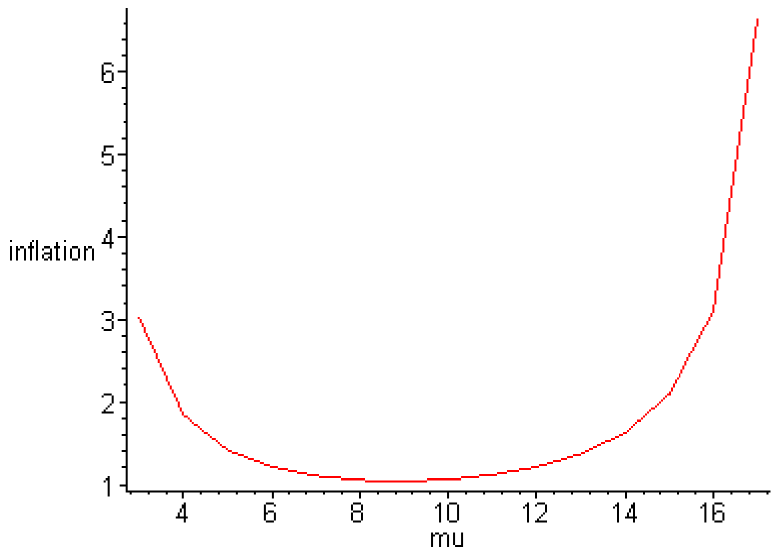


(5a) Effect of change in μ on k

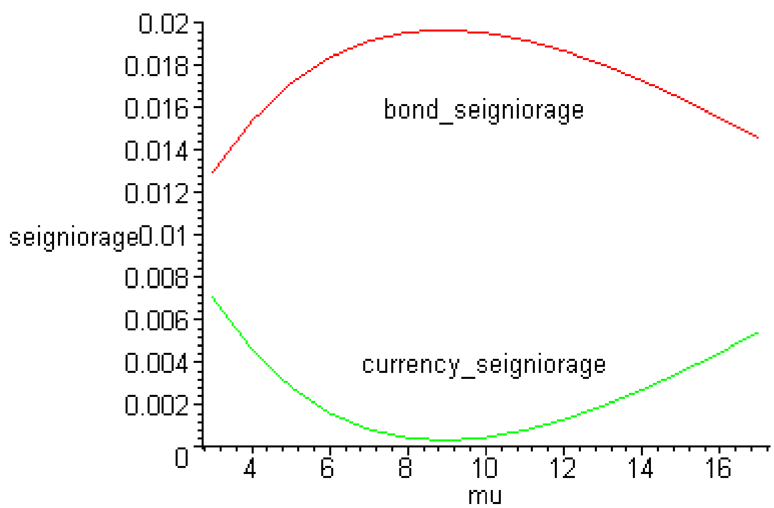


(5b) Effect of change in μ on ρ

[29] For the U.S., μ has steadily increased from around 5 in the 1960s to around 12 in the mid-90's.



(5c) Effect of change in μ on inflation



(5d) Effects of change in μ on seigniorages

3.5 Concluding Remarks

Does creating money to finance a permanent government deficit of fixed size always lead to more inflation than bond financing of the same deficit? Even two decades ago, the answer to this question would have been unequivocally in the affirmative. Following the publication of Sargent and Wallace's classic paper "Some Unpleasant Monetarist Arithmetic" in 1981, the answer has ceased to be so clear-cut. In that paper, Sargent and Wallace produced a dynamic pure-exchange general equilibrium model and used it to generate simple examples where an increased reliance on bond finance (tight money policies) caused a permanent increase in the inflation rate.

This chapter revisits the link between monetary policies and inflation in the spirit of Sargent and Wallace (1981). To that end, it employed the structure laid out in Sargent and Wallace (1981) into a standard overlapping generations model with production and capital accumulation. Under fairly general specifications of technology and preferences, this chapter generalizes the conditions under which tight money through open market operations leads to a higher steady state inflation. It discovers the "interest rate effect," a new channel through which the UMA proposition holds even if the real interest rate is less than the growth rate.

A complete characterization of the UMA proposition leads to an important implication for the long-run monetary policy making. The UMA proposition tells us that raising the bond-money ratio does not necessarily reduce the steady state inflation. In addition, Miller and Sargent (1984), Bhattacharya, Guzman, and Smith (1998) demonstrate that comparing the real interest rate and the growth rate is not enough to know correctly whether a tight money reduces inflation. Then, what should the central bank do to achieve *sustainable* low inflation? The analysis in this chapter reveals that in the long run, the lowest (positive) rate of inflation is achieved when the central bank chooses the bond-to-money ratio so as to maximize bond seigniorage.

In the analysis, we simply assumed that the central bank *has to* raise seigniorage revenue. It is therefore important to incorporate why and how the central bank is forced to raise seigniorage revenue. There are three potential explanations. First, it is hard to collect direct taxes and an inflation tax might be a more efficient way to collect tax revenue in some countries.[30] Second, the government may not wish to raise tax rates. Finally, tax rates in a country might be at the point in which tax revenue is already maximized (i.e., the economy is at the peak of the tax revenue Laffer curve). Potential future work is to model tax revenue and to investigate the interaction between the tax-revenue Laffer curve, the currency Laffer curve, and the bond-seigniorage Laffer curve.

30 Al-Marhubi (2000) found a positive relationship between corruption and inflation.

3.A Proof of Proposition 7

To study the local dynamical properties of this system, it is necessary to linearize it in a neighborhood of any steady state equilibrium (k, ρ) characterized by (3.17) and (3.18). After some algebra, it is easily established that

$$\begin{pmatrix} 1 & \frac{-\phi}{A} S_\rho \\ A-\phi & 0 \end{pmatrix} \begin{pmatrix} \hat{k}_{t+1} \\ \hat{\rho}_{t+1} \end{pmatrix} = \begin{pmatrix} \frac{\phi}{A} S_\omega \omega' & 0 \\ A-\phi+\phi\omega'-\phi\frac{g}{k} & Ak \end{pmatrix} \begin{pmatrix} \hat{k}_t \\ \hat{\rho}_t \end{pmatrix} \tag{3.49}$$

where $\hat{k}_t = k_t - k$ and $\hat{\rho}_t = \rho_t - \rho$. Purely for notational convenience, we denote

$$X \equiv \begin{pmatrix} 1 & \frac{-\phi}{A} S_\rho \\ A-\phi & 0 \end{pmatrix},$$

$$Y \equiv \begin{pmatrix} \frac{\phi}{A} S_\omega \omega' & 0 \\ A-\phi+\phi\omega'-\phi\frac{g}{k} & Ak \end{pmatrix},$$

and

$$z_t \equiv \begin{pmatrix} \hat{k}_t \\ \hat{\rho}_t \end{pmatrix}.$$

Then (3.49) may be rewritten as

$$X z_{t+1} = Y z_t$$

or

$$z_{t+1} = \left(X^{-1} Y\right) z_t = J z_t.$$

Straightforward matrix algebra establishes that J, the Jacobian matrix, is

$$J = \frac{A}{\phi S_\rho (A-\phi)} \begin{pmatrix} \frac{\phi S_\rho}{A}\left\{A-\phi+\phi\left(\omega'-\frac{g}{k}\right)\right\} & \phi k S_\rho \\ (A-\phi)\left(1-\frac{\phi}{A} S_\omega \omega'\right)+\phi\left(\omega'-\frac{g}{k}\right) & Ak \end{pmatrix}.$$

Let $D(k)$ denote the determinant of J as a function of the steady state capital stock and $T(k)$ denote its trace. Then it is easily checked that

$$D(k) = \frac{A\phi k S_\omega \omega'}{\phi S_\rho (A-\phi)} > 0 \tag{3.50}$$

and

$$T(k)=\left[1-\frac{\phi S_\rho A k\rho'-A^2k}{\phi S_\rho(A-\phi)}\right]. \tag{3.51}$$

It is now possible to describe the stability properties of the various steady states.[31]

Let us first prove that steady state equilibria characterized by $\Theta\left(k\right)<1$ are saddle-point stable. Simple calculation gives

$$1-T\left(k\right)+D\left(k\right)=-\frac{AS\left[1-\Theta\left(k\right)\right]}{\left(A-\phi\right)S_\rho}, \tag{3.52}$$

and

$$1+T\left(k\right)+D\left(k\right)=\frac{2AS_\rho\left[1-\frac{\phi}{A}-\frac{\phi}{A}S\rho'\left(k\right)\right]+AS\left[1+\Theta\left(k\right)\right]}{\left(A-\phi\right)S_\rho}. \tag{3.53}$$

To prove $1+T\left(k\right)+D\left(k\right)>0$, it is sufficient to show that $1-\frac{\phi}{A}-\frac{\phi}{A}S\rho'\left(k\right)$ is positive. Suppose it is negative. Then we have

$$1-\frac{\phi}{A}-\frac{\phi}{A}k\left[f''\left(k\right)+\frac{g}{k^2}\right]<0.$$

This can be written as

$$\frac{\phi kf'\left(k\right)+\left(1-\phi\right)\left(1+\mu\right)k-\phi g}{Ak}<\frac{\phi k\left[f'\left(k\right)+kf''\left(k\right)\right]}{Ak}.$$

Since the left hand side of this inequality is nothing but ρ, we can write

$$\rho<\frac{\phi k\left[f'\left(k\right)+kf''\left(k\right)\right]}{Ak}.$$

Finally, using (3.7), we get $\mu\phi\left(1-\phi\right)f'\left(k\right)+\left(1-\phi\right)AR-\phi kf''\left(k\right)<0$, the desired contradiction.

31 See Azariadis (1993; Chapter 6) for a summary of the stability results for discrete-time linear dynamical systems.

Let us now prove that steady state equilibria characterized by $\Theta(k) > 1$ are sources. From (3.52), we can immediately verify that $1 - T(k) + D(k) > 0$. Since we have proved that $1 - \frac{\phi}{A} - \frac{\phi}{A} S\rho'(k)$ is positive, it is easy to verify that $1 + T(k) + D(k) > 0$.

Chapter 4
Government Employment Programs, Unemployment, and Capital Accumulation

4.1 Introduction

4.1.1 Public Sector Employment: Too Big to Ignore

In most of the standard growth models, public sector employment is largely ignored. Barro (1991), for instance, introduces government expenditure into an endogenous growth theory to study the impact of public goods production on the growth rate of an economy by assuming that one unit of government expenditure is costlessly transformed into one unit of public good. A natural question arises: Are no labor inputs required to produce public goods and services? More generally, how important is the government in understanding labor market?

According to Heller and Tait (1983), the share of public sector employment in total employment averaged 44% in 23 developing countries and 24% in 14 industrialized countries (Gelb et. al., 1991, p.1186). Extreme examples are India (72%), Ghana (74%), Tanzania (78%), and Zambia (81%) (Gelb et el., 1991, p.1186). A notable example is India. Bhagwati (1993, pp.63-64) reports:

> ...the overwhelming presence of the public sector in India must be spelled out to see why the matter of its functioning is of great importance to Indian productivity and economic performance. Thus, the 244 economic enterprises of the central government alone, excluding the railways and the utilities, employed as many as 2.3 million

workers in 1990. In manufacturing, if the small ‘unorganized’ sector is excluded, their employment was 40 per cent of that provided by the private sector firms. In fact, the public sector enterprises in manufacturing, mining, construction, transport and communications, banking and insurance, when state-level enterprises are counted in, provided nearly 70 per cent of the 26 million jobs in the large-scale ‘organized’ sector in 1989.

4.1.2 Needed: A Theory of Government Employment

An analytical framework is a vital part of the process of assessing government employment. In contrast to other well-established fields of macroeconomics, studies of public sector employment suffer from the absence of an analytical framework. Schiavo-Campo et. al. (1997), too, admit the need for an analytical framework on which policy analysis can be based. Although public employment programs have been studied in the labor economics literature at a micro level, there is very little work investigating their macroeconomic impact. The (only) existing macroeconomic literature dates back to the studies of “pump-priming” by Kahn (1931), Clark (1935) and Keynes (1936). Their advocacy of such employment programs was based on an “multiplier” view of government investment. The principal limitations of this line of inquiry are the following. First, in the old-style Keynesian models, unemployment is purely a consequence of an *ad hoc* assumption of rigid wages. As such, these models are ill-suited to study the effects of employment generation policies. Second, these models are not dynamic and the equilibrium relationships in them are not derived from primitives. The value-added of this chapter is that it tackles these *lacunae* within the context of a dynamic optimizing framework.

4.1.3 Defining Government Employment: A Problem

Before constructing a model of government employment, the term "government employment" has to be defined. Unfortunately, it has to be pointed out that there are serious measurement and definition problems. Schiavo-Campo *et. al.* (1997a, p4) put it nicely:

> In the first place, statistics of any reliability simply don't exist in many countries. When reasonable data are available, employment comparisons are complicated by the fact that some countries include teachers and/or health workers in the civil service, others don't; some countries include contractual and seasonal (sometimes even daily) workers in government employment, others don't; local government employment may or may not include employees paid out of the central government budget but are not listed among central personnel; paramilitary personnel (*gendarmes*, *carabinieri*, etc.) may be included in civilian personnel because of their public order function, or in the armed forces because of their military status; employees of legislative bodies are sometimes included in government personnel, etc.

For the purpose of *macroeconomic policy evaluations*, it is sufficient to consider only two types of government employment: government employment as a by-product of public goods production, and government as an employer of last resort.

In the former case, the objective of the government has to be improvement of social welfare through providing (productive, or welfare-improving) public goods. Since labor inputs are required to produce a desired level of public goods, the government needs to hire workers. An important task is to ask the optimal level of public goods in the spirit of Barro (1991). Because of the presence of the public sector labor market, the interaction between the government's wage policy and optimal public goods provision would add a new insight into the conventional analysis.

In the latter case, the provision of government employment is itself the primary purpose. Thus, the government does *not* have to provide public goods to the economy. In this view, government employment is important for two reasons. One is because it is a form of fiscal policy. As the text-book macroeconomic theory suggests, government spending may increase aggregate demand. In a sense, this is very “Keynesian.” Another is because the government is providing a form of “income maintenance programs” The question is whether such government intervention in the labor market can really improve the labor market outcome. The answer to the question depends crucially on how the labor market is modeled. If frictionless model is assumed, for example, then such government employment will result only in a partial re-allocation of the labor force from the private to the public sector. If, on the other hand, some market imperfections are assumed, then it is easy to imagine that there is room for government interventions.

As Schiavo-Campo et al (1997) argue, it is often the case that government employment has multiple purposes. Because of that, it is quite hard to distinguish the “productive” part and the “unproductive” part of government employment. Thus in this chapter we consider the two cases separately.

4.1.4 An OG Model with Adverse Selection

The framework employed here is a standard overlapping generations model with production along the lines of Diamond (1965) with one exception; agents are heterogeneous in terms of their intrinsic productive abilities, and this is private information. Private sector firms use capital and labor to produce a single good via a standard neoclassical production technology. They use equilibrium unemployment as a sorting device offering a menu of wages and unemployment probabilities that entice only the high ability people to seek employment with them. Thus, the very presence of judiciously chosen unemployment rates keeps the low ability people (and some high ability people) out of the private sector. This is where the government steps in and sets up a publicly-funded employment program which indiscriminately employs a given fraction of those unemployed in the private sector and pays them a real wage which is a fraction of the current private sector wage. The wage bill is funded by a lump-sum tax on all agents. The public sector employees produce a "government good" which is either completely useless to private agents or is the source of a positive externality.

In this setting, it is apparent that the very involvement of the government affects the information friction in the labor market in that it increases the payoff to the low and high ability people from not seeking private sector employment. Private sector firms react by altering the vector of unemployment probabilities and wage rates they offer which in turn has important effects on capital accumulation.

Multiple long-run equilibria are easily possible here. The existing level of government involvement matters for capital accumulation in a precise and crucial sense. Indeed, below (above) a critical level, further increases in the volume of the public employment programs improves (lowers) long-run real activity for countries stuck at a low-activity steady state. If there are two steady state equilibria, then the high real-activity steady state is dynamically stable. It is possible for the low activity steady state to be stable too. In this case, countries stuck at the stable low activity steady state may be thought of as being caught in a development trap. Again, an increase in the volume of these employment programs can get the economy out of this trap. In other words, the paper provides a dynamic justification for the pursuance of Keynesian pump-priming policies in less developed countries. However, at the high-activity steady state, the provision of public employment is harmful.

The model is then extended in two directions. First, the competing policy of unemployment insurance is considered. To be eligible for unemployment insurance, an agent has to seek private sector employment and then not find any. In contrast, in the public employment programs considered above, eligibility is not an issue: anyone can potentially get employment in the public sector without even *trying* to obtain private sector employment. This seemingly small difference has dramatic effects on the results. In general, unemployment insurance improves (reduces) long-run real activity for countries at the high (low) real-activity steady state. Such a result (alongside the earlier results) therefore provides a partial explanation as to why less-developed economies shy away from unemployment insurance programs and yet adopt public employment programs with readiness. Second, the model is extended to include a government that employs people in the public sector to produce a good that enters into the production function of private agents as a beneficial externality. In this latter case, numerical computations reveal the surprising possibility that an increase in the volume of the government employment program may actually *increase* the unemployment rate.

The rest of the paper is organized as follows. Section 4.2 lays out the model environment and Section 4.3 looks at trade in labor markets. The general equilibrium is characterized in Section 4.4. Section 4.5 presents policy analyses and Section 4.6 discusses an economy with unemployment insurance. The case with productive government is considered in Section 4.7. Section 4.8 concludes.

4.2 Environment

The economy is populated by an infinite sequence of two period lived overlapping generations, plus an initial old generation.[32] At any date $t > 0$, a generation of unit measure is born. Each such generation consists of two types of agents. A fixed fraction $\lambda \in (0, 1)$ of the population is type L, or "unskilled worker," and the remainder, $1 - \lambda$ is of type H, or "skilled worker." These types reflect inherent productivities of the workers which are described more fully below. All agents know exactly the distribution of types in the population, but an individual agent's type remains strictly as private information to him.

