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Persistent Inequality and Private Provision of Public Goods*

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This paper examines the relationship that may be held to exist between the incentive to free ride and the persistence of inequality by studying those situations in which agents invest in human capital and then provide public goods privately. An agent's stock of human capital is affected by his parental stock; the more human capital a parent has, the more effectively his child can learn. Then, the incentives to free ride at provision of public goods in the old period are different among agents; those born of a well-educated parent studies harder than an agent born of a less-educated parent, and this lead to the persistence of inequality.

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1. Introduction

In the standard neoclassical growth model, inequality among individuals will disappear in the long run. Because the poor grow faster than the rich due to concavity of the neoclassical production function. In reality, however, inequality does not in general decline over time.[1] Why, therefore, does such inequality persist? This paper seeks to analyze the persistence of inequality in human capital by introducing private provision of public goods into a simple overlapping generations model.

Because standard growth models assume perfect markets, differences in income and wealth among individuals are generally ignored, and does not have any effects on macroeconomic performance. We are consequently justified in using representative agent models. Thus, models with imperfect markets are necessary to analyze the effects of inequality on macroeconomy. Galor and Zeira (1993), Banerjee and Newman (1993), and Aghion and Bolton (1997) focus on capital market imperfection and technological indivisibility, and argue that since the presence of imperfection prevents the poor from borrowing to invest, the poor remain poor.

Another approach emphasizes human capital formation which is af-

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[1] There are a lot of empirical studies on inequality. For example, see Gottschalk and Smeeding (2000) and references therein.

fected by public education or externality. Bénabou (1993) and Durlauf (1996) study the effect of local externalities on education and show the emergence of endogenous segregation.

In this paper, we focus on the growth of human capital as it is affected by the incentive to free ride. The poorer an individual is, the stronger is his incentive to free ride, and thus the less he invests in human capital. That is, the poor invest less than the rich, which means that inequality persists. The basic mechanism works like this. When young, an individual learns to accumulate human capital, and when old, he voluntarily provides public goods. If a (fortunate) child born of well-educated parents is given a better environment in which to study than a (less fortunate) child born of less-educated parent, he can provide more public goods than a less fortunate child. Then, a fortunate child expects that a less fortunate child will not provide public goods enough when he is old. As a result, a fortunate child studies harder to provide for future public activities. On the other hand, a less fortunate child has a prospect that a fortunate child will be a highly educated adult and do public activities a lot, and thus he will need not to do so. Therefore, a less fortunate child enjoys much leisure and does not study harder than a fortunate child. This difference of investment rates induces the disparity in the growth rates of their human capitals, and this in turn will create an unequal economy.

In the model, a public good is a factor which is provided voluntarily by old agents and which benefits the learning environments for young agents; e.g. knowledge, public order, and education which is received by all children. That is, the learning technologies have a common factor among all young agents. A common factor in production function has an equalizing force. In Tamura (1991), a spillover effect of human capital in the investment technology provides the below-average human capital agents with a higher rate of return on investment than the above-average human capital agents. Thus the below-average human capital agents grow faster than the above-average human capital agents. In our model, a public good to the young has a similar effect. The long-run states of the economy, therefore, are determined by the balance of the equalizing force and the unequalizing force.

This paper has the same implication as Eeckhout and Jovanovic (2002), which claims that knowledge spillovers promote inequality by inducing the technological followers to free ride. They assume that the less knowledge a firm has, the more knowledge it can access from others. Thus the technological followers invest less than the leaders.

This paper regards the disparity between two classes, the rich and the poor, as a definition of inequality, and in our model, therefore, the persistence of inequality means that over time the poor dynasties remain relatively poor and the rich dynasties remain relatively rich. What this means is that in such an economic environment intergenerational mobility may be very weak. Indeed, recent empirical researches in the United States has found that intergenerational mobility is much lower than is generally thought (see e.g. Solon, 1992, 1999; Zimmerman, 1992). Our model supports these empirical findings.

We follow Glomm and Ravikumar (1992) in using functional forms for preferences and learning technologies. Preferences are logarithmic and learning technologies are Cobb-Douglas. Such functional forms, however, generally imply that all agents invest at the same rate (Glomm and Ravikumar, 1992; Tamura, 1991; Bénabou, 1996), but our analysis reveals a disparity in investment rates between the rich and the poor. Such a specification enables us to highlight the influence of incentives to free ride on investment decisions.

We describe our model in Section 2. Section 3 analyses individuals' behaviors, and shows the differences of investment decisions among agents. Section 4 describes the long-run dynamics of the economy, and Section 5 offers a conclusion.

2. Model

Our analysis focuses on the effect made by the private provision of public goods upon the accumulation of human capital. Consider an overlapping generations model in which agents accumulate human capital when they are young and produce private and public goods when they are old.