All agents are endowed with one unit of labor only when young. In addition, the initial generation is endowed with an aggregate stock of $K_0 > 0$ units of capital. No young agent has any endowment of capital or goods at any other date.

All agents are risk-neutral. In addition, they care only about their old-age consumption. Consequently, all first-period income will be saved.

In each period, there is a single perishable good in the economy. The final good may be consumed (by old agents), or converted into productive capital. This good may be produced in one of three "sectors."[33] The first is a "formal" private sector, in which production is organized by firms (old capitalists). Here, firms have access to a constant returns to scale production function $F(.)$ which uses capital, K, and effective labor, $(1 - \epsilon)N$, as inputs to produce output $F(K, (1 - \epsilon)N)$, where ϵ is the fraction of employees that

[32] The setup follows Bencivenga and Smith (1997), and more specifically, Betts and Bhattacharya (1998).

[33] For lack of a better terminology, the word "sectors" is used here. The final products from each of these sectors is exactly identical and are perceived to be so by all agents in the goods market.

are L types, and N is the total number of employees. L types have zero productivity in the private sector. As such, private sector employers would like to employ only H types.

Let $k_t \equiv \frac{K_t}{(1-\epsilon)N_t}$ denote the capital to effective-labor unit ratio and let $f(k_t) \equiv F(\frac{K_t}{(1-\epsilon)N_t}, 1)$, denote the associated intensive production function where $f(.)$ satisfies $f(0) = 0$, $f' > 0 > f''$, and standard Inada conditions. At several points in the analysis below, a specific Cobb-Douglas form is used. This is,

$$f(k) = Ak^{\alpha} \tag{4.1}$$

where $A > 0$ and $\alpha \in (0, 1)$.

The second sector in which the single final good can be produced is an "informal," or household production sector. Here, an individual young agent can produce the good at home with a simple linear technology that uses only labor an input. There are no information frictions here. Let $\beta_i \geq 0$ be a parameter that captures the productivity of a type i young agent in household production. Then, a young type i agent who supplies $n_t^i \leq 1$ units of labor in household production at date t can produce $\beta_i n_t^i$ units of the final good. Note that household production is limited by the unit labor endowment and is never organized at higher scales of production. In addition,

Assumption 1

$$\beta_H > \beta_L > 0. \tag{A.1}$$

That is, even in the household production sector, the labor productivity of L types is lower than that of the H types.[34] Additionally, household production is relatively inefficient and hence we focus only on equilibria that satisfy

Assumption 2

$$F_2(k,1) > \beta_H. \tag{A.2}$$

Finally, the final good may also be produced in the government, or public sector. Once produced, it gets costlessly transformed into a government good which is completely useless to private agents. As in the informal sector, no capital is used here. For analytical simplicity, we assume a linear production technology: one unit of labor employed in the public sector at t returns one unit of the government good at t.

The capital production technology is the same as in Diamond (1965); all savings today get converted into productive capital tomorrow. The depreciation rate of the capital stock within a period is assumed to be 100%.

4.3 Labor Market Equilibrium

4.3.1 Preliminaries

The activities of young agents in the labor market is as follows. Let ω_t denote the real wage rate paid in the private sector. Each young agent faces the following set of options. Imagine that a period of time is split into two stages. In the first stage, a type $i = L, H$ agent may

34 This assumption, however, is *sufficient* for the existence of a separating Nash equilibrium contract and is *not* necessary.

choose to seek employment in the private sector; in that case, he incurs an utility/search cost s_i regardless of whether he is successful or not in this endeavour. If he finds employment in the private sector, he will earn ω_t. If he fails, then he proceeds to the second stage. If however he chooses *not* to seek private sector employment, then he will proceed to the second stage without incurring the search cost. At the end of the first stage, there will be agents without a job. At the second stage, these agents will seek public sector employment for sure (i.e., it is assumed that seeking public sector employment is always preferred to "staying at home"). The public sector employment program will employ only a fraction of these people; the rest will simply produce at home.

4.3.2 The Government

Private sector firms wish to hire from among the H types and offer contracts that keep the L types away. The government is assumed to be either not endowed with any screening technology or to choose not to make use of one. Consequently, it offers the same wage to all types. Let $\overline{\omega}_t$ denote the real wage rate in the public sector. It will be convenient to assume that the wage rate in the public sector is set such that $\overline{\omega}_t = \theta\omega_t$.[35] Note that the public sector wage rate is determined exogenously. This is because the government is not modeled as a profit-maximizer.

At the wage rate $\overline{\omega}_t$, the government chooses how many workers it wishes to employ in the public sector.[36] Let ϕ_t be the probability that a worker who is not employed in the

35 See Table 2 in Chapter 1 for public to private wage ratio across regions. Also see Ehrenberg and Schwarz (1986).

36 This choice is arbitrary and, in particular, is not motivated by any explicit welfare considerations.

private sector is employed in the public sector. u_t denotes the private sector unemployment rate (defined as the fraction of workers who seek for a private sector employment and fails to find one) and L_t denotes the total number of workers employed in the public sector. Then the government's conduct of its public employment program is entirely described by the simple policy rule

$$\phi_t\left[\lambda + (1-\lambda)\,u_t\right] = L_t. \tag{4.2}$$

The parameter ϕ_t therefore captures the volume of the government's employment programs. Throughout, we shall impose $\phi_t = \phi \in [0,1]$. That is, the probability of being employed in the public sector is constant. As an aside, note that since u_t will be time-varying, a policy that keeps ϕ constant may be thought of as being "countercyclical."[37] Also, note that the number of agents who are not employed either in the private or the public sector is $U_t \equiv (1-\phi)\left[\lambda + (1-\lambda)\,u_t\right]$, which measures the unemployment after the provision of government employment. Since U_t is a linear transformation of $u_t,$ the law of motion for U_t is determined once u_t is known.

Let us now turn to the issue of funding of such employment programs. To keep matters simple, assume that the government finances the employment programs by imposing a lump-sum tax, τ, on *all* agents. Then the government budget constraint is

$$\phi\left[\lambda + (1-\lambda)\,u_t\right]\overline{\omega}_t = \tau_t. \tag{4.3}$$

[37] More precisely, public employment is high when the private sector unemployment rate is high. The term 'countercyclical' is being used casually here. In fact, it turns out that a policy of keeping ϕ fixed implies that the government *raises* the volume of public employment when the capital-labor ratio is relatively *high*.

4.3.3 The disutility/search costs

If an agent of type i were to seek private sector employment, then he first incurs a real disutility/search cost of s_i units of the final good. We impose the following:

Assumption 3

$$s_L > s_H. \tag{A.3}$$

In order to ensure that all agents prefer seeking public sector employment to producing at home, let us also impose the following:

Assumption 4

$$\overline{\omega}_t > \beta_{H.} \tag{A.4}$$

Assumptions (A.1)-(A.4) will be maintained henceforth. Now, under our assumptions so far, it is the case that only H types will be productive in the private sector. Thus, the strategy here is to look for an equilibrium contract which private sector firms can offer that will attract all H types to seek private employment and dissuade all L types. To presage, in equilibrium no L type agent will seek a private sector job. The total private sector labor force is therefore $N_t = (1-\lambda)(1-u_t)$ and so the capital-labor ratio satisfies

$$k_t \equiv \frac{K_t}{N_t} = \frac{K_t}{(1-\lambda)(1-u_t)}. \tag{4.4}$$

4.3.4 Self-Selection

Recall that the intrinsic labor productivities of agents are private information. As such, any agent *may* misrepresent his type. In this setting, if firms are to recruit only the H types, they must offer employment contracts which will be accepted by H types and rejected by L types. Such contracts will maximize the lifetime expected utility of a type-H agent subject to the restriction that no type-L agent finds it in his best interest to misrepresent his type in order to obtain employment at type-H contractual terms.

A type H agent, if he seeks private sector employment, will find a job in the private sector with probability $(1-u_t)$ and will earn ω_t in that case. As described earlier, he must incur a real cost of s_H. If he fails to find a private sector job, an event which occurs with probability u_t, then he will seek employment in the public sector. If he successfully finds a job in the public sector, an event which happens with probability ϕ, then he earns the wage $\overline{\omega}$. If not, then he produces β_H at home. In any event, given that he cares only about old period consumption, he saves all of his young period income. All H types prefer seeking private sector employment to not seeking it only if the expected after-tax income from the former exceeds the latter; that is, if

$$\begin{aligned} & (1-u_t)(\omega_t-\tau_t-s_H)+u_t\left[\phi\left(\overline{\omega}_t-\tau_t-s_H\right)+(1-\phi)\left(\beta_H-\tau_t-s_H\right)\right] \\ > \; & \phi\left(\overline{\omega}_t-\tau_t\right)+(1-\phi)\left(\beta_H-\tau_t\right) \end{aligned}$$

holds, or equivalently, if

$$\omega_t > \phi\overline{\omega}_t+(1-\phi)\,\beta_H+\frac{s_H}{1-u_t}>0. \tag{4.5}$$

Similarly, if firms are to dissuade a type L agent from seeking a private sector job, employment contracts must satisfy

$$\omega_t \leq \phi\overline{\omega}_t+(1-\phi)\,\beta_L+\frac{s_L}{1-u_t}. \tag{4.6}$$

In order to rule out the possibility that an agent may be bankrupted in some state of the world by the lump-sum tax, we shall impose the following:

Assumption 5

$$\omega_t - s_i > \overline{\omega}_t - s_i > \tau_t \text{ and } \beta_i > \tau_t. \tag{A.5}$$

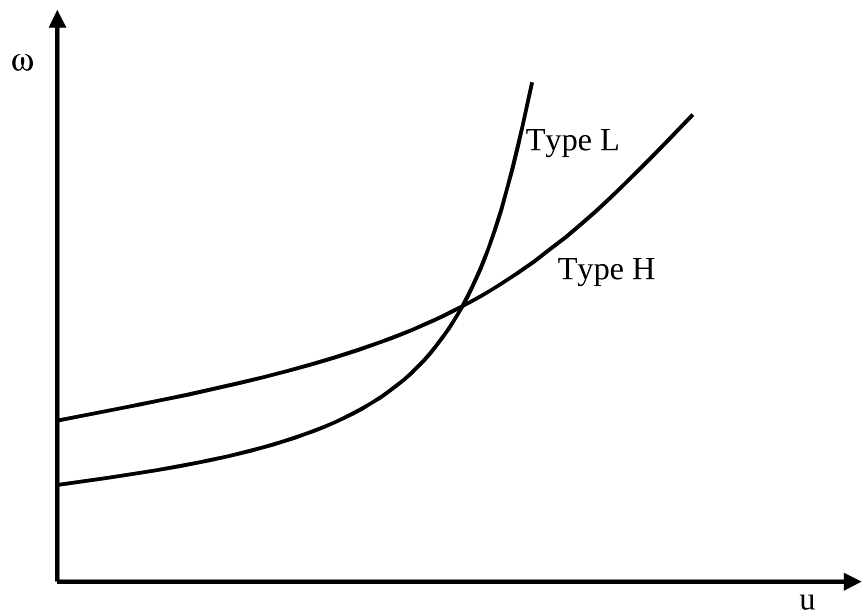
As is well known from Rothschild and Stiglitz (1976), a single-crossing property on preferences is required to ensure the existence of an equilibrium in which agents self-select. The necessary conditions for the single-crossing of the preference are (A.1) and

Assumption 6

$$\phi < \overline{\phi} \equiv 1 - \frac{s_L - s_H}{\beta_H - \beta_L}. \tag{A.6}$$

It is routine to check that under (A.1)-(A.6), there is a single-crossing on the (u_t, ω_t) space. (See Figure 4-1.) The single-crossing in this context means that to obtain the same amount of additional wage, a type-H worker is willing to accept a larger increase in the unemployment rate than a type-L does.

Figure 4-1 Indifference Curves



4.3.5 Contracts

The equilibrium concept employed here is Rothschild and Stiglitz's (1976) notion of competitive screening. Firms are Nash competitors in labor market. Each firm announces labor contracts which specify a wage rate ω_t and the probability of employment $1-u_t$, taking the contracts offered by other firms and all the policy parameters as given. Firms also choose how much capital to use.

Any contract must satisfy (4.5) and (4.6). Also, if a firm hires a total of $N_t = N_t^L + N_t^H \equiv \epsilon_t N_t + (1-\epsilon_t) N_t$ units of labor and rents K_t units of capital at the rental rate r_t, then the maximized profit, Π, must satisfy

$$\Pi \equiv \max_{K_t, N_t} \left[F\left(K_t, (1-\epsilon_t) N_t\right) - \omega_t N_t - r_t K_t \right] \geq 0. \tag{4.7}$$

As in Rothschild and Stiglitz (1976), or more specifically, Bencivenga and Smith (1997), any Nash equilibrium contract announcement must maximize the expected utility of type H agents, subject to the participation constraint for the type L agents (4.6) and the non-negative profit constraint (4.7), with $u_t \in (0,1)\,\forall t$ and r_t given. Under assumptions (A.1)-(A.6), a contract that induces a separating Nash equilibrium satisfies (4.5), (4.6),[38] $\Pi = 0$, $u_t \in (0,1)$, $\omega_t > 0$ and $K_t > 0 \forall t$. Equation (4.7) as an equality therefore specifies the wage which will be offered in a Nash equilibrium by all firms. At this wage anyone who gets a private sector job will be from among the H types; *i.e.* $\epsilon_t = 0$ will hold. The details are spelt out below.

Proposition 10 *Under (A.1)-(A.6), a separating Nash equilibrium is a menu of contracts that satisfies* $\Pi = 0$, $0 < u_t < 0$, $\omega_t > 0$ *and* $K_t > 0$. *In addition, under the specification (4.1), the following relationships hold:*

$$r = f'(k_t) = \alpha A k_t^{\alpha-1} \tag{4.8}$$

$$\omega_t = f(k_t) - k_t f'(k_t) \equiv \omega(k_t) = (1-\alpha) A k_t^{\alpha} \tag{4.9}$$

$$u_t = 1 - \frac{s_L}{\omega_t - [\phi\overline{\omega}_t + (1-\phi)\beta_L]}. \tag{4.10}$$

Under the specification $\overline{\omega}_t = \theta\omega_t$, (4.5) and (4.10) imply

$$\omega(k_t) \geq \frac{(1-\phi)(s_L\beta_H - s_H\beta_L)}{(1-\phi\theta)(s_L - s_H)} > 0. \tag{4.11}$$

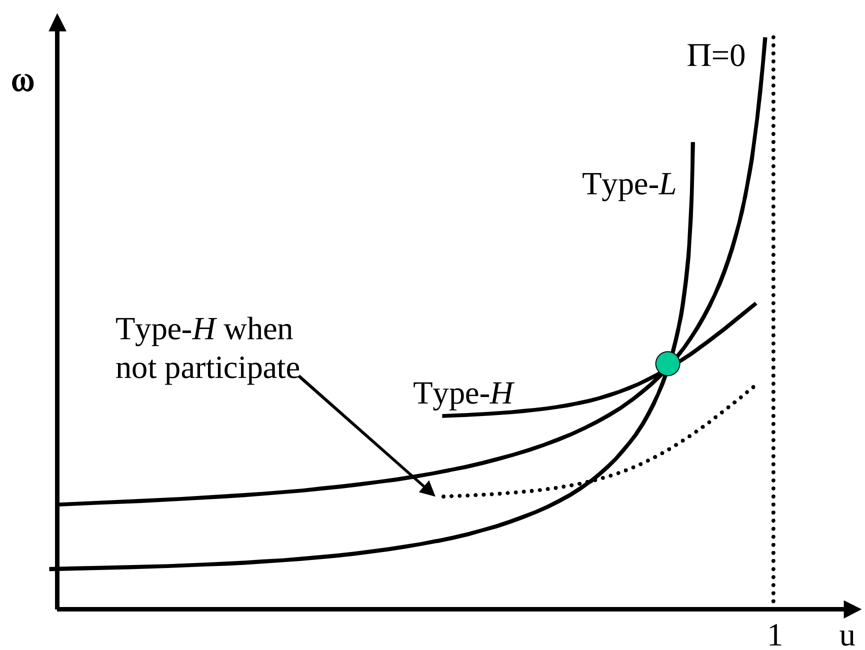
[38] When (4.6) holds with equality, all type L agents are *indifferent* between seeking private employment and not seeking it. In order to completely dissuade the type L agents, it is necessary to specify the Nash conjecture such that whenever an agent is indifferent between seeking private employment and not seeking it, he will choose not to seek it with probability one.

In addition, $u_t \in (0,1)$ requires that (from (4.10))

$$\omega(k_t) > \frac{s_L + (1-\phi)\beta_L}{(1-\phi\theta)}. \tag{4.12}$$

The separating Nash equilibrium contract is shown in Figure 4-2. Note that the firms' zero-profit locus is convex. Under the equilibrium contract, the firms earn zero profit, a type-L worker is indifferent between participation and non-participation and a type-H strictly prefers participation.

Figure 4-2 Equilibrium Contract



4.3.6 Existence of a Nash Equilibrium

As is well known, a Nash equilibrium does not exist if there is a pooling contract that attracts all H and L types and yet brings in non-negative profits to the firm. This subsection writes down conditions that rule out such possibilities.

Let $(\hat{\omega}_t, \hat{u}_t)$ denote a potential pooling contract and $\hat{K}_t$ denote the capital stock employed by the firm that offers the pooling contract. Obviously, $\hat{K}_t \leq K_t$ must hold since a firm cannot employ capital more than the aggregate capital stock available at time t. Let $x \equiv \phi\overline{\omega}_t + (1-\phi)\beta_H$. Then type-$H$ agents prefer the pooling contract if

$$(1-\hat{u}_t)\,\hat{\omega}_t + \hat{u}_t x > (1-u_t)\,\omega_t + u_t x,$$

or, equivalently,

$$(1-\hat{u}_t)\,(\hat{\omega}_t - x) > (1-u_t)\,(\omega_t - x)\,. \tag{4.13}$$

The following lemma (a proof of which appears in Appendix B) sketches the conditions required to sustain the separating Nash contract.

Lemma 7

(a) If $K_t > \hat{K}_t$, then there is no pooling contract that satisfies (4.13).

(b) If $K_t = \hat{K}_t$ and

$$(1-\lambda)\,F_2\,(k_t\,(1-\lambda)\,(1-u_t)\,,1) \geq x,$$

then there does not exist a pooling contract that satisfies (4.13) if

$$\begin{aligned} & F\,(k_t\,(1-\lambda)\,(1-u_t)\,,(1-\lambda)) - f'\,(k_t)\,k_t\,(1-\lambda)\,(1-u_t) \\ \leq\ & (1-u_t)\,\omega_t + u_t x. \end{aligned} \tag{4.14}$$

(c) If $K_t = \hat{K}_t$ and

$$(1-\lambda)\,F_2\,(k_t\,(1-\lambda)\,(1-u_t)\,,1) < x$$

then there does not exist a pooling contract that satisfies (4.13) if

$$\begin{aligned} & k_t\left(1-\lambda\right)\left\{f\left(\omega^{-1}\left(\frac{x}{1-\lambda}\right)\right)-f'\left(k_t\right)\omega^{-1}\left(\frac{x}{1-\lambda}\right)\right\} \\ \leq\ & \omega^{-1}\left(\frac{x}{1-\lambda}\right)\omega_t+\left(k_t-\omega^{-1}\left(\frac{x}{1-\lambda}\right)\right)x. \end{aligned} \tag{4.15}$$

Lemma 7 states restrictions on the equilibrium sequence $\{k_t\}$ that must be satisfied in order for a nontrivial equilibrium to exist. Throughout we focus solely on equilibria for which these conditions are met.

4.4 General Equilibrium

4.4.1 Characterization

Let us now proceed to characterize the equilibrium sequence $\{k_t\}$. Since all young-period after-tax income is invested, the capital stock at date $t+1$ is

$$\begin{aligned} K_{t+1} &= \left(1-\lambda\right)\left(1-u_t\right)\left(\omega_t-\tau_t-s_H\right) \\ &\quad +\left(1-\lambda\right)u_t\phi\left(\overline{\omega}_t-\tau_t-s_H\right)+u_t\left(1-\phi\right)\left(\beta_H-\tau_t-s_H\right) \\ &\quad +\lambda\left[\phi\left(\overline{\omega}_t-\tau_t\right)+\left(1-\phi\right)\left(\beta_L-\tau_t\right)\right]. \end{aligned} \tag{4.16}$$

Use (4.3) to eliminate τ_t from (4.16) to obtain

$$K_{t+1}=\left(1-\lambda\right)\left[\left(1-u_t\right)\omega_t+u_t\left(1-\phi\right)\beta_H-s_H\right]+\lambda\left(1-\phi\right)\beta_L. \tag{4.17}$$

As an aside, note that, given ω_t and u_t, an increase in ϕ *reduces* the aggregate capital stock.[39] Using (4.14) and (4.10), it is possible to rewrite (4.17) as

$$H(k_{t+1}) = \frac{(1-\lambda)\, s_L \left[\omega(k_t) - (1-\phi)\,\beta_H\right]}{(1-\phi\theta)\,\omega(k_t) - (1-\phi)\,\beta_L} + \psi, \tag{4.18}$$

where

$$H(k) \equiv \frac{(1-\lambda)\, s_L k}{(1-\phi\theta)\,\omega(k) - (1-\phi)\,\beta_L} \tag{4.19}$$

and

$$\psi \equiv (1-\lambda)\left[(1-\phi)\,\beta_H - s_H\right] + \lambda(1-\phi)\,\beta_L. \tag{4.20}$$

Equation (4.18) describes the equilibrium law of motion for k_t. Given an initial k_0, (4.18) describes the subsequent equilibrium evolution of the capital-labor ratio for this economy. Of course, valid equilibrium sequences will satisfy also all of the stipulations imposed earlier. In particular, a valid equilibrium $\{k_t\}$ must satisfy Assumptions (A.1)-(A.6), (4.11), (4.12) and the restrictions embedded in Lemma 7. The following lemma establishes some properties of the functions H.

Lemma 8

(a) $H(0) = 0$,

(b) $\lim_{k\to\infty} H(k) = \infty$ *and*

(c) $H'(k) > 0$ *holds for all* k *that satisfies*

$$(1-\phi\theta)\left[\omega(k) - k\omega'(k)\right] - (1-\phi)\,\beta_L > 0.$$

[39] This is because all the wage income from public employment $\phi\left[\lambda + (1-\lambda)\,u_t\right]\overline{\omega}_t$ does not contribute to formation of new capital stock since, in the aggregate, it is offset by the lump-sum tax. Therefore, an increase in ϕ results in a *decrease* in value-added by home production, which amounts to $(1-\lambda)\,u_t\beta_H + \lambda\beta_L$.

Proof. (c): From (4.19),

$$H'(k) = \frac{(1-\lambda)\, s_L \{(1-\phi\theta)[\omega(k) - k\omega'(k)] - (1-\phi)\beta_L\}}{[(1-\phi\theta)\,\omega(k) - (1-\phi)\beta_L]^2}. \tag{4.21}$$

Since the denominator of (4.21) is positive, $H'(k) > 0$ holds iff $(1-\phi\theta)[\omega(k) - k\omega'(k)] - (1-\phi)\beta_L > 0$. ■

Under (4.1), (c) reduces to the requirement that $\omega(k) > \frac{(1-\phi)\beta_L}{(1-\phi\theta)(1-\alpha)}$ holds. For future reference, note that $H'(k) \equiv \partial K/\partial k > 0$ implies that the capital-labor ratio k and the aggregate capital stock K are positively correlated in equilibrium. Since u and k are positively related, u and K are positively related if $\omega(k) > \frac{(1-\phi)\beta_L}{(1-\phi\theta)(1-\alpha)}$ holds.

It turns out that it is more convenient to transform (4.18) and study the "forward dynamics" since (4.18) is typically a correspondence rather than a function. Rewrite (4.18) as follows:

$$\omega(k_t) = \frac{\xi(1-\lambda)\, s_L - x_L H(k_{t+1})}{(1-\lambda)\, s_L + \psi(1-\phi\theta) - (1-\phi\theta)\, H(k_{t+1})}. \tag{4.22}$$

Since $\omega'(k) > 0$, the law of motion for k_t as a function of k_{t+1} may be expressed as

$$k_t = \omega^{-1}\left(\frac{\xi(1-\lambda)\, s_L - x_L H(k_{t+1})}{(1-\lambda)\, s_L + \psi(1-\phi\theta) - (1-\phi\theta)\, H(k_{t+1})}\right) \equiv W(k_{t+1}). \tag{4.23}$$

It is important to establish some properties of the function W. Totally differentiate (4.22) to obtain

$$\frac{dk_t}{dk_{t+1}} = \frac{H'(k_{t+1})(1-\lambda)\, s_L(1-\phi)((1-\phi\theta)\beta_H - \beta_L)}{[(1-\lambda)\, s_L + \psi(1-\phi\theta) - (1-\phi\theta)\, H(k_{t+1})]^2\, \omega'(k_t)}, \tag{4.24}$$

where, from (4.19),

$$H'(k) = \frac{(1-\lambda)\, s_L \{(1-\phi\theta)[\omega(k) - k\omega'(k)] - (1-\phi)\beta_L\}}{[(1-\phi\theta)\,\omega(k) - (1-\phi)\beta_L]^2}.$$

The following lemma outlines some limiting behavior of the function W and its slope.

Lemma 9

(a) Let $\overline{k}_1$ *and* $\overline{k}_2$ *be the solutions to*

$$(1-\lambda)\,s_L + \psi\,(1-\phi\theta) - (1-\phi\theta)\,H\,(k) = 0.$$

Then

$$\lim_{k\to\overline{k}_1} W'\,(k) = \infty \text{ and } \lim_{k\to\overline{k}_2} W'\,(k) = \infty.$$

(b) Suppose $k_t = W\,(k_{t+1})$ *has two stationary solutions,* $k = k_l$ *and* $k = k_h$*. Then* $\overline{k}_1 < k_l$ *and* $k_h < \overline{k}_2$ *hold iff* $\phi < \frac{1}{\theta}\left[1-\frac{\beta_L}{\beta_H}\right]$.

Proof. Consider the denominator of (4.24). Since $(1-\phi\theta)\,\omega\,(k) - (1-\phi)\,\beta_L > 0$, (from (4.12)) the denominator will be zero iff

$$(1-\lambda)\,s_L + \psi\,(1-\phi\theta) - (1-\phi\theta)\,H\,(k) = 0. \tag{4.25}$$

(4.25) can be reduced to

$$k = \frac{[(1-\lambda)\,s_L + \psi\,(1-\phi\theta)]\,\omega\,(k)}{(1-\lambda)\,s_L} - \delta, \tag{4.26}$$

where

$$\delta \equiv \frac{\frac{(1-\lambda)s_L}{(1-\phi\theta)}\,(1-\phi)\,\beta_L + \psi\,(1-\phi)\,\beta_L}{(1-\lambda)\,s_L}.$$

Notice that (4.26) is very similar to (4.28). Let $\overline{k}_1$ and $\overline{k}_2$ be the solutions to (4.26). Then $dk_t/dk_{t+1} \to \infty$ as $k \to \overline{k}_1$ or $k \to \overline{k}_2$. Notice that $\overline{k}_1 < k_l$ and $k_h < \overline{k}_2$ hold if $\xi > \delta$ holds, which is true iff

$$(1-\phi\theta)\,\beta_H - \beta_L > 0$$

or, equivalently,

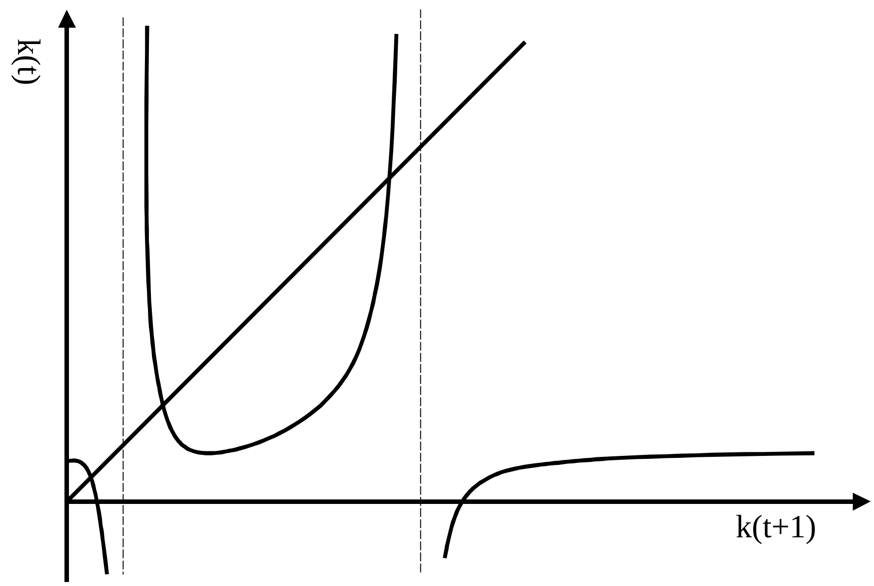
$$\phi < \frac{1}{\theta}\left(1-\frac{\beta_L}{\beta_H}\right). \tag{4.27}$$

holds. In words, the all steady states (if any) will be located inside the asymptotes $\overline{k}_1$ and $\overline{k}_2$. If, on the other hand, (4.27) is violated, then $k_l < \overline{k}_1$ and $\overline{k}_2 < k_h$ hold. In this case, all steady states (if any) will be outside of the asymptotes. ■

To keep the analysis simple, henceforth we shall restrict attention to the specification of the production function as in (4.1). Let us now proceed to determining the configuration of the phase diagram. To that end, focus on the numerator of (4.24).

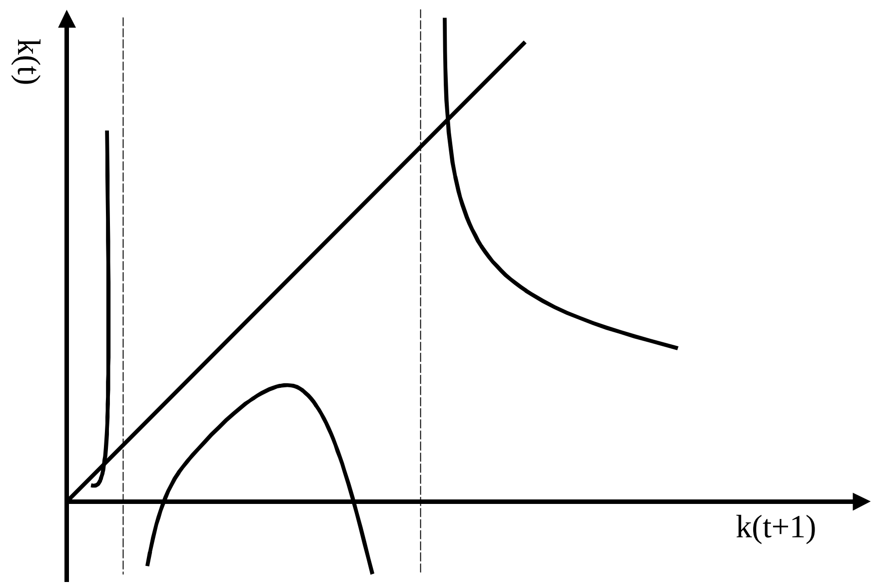
First, consider the case with $(1-\phi\theta)\,\beta_H - \beta_L > 0$. From (4.24), it is easy to show that $dk_t/dk_{t+1} > 0$ holds iff $\omega(k) > \frac{(1-\phi)\beta_L}{(1-\phi\theta)(1-\alpha)}$. Let $k = \overline{k}$ be the solution to $\omega(k) = \frac{(1-\phi)\beta_L}{(1-\phi\theta)(1-\alpha)}$. The function W is upward sloping for $k \geq \overline{k}$ and downward sloping for $k < \overline{k}$. There is a U-shaped portion of the function W between the asymptotes $\overline{k}_1$ and $\overline{k}_2$. This U-shaped portion has a trough at $\overline{k}$. From Lemma 9, two of the steady states are *inside* the asymptotes, suggesting that the two steady states are on the U-shaped portion of the function W. Therefore, the function W should have the configuration shown in Figure 4-3a.

Figure 4-3a. Function W : Case with $(1-\phi\theta)\,\beta_H - \beta_L > 0$



Now consider the case with: $(1-\phi\theta)\,\beta_H - \beta_L < 0$. From (4.24), it is easy to check that $dk_t/dk_{t+1} > 0$ holds iff $\omega\left(k\right) < \frac{(1-\phi)\beta_L}{(1-\phi\theta)(1-\alpha)}$. In this case, the function W is upward sloping for $k < \overline{k}$ and downward sloping for $k \geq \overline{k}$. There is an inverse U-shaped portion of the function W between the asymptotes $\overline{k}_1$ and $\overline{k}_2$. From Lemma 9, two of the steady states are *outside* of the asymptotes. This suggests that a steady state cannot be found on the inverse U-shaped portion of the function W. Therefore the function W has the configuration displayed in Figure 4-3b.

Figure 4-3b. Function W : Case 2



The rest of the analysis focuses on equilibria in which case 1 holds, or the size of public employment programs is relatively small, i.e., $\phi < \frac{1}{\theta}\left[1 - \frac{\beta_L}{\beta_H}\right]$. The following lemma is apparent from Figure 4-3a.

Lemma 10 *Let $\tilde{k}$ such that $W'(\tilde{k}) = 0$. Then (4.23) has no fixed points, a single fixed point, or two fixed points when $W(\tilde{k})$ is greater than, equal to, and less than $\tilde{k}$ respectively.*

Henceforth, we shall mainly focus on cases in which there are exactly two steady states. Of course, any valid k must satisfy Assumptions (A.1)-(A.6), (4.11), (4.12) and the restrictions embedded in Lemma 7. Let k_h denote the high-capital-stock steady state and k_l denote the low-capital-stock steady state. The following two examples illustrate the existence of two valid steady state equilibria.[40]

[40] It is easy to check that in the examples below, equilibrium unemployment rates in the private sector are rather high. Several models in which firms use equilibrium unemployment as a sorting device share the same

Example 1 (a) Let $A = 1$, $\alpha = 0.6$, $\lambda = 0.3$, $\beta_H = 0.8$, $\beta_L = 0.6$, $s_L = 0.1$, $s_H = 0$ and $\phi = 0$. Then $k_l = 4.97$ and $k_h = 24.07$. (b) Let $A = 1.3$, $\alpha = 0.6$, $\lambda = 0.3$, $\beta_H = 0.8$, $\beta_L = 0.6$, $s_L = 0.1$, $s_H = 0$, $\theta = 0.8$ and $\phi = 0.3$. Then $k_l = 3.03$ and $k_h = 10.25$.