We suppose that there are M agents in the economy, and that each agent has a parent and a child, so that there is no population growth. The economy is divided into two classes; a well-educated and a less-educated class. The well-educated class contains N agents and the less-educated class $M-N$, where $M>N$, $M\geq 2$, and $N\geq 1$. The difference between the two classes is the stock of human capital, which, in this economy, is the only factor of production. Initially, an agent from a well-educated class has more human capital than his less-educated coeval. We use subscript $i=w$ for a well-educated agent and $i=l$ for a less-educated agent. Following Glomm and Ravikumar (1992), we assume that the preferences of an agent i born at time t are represented by

$$U_{it}=\ln n_{it}+\ln c_{it+1}+\ln E_{t+1}, \tag{1}$$

where n_{it} is leisure at time t, c_{it+1} is consumption at time $t+1$, and E_{t+1} is an educational activity to children at time $t+1$. The term $\ln E_{t+1}$ represents intergenerational altruism.[2] An old agent's utility depends on the level of

[2] This altruism can be thought as a kind of the "warm glow" preferences (see Andreoni, 1989). Generally, it means that parent's utility depends on the amount of bequests he gives to his child. In Glomm and Ravikumar (1992), parent's utility depends on the quality of education his child receives, which is provided by government or by himself. In our model, the quality of education as privately provided public goods. Other specifications are also possible. For example, parent's altruism may be a function of the level of child's human capital. But, in our model, such a specification doesn't alter qualitative results.

education received by his child. In this community, all children are taught in one place and all receive the same educational contents (E_{t+1}) by old agents who take part in this activity voluntarily. If an old agent think that his child is not receiving adequate education (if E_{t+1} is too small), he will himself partake in education. If he is satisfied with education his child receives, which is provided by other old agents, he does nothing himself. So, E_{t+1} is a public good for each old agent.[3)]

Each agent is endowed a unit time when young, and allocates n_{it} units of time for leisure and $1-n_{it}$ units for accumulating human capital. The stock of human capital of an agent i born at time t, h_{it+1}, is determined not only by the learning time but also by the parental stock of human capital, h_{it}, and an educational activity of the old generation, E_t.[4)] The dependence on h_{it} means that a child born of a well-educated parents is given better opportunities to learn, and has better environments in which to study. The learning technology is represented by

$$h_{it+1} = A(1-n_{it})h_{it}^{\beta}E_t^{1-\beta}, \quad 0<\beta<1, \tag{2}$$

where A is a constant parameter. In this learning technology, community education, E_t, is a common factor among agents, and entails that less-educated agents have a greater rate of return to human capital than well-educated agents.[5)] That is, a common factor implies the existence of the equalizing force.

When old, each agent i produces private and public goods from human capital accumulated in his youth. That is, the old agent's activities are to produce private consumption goods, c_{it+1}, and to provide the education of children in the community, e_{it+1}. Since an individual's income is the same as his human capital h_{t+1}, so his budget constraint is as follows;

$$c_{it+1}+e_{it+1}=h_{it+1}, \quad c_{it+1}, \quad e_{it+1} \geq 0 \tag{3}$$

Total education, E_{t+1}, is equal to the sum of all individuals' educational activities.

$$E_{t+1} = \sum_{k=1}^{M} e_{kt+1} \tag{4}$$

Given h_{it}, E_t and e_{-it+1}, an agent i born at time t chooses n_{it}, c_{it+1}, and e_{it+1} to maximize (1) subject to (2), (3), and (4).

[3)] Et is a public factor which affects the learning environment for children. So, we can think of Et not only as education but also as a stock of knowledge or public order, and so on.

[4)] In reality, education is provided by the government. Or parents educate their child at home. Glomm and Ravikumar (1992) analyses such situations. In this paper, we focus on alternative factors which affect human capital accumulation and which have public goods properties, and study the effects on inequality.

[5)] In order to see this effect, rewrite (2) as $h_{it+1}/h_{it} = A(1-n_{it})(E_t/h_{it})^{1-\beta}$. See Tamura (1991).

3. Short-Run Equilibrium

We solve agent i's optimization problem in two steps. First, we solve for optimal consumption and educational activity: choices when an individual is old. Substituting (3) into objective function U_{it} and solving for c_{it+1}, we have the following first order condition;

$$c_{it+1} = \frac{1}{2}(h_{it+1} + e_{-it+1}), \tag{5}$$

where $e_{-it+1} = E_{t+1} - e_{it+1}$. This condition implies $c_{it+1} = E_{t+1}$. That is, given educational activities by others e_{-it+1}, an old agent i allocate human capital to equate consumption and total education. With this relationship, we can rewrite the objective function as

$$U_{it} = \ln n_{it} + 2\ln\left[\frac{1}{2}(h_{it+1} + e_{-it+1})\right]. \tag{6}$$

In the next step, substituting (2) into (6), we solve for optimal leisure n_{it}: a choice when an individual is young.

$$n_{it} = \frac{1}{3} + \frac{e_{-it+1}}{3Ah_{it}^{\beta}E_t^{1-\beta}} \tag{7}$$

We see from equation (7) the situation in which agent i reduces leisure: (i) when other agents supply less education;[6] and (ii) when the stock of human capital of a parent is large. That is, an agent born of a well-educated family studies harder than an agent born of a less-educated family. These two are obviously related. In our model, education is a public good, i.e. an agent's utility depends on not only his own contribution but also on the contributions by the others. Then, we can rewrite the utility function as $\ln(e_i + e_{-i})$. From this representation, it is clear that the larger is e_{-i}, the lower is the marginal utility of e_i. Then, agent i reduces his educational activity and increases leisure and private consumption.

That is, the more other agents supply public goods, the stronger are the incentives to free ride. Well-educated agents supply more education than less-educated agents, so that $e_{-w} < e_{-l}$. Less-educated agents, therefore, have a stronger incentive to free ride and thus study less than well-educated agents. Because of this mechanism, inequality persists.