4.4.2 Dynamic Properties of the Model

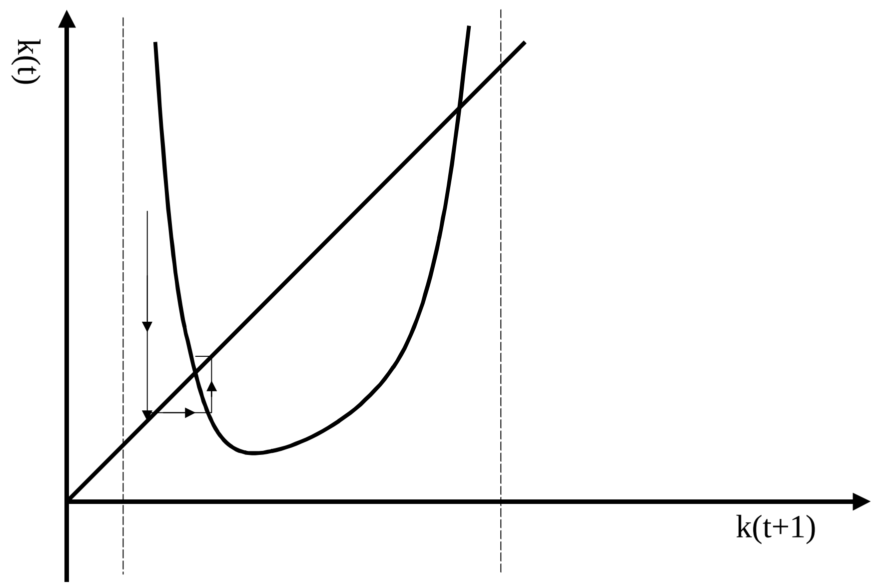
The local stability properties of the two steady states are illustrated by the examples below.

Example 2 Let $A = 1.1$, $\alpha = 0.6$, $\lambda = 0.3$, $\beta_H = 0.8$, $\beta_L = 0.6$, $s_L = 0.1$, $s_H = 0$, $\theta = 0.9$ $\phi = 0.1$. With this specification, there are two valid steady states, $k_l = 4.21$, and $k_h = 19.03$. Then $W'(k_l) = -12.89 < -1$ and $W'(k_h) = 45.52 > 1$. In this case, the low-capital steady state k_l is unstable in the forward dynamics and hence it is stable in the normal backward dynamics. The high-capital steady state is stable in the normal backward dynamics. (See Figure 4-4a.)

---

discomforting feature. Presumably this is due to the simplifying assumption that the firms have no access to other "more sophisticated" screening technologies. See Bencivenga and Smith (1995) for a discussion.

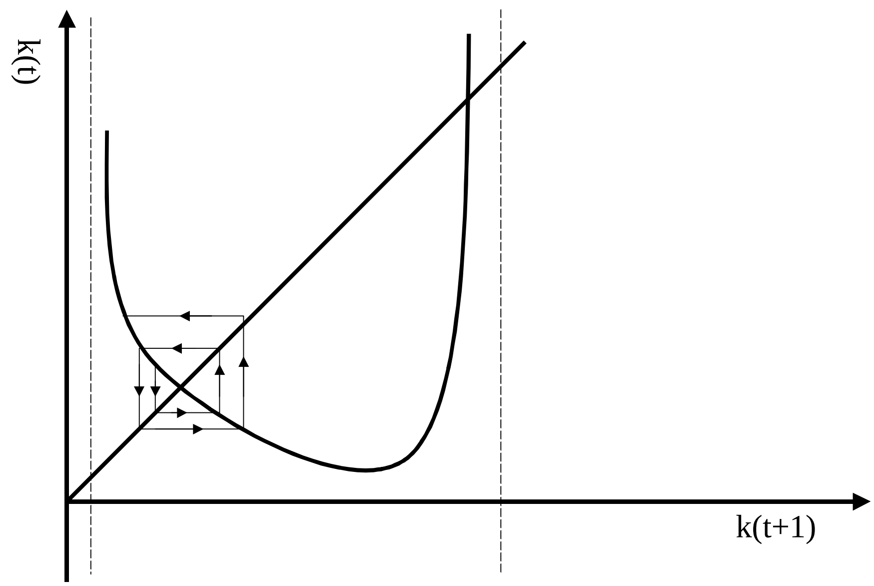
Figure 4-4a. Dynamics: The Low Activity Steady State is Stable



The above example illustrates the possibility of multiple asymptotically stable steady states. Depending on the initial capital-labor ratio, k_0, a country may approach either the high capital or the low capital stationary state. Countries that start off near the low capital steady state will be attracted to it and will stay permanently stuck at a low level of real activity. In other words, development traps may arise easily. How might a country get out of such a trap? The next example illustrates.

Example 3 Let $A = 1.1$, $\alpha = 0.6$, $\lambda = 0.3$, $\beta_H = 0.8$, $\beta_L = 0.6$, $s_L = 0.1$, $s_H = 0$, $\theta = 0.9$ and $\phi = 0.1621$. With this specification, there are two valid steady states, $k_l = 7.36$, $k_h = 8.02$. In addition, $W'(k_l) = -0.75 > -1$ and $W'(k_h) = 3.48 > 1$. In this case the low-capital steady state k_l is stable in the forward dynamics and hence it is unstable in the normal backward dynamics. (See Figure 4-4b.)

Figure 4-4b. Dynamics: The Low Activity Steady State is Unstable



The parameters of the two economies are identical except for the value of ϕ. In the second example, ϕ is higher by 62%. This increase has dramatic effects in that the low capital steady state is no longer stable in the normal backward dynamics. In other words, an increase in the level of public employment programs renders the low capital steady state unstable, thereby allowing countries to break out of the development trap. Such examples illustrate the immense potency of these public employment programs. Of course, enroute to the high capital steady state, the economy may get stuck in periodic orbits around k_h and never attain such a permanently high level of real activity as at k_h.[41]

4.5 Policy Analysis

41 Betts and Bhattacharya (1998) study the possibility of generating chaotic orbits in such settings.

4.5.1 Wage Policy

A major focus of the present analysis is to investigate the likely long-run effects of the government's job-creation policies. Two parameters both under the control of public policy are of paramount importance here; θ, which measures the private to public sector wage rate and ϕ, which measures the size of public sector employment. Let us now proceed to study the effects of changes in these parameters on the steady-state capital stock and hence on long-run real activity.

Proposition 11

$$\left.\frac{dk}{d\theta}\right|_{k=k_l} \geq 0, \text{ and } \left.\frac{dk}{d\theta}\right|_{k=k_h} < 0.$$

Proof. In a steady state, $k_{t+1} = k_t$ and $u_{t+1} = u_t$ for all t. From the equilibrium law of motion (4.18) we get

$$k = \left(1 + \frac{\psi\left(1 - \phi\theta\right)}{\left(1 - \lambda\right) s_L}\right) \omega\left(k\right) - \xi \equiv \Phi\left(k\right), \tag{4.28}$$

where

$$\xi \equiv \frac{\left(1 - \lambda\right) s_L \left(1 - \phi\right) \beta_H + \psi\left(1 - \phi\right) \beta_L}{\left(1 - \lambda\right) s_L}.$$

$\Phi\left(k\right)$ is monotone, increasing and concave. Totally differentiating (4.28) yields

$$\frac{dk}{d\theta} = \frac{-\psi\phi\omega\left(k\right)}{1 - \Phi'\left(k\right)}.$$

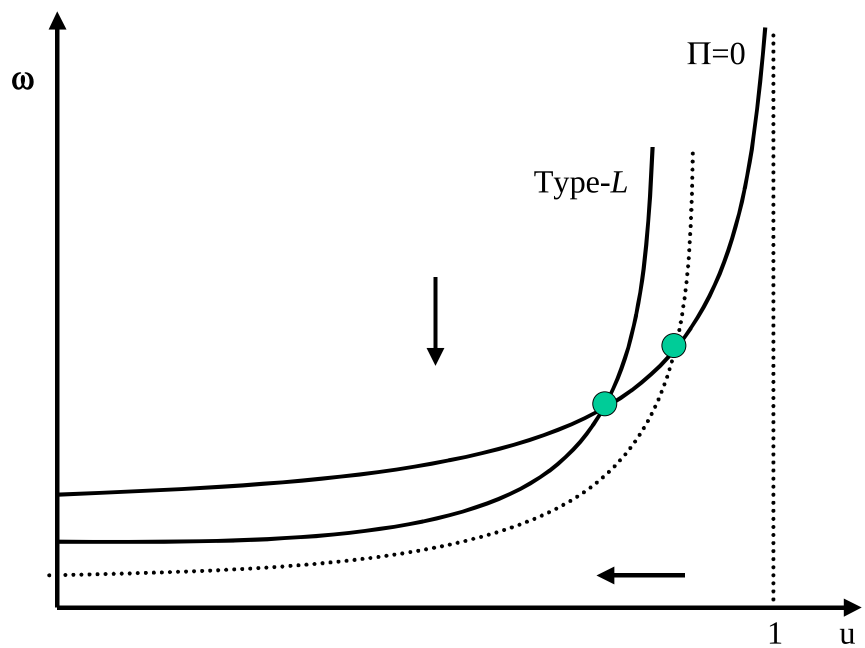
Since $\Phi'\left(k_l\right) \geq 1 \geq \Phi'\left(k_h\right)$ holds, the rest of the proof is immediate. ■

Proposition 11 states that an increase in θ raises (reduces) the long-run capital-labor ratio at the low (high) activity steady state. The intuition for this result is as follows. The Nash equilibrium contract must satisfy the self-selection constraint for type-L; *i.e.*, (4.6) must hold with equality. For convenience, we reproduce the equation here:

$$(1-u)\,\omega - s_L + ux = x, \tag{4.29}$$

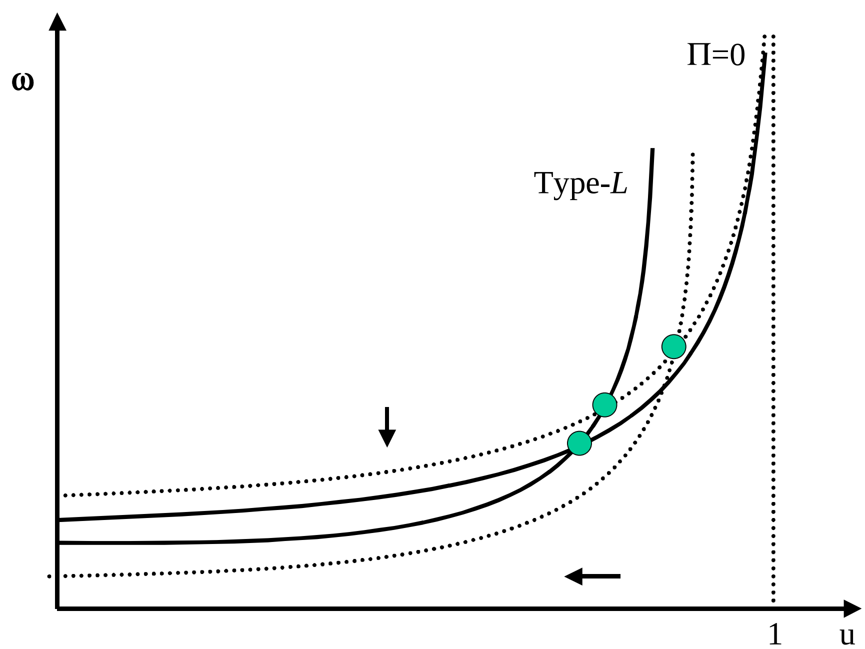
where $x \equiv \phi\overline{\omega} + (1-\phi)\,\beta_L$. Consider a small increase in θ. Other things equal, this raises the right-hand-side of (4.29) relative to the left-hand-side. That is, an increase in the public-to-private wage makes "participation" in the private sector less attractive. Thus the indifference curve for a type-L shifts up. (See Figure 4-5a). The firms have to react this by offering a low unemployment and a low wage rage, as shown in Figure 4-5a.

Figure 4-5a. Effect of an Increase in θ in Period t



Under the new contract, more type-H are employed under a lower wage rate. At the high-k steady state, this leads to a lower income and hence a lower national savings. Therefore the next-period capital stock K_{t+1} will be *lower*. In period $t+1$ the marginal product of labor *decreases* because of the reduction in capital stock. This shifts the zero-profit locus down. Thus in period $t+1$ the firms offer a new contract, under which u_{t+1} and ω_{t+1} are even lower. This process continues until the economy reaches a new steady state equilibrium, since the transitional dynamics at the high-activity steady state is monotonic.[42] (See Figure 4-5b.)

Figure 4-5b. Effect of an Increase in θ in Period $t+1$ at the High-k Equilibrium



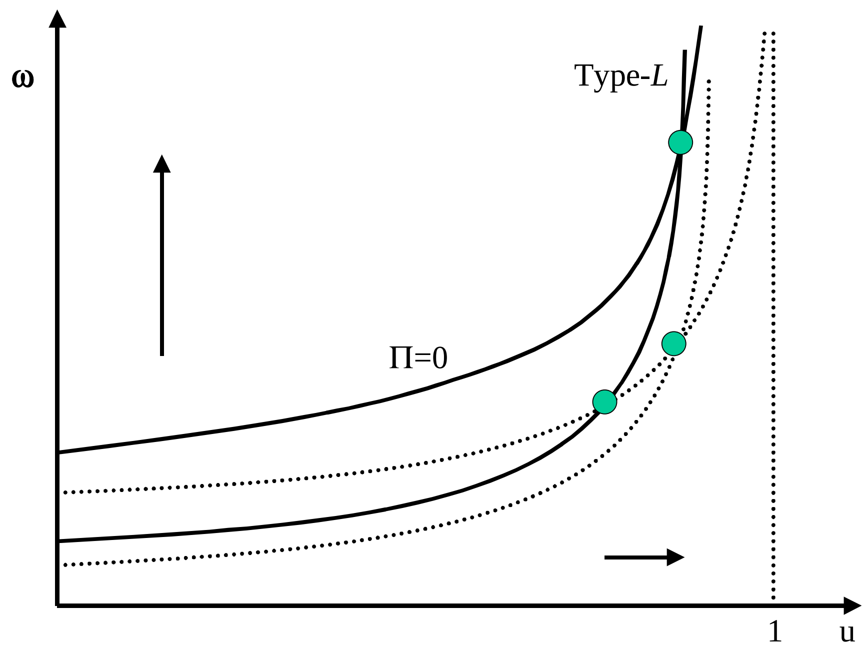
A comparative statics exercise for the low-activity steady state, however, is more complex since the transitional dynamics is non-monotonic. Consider again an increase in θ. This policy makes participation less attractive. The private sector firms react this by reducing both u_t and ω_t in order to screen type-L workers out. (See Figure 4-5a.) Under the new contract, more type-H are employed under a lower wage rate. At the low-k equilibrium, this leads to a *higher* national income and savings because the increase in

42 Notice that the result depends crucially on the concavity of the production technology. Thus, there is a possibility that the policy raises the long-run capital-labor ratio if the production function exhibits convexity.

employment is greater than the decrease in wage rate.[43] Therefore the next-period capital stock K_{t+1} will be *higher*. In period $t+1$, the marginal product of labor is higher. So the firms *raise* both u_{t+1} and ω_{t+1} because the zero-profit locus shifts *up*. (See Figure 4-5c.) Under the new contract, less type-H are employed under a higher wage rate. Thus the national income and savings are higher. This leads to a *lower* next-period capital stock K_{t+2}. Thus, in period $t+2$ the marginal product of labor decreases. This process continues until the economy reaches a new steady state, if the system is locally stable. If it is, then the new steady state capital-labor ratio is higher. As demonstrated above (and proved in the previous section), the transitional dynamics near the low-activity steady state involves oscillations. The policy implication is that even though the government interventions in the labor market at the low-activity steady state increase the long-run capital-labor ratio, such policies are highly destabilizing.

43 This is because the zero-profit locus is steeper at the high-k steady state than at the low-k steady state.

Figure 4-5c. Effect of an Increase in θ in Period $t+1$ at the Low-k Equilibrium



4.5.2 Size of the Government Employment

Now consider the effect of a change in ϕ (the size of public employment programs) on the long-run capital-labor ratio.

Proposition 12
Define

$$\widetilde{\phi} \equiv 1 - \frac{(1-\lambda)\, s_H}{2\beta} \text{ and } \widetilde{\theta} \equiv \frac{\beta}{(1-\lambda)\, s_H - (1-2\phi)\,\beta}.$$

a) Suppose $\phi > \widetilde{\phi}$ *and* $\theta > \widetilde{\theta}$*. Then*

$$\left.\frac{dk}{d\phi}\right|_{k=k_l} < 0 \text{ and } \left.\frac{dk}{d\phi}\right|_{k=k_h} > 0.$$

b) Suppose $\phi < \widetilde{\phi}$ or $\theta < \widetilde{\theta}$. In addition, suppose

$$\frac{(1-\phi)(s_L\beta_H - s_H\beta_L)}{(1-\phi\theta)(s_L - s_H)} \geq \frac{(1-\lambda)s_L\beta_H + \psi\beta_L + (1-\phi)\beta_L\beta}{\psi\theta + (1-\phi\theta)\beta}.$$

Then,

$$\left.\frac{dk}{d\phi}\right|_{k=k_l} > 0 \text{ and } \left.\frac{dk}{d\phi}\right|_{k=k_h} < 0.$$

Proof. Totally differentiate (4.28) to yield

$$(1-\Phi'(k))\,dk = \left(-\frac{\psi\theta\omega(k)}{(1-\lambda)s_L} + \frac{(1-\phi\theta)\omega(k)}{(1-\lambda)s_L}\frac{d\psi}{d\phi} - \frac{d\xi}{d\phi}\right)d\phi,$$

where

$$\frac{d\psi}{d\phi} = -(1-\lambda)\beta_H - \lambda\beta_L \equiv -\beta$$

and

$$\frac{d\xi}{d\phi} = -\frac{(1-\lambda)s_L\beta_H + \psi\beta_L + (1-\phi)\beta_L[(1-\lambda)\beta_H + \lambda\beta_L]}{(1-\lambda)s_L}.$$

Combine these equations to write

$$\begin{aligned} &(1-\Phi'(k))\,dk \\ = &\left\{\begin{array}{c} -\psi\theta\omega(k) - [(1-\lambda)\beta_H + \lambda\beta_L](1-\phi\theta)\omega(k) \\ +[(1-\lambda)s_L\beta_H + \psi\beta_L + (1-\phi)\beta_L[(1-\lambda)\beta_H + \lambda\beta_L]] \end{array}\right\}\frac{d\phi}{(1-\lambda)s_L}, \end{aligned}$$

which can be rewritten as

$$\frac{dk}{d\phi} = \frac{-[\psi\theta + \beta(1-\phi\theta)]\omega(k) + [(1-\lambda)s_L\beta_H + \psi\beta_L + (1-\phi)\beta_L\beta]}{[1-\Phi'(k)](1-\lambda)s_L}. \tag{4.30}$$

First let us consider the case with $\psi\theta + \beta(1-\phi\theta) < 0$. This happens when

$$\theta[(1-2\phi)\beta - (1-\lambda)s_H] + \beta < 0, \tag{4.31}$$

which occurs iff

$$\begin{aligned}\phi &> \widetilde{\phi} \equiv 1 - \frac{(1-\lambda)\, s_H}{2\beta} \text{ and} \\ \theta &> \widetilde{\theta} \equiv \frac{\beta}{(1-\lambda)\, s_H - (1-2\phi)\,\beta}.\end{aligned}$$

In this case, the numerator of (4.30) is unambiguously positive. This proves (a).

Now consider the case with $\psi\theta + \beta(1-\phi\theta) > 0$. This happens when

$$\theta\left[(1-2\phi)\,\beta - (1-\lambda)\, s_H\right] + \beta > 0, \tag{4.32}$$

which occurs iff

$$\phi < \widetilde{\phi} \equiv 1 - \frac{(1-\lambda)\, s_H}{2\beta} \tag{4.33}$$

or (4.33) is violated and

$$\theta < \widetilde{\theta} \equiv \frac{\beta}{(1-\lambda)\, s_H - (1-2\phi)\,\beta}. \tag{4.34}$$

In this case,

$$\begin{aligned}\left.\frac{dk}{d\phi}\right|_{k=k_l} &> 0 \text{ iff } \omega(k) > \frac{(1-\lambda)\, s_L\beta_H + \psi\beta_L + (1-\phi)\,\beta_L\beta}{\psi\theta + (1-\phi\theta)\,\beta} \text{ and} \\ \left.\frac{dk}{d\phi}\right|_{k=k_h} &< 0 \text{ iff } \omega(k) > \frac{(1-\lambda)\, s_L\beta_H + \psi\beta_L + (1-\phi)\,\beta_L\beta}{\psi\theta + (1-\phi\theta)\,\beta}.\end{aligned}$$

Since any valid equilibrium has to satisfy

$$\omega(k) > \frac{(1-\phi)\,(s_L\beta_H - s_H\beta_L)}{(1-\phi\theta)\,(s_L - s_H)},$$

a sufficient condition for $\omega(k) > \frac{(1-\lambda)s_L\beta_H + \psi\beta_L + (1-\phi)\beta_L\beta}{\psi\theta + (1-\phi\theta)\beta}$ is

$$\frac{(1-\phi)\,(s_L\beta_H - s_H\beta_L)}{(1-\phi\theta)\,(s_L - s_H)} \geq \frac{(1-\lambda)\, s_L\beta_H + \psi\beta_L + (1-\phi)\,\beta_L\beta}{\psi\theta + (1-\phi\theta)\,\beta}. \tag{4.35}$$

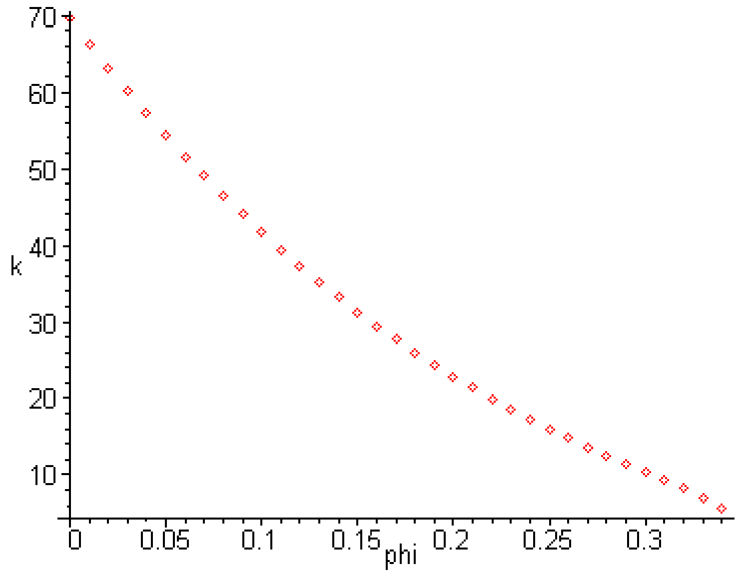
Therefore (b) is proved. ■

To begin with, let us focus on case (b).[44] Condition (4.35) is easily satisfied if the difference in home productivities between a type-H and a type-L is large and the difference in search costs is small. Here, the intuition is basically the same as that provided above. An increase in ϕ makes participation less attractive. In contrast to Proposition 11, the result obtained here is complex because a change in the size of public sector employment changes directly the next-period capital stock. Case (b) is a situation where the level of the government's involvement in the labor market via the employment programs *and* the ratio of the real wage rate offered in the public sector relative to the private sector are relatively low. If the situation were to be reversed, then the proposition makes clear that the effect on the long-run capital stock would be reversed too. In other words, the model predicts that developing countries (those stuck at k_l) with small public employment programs will see a rise in long-run per capita incomes following an increase in the size of their government's involvement in the labor market. However, such countries if they *already* have large government employment programs in place and where the public sector wage rate is "sufficiently competitive", would suffer declines in long-run real activity were they to increase *further* the size of the public employment programs. In short, the existing *level* of government involvement matters.

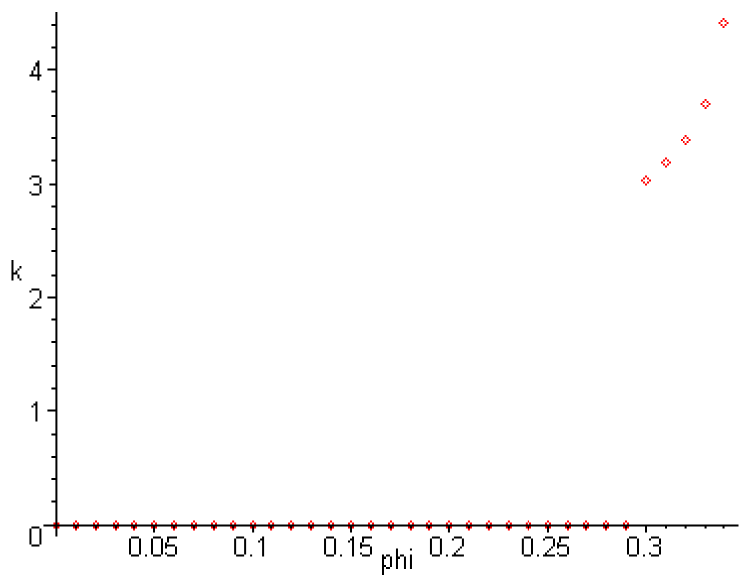
Figure 4-6 demonstrates the above proposition (case (b)) using a parameterized model. At the high-activity steady state, an increase in the size of government employment reduces the long-run capital-labor ratio. At the low-activity steady state, the result is reversed.

44 Assuming $s_H = 0$ would obviously rule out case (a).

Figure 4-6. Effect of an increase in ϕ on k
($A = 1.3$, $\alpha = 0.6$, $\lambda = 0.3$, $\beta_H = 0.8$, $\beta_L = 0.6$, $s_L = 0.1$, $s_H = 0$, $\theta = 0.8$)



Effect at the High-Activity Steady State



Effect at the Low-Activity Steady State

Note that some range of policy parameters does not support the low-activity steady state equilibrium. This is because the capital-labor ratio is so low that the corresponding wage rate is lower than the home productivity. Thus, there will be no market activity (autarky). A literal implication of the numerical study is that the government intervention could push an autarky economy to a market economy if the size of government intervention is large enough.

Another interesting implication of Proposition 12 is that these results hold even though the aforementioned public employment programs are completely *unproductive* in the sense that the government good produced from them is of no value whatsoever to private agents. In other words, these employment programs conform closely to the classic Keynesian pump-priming policies followed in the U.S and elsewhere in the 40's and the 50's

4.6 Unemployment Insurance

One of the most commonly used tool for the government intervention in the labor market is to provide unemployment insurance (hereafter UI). Both UI and public employment provide some insurance against being out of job. It is natural to ask whether a UI scheme and public sector employment are substitutes.[45] This section sketches a model of UI and briefly discusses how the provision of UI and public employment programs differ *even* when government production is completely useless. It turns out that only mild modifications to the current framework are needed.

[45] Casual empiricism suggests that the provision of publicly funded unemployment insurance is not common in developing economies, even though public employment programs are widely and heavily used. This suggests that unemployment insurance programs are not perfect substitutes for public employment programs. Low income countries seem to adopt the latter and shy away from the former. On the other hand, developed economies have significant size of unemployment insurance systems.

Suppose that the government provides a UI benefit, instead of public sector employment. If the eligibility for the UI benefit is such that anyone who has no job receives the benefit, then the analysis is exactly the same as the one for government employment studied in the previous sections. The problem, however, is that in reality, one has to be in the labor force in order to be eligible for the UI benefit because otherwise UI would discourage people from work. In fact, it has been shown that the provision of government employment makes participation less attractive. Therefore we modify the model slightly so as to capture the actual UI policy better.

The main idea here is that a worker has to "participate" into the labor market in order to be eligible for the UI benefit. Thus, in contrast to public sector employment, UI makes "participation" *more* attractive. Let b denote the amount of UI benefit. In the current setup, only $(1-\lambda)\,u_t$ workers will be eligible for the UI benefit, while $\lambda+(1-\lambda)\,u_t$ are eligible for public employment in the previous section. Accordingly, the self-selection constraint for type-H is

$$(1-u_t)\,(\omega_t-s_H)+u_t\,(\beta_H+b-s_H)>\beta_H. \tag{4.36}$$

The right-hand-side of (4.36) is just β_H, capturing the fact that a worker is eligible for the UI benefit only if he seeks for private sector employment and fails to find any. Similarly, the self-selection condition for a type-L is

$$(1-u_t)\,(\omega_t-s_L)+u_t\,(\beta_L+b-s_L)\leq\beta_L. \tag{4.37}$$

Notice that, *ceteris paribus*, an increase in the benefit b would simply make participation attractive to all types of workers. The Nash equilibrium separating contract gets altered to

$$u_t = 1 - \frac{s_L - b}{\omega(k_t) - (\beta_L + b)}.$$

Since $u_t \in (0, 1)$ must hold,

$$\omega(k_t) > \beta_L + b \text{ and } \omega(k_t) > s_L + \beta_L \tag{4.38}$$

must obtain. Analogous to (4.16), the next period's capital stock evolves according to

$$K_{t+1} = (1-\lambda)\left[(1-u_t)(\omega_t - \tau_t - s_H) + u_t(\beta_H + b - \tau_t - s_H)\right] + \lambda(\beta_L - \tau_t).$$

Since the government finances the UI program by a lump-sum tax on everyone, the government budget constraint is

$$(1-\lambda) u_t b = \tau_t. \tag{4.39}$$

The equilibrium law of motion for the capital-labor ratio is

$$\frac{(1-\lambda)(s_L - b)}{\omega(k_{t+1}) - (\beta_L + b)} k_{t+1} = (1-\lambda)\frac{(s_L - b)(\omega(k_t) - \beta_H)}{\omega(k_t) - (\beta_L + b)} + (1-\lambda)(\beta_H - s_H) + \lambda\beta_L.$$

At a steady state,

$$\begin{aligned} k &= \left(1 + \frac{(1-\lambda)(\beta_H - s_H) + \lambda\beta_L}{(1-\lambda)(s_L - b)}\right)\omega(k) \\ &\quad - \left(\beta_H + \frac{[(1-\lambda)(\beta_H - s_H) + \lambda\beta_L](\beta_L + b)}{(1-\lambda)(s_L - b)}\right) \\ &\equiv \Gamma(k). \end{aligned} \tag{4.40}$$

As before, it is easy to demonstrate the possibility of two steady state equilibria. Let $k = k_l$ and $k = k_h$ denote the low (high) capital steady states. The central result of this section is stated below.

Proposition 13

$$\left.\frac{dk}{db}\right|_{k=k_l} < 0 \text{ and } \left.\frac{dk}{db}\right|_{k=k_h} > 0.$$

Proof. From (4.40), it is easy to check that

$$\frac{dk}{db} = \frac{\partial\Gamma/\partial b}{1-\Gamma'(k)},$$

where

$$\frac{\partial\Gamma}{\partial b} = \frac{(1-\lambda)(\omega(k)-s_L-\beta_L)[(1-\lambda)(\beta_H-s_H)+\lambda\beta_L]}{[(1-\lambda)(s_L-b)]^2} > 0$$

since $\omega(k)-s_L-\beta_L > 0$ from (4.38). The rest of the proof is immediate. ■

Proposition 13 asserts that an increase in the amount of UI benefit reduces (raises) the capital-labor ratio at the low-k (high-k) steady state. Somewhat surprisingly, such a result has a flavor that is diametrically opposite to the one stated in say Proposition 11. Here is the reason why. In equilibrium, (4.37) holds with equality. Consider a small increase in b. An increase in the UI benefit makes participation *more* attractive.[46] At the high-k equilibrium, the firms have to react this by *raising* u_t and ω_t. Under the new contract, less type-H are employed under a higher wage rate. At the high-k equilibrium, this leads to a greater national income and savings. Thus the next-period capital stock K_{t+1} will be *higher*. In period $t+1$ the marginal product of labor increases. As a result, the firms raise both u_t and ω_t even more. This process continues until the economy reaches a new steady state equilibrium. Thus an increase in the UI benefit *raises* the long-run capital-labor ratio at the high-k equilibrium.

[46] In the case with public employment programs, an increase in ϕ or θ raises the payoff of *non-participation* relative to participation.

Consider again an increase in b, now at the low-activity steady state. This policy makes participation more attractive. Thus the firms *raise* u_t and ω_t. Under the new contract, less type-H are employed under a higher wage rate. At the low-k equilibrium, this leads to a *lower* national income and savings. Thus the next-period capital stock K_{t+1} will be *lower*. Therefore the marginal product of labor in period $t+1$ is lower. The firms react this by *reducing* u_{t+1} and ω_{t+1}. Under the new contract, more type-H are employed under a lower wage rate. This leads to a greater national income and savings. Thus the next-period capital stock K_{t+2} will be *higher*. So u_{t+2} and ω_{t+2} are *higher*. This process continues until the economy reaches a new steady state, if the system is locally stable. If it is, then the new steady state capital-labor ratio is *lower*.

In short, the principal difference between public employment programs and UI programs to private agents is one of eligibility or non-eligibility and the payoffs in each case. As shown above, this simple fact produces an important policy implication: UI policy, characterized as an income maintenance policy in which only workers who seek employment *and* fail to find any are eligible, improves long-run real activity *only* for "developed" countries (at the high capital steady state). Such a result (along with Proposition 12 earlier) therefore provides a partial explanation as to why less-developed economies shy away from unemployment insurance programs and yet adopt public employment programs with readiness.[47]

[47] These remarks come with the usual caveat that the comparisons here are being made at steady states and so no welfare calculations for the transition path from one steady state to another are being presented.

4.7 Productive Government

So far we have assumed that the "government good" created by the public employment programs do not contribute to the economy in any way. Of course this is unrealistic – after all the public sectors in many countries around the world play a major role in their country's development. In fact, a major source of their influence lies in the fact that they are often the sole producers of "social overhead capital" or infrastructure.[48] The creation, extension, and maintenance of this capital creates jobs as well as provides beneficial externalities to the private sector. Schmitz (1996) documents that the "public enterprise share of national output" has often exceeded 30% in many countries. This section extends the earlier setup to include a government that is "productive". In particular, in what follows it is assumed that the government good produced by people in the public employment programs enters the production function of private sector firms. In such a case, the difference between provision of public employment and UI becomes stark.

4.7.1 Technology

In the case with productive government, the public goods produced at time t are assumed to generate a beneficial external effect to private production. Following Barro (1990), the production function is specified as $Y_t = F\left(K_t, \left(1-\epsilon_t\right)N_t, g_t\right) \equiv AK_t^{\alpha}\left[\left(1-\epsilon_t\right)N_t\right]^{1-\alpha} g_t^{\gamma}$. Its intensive form is

$$f\left(k\right) = Ak_t^{\alpha} g_t^{\gamma}. \tag{4.41}$$

48 See, for example, Aschauer (1989).

In what follows, we impose $\gamma = \alpha$ solely for computational simplicity. I also assume that this "public capital" depreciates 100% within a period.[49] The amount of public good produced at date t is described by the function $g_t = G(L_t)$. It will be convenient to specify this production function in its simplest possible form as:

$$G(L_t) = L_t = \phi\left[\lambda + (1-\lambda) u_t\right]. \tag{4.42}$$

Note that the assumption of a linear public sector production technology does *not* mean that its contribution to the private sector production is linear: the decreasing returns are captured by $\gamma \in (0,1)$.