Generally, in models with log preference and Cobb-Douglas production function, investment rates are equal among agents.[7] That is, every agent studies for the same number of hours. But, in our model, the times de-

[6] If $e_{-it+1} = 0$, then $n_{it} = 1/3$, which agents enjoy when education is privately supplied.

[7] For example of this specification, see Tamura (1991), Glomm and Ravikumar (1992), and Bénabou (1996).

voted to learning are different among agents because of presence of public goods.[8)]

Using (2), (5), and (7), we obtain e_{it+1} as a reaction function to the education supplied by the others.

$$e_{it+1} = -\frac{2}{3}e_{-it+1} + \frac{Ah_{it}^{\beta}E_t^{1-\beta}}{3} \tag{8}$$

Because M agents exist, we must solve M equations to determine their optimal contributions; the Cournot-Nash equilibrium of this economy.

$$e_{it+1} = \frac{AE_t^{1-\beta}}{2M+1}\left[(2M+1)h_{it}^{\beta} - 2\sum\nolimits_{k=1}^{M} h_{kt}^{\beta}\right] \tag{9}$$

Considering that a well-educated class has N agents and a lower has $M-N$ agents, we can rewrite terms in the square brackets as $(2M+1)h_{it}^{\beta}-2[Nh_{wt}^{\beta}+(M-N)h_{lt}^{\beta}]$. Substituting h_{wt} and h_{lt} into h_{it}, we obtain the optimal education of well-educated and less-educated agents.

$$e_{wt+1} = \frac{AE_t^{1-\beta}}{2M+1}\left[2(M-N)\left(h_{wt}^{\beta}-h_{lt}^{\beta}\right)+h_{wt}^{\beta}\right] \tag{10}$$

$$e_{lt+1} = \frac{AE_t^{1-\beta}}{2M+1}\left[\left(2N+1\right)h_{lt}^{\beta}-2Nh_{wt}^{\beta}\right] \tag{11}$$

If the sign of terms in the square brackets is not positive, then the non-negativity constraint is binding, so that $e_{it+1}=0$: that is, an agent i free rides perfectly. Clearly, because $h_{wt} \geq h_{lt}$, e_{wt+1} is positive, i.e. well-educated agents never become perfect free riders. On the other hand, terms in the square brackets of (11) are negative when the following relationship between h_{wt} and h_{lt} holds.

$$\frac{h_{wt}}{h_{lt}} = \left(\frac{2N+1}{2N}\right)^{\frac{1}{\beta}} \tag{12}$$

Define L.H.S. of this inequality as I_t, and R.H.S. as $\bar{I}$. I_t means a degree of inequality. Since $h_{wt} \geq h_{lt}$ by definition, the minimum value of I_t is unity, and the more unequal is the community, the larger becomes I_t. $\bar{I}$ represents a critical value of inequality. If the community is so unequal that

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8) Even if there are no public goods, investment rates vary among agents with general preferences of constant relative risk aversion. For example, when utility function is $U_{it} = n_{it}^{1-\sigma}/(1-\sigma)+c_{it+1}^{1-\sigma}/(1-\sigma)$ and production function is $h_{it+1} = (1-n_{it})h_{it}^{\beta}$, then optimal leisure becomes

$$n_{it} = h_{it}^{\beta\frac{1-\sigma}{\sigma}} \Big/ \left[1+h_{it}^{\beta\frac{1-\sigma}{\sigma}}\right].$$

We can see that $\partial n_{it}/\partial h_{it}<0$ when $\sigma<1$ and $\partial n_{it}/\partial h_{it}>0$ when $\sigma>1$.

$I_t \geq \bar{I}$, then agents in a less-educated class become perfect free-riders. We call such a situation the *free-ride phase*, and the another situation the *normal phase*, in which less-educated agents also supply public goods. It follows from (12) that $\partial\bar{I}/\partial N < 0$ and $\partial\bar{I}/\partial\beta < 0$. That is, in an economy in which a number of well-educated agents or β is large, less-educated agents are more likely to be perfect free riders. That a number of well-educated agents is large means that contributions by others are large for a less-educated agent, he, therefore, has a large incentive to free ride.

Now, we derive human capital accumulation in the normal phase. Using (9) to obtain e_{-ut+1} and e_{-lt+1}, the substitution of (7) into (2) yields

$$h_{wt+1} = \frac{1}{2M+1} AE_t^{1-\beta}\left[(2M-N+1)h_{wt}^{\beta} - (M-N)h_{lt}^{\beta}\right] \tag{13}$$

$$h_{lt+1} = \frac{1}{2M+1} AE_t^{1-\beta}\left[(M+N+1)h_{lt}^{\beta} - Nh_{wt}^{\beta}\right] \tag{14}$$

These equations describe the transition of human capitals and we use them in order to analyze the aggregate behavior of the economy.

Optimal Decisions in Free-Ride Phase

In the free-ride phase, since less-educated agents do not contribute at all, well-educated agents' choices are not affected by less-educated agents' behavior. Therefore each agent in the well-educated class considers interaction among N well-educated agents. That is, in this case, we can discover well-educated agents' behavior by solving a voluntary contribution model with N homogeneous agents described by (1), (2), (3), and (4). The contribution of well-educated agents and the accumulation of their human capital are as follows.

$$e_{wt+1} = \frac{1}{2N+1} Ah_{wt}^{\beta}E_t^{1-\beta} \tag{15}$$

$$h_{wt+1} = \frac{N+1}{2N+1} Ah_{wt}^{\beta}E_t^{1-\beta} \tag{16}$$

These equations provide us with a standard result of the voluntary provision of public goods. The larger is N, the less is the supply of education by each agent. That is, as the number of contributers grows larger, each agent free rides more strongly.[9]

Next, we describe the optimal decisions of less-educated agents. In the free-ride phase, since $e_{lt+1}=0$, less-educated agents' utility functions are as follows.

$$U_{lt} = \ln n_{lt} + \ln c_{lt+1} + \ln Ne_{wt+1} \tag{17}$$

[9] For example, see Chamberlin (1974), McGuire (1974), and Andreoni (1988).

Since less-educated agents can not control the third term in R.H.S., they maximize the first two terms with respect to leisure and consumption. Since they do not provide education, they use human capital only to produce consumption goods; $c_{lt+1}=h_{lt+1}$. Then agents choose only n_{lt}. From f.o.c., we can see that optimal leisure is $n_{lt}=1/2$. Substituting this value into (2), the stock of human capital is determined.

$$h_{lt+1}=\frac{1}{2}Ah_{lt}^{\beta}E_t^{1-\beta} \tag{18}$$

Comparing (16) and (18), it is clear that less-educated agents study less than well-educated agents: $(N+1)/(2N+1)>1/2$. This is the same result as for the normal phase.

4. Dynamics

In this section, we will analyze the dynamic behavior of the economy, the transition, that is, of the degree of inequality and long-run growth rates.

4.1 Transition of the degree of inequality

First, we will derive transition equations of I_t in normal and free-ride phases. We shall analyze the global dynamics of I_t.

In order to determine the transition of inequality, we must know how the agents in each class accumulate human capital. Dividing (13) by (14), we have the dynamic equation of I_t in normal phase:

$$I_{t+1}=\frac{(2M-N+1)I_t^{\beta}-(M-N)}{M+N+1-NI_t^{\beta}}\equiv f(I_t) \tag{19}$$

It follows that $f'(I)>0$ and $f(1)=1$: that is, $f(I_t)$ is monotonically increasing and has a stationary point: $I_{t+1}=I_t=1$. If this stationary point is stable, we call it *equal equilibrium*.

From a second derivative of $f(I_t)$, we find that the graph of $f(I_t)$ may have an inflection point. If an inflection point exists, $f(I_t)$ is concave when I_t is small, and then becomes convex. Therefore, $f(I_t)$ may have a stable stationary point which is larger than unity as in Figure 1-(b), and we call this *unequal equilibrium*. From the form of $f(I_t)$, we can see that $f(I_t)$ has at most one stable equilibrium (see Figure 1).

Next, the law of motion of I_t in the free-ride phase can be represented as a ratio of (16) and (18).

$$I_{t+1}=\frac{2N+2}{2N+1}I_t^{\beta}\equiv g(I_t) \tag{20}$$

Clearly from Figure 1-(f), this dynamic equation has a stable stationary point as

$$I_\infty^g \equiv \left(\frac{2N+2}{2N+1}\right)^{\frac{1}{1-\beta}} \tag{21}$$

We call this stationary point *free-ride equilibrium*.

Global motion of I_t is determined by $f(I_t)$ where $I_t \leq \bar{I}$, and by $g(I_t)$ where $I_t > \bar{I}$. Define the function $Q(I_t)$ as a global dynamic equation of I_t.

$$Q(I_t) = \begin{cases} f(I_t), & if \ I_t < \bar{I}, \\ g(I_t), & if \ I_t \geq \bar{I}. \end{cases} \tag{22}$$

The forms of $Q(I_t)$ are described in Figure 2, 3, 4. We have the following lemma concerning a fundamental property of $Q(I_t)$.

Lemma 1 *$Q(I_t)$ has at least one globally stable equilibrium.*
Proof See Appendix.

Lemma 1 states that the long-run degree of inequality does not diverge, but converges upon a certain value and remains constant over time.

We focus on $f'(1)$ and the relationship between I_∞^g and $\bar{I}$, which characterize the dynamics of the economy. First, whether $f'(1)$ is larger than unity is related to the stability of a stationary point $I_t=1$; that is, the existence of equal equilibrium. Second, the relationship between I_∞^g and $\bar{I}$ determines the existence of free-ride equilibrium. Because $f(It)$ describes the motion of I_t in the region $I_t < \bar{I}$, if $I_\infty^g < \bar{I}$ as in Figure 2, I_∞^g is not a solution of $Q(I_t)=1$. From these conditions, we obtain the following proposition.

Proposition 1 *Define* $\beta_s(M) \equiv (M+1)/(2M+1)$ *and* $\beta_g(N) \equiv [\ln(2N+1) - \ln 2N]/[\ln(N+1) - \ln N]$. (i) *If* $\beta < \beta_s(M)$, *$Q(I_t)$ has the equal equilibrium.* (ii) *If* $\beta > \beta_g(N)$, *$Q(I_t)$ has the free-ride equilibrium.*
Proof(i) Rewriting a condition $f'(1) < 1$, which means that a stationary point $I_t=1$ is stable, we get $\beta < (M+1)/(2M+1)$. Therefore, $\beta < \beta_s(M)$ is the condition which ensures the existence of equal equilibrium. (ii) Rewriting the condition $I_\infty^g > \bar{I}$, we get $\beta > [\ln(2N+1) - \ln 2N]/[\ln(N+1) - \ln N]$.[10]
Q.E.D.