4.7.2 General Equilibrium

In the case with productive government, (4.8)-(4.10) are replaced by

$$\begin{aligned} r_t &= f'(k_t; g_t) = \alpha A k_t^{\alpha-1} g_t^{\gamma} \\ \omega_t &= f(k_t) - k_t f'(k_t) \equiv \omega(k_t; g_t) = (1-\alpha) A k_t^{\alpha} g_t^{\gamma} && (4.43) \\ u_t &= 1 - \frac{s_L}{\omega_t - \left[\overline{\omega} + (1-\phi)\beta_L\right]} \equiv 1 - \frac{s_L}{(1-\phi\theta)\,\omega_t - (1-\phi)\beta_L}. && (4.44) \end{aligned}$$

In equilibrium, analogous to (4.18),

$$(1-\lambda)(1-u_{t+1}) k_{t+1} = (1-\lambda)\left[(1-u_t)\omega_t + u_t(1-\phi)\beta_H - s_H\right] + \lambda(1-\phi)\beta_L. \tag{4.45}$$

The equations (4.44) and (4.45) along with (4.42) jointly describe the equilibrium law of motions for k_t and u_t. In contrast to the previous sections, it is no longer possible to write

49 For a model of (endogenous) growth with durable public capital goods, see Futagami et al. (1993).

the equilibrium law of motion solely in terms of k_t. In order to make further progress, we shall work with the production function in (4.41) and obtain the equilibrium law of motion in terms of u_t as follows:

$$k_t = \left(\frac{\frac{s_L}{1-u_t} + (1-\phi)\beta_L}{(1-\phi\theta)(1-\alpha)A\{\phi\lambda + (1-\lambda)u_t\}^{\gamma}} \right)^{1/\alpha} \equiv Q(u_t). \tag{4.46}$$

The relationship between u and k in equilibrium is described by (4.46). It is informative to study the properties of the function Q. Define

$$\widetilde{u}_1 \equiv 1 + \frac{(\gamma+1)s_L - \sqrt{\begin{array}{c}(2(1-\phi)\beta_L\gamma + \gamma s_L + s_L)^2 \\ -4\gamma(1-\phi)\beta_L\left[\gamma((1-\phi)\beta_L + s_L) - \frac{\lambda s_L}{1-\lambda}\right]\end{array}}}{2(1-\phi)\beta_L\gamma}.$$

Lemma 11

(a) $Q(0) > 0$

(b) $Q'(u) > 0$ *iff* $u > \widetilde{u}_1$.

Proof. Differentiate (4.46) with respect to u to give

$$\begin{aligned} Q'(u) &= \frac{1}{\alpha}\left(\frac{\frac{s_L}{1-u} + (1-\phi)\beta_L}{(1-\phi\theta)(1-\alpha)Ag^{\gamma}}\right)^{1/\alpha-1} \\ &\quad \times \left\{\frac{\frac{s_L}{(1-u)^2} - \left(\frac{s_L}{1-u} + (1-\phi)\beta_L\right)\gamma g^{-1}G'(\cdot)\phi(1-\lambda)}{(1-\phi\theta)(1-\alpha)Ag^{\gamma}}\right\}. \end{aligned}$$

This implies that $Q'(u) < 0$ iff

$$\frac{s_L}{(1-u)^2} - \left(\frac{s_L}{1-u} + (1-\phi)\beta_L\right)\gamma g^{-1}G'(\cdot)\phi(1-\lambda) < 0,$$

which can be reduced to

$$(1-u)\left[(1-u)(1-\phi)\beta_L + s_L\right]\frac{\gamma(1-\lambda)}{\lambda + (1-\lambda)u} - s_L > 0.$$

The above inequality can be written as the quadratic form

$$\gamma(1-\phi)\beta_L u^2 - (2(1-\phi)\beta_L\gamma + \gamma s_L + s_L)u + \gamma((1-\phi)\beta_L + s_L) - \frac{\lambda s_L}{1-\lambda} > 0.$$

Therefore, $Q'(u) < 0$ iff $u < \widetilde{u}_1$ or $u < \widetilde{u}_2$, where

$$\widetilde{u}_1 \equiv 1 + \frac{(\gamma+1)s_L - \sqrt{\begin{array}{c}(2(1-\phi)\beta_L\gamma + \gamma s_L + s_L)^2 \\ -4\gamma(1-\phi)\beta_L\left(\gamma((1-\phi)\beta_L + s_L) - \frac{\lambda s_L}{1-\lambda}\right)\end{array}}}{2(1-\phi)\beta_L\gamma} \text{ and}$$

$$\widetilde{u}_2 \equiv 1 + \frac{(\gamma+1)s_L + \sqrt{\begin{array}{c}(2(1-\phi)\beta_L\gamma + \gamma s_L + s_L)^2 \\ -4\gamma(1-\phi)\beta_L\left(\gamma((1-\phi)\beta_L + s_L) - \frac{\lambda s_L}{1-\lambda}\right)\end{array}}}{2(1-\phi)\beta_L\gamma}.$$

However, $\widetilde{u}_2$ is unambiguously greater than unity. Therefore only $\widetilde{u}_1$ can be valid. ■

Lemma 11 establishes that the relation between u and k is not monotonic. In fact, Q is U-shaped with one trough if $\widetilde{u}_1$ lies between zero and unity. The map from k to u is a correspondence rather than a function. Thus, instead of characterizing the equilibrium in term of the capital-labor ratio, we derive the equilibrium law of motion in terms of u.

4.7.3 Equilibrium Law of Motion

From (4.44) it is easily verified that

$$\omega_t = \frac{\frac{s_L}{1-u_t} + (1-\phi)\beta_L}{1-\phi\theta}. \tag{4.47}$$

Substitute (4.46) and (4.47) into (4.45) to discover

$$\begin{aligned}(1-\lambda)(1-u_{t+1})Q(u_{t+1}) &= (1-\lambda)\left((1-\phi)\beta_H - \frac{(1-\phi)\beta_L}{1-\phi\theta}\right)u_t \\ &\quad + (1-\lambda)\left(\frac{s_L + (1-\phi)\beta_L}{1-\phi\theta} - s_H\right) \\ &\quad + \lambda(1-\phi)\beta_L. \end{aligned} \tag{4.48}$$

As is clear from (4.48), it is not possible to find the backward dynamics. Thus, rewrite (4.48) as the forward dynamical relationship

$$u_t = \frac{(1-u_{t+1})\,Q\,(u_{t+1})}{(1-\phi)\,\beta_H - \frac{(1-\phi)\beta_L}{1-\phi\theta}} - \eta \equiv M\,(u_{t+1})\,, \tag{4.49}$$

where

$$\eta \equiv \frac{(1-\lambda)\left[\frac{s_L+(1-\phi)\beta_L}{1-\phi\theta} - s_H\right] + \lambda\,(1-\phi)\,\beta_L}{(1-\lambda)\left[(1-\phi)\,\beta_H - \frac{(1-\phi)\beta_L}{1-\phi\theta}\right]}. \tag{4.50}$$

Define

$$\hat{u} \equiv \frac{(\alpha-\lambda)\,s_L + \alpha\,(1-\phi)\,\beta_L}{(1-\lambda)\,s_L + \alpha\,(1-\phi)\,\beta_L}.$$

We are now in a position to establish some properties of the function M.

Lemma 12 *Let $\gamma = \alpha$. Then the function M satisfies that*

(a) $M'\,(0) < 0$ and

(b) if $(1-\phi\theta)\,\beta_H - \beta_L > 0$, *then* $M'\,(u) \geq 0$ *holds iff* $u \geq \widehat{u}$ *and*

(c) if $(1-\phi\theta)\,\beta_H - \beta_L < 0$, *then* $M'\,(u) \geq 0$ *holds iff* $u \leq \widehat{u}$.

Proof. It is easy to show that from (4.49)

$$M'\,(u_{t+1}) = \frac{(1-u_{t+1})\,Q'\,(u_{t+1}) - Q\,(u_{t+1})}{(1-\phi)\,\beta_H - \frac{(1-\phi)\beta_L}{1-\phi\theta}}. \tag{4.51}$$

From (4.51), $M'\,(u_{t+1}) > 0$ holds iff $(1-u_{t+1})\,Q'\,(u_{t+1}) - Q\,(u_{t+1}) > 0$ and $(1-\phi)\,\beta_H - \frac{(1-\phi)\beta_L}{1-\phi\theta} > 0$. There are two cases to be considered. First, consider the case with $(1-\phi\theta)\,\beta_H - \beta_L > 0$. From (4.51), it is easy to check that $M'\,(u_{t+1}) > 0$ holds iff $(1-u_{t+1})\,Q'\,(u_{t+1}) -$

$Q(u_{t+1}) > 0$. Use the expressions for $Q(u)$ and $Q'(u)$ to rewrite the condition as

$$\begin{aligned} & \frac{1}{\alpha}\left\{\frac{s_L}{1-u_{t+1}} - \left(s_L + (1-u_{t+1})(1-\phi)\beta_L\right)\frac{\gamma(1-\lambda)}{\lambda+(1-\lambda)u}\right\} \\ > & \left(\frac{s_L}{1-u_{t+1}} + (1-\phi)\beta_L\right), \end{aligned}$$

which can be written as the quadratic form

$$\begin{aligned} & (1-\lambda)(\alpha-\gamma)(1-\phi)\beta_L u^2 \\ & + \left[(1+\gamma-\alpha)(1-\lambda)s_L - 2(1-\lambda)(\alpha-\gamma)(1-\phi)\beta_L + \alpha(1-\phi)\beta_L\right]u \\ & + (\lambda(1-\alpha) - (1-\lambda)\gamma)s_L - (\gamma(1-\lambda)+\alpha\lambda)(1-\phi)\beta_L \\ > & \ 0. \end{aligned}$$

Assuming $\alpha \equiv \gamma$, the above inequality holds if

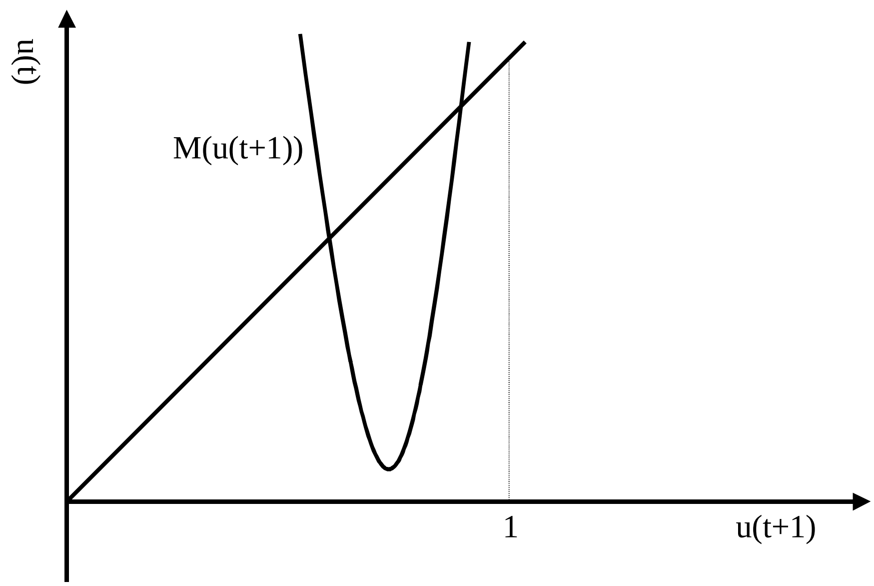
$$u > \frac{(\alpha-\lambda)s_L + \alpha(1-\phi)\beta_L}{(1-\lambda)s_L + \alpha(1-\phi)\beta_L} \equiv \hat{u}.$$

If $0 < \hat{u} < 1$, then the function M is U-shaped with one trough.

Next consider the case with $(1-\phi\theta)\beta_H - \beta_L < 0$. From (4.51), $M'(u_{t+1}) > 0$ holds iff $(1-u_{t+1})Q'(u_{t+1}) - Q(u_{t+1}) < 0$. Therefore, with the assumption that $\alpha \equiv \gamma$, $M'(u_{t+1}) > 0$ holds if $u < \hat{u}$. If $0 < \hat{u} < 1$, then the function M is inverse U-shaped with one peak. ■

Lemma 12 states that if the condition $(1-\phi\theta)\beta_H - \beta_L > 0$ is satisfied, the function M is U-shaped with one trough. Otherwise the function is inverse U-shaped with one peak. A typical configuration of the function is presented in Figure 4-7.

Figure 4-7. Function M



4.7.4 Steady State

In steady state, $u_{t+1} = u_t \equiv u$ and $k_{t+1} = k_t \equiv k$ hold in equation (4.49). This yields

$$u = M(u) \equiv \frac{(1-u)\,Q(u)}{(1-\phi)\,\beta_H - \frac{(1-\phi)\beta_L}{1-\phi\theta}} - \eta. \tag{4.52}$$

It is easy to show that the map $u = M(u)$ may have two fixed points, denoted $u = u_l$ and $u = u_h$. The central result of this subsection is stated below.

Proposition 14 *If* $(1-\phi\theta)\,\beta_H - \beta_L > 0$*, then*

$$\left.\frac{du}{d\theta}\right|_{u=u_l} > 0 \text{ and } \left.\frac{du}{d\theta}\right|_{u=u_h} < 0.$$

Proof. From (4.52),

$$\frac{du}{d\theta} = \frac{\partial M/\partial\theta}{1 - M'(u)}, \tag{4.53}$$

where

$$\frac{\partial M}{\partial\theta} = \frac{(1-u)\frac{\partial Q(u)}{\partial\theta}\left[(1-\phi)\beta_H - \frac{(1-\phi)\beta_L}{1-\phi\theta}\right] + (1-u)Q(u)\frac{(1-\phi)\beta_L}{(1-\phi\theta)^2}\phi}{\left[(1-\phi)\beta_H - \frac{(1-\phi)\beta_L}{1-\phi\theta}\right]^2} - \frac{\partial\eta}{\partial\theta}. \tag{4.54}$$

From (4.46) and (4.50),

$$\begin{aligned}\frac{\partial Q(u)}{\partial\theta} &= \frac{1}{\alpha}\left(\frac{\frac{s_L}{1-u} + (1-\phi)\beta_L}{(1-\phi\theta)(1-\alpha)A(\phi[\lambda+(1-\lambda)u])^\gamma}\right)^{1/\alpha-1} \\ &\quad\times\frac{\left[\frac{s_L}{1-u} + (1-\phi)\beta_L\right]\phi(1-\alpha)A(\phi[\lambda+(1-\lambda)u])^\gamma}{[(1-\phi\theta)(1-\alpha)A(\phi[\lambda+(1-\lambda)u])^\gamma]^2} \\ &= \frac{1}{\alpha}\frac{\phi}{(1-\phi\theta)}Q(u)\end{aligned} \tag{4.55}$$

and

$$\begin{aligned}\frac{\partial\eta}{\partial\theta} &= \frac{(s_L + (1-\phi)\beta_L)\phi}{(1-\lambda)\left[(1-\phi)\beta_H - \frac{(1-\phi)\beta_L}{1-\phi\theta}\right](1-\phi\theta)^2} \\ &\quad+\frac{\left[(1-\lambda)\left[\frac{s_L+(1-\phi)\beta_L}{1-\phi\theta} - s_H\right] + \lambda(1-\phi)\beta_L\right](1-\phi)\beta_L\phi}{(1-\lambda)\left[(1-\phi)\beta_H - \frac{(1-\phi)\beta_L}{1-\phi\theta}\right]^2(1-\phi\theta)^2}.\end{aligned} \tag{4.56}$$

Substitute (4.55) and (4.56) into (4.54) to discover

$$\begin{aligned}\frac{\partial M}{\partial\theta} &= \frac{(1-u)Q(u)}{\left((1-\phi)\beta_H - \frac{(1-\phi)\beta_L}{1-\phi\theta}\right)^2}\left(\frac{1}{\alpha}\frac{\phi}{(1-\phi\theta)}\left((1-\phi)\beta_H - \frac{(1-\phi)\beta_L}{1-\phi\theta}\right) + \frac{(1-\phi)\beta_L}{(1-\phi\theta)^2}\phi\right) \\ &\quad-\frac{(s_L + (1-\phi)\beta_L)\phi}{(1-\lambda)\left[(1-\phi)\beta_H - \frac{(1-\phi)\beta_L}{1-\phi\theta}\right](1-\phi\theta)^2} \\ &\quad-\frac{\left[(1-\lambda)\left[\frac{s_L+(1-\phi)\beta_L}{1-\phi\theta} - s_H\right] + \lambda(1-\phi)\beta_L\right](1-\phi)\beta_L\phi}{(1-\lambda)\left[(1-\phi)\beta_H - \frac{(1-\phi)\beta_L}{1-\phi\theta}\right]^2(1-\phi\theta)^2}\end{aligned}$$

Therefore, $\partial M/\partial\theta > 0$ holds iff

$$(1-\lambda)(1-u)Q(u)\left(\frac{1}{\alpha}\left(\beta_H - \frac{\beta_L}{1-\phi\theta}\right) + \frac{\beta_L}{(1-\phi\theta)}\right)$$

$$
\begin{aligned}
& -\frac{s_L + (1-\phi)\,\beta_L}{(1-\phi\theta)}\left(\beta_H - \frac{\beta_L}{1-\phi\theta}\right) \\
& -\left((1-\lambda)\left(\frac{s_L + (1-\phi)\,\beta_L}{1-\phi\theta} - s_H\right) + \lambda\,(1-\phi)\,\beta_L\right)\frac{\beta_L}{(1-\phi\theta)} \\
> \;& 0. \qquad (4.57)
\end{aligned}
$$