Proposition 1-(ii) is a chief result of this paper; the persistence of inequality in a long-run equilibrium. Proposition 1 asserts that, as β gets larger, the equal equilibrium disappears and the free-ride equilibrium appears, that is, roughly speaking, the economy grows more unequal. Also, we can see similar properties concerning the degree of inequality at the

[10] We also obtain this inequality from rewriting $\bar{I} < f(\bar{I})$, which means that an intersection of $f(I_t)$ with $g(I_t)$ is above 45 degrees line.

unequal and free-ride equilibria. Equation (21) implies $\partial I^g_\infty/\partial\beta>0$; that is, the larger is β, the more unequal is the free-ride equilibrium. Moreover, the first derivative of $f(I_t)$ with respect to β is strictly positive, except at $I_t=1$, which means that $f(I_t)$ shifts upward with an increase in β. Therefore, if the unequal equilibrium exists, an increase in β makes it more unequal. These implications follow. $(1-\beta)$ is a weight of community education in production functions. Therefore, an increase in β implies the decrease in the weight of community education and thus weakens the equalizing force, which makes the economy more unequal.

Remark 1 An increase in β makes the economy more unequal.

Now we classify the long-run states of the economy with various configurations of the parameters. Obviously, by definition, $\beta_s(M)$ and $\beta_g(N)$ depend on M and N, respectively. Therefore, as M and N vary, the relationship between $\beta_s(M)$ and $\beta_g(N)$ also changes. We see from Figure 5, when the difference between M and N is small, $\beta_s(M)$ is larger than $\beta_g(N)$ and vice versa. So, in order to characterize the dynamics of the economy, we focus on the two cases in which $\beta_g(N)<\beta_s(M)$ and which $\beta_s(M)<\beta_g(N)$.[11)] When $\beta_g(N)<\beta_s(M)$, there are three possibilities as follows.
・ $\beta<\beta_g(N)$; Equal equilibrium only prevails. That is, in the long run, inequality will vanish from each initial position of I_t (See Figure 2).
・ $\beta_g(N)\leq\beta\leq\beta_s(M)$;[12)] Equal and free-ride equilibria prevail. That is, differences in initial inequalities lead the economy towards different steady states. (See Figure 4.)
・ $\beta_s(M)<\beta$;[13)] Where free-ride equilibrium prevails, while unequal equilibrium may also occur. (See Figure 3.)

When $\beta_s(M)<\beta_g(N)$, the following cases obtain.
・ $\beta\leq\beta_s(M)$; equal equilibrium only.
・ $\beta_s(M)<\beta\leq\beta_g(N)$; neither equal nor free-ride equilibria, and I^f_∞, which is greater than unity, is therefore the only stable equilibrium.
・ $\beta_g(N)<\beta$; free-ride equilibrium, while unequal equilibrium may also occur. (See Figure 3.)
To sum up, the long-run dynamics are classified into five cases: the economy has (i) equal equilibrium, (ii) unequal equilibrium, (iii) free-ride equilibrium, (iv) equal and free-ride equilibria, or (v) unequal and free-ride equilibria.

[11)] Both $\beta_g(N)$ and $\beta_s(M)$ have the same limits; $\lim_{N\to\infty}\beta_g(N)=\lim_{M\to\infty}\beta_s(M)=0.5$. Therefore, when M and N are large enough, knowing whether β is larger than 0.5 makes it possible to characterize the dynamics of the economy.

[12)] When $f''(1)>0$, this condition is replaced with $\beta_g(N)\leq\beta<\beta_s(M)$.

[13)] When $f''(1)>0$, this condition is replaced with $\beta_s(M)\leq\beta$.

4.2 Long-run growth

This subsection studies the long-term growth rates. First, we consider growth rates in the normal phase. The individuals' stocks of human capital are determined by (13) and (14). The well-educated class has N members and the less-educated class $M-N$, so the total stock of human capital in this economy is $H_{t+1}=Nh_{wt+1}+(M-N)h_{lt+1}$. Substituting (13), (14), and $E_t=H_t/(M+1)$, which is made up of (5), into H_{t+1}, we get

$$H_{t+1}=\frac{(M+1)^{\beta}}{2M+1}\frac{NI_t^{\beta}+M-N}{(NI_t+M-N)^{\beta}}AH_t \tag{23}$$

Let $X_f(I_t)A$ denote the coefficient of H_t, which is the growth rate of H.[14) Next, we derive the growth rate in the free-ride phase. In the long run, the degree of inequality is constant and equal to I_{∞}^{g}, which means that h_{wt} and h_{lt} grows at the same rate in the free-ride equilibrium. Therefore, the growth rate of h_{wt} is equal to that of H_t. Then, we derive the growth rate of h_{wt}. In the free-ride phase, $E_t=Nh_{wt}/(N+1)$, with which we can rewrite (16) as follows.

$$h_{wt+1}=\frac{(N+1)^{\beta}}{2N+1}N^{1-\beta}Ah_{wt} \tag{24}$$

Define X_gA as the growth rate of h_{wt} in the free-ride equilibrium.

Now we consider growth rates in the equal and free-ride equilibria. Define $Y(x,\beta)\equiv[(x+1)^{\beta}/(2x+1)]x^{1-\beta}$ where x is a finite natural number, so that $Y(M,\ \cdot)=X_f(1)$ and $Y(N,\ \cdot)=X_g$. $Y(x,\ \cdot)$ represents the growth rate at an equilibrium in which all x contributors are identical such as in state of equal or free-ride equilibrium.[15) Rewriting a condition $\partial Y(x,\ \cdot)/\partial x>0$, we have the following inequality.

$$\beta<(x+1)/(2x+1) \tag{25}$$

The maximum of R.H.S. is 2/3 and the minimum 1/2. Therefore, (i) if $\beta<1/2$, an inequality (25) holds for any value of x and thus $Y(x,\ \cdot)$ is a monotonically increasing function of x. In this case, β is small enough for the economy to possess equal equilibrium. Therefore, an economy with many contributors grows faster than that with few contributors in conditions of equal equilibrium. (ii) if $\beta>2/3$, an inequality (25) never hold for any value of x and thus $Y(x,\ \cdot)$ is a monotonically decreasing function of x. In this case, β is large enough for the economy to permit free-ride equilibrium. Therefore, an economy with many contributors grows slower than

14)We assume that A is large enough for the growth rate to be larger than unity.

15)In the free-ride equilibrium, less-educated agents are not the contributors of community education. In the unequal equilibrium, two types of contributors exists: well- and less-educated agents.

that with few contributors in conditions of free-ride equilibrium. (iii) If $1/2<\beta<2/3$, $Y(x,\ \cdot)$ has a maximum value, as in Figure 6.

Remark 2 (i) When β is small enough, an economy with many contributors grows faster than that with few contributors. (ii) When β is large enough, an economy with many contributors grows more slowly than that with few contributors.

The increase in the number of the contributors, M or N, has two consequences. First, the incentive to free ride rises with the number of contributors. So, agents reduce time devoted to learning, which reduces the growth rate. Second, an increase in the number of contributors enlarges the total supply of community education. In the normal phase, $E_t=M\hat{h}_t/(M+1)$, where $\hat{h}_t$ is average level of human capital of contributors. Similarly, $E_t = N\hat{h}_t/(N+1)$ in the free-ride phase. From these equations, if the average level of human capital is constant, E_t increases with M or N. This effect raises the growth rate. β determines which of these two effects is larger. That β is large means that the weight of E_t at human capital formation is small, and that the weight of learning $(1-n_{it})$ is relatively large. So, when β is large, the increase in the number of contributors reduces the growth rate.

Next, we consider the effect of β on the growth rate. From (23) and (24), it follows that $dX_f(1)/d\beta>0$ and $dX_g/d\beta>0$. Therefore, the larger is β, the higher is the growth rate. We can see this fact from Figure 6.

Remark 3 A long-run growth rate rises with β.

From Remarks 1 and 3 entail that in the long run the economy with large β is more unequal and grows faster than that with small β.

4.3 Multiple equilibria

When $\beta_g(N)<\beta<\beta_s(M)$, the economy manages both equal and free-ride equilibria, and an unstable equilibrium $\hat{I}\in(1,\ \hat{I})$ obtains, as in Figure 4. Therefore, when an initial value of I_t is less than $\hat{I}$, the long-run inequality converges upon the equal equilibrium, and when an initial value is larger than $\hat{I}$, the long-run inequality converges to the free-ride equilibrium. Generally, $X_f(1)$ is different from X_g, so the long-term growth rate depends on an initial value of inequality. For example, when $M=2$ and $N=1$, we see that $\beta_g(N)\approx 0.585$ and $\beta_s(M)=0.6$. If $\beta=0.59$, then the relationship that $\beta_g(N)\leq\beta\leq\beta_s(M)$ holds so that there exist multiple equilibria and growth rates are $X_f(1)\approx 0.508$ and $X_g\approx 0.502$. In this numerical example, therefore, an initially equal economy grows faster in the long run than an initially unequal economy.

In general, models with multiple equilibria contain the feature that a one-time intervention by the government has a permanent effect, which holds also in our model. Let an economy be highly unequal so that $I_t>\hat{I}$.

If the government does nothing, then the economy will converge upon unequal equilibrium. If the government, in the initial period, redistributes income from the well-educated to the less-educated so that $I_t < \hat{I}$, then the economy will converge upon equal equilibrium.

5. Conclusion

We have here offered a model that links the formation of human capital and the private provision of public goods. A child allocates his time to acquire knowledge for the future production of private and public goods. The learning technology is affected by the parental stock of human capital; the more human capital a parent has, the more effectively his child can learn. All other things being equal, an individual born of a well-educated parent accumulates more human capital and thus provides more public goods. Incentive to free ride for an individual is strong when contributions by the others are large. Since an individual born of a less-educated parent accumulates less human capital and provides less public goods, contributions by others are large for him. So, he has stronger incentive to free ride and hence allocates less time to learn than an individual born of a well-educated parent. This difference of investment rates makes the economy unequal.