Use (4.52) to reduce (4.57) to

$$
u+\eta > \frac{\left[\frac{s_L+(1-\phi)\beta_L}{(1-\phi\theta)}\left[\beta_H - \frac{\beta_L}{1-\phi\theta}\right] + \left[(1-\lambda)\left[\frac{s_L+(1-\phi)\beta_L}{1-\phi\theta} - s_H\right] + \lambda\,(1-\phi)\,\beta_L\right]\frac{\beta_L}{(1-\phi\theta)}\right]}{(1-\lambda)\left[(1-\phi)\,\beta_H - \frac{(1-\phi)\beta_L}{1-\phi\theta}\right]\left[\frac{1}{\alpha}\left[\beta_H - \frac{\beta_L}{1-\phi\theta}\right] + \frac{\beta_L}{(1-\phi\theta)}\right]},
$$

so

$$
u > \frac{-\left[\beta_H - \frac{\beta_L}{1-\phi\theta}\right]\frac{1}{\alpha}\left[(1-\lambda)\,(1-\alpha)\,\frac{s_L+(1-\phi)\beta_L}{1-\phi\theta} - (1-\lambda)\,s_H + \lambda\,(1-\phi)\,\beta_L\right]}{(1-\lambda)\left[(1-\phi)\,\beta_H - \frac{(1-\phi)\beta_L}{1-\phi\theta}\right]\left[\frac{1}{\alpha}\left[\beta_H - \frac{\beta_L}{1-\phi\theta}\right] + \frac{\beta_L}{(1-\phi\theta)}\right]}. \qquad (4.58)
$$

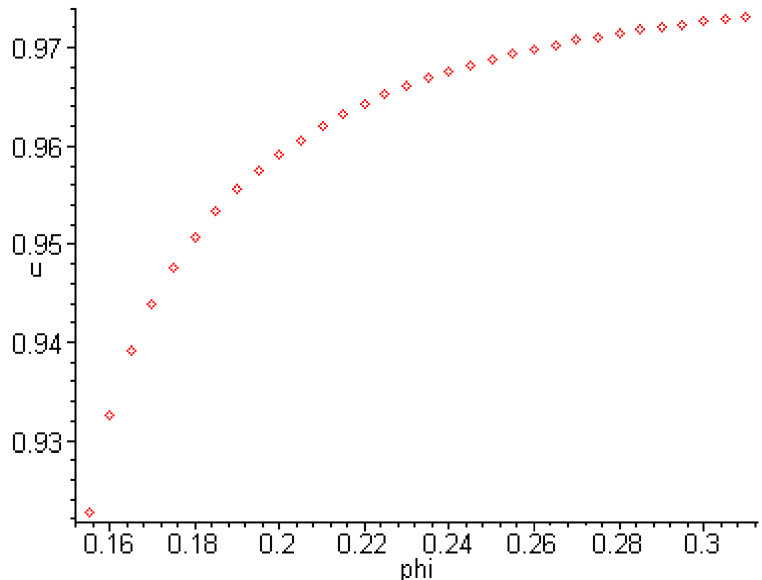
This is satisfied for policy parameters that satisfy $(1-\phi\theta)\,\beta_H - \beta_L > 0$ since the right-hand-side of the expression (4.58) becomes negative. Therefore, $\partial M/\partial\theta > 0$. Since $M'(u_l) < 1 < M'(u_h)$, the rest of the proof is immediate from (4.53). ■

Proposition 14 asserts that in the case where public employment programs create a positive externality to private production and where the volume of the program (as captured by ϕ) is not too high, a small decrease in the gap between private and public sector wages raises (reduces) the unemployment rate and the capital-labor ratio at the low-u (high-u) steady state. The intuition underlying this result follows the same logic as in Proposition 11. In equilibrium, (4.6) must hold in equality. An increase in θ raises the payoff for type-L workers of non-participation. To keep (4.6) in equality, the firms react and raise the left hand side of (4.6). At the low-u (high-u) steady state this can be done by raising (reducing) u. Therefore, an increase in θ results in an increase (decrease) in u at the low-u (high-u) steady state.

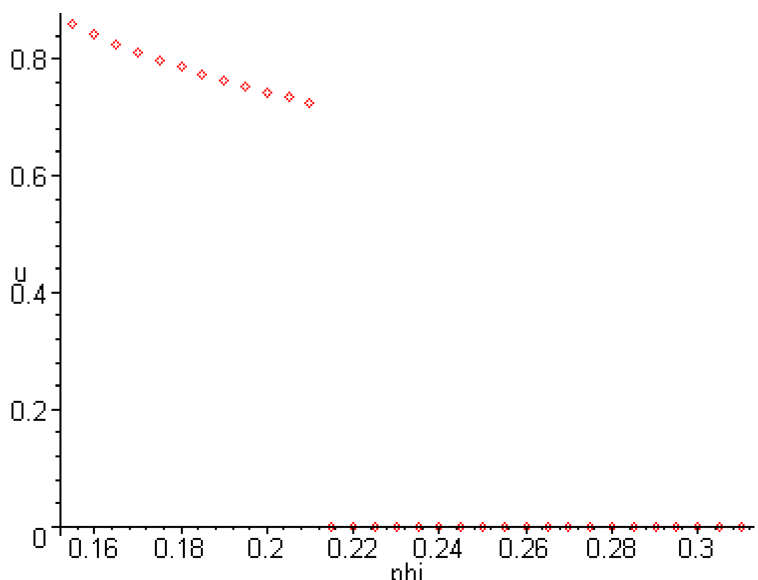
The effect of an increase in ϕ on u is analytically ambiguous and messy. To provide a feel for the direction of the effects, two examples are presented below.

Example 4 Let the parameters of the economy be $A = 3.5$, $\alpha = 0.6$, $\gamma = 0.6$, $\lambda = 0.3$, $\beta_H = 0.8$, $\beta_L = 0.6$, $s_L = 0.1$, $s_H = 0$ and $\theta = 0.8$. For this specification, Figure 4-8 illustrates that at the high (low) capital steady state an increase in ϕ raises (reduces) the unemployment rate.

Figure 4-8. Effect of an increase in ϕ on u
($A = 3.5$, $\alpha = \gamma = 0.6$, $\lambda = 0.3$, $\beta_H = 0.8$, $\beta_L = 0.6$, $s_L = 0.1$, $s_H = 0$, $\theta = 0.8$)



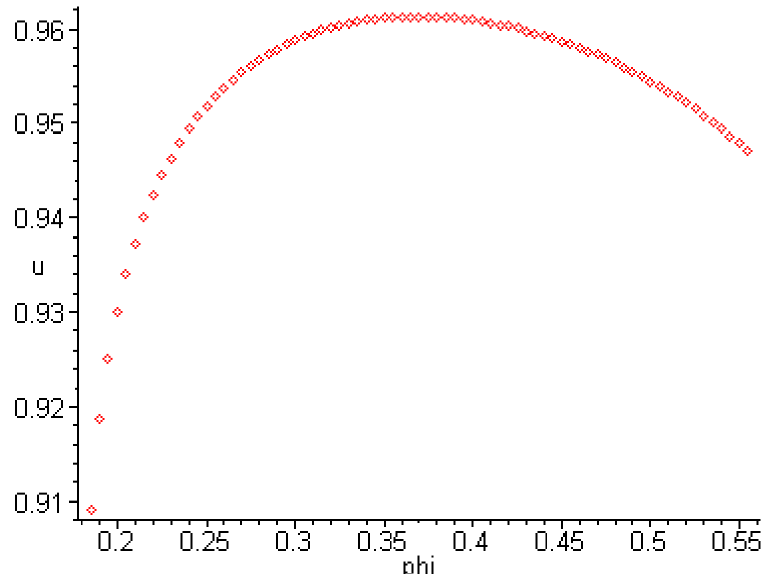
Effect at the High-Activity Steady State



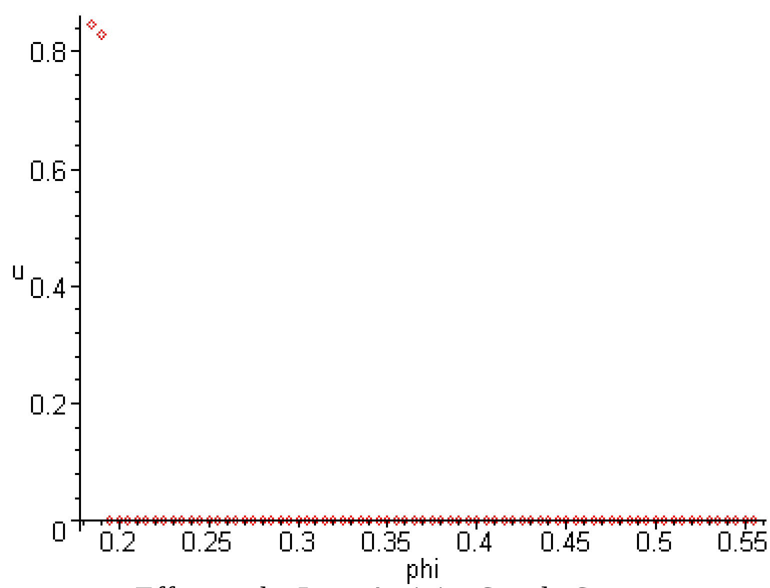
Effect at the Low-Activity Steady State

Example 5 Let the parameters of the economy be $A = 3$, $\alpha = 0.6$, $\gamma = 0.6$, $\lambda = 0.3$, $\beta_H = 0.9$, $\beta_L = 0.5$, $s_L = 0.1$, $s_H = 0$ and $\theta = 0.8$. For this specification, Figure 4-9 illustrates that at the high capital steady state the effect of an increase in ϕ on u may be non-monotonic.

Figure 4-9. Effect of an increase in ϕ on u
($A = 3$, $\alpha = \gamma = 0.6$, $\lambda = 0.3$, $\beta_H = 0.9$, $\beta_L = 0.5$, $s_L = 0.1$, $s_H = 0$, $\theta = 0.8$)



Effect at the High-Activity Steady State



Effect at the Low-Activity Steady State

The latter example captures the essence of an interesting tension. On the one hand, increased employment in public programs increases the beneficial externality entering private production functions while, on the other hand, society pays a cost in the sense that "income maintenance" worsens the severity of the information friction in the labor market.

4.7.5 Dynamics

Dynamical equilibria of the model may be studied by examining (4.49). Note that (4.49) represents the forward dynamics. The following examples illustrate the local stability properties of the two steady states.

Example 6 Let the parameters of the economy be $A = 3.15$, $\alpha = 0.6$, $\gamma = 0.6$, $\lambda = 0.3$, $\beta_H = 0.8$, $\beta_L = 0.6$, $s_L = 0.1$, $s_H = 0$, $\theta = 0.8$ and $\phi = 0.2$. With this specification, $u_l = 0.85$, $u_h = 0.92$, $M'(u_l) = -12.96 < -1$ and $M'(u_h) = 25.06 > 1$. In this case, the low-capital steady state u_l is unstable in the forward dynamics and hence is stable in the normal backward dynamics.

Again, the possibility of multiple asymptotically stable steady states emerges along with the possibility of observing development trap phenomena. Countries may succeed in getting "unstuck" by closing the gap between private and public sector wages, as the next example illustrates.

Example 7 Let $A = 3.15$, $\alpha = 0.6$, $\gamma = 0.6$, $\lambda = 0.3$, $\beta_H = 0.8$, $\beta_L = 0.6$, $s_L = 0.1$, $s_H = 0$ and $\phi = 0.2$ as above, but now $\theta = 0.8585$. With this specification, $u_l = 0.887$, $u_h = 0.89$, $M'(u_l) = -0.62 > -1$ and $M'(u_h) = 2.69 > 1$. In this case, the low-capital steady state u_l is stable in the forward dynamics and hence is unstable in the normal backward dynamics.

4.8 Concluding Remarks

This chapter presents a structure where unemployment arises endogenously and then uses it to study government-funded employment-generation programs. The main contribution of this chapter is to recognize that government employment is largely ignored in the standard macroeconomic models and to make a first attempt to present an analytical framework for studying government employment. The model employed is a standard overlapping generations model with production where agents are heterogenous in terms of their intrinsic productive abilities, and this is private information. Private sector firms use equilibrium unemployment as a sorting device, offering a menu of wages and unemployment probabilities that entice only the high ability people to seek employment with them. The government steps in and sets up a publicly-funded employment program which indiscriminately employs a given fraction of those unemployed in the private sector and funds the wage bill by imposing a lump-sum tax on all agents. The very involvement of the government affects the information friction in the labor market which, in turn, has important effects on capital accumulation. Multiple long-run equilibria are shown to be possible. The existing level of government involvement matters in that below (above) a critical level, further increases in the volume of the public employment programs improves (lowers) long-run real activity for countries stuck at the low-activity steady state. If there are exactly two steady state equilibria, then the high real-activity steady state is dynamically stable. It is possible for the low activity steady state to be stable too. An increase in the size of the employment program can get the economy out of this poverty trap.

In the current framework, the government is assumed not to have access to *any* screening technologies when it comes to hiring people for the public sector. Such a stance on government action may be relevant in instances of pure pump-priming; *i.e.*, where the sole purpose of the public employment program is to reduce unemployment with no attention paid to who gets hired. However, there are many countries around the world where the public sector competes one on one with the private sector to hire from among the very best. Indeed, Panizza (1999) reports on the presence of a "public sector wage premium" in some countries. This suggests that employment in the public sector may be more attractive than in the private sector in some cases. A direction for future research would be to extend the current setup to allow for this possibility.

Another such direction is to incorporate a more realistic screening technology in the labor market. As noted, the model generates unusually high equilibrium unemployment rates. This is due to the simplifying assumption that the firms have to use the wage-unemployment rate contracts to screen out worker types. Presumably, introduction of a more realistic screening technology would generate a realistic level of equilibrium unemployment. Bose and Cothren (1996) develop an endogenous growth model with asymmetric information in the credit market in which both credit rationing and costly screening technology co-exist. One direction of future work is to adopt Bose and Cothren's screening device. Such a modification may change the qualitative as well as quantitative properties of unemployment rate.

Nevertheless, the adverse selection model employed in this paper has an important advantage. It makes it possible to study how government intervention in the labor market influences agents' incentive to work. In their recent paper, Cole and Ohanian (2000) revisit New Deal policies. They document that there are two facts about the Great Depression that cannot be explained by the standard growth model: slow recovery and high wage rate. An interesting future work is to consider how New Deal policies, especially direct job creation policies, might have affected the economy through changing workers' attitude toward work.

4.A Proof of Proposition 10

We begin the proof by considering all of the constraints, including those that are irrelevant. Let (ω_t^H, u_t^H) and (ω_t^L, u_t^L) denote (separating) contracts for type-H and type-L, respectively. The number of type H workers employed in the private sector is $(1-\lambda)\left(1-u_t^H\right)$, while the number of type L is $\lambda\left(1-u_t^L\right)$. Then the firm's maximization problem is[50]:

$$\max F\left(K_t,(1-\lambda)\left(1-u_t^H\right)\right)-r_tK_t-\omega_t^H(1-\lambda)\left(1-u_t^H\right)-\omega_t^L\lambda\left(1-u_t^L\right)$$

subject to

$$\left(1-u_t^H\right)\left(\omega_t^H-s_H\right)+u_t^H\left(x_t^H-s_H\right) \geq x_t^H \tag{PH}$$

$$\left(1-u_t^L\right)\left(\omega_t^L-s_L\right)+u_t^L\left(x_t^L-s_L\right) \leq x_t^L \tag{NPL}$$

$$\left(1-u_t^H\right)\left(\omega_t^H-s_H\right)+u_t^H\left(x_t^H-s_H\right) \geq \left(1-u_t^L\right)\left(\omega_t^L-s_H\right)+u_t^L\left(x_t^H-s_H\right) \tag{IH}$$

$$\left(1-u_t^L\right)\left(\omega_t^L-s_L\right)+u_t^L\left(x_t^L-s_L\right) \geq \left(1-u_t^H\right)\left(\omega_t^H-s_L\right)+u_t^H\left(x_t^L-s_L\right) \tag{IL}$$

where $x_t^H \equiv \phi\overline{\omega}_t+(1-\phi)\beta_H$ and $x_t^L \equiv \phi\overline{\omega}_t+(1-\phi)\beta_L$. (PH) is the participation constraint for type-H workers while (NPL) is the non-participation constraint for type-L workers. (IH) ((IL)) is the incentive compatibility, or self-selection, constraint for type-H (type-L) workers. Combine (PL) and (IL) to give

$$\left(1-u_t^H\right)\left(\omega_t^H-s_L\right)+u_t^H\left(x_t^L-s_L\right)\leq x_t^L. \tag{4.59}$$

Since a type L worker has zero productivity, either $\omega_t^L=0$ or $u_t^L=1$. Suppose $\omega_t^L>0$ or $u_t^L=1$. Then the right-hand-side of (IH) is $x_t^H-s_H$, which is strictly less than x_t^H, the right-hand-side of (PH). Hence (IH) is redundant. Suppose, on the other hand, that $\omega_t^L=0$

[50] Note that we omit the lump-sum tax τ_t and the interest on savings r_{t+1} since they will be canceled out.

or $u_t^L < 1$. In this case, the right-hand-side of (IH) is $u_t^L x_t^H - s_H$, which is strictly less than x_t^H since $\left(1 - u_t^L\right) x_t^H + s_H > 0$. Thus, (IH) is redundant. Therefore, only (PH) and (4.59) are valid constraints. Note that the pair (ω_t^L, u_t^L) does not appear in (PH) or (4.59). The rest of the proof is standard.[51]

[51] See, for example, Kreps (1990) and Mas-Colell *et. al.* (1995).