In this paper, we have assumed a discrete distribution in supposing initially that the economy is divided into two classes. A natural extension of the model adopts a continuous distribution of human capital. As a result, the number of well-educated agents will be determined endogenously, and, consequently, the long-term growth rate must be considered as a function of the initial inequality. That is to say, the economy supports multiple equilibria, with broader configurations, than that presupposed by the terms of this paper.

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Appendix

Proof of Lemma 1 Because $f(I_t)$ and $g(I_t)$ are monotonically increasing and continuous, and $f(\bar{I})=g(\bar{I})$, $Q(I_t)$ is also monotonically increasing and continuous. $Q(I_t)$ is monotonically increasing, so I_t converge or diverge without cyclical movement in neighborhood of equilibria. Therefore, let $I_\infty^{\max}$ denote the maximum I_t which satisfies $Q(I_t)=I_t$, if following relationship hold, I_t converge to an equilibrium globally.

$$Q(I_t) < I_t, \forall I_t > I_\infty^{\max} \tag{26}$$

First, we consider a case in which $I_\infty^g \geq \bar{I}$. In this case, $I_\infty^{\max} = I_\infty^g$. I_∞^g is a stable equilibrium, so that the condition (26) is satisfied. Next, when $I_\infty^g < \bar{I}$, $I_\infty^{\max}$ exists in $[1, \bar{I}]$. Rewriting the inequality, $I_\infty^g > \bar{I}$, we get $Q(\bar{I}) < \bar{I}$, which means

$Q(I_t) < I_t, \forall I_t \geq \bar{I}$ from the shape of $g(I_t)$. Therefore, if we find that $Q(I_t) < I_t$ in $[I_\infty^{\max}, \bar{I})$, the proof will be finished. We derive contradiction by assuming that there exists some $I' \in (I_\infty^{\max}, \bar{I})$ such that $Q(I') > I'$. That $Q(I') > I'$ and $Q(\bar{I}) < \bar{I}$ means $Q(I_t)$ must intersect 45 degrees line in $(I', \bar{I})$ from the intermediate value theorem. Then, there exists an equilibrium which is larger than $I_\infty^{\max}$. This is the contradiction. Q.E.D.

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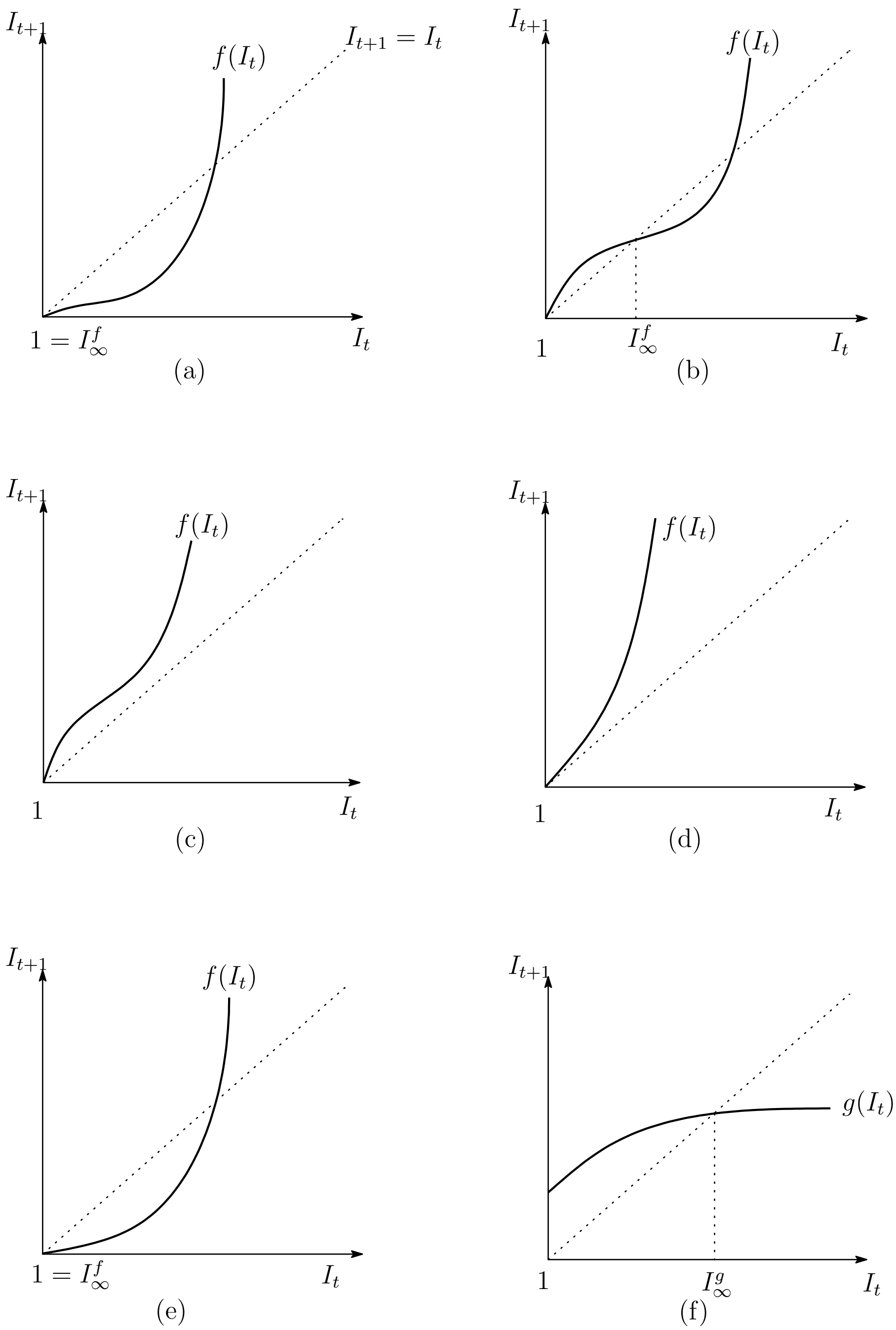