4.B Proof of Lemma 7

The basic strategy here is to find the condition that violates (4.13). Consider $\widehat{\omega}_t$. Manipulate (4.7) to yield

$$\widehat{\omega}_t \leq \max_{\widehat{K}_t \leq K_t} \left[F\left(\widehat{K}_t, (1-\lambda)(1-\widehat{u}_t)\right) - r_t \widehat{K}_t \right] \frac{1}{1-\widehat{u}_t}. \tag{4.60}$$

$\widehat{K}_t$ must satisfy the first order condition

$$F_2\left(\widehat{K}_t, (1-\lambda)(1-\widehat{u}_t)\right) \equiv f'\left(\frac{\widehat{K}_t}{(1-\lambda)(1-\widehat{u}_t)}\right) \geq r_t.$$

The inequality above captures the possibility that the interior maximum may not exist for the problem (4.60). Since $K_t = (1-\lambda)(1-u_t)k_t$ and $f'(k_t) = r_t$,

$$f'\left(\frac{K_t}{(1-\lambda)(1-\widehat{u}_t)}\right) \equiv f'\left(\frac{(1-u_t)k_t}{(1-\widehat{u}_t)}\right) \geq f'((1-u_t)k_t) > f'(k_t) = r_t.$$

Thus $\widehat{K}_t = K_t$. Then (4.60) can be written as

$$\widehat{\omega}_t \equiv \left[F(k_t(1-\lambda)(1-u_t), (1-\lambda)(1-\widehat{u}_t)) - f'(k_t)k_t(1-\lambda)(1-u_t)\right]\frac{1}{1-\widehat{u}_t}, \tag{4.61}$$

so that

$$\begin{aligned}(1-\widehat{u}_t)(\widehat{\omega}_t - x) \quad &\equiv \quad F(k_t(1-\lambda)(1-u_t), (1-\lambda)(1-\widehat{u}_t)) \\ &\quad -f'(k_t)k_t(1-\lambda)(1-u_t) - (1-\widehat{u}_t)x.\end{aligned}$$

Therefore, the maximum of $(1-\widehat{u}_t)(\widehat{\omega}_t - x)$ is the solution to

$$\max_{(1-\widehat{u}_t)} \left[F(k_t(1-\lambda)(1-u_t), (1-\lambda)(1-\widehat{u}_t)) - f'(k_t)k_t(1-\lambda)(1-u_t) - (1-\widehat{u}_t)x\right]. \tag{P}$$

We need to find the conditions under which (4.13) is violated. In other words, we need to find the conditions under which the maximum of $(1-\widehat{u}_t)(\widehat{\omega}_t - x)$ is less than or equal to $(1-u_t)(\omega_t - x)$.

Suppose

$$(1-\lambda) F_2 (k_t (1-u_t), 1) \equiv (1-\lambda)\, \omega (k_t (1-u_t)) \geq x. \tag{4.62}$$

Then (P) does not have an interior maximum. Thus, it is maximized by $\widehat{u}_t = 0$. From (4.13),

$$\widehat{\omega}_t \leq (1-u_t)\, \omega_t + u_t x. \tag{4.63}$$

We need to find $\widehat{\omega}_t$. When $\widehat{u}_t = 0$, from (4.61) $\widehat{\omega}_t$ satisfies

$$\widehat{\omega}_t = F (k_t (1-\lambda)(1-u_t), (1-\lambda)) - f'(k_t)\, k_t (1-\lambda)(1-u_t). \tag{4.64}$$

Therefore, (4.63) and (4.64) imply that there does not exist a profitable pooling equilibrium if

$$\begin{aligned} & F (k_t (1-\lambda)(1-u_t), (1-\lambda)) - f'(k_t)\, k_t (1-\lambda)(1-u_t) \\ \leq\ & (1-u_t)\, \omega_t + u_t (\phi \overline{\omega}_t + (1-\phi)\, \beta_H). \end{aligned} \tag{4.65}$$

Suppose (4.62) is not satisfied. (P) is maximized by

$$(1-\lambda) F_2 (k_t (1-\lambda)(1-u_t), (1-\lambda)(1-\widehat{u}_t)) = x,$$

which can be reduced to

$$(1-\lambda)\, \omega \left(k_t \frac{1-u_t}{1-\widehat{u}_t}, 1 \right) = x. \tag{4.66}$$

Solve (4.66) for $(1-\widehat{u}_t)$ to obtain

$$1-\widehat{u}_t = \frac{k_t (1-u_t)}{\omega^{-1}\left(\frac{x}{1-\lambda}\right)}. \tag{4.67}$$

Thus, the maximum of $(1-\widehat{u}_t)(\widehat{\omega}_t - x)$ is given by

$$\frac{k_t(1-u_t)}{\omega^{-1}\left(\frac{x}{1-\lambda}\right)}(\widehat{\omega}_t - x).$$

(4.13) can then be written as

$$\frac{k_t}{\omega^{-1}\left(\frac{x}{1-\lambda}\right)}(\widehat{\omega}_t - x) \le \omega_t - x. \tag{4.68}$$

To find $\widehat{\omega}_t$, from (4.61), compute

$$\begin{aligned}\widehat{\omega}_t &= (1-\lambda)\left(F\left(k_t\frac{(1-u_t)}{(1-\widehat{u}_t)},1\right) - f'(k_t)\,k_t\frac{(1-u_t)}{(1-\widehat{u}_t)}\right)\\ &= (1-\lambda)\left(f\left(\omega^{-1}\left(\frac{x}{1-\lambda}\right)\right) - f'(k_t)\,\omega^{-1}\left(\frac{x}{1-\lambda}\right)\right).\end{aligned} \tag{4.69}$$

Therefore, (4.68) and (4.69) imply that there does not exist a profitable pooling equilibrium if

$$\begin{aligned}&k_t(1-\lambda)\left(f\left(\omega^{-1}\left(\frac{x}{1-\lambda}\right)\right) - f'(k_t)\,\omega^{-1}\left(\frac{x}{1-\lambda}\right)\right)\\ \le\ &\omega^{-1}\left(\frac{x}{1-\lambda}\right)\omega_t + \left(k_t - \omega^{-1}\left(\frac{x}{1-\lambda}\right)\right)x.\end{aligned} \tag{4.70}$$

References

Adachi, Hiroyuki. "A Search-Theoretic Approach to Two-Sided Matching." Ph.D. dissertation, State University of New York at Buffalo (1998)

Al-Marhubi, Fahim A. "Corruption and Inflation." *Economics Letters*, 66 (2000) 199-202.

Aschauer, David Alan. "Is Public Expenditure Productive?" *Journal of Public Economics*, 23 (1989) 177-200.

Azariadis, Costas. *Intertemporal Macroeconomics*, Basil Blackwell, Cambridge (1993).

Baily, Martin Neil, and James Tobin. "Inflation-Unemployment Consequences of Job Creation Policies." In *Creating Jobs: Public Employment Programs and Wage Subsidies*, edited by John L. Palmer, Brookings Institution, Washington, D.C. (1978)

Barro, Robert J. "Government Spending in a Simple Model of Endogenous Growth." *Journal of Political Economy* 98 (1990) s103-125.

Bencivenga, Valerie R., and Bruce D. Smith. "Unemployment, Migration, and Growth." CAE Working Paper #95-17 (1995)

Bencivenga, Valerie R., and Bruce D. Smith. "Unemployment, Migration, and Growth." *Journal of Political Economy* 105 (1997) 582-608.

Bester, Helmut. "Bargaining, Search Costs and Equilibrium Price Distribution." *Review of Economic Studies* (1988) 201-214.

Betts, Caroline, and Joydeep Bhattacharya. "Unemployment, Credit Rationing, and Capital Accumulation: A Tale of Two Frictions." *Economic Theory*, 12 (1998) 489-517.

Bhagwati, Jagdish. *India in Transition: Freeing The Economy*, Clarendon Press, Oxford (1993)

Bhattacharya, Joydeep, Guzman, Mark G., Smith, Bruce D. "Some Even More Unpleasant Monetarist Arithmetic." *Canadian Journal of Economics 31* (1998) 596-623.

Bhattacharya, Joydeep, and Joseph H. Haslag. "Seigniorage in a Neoclassical Economy: Some Computational Results." mimeo, (1998) Bhattacharya, Joydeep, and Joseph H. Haslag. "Monetary Policy Arithmetic: Some Recent Contributions." Federal Reserve Bank of Dallas *Economic and Financial Review* (1999) 26-36.

Binmore, Ken, Ariel Rubinstein, and Asher Wolinsky. "The Nash Bargaining Solution in Economic Modeling." *Rand Journal of Economics* 17 (1986) 176-188.

Bose, Niloy, and Richard Cothren. "Equilibrium Loan Contracts and Endogenous Growth in the Presence of Asymmetric Information." *Journal of Monetary Economics* 38 (1996) 363-376.

Bullard, James, and Steven Russell. "How Costly is Sustained Low Inflation for the U.S. Economy?" mimeo, (1997)

Bullard, James, and Steven Russell. "An Empirically plausible model of low real interest rates and unbacked government debt." *Journal of Monetary Economics* 44 (1999) 477-508.

Burdett, Kenneth, Melvyn G. Coles, Nobuhiro Kiyotaki, and Randall Wright. "Buyers and Sellers: Should I Stay or Should I Go?" *American Economic Review* 85 (1995) 281-286.

Camera, Gabriele, and Dean Corbae. "Money and Price Dispersion." *International Economic Review* 40 (1999)

Canzoneri, Matthew B., Robert E. Cumby, and Behzad T. Diba. "Is the Price Level Determined by the Needs of Fiscal Solvency?" NBER working paper 6472 (1998)

Clark, John M. *Economics of Planning Public Works.* United States Government Printing Office, Washington, D.C., (1935)

Click, Reid W. "Seignorage in a Cross-Section of Countries." *Journal of Money, Credit, and Banking,* (1998) 30 (2), 154-170

Cochrane, John H. "Frictionless View of U.S. Inflation." in NBER Macroeconomics Annual (1998)

Cochrane, John H. "Money as Stock: Price Level Determination With No Money Demand." NBER Working Paper 7498 (2000)

Cole, Harold L., and Lee E. Ohanian. "New Deal Policies and the Persistence of the Great Depression: AGeneral Equilibrium Analysis." Federal Reserve Bank of Minneapolis, Working Paper 597, (2000) Coles, Melvyn G., and Randall Wright. “A Dynamic Equilibrium Model of Search, Bargaining, and Money.” *Journal of Economic Theory* 78 (1998) 32-54.

Diamond, Peter. “National Debt in a Neoclassical Growth Model.” *American Economic Review*, 55, (1965) 1126-50.

Drazen, Allan. “Tight Money and Inflation: Further Results.” *Journal of Monetary Economics*, 15 (1985) 113-20.

Darby, Michael R. “Some Pleasant Monetarist Arithmetic.” *Federal Reserve Bank of Minneapolis Quarterly Review,* (1984)

Ehrenberg, Ronald, and Joshua Schwarz. “Public Sector Labor Markets.” in O. Ashenfelter and R. Layard, eds., *Handbook of Labor Economics*, Amsterdam, North Holland (1965)

Espinosa, Marco, and Stephen Russell. “Open Market Operations with Conventional Lasting Real Effects.” *Canadian Journal of Economics*, (1998) 92-103.

Freeman, Scott. “Reserve requirements and optimal seigniorage”, *Journal of Monetary Economics* 19 (1987) 307-14

Futagami, Koichi, Yuichi Morita, and Akihisa Shibata. “Dynamic Analysis of an Endogenous Growth Model with Public Capital.” *Scandinavian Journal of Economics* 95 (1993) 607-625.

Galor, Oded, and Harl E. Ryder. “Existence, Uniqueness, and Stability of Equilibrium in an Overlapping-Generations Model with Productive Capital.” *Journal of Economic Theory,* 49 (1989) 360-385.

Gelb, A, J. B. Knight, and R. H. Sabot. “Public Sector Employment, Rent Seeking and Economic Growth.” *Economic Journal,* 101 (1991) 1186-1199.

Greenwood, John “Tight Money Strangles Japan’s Recovery” *The Wall Street Journal*, July 16 (1998)

Hayakawa, Hiroaki, and Yiannis Venieris. “Consumer Interdependence via Reference Groups.” *Journal of Political Economy,* 85 (1977) 599-615.

Hetzel, Robert L. “A Quantity Theory Framework for Monetary Policy.” Federal Reserve Bank of Richmond *Economic Review*, Summer (1993) 35-47.

Highfield, Richard A., Haureen O'Hara, and Bruce Smith. "Do Open Market Operation Matter? Theory and Evidence from the Second Bank of the United States." *Journal of Economic Dynamics and Control*, 20 (1996) 479-519.

Kahn, R. F. "The Relation of Home Investment to Unemployment." *Economic Journal*, (1931)

Keynes, John. M *The General Theory of Employment, Interest, and Money*, Harcourt, Brace & World Inc. (1936)

Kiyotaki, Nobuhiro, and Randall Wright. "On Money as a Medium of Exchange." *Journal of Political Economy*, 97 (1989) 927-954.

Kiyotaki, Nobuhiro, and Randall Wright. "A Contribution to the Pure Theory of Money." *Journal of Economic Theory*, 53 (1991) 215-235.

Kiyotaki, Nobuhiro, and Randall Wright. "A Search Theoretic Approach to Monetary Economics." *American Economic Review*, 83 (1993) 63-77.

Kocherlakota, Narayana, and Christopher Phelan. "Explaining the Fiscal Theory of the Price Level." *Federal Reserve Bank of Minneapolis Quarterly Review*, 23 (1999) 14-23.

Kreps, David M. *A Course in Microeconomic Theory*, Princeton University Press, Princeton (1990)

Li, Victor E. "The Optimal Taxation of Fiat Money in Search Equilibrium," *International Economic Review*, 36 (1995) 927-942.

Mas-Colell, Andrew, Michael D. Whinston, and Jerry R. Green. *Microeconomic Theory*, Oxford University Press, New York (1995)

Mazumdar, Dipak. "Microeconomic Issues of Labor Markets in Developing Countries: Analysis and Policy Implications." (1989), EDI paper no. 40, World Bank

Miller, Preston J. (ed.) *Rational Expectations Revolution*, MIT press, Cambridge, Massachusetts, (1994)

Miller Preston J., and Thomas J. Sargent. "A Reply to Darby." *Federal Reserve Bank of Minneapolis Quarterly Review* (1984)

Mortensen, Dale T. “The Matching Process as a Noncooperative Bargaining Game.” in *The Economics of Information and Uncertainty* (J. J. McCall, Ed.), University of Chicago Press, Chicago, (1982)

Osborne, Martin J., and Ariel Rubinstein. *Bargaining and Markets*. San Diego: Academic Press, (1990)

Panizza, Ugo. “Why Do Lazy People Make More Money? The Strange Case of the Public Sector Premium.” Inter-American Development Bank (1999)

Rothschild, Michael, and Joseph Stiglitz. “Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information.” *Quarterly Journal of Economics,* 91 (1976) 629-649.

Rubinstein, Ariel. “Perfect Equilibrium in a Bargaining Model.” *Econometrica,* 50 (1982) 97-109.

Rubinstein, Ariel, and Asher Wolinsky. “Equilibrium in a Market with Sequential Bargaining.” *Econometrica,* 53 (1985) 1133-1150.

Sargent, Thomas J. *Dynamic Macroeconomic Theory*, Harvard University Press*:* Cambridge, (1987)

Sargent, Thomas J. “A Primer on Monetary and Fiscal Policy.” *Journal of Banking & Finance* 23 (1999) 1463-1482.

Sargent, Thomas J. and Neil Wallace. “Some Unpleasant Monetarist Arithmetic.” *Federal Reserve Bank of Minneapolis Quarterly Review,* (1981) 1-17.

Sargent, Thomas J. and Bruce Smith. “Irrelevance of Open Market Operations in Some Economies with Government Currencies Being Dominated in Rate of Return.” *American Economic Review,* (1987) 78-92.

Schiavo-Campo, Salvatore, Giulio de Tommaso, and Amitabha Mukherjee. “Government Employment and Pay in Global Perspective: A Selective Synthesis of International Facts, Policies and Experience.” mimeo, World Bank (1997a)

Schiavo-Campo, Salvatore, Giulio de Tommaso, and Amitabha Mukherjee. “International Statistical Survey of Government Employment and Wages.” mimeo, World Bank (1997b)

Schmitz, James, A “The Role Played by Public Enterprises: How Much Does it Differ Across Countries?” *Federal Reserve Bank of Minneapolis Quarterly Review*, (1996), 2-15

Shi, Shouyoung. “Money and Prices: A Model of Search and Bargaining.” *Journal of Economic Theory,* 67 (1995) 467-496.

Smith, Bruce D. “Interest on Reserves and Sunspot Equilibria: Friedman’s Proposal Reconsidered.” *Review of Economic Studies,* 58 (1991) 93-105.

Smith, Bruce D.“Efficiency and Determinacy of Equilibrium Under Inflation Targeting.” *Economic Theory,* 4(3) (1994) 327-344.

Trejos, Alberto, and Randall Wright. “Search, Bargaining, Money and Prices: Recent Results and Policy Implications.” *Journal of Money, Credit, and Banking,* 25 (1993) 558-576.

Trejos, Alberto, and Randall Wright. “Search, Bargaining, Money and Prices.” *Journal of Political Economy,* 103 (1995) 118-141.

Wallace, Neil. “A Modigliani-Miller Theorem for Open-Market Operations.” *American Economic Review*, (1981) 267-274.

Wallace, Neil. “A Legal Restrictions Theory of the Demand for “Money” and the Role of Monetary Policy.” *Federal Reserve Bank of Minneapolis Quarterly Review,* (1983).

Wallace, Neil. “Some of the Choices for Monetary Policy.” *Federal Reserve Bank of Minneapolis Quarterly Review,* (1984).

Wallace, Neil. “Some Alternative Monetary Models and Their Implications for the Role of Open-Market Policy.” in Barro ed. *Modern Business Cycle Theory*, Harvard University Press, Cambridge, Massachusetts (1989)

Wallace, Neil. “Absence-of-Double-Coincidence Models of Money: A Progress Report.” *Federal Reserve Bank of Minneapolis Quarterly Review* Winter (1997) 2-20.

Walsh, Carl E. *Monetary Theory and Policy*, MIT press, Cambridge, Massachusetts (1998)

Weil, Philippe. “Permanent Budget Deficits and Inflation.” *Journal of Monetary Economics*, (1987) 393-410

Wolinsky, Asher. “Matching, Search, and Bargaining.” *Journal of Economic Theory,* 42 (1987) 311-333.

World Bank. *World Development Report 1995*, Washington, D.C. (1995).

World Bank. *World Development Report 1997*, Washington, D.C. (1995).