Figure 1: Forms of $f(I_t)$ and $g(I_t)$.

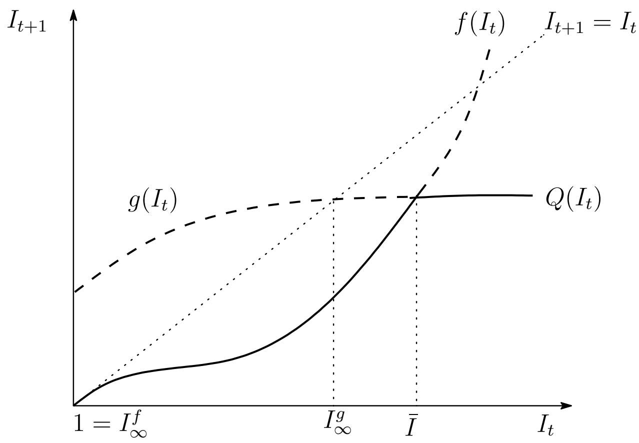

Figure 2: (a) only equal equilibrium exists.

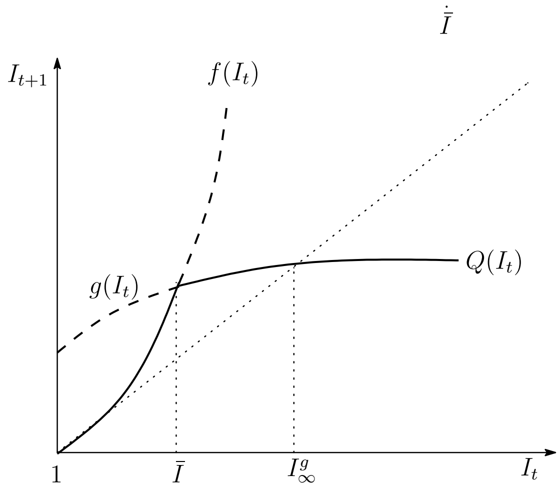

Figure 3: (b) only free-ride equilibrium exists.

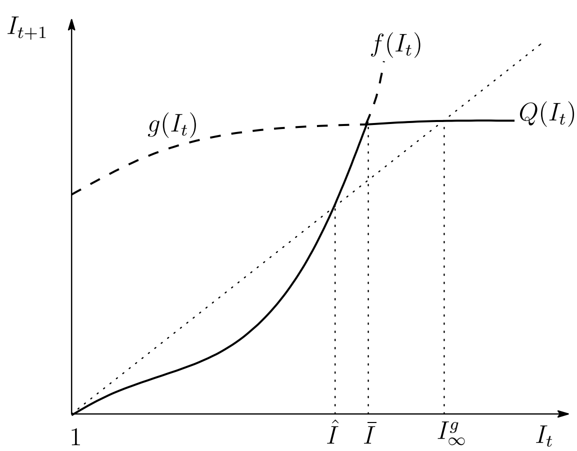

Figure 4: (c) Multiple equilibria.

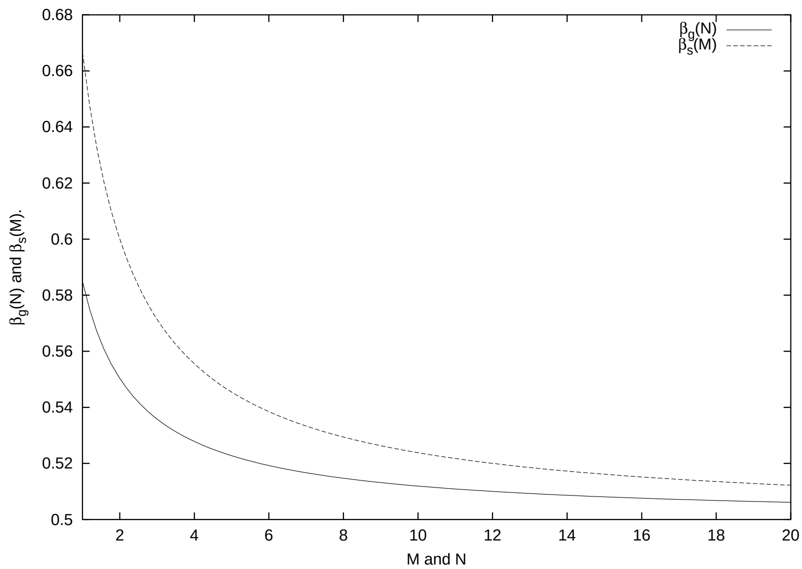

Figure 5: Critical values of β; β_g and β_s.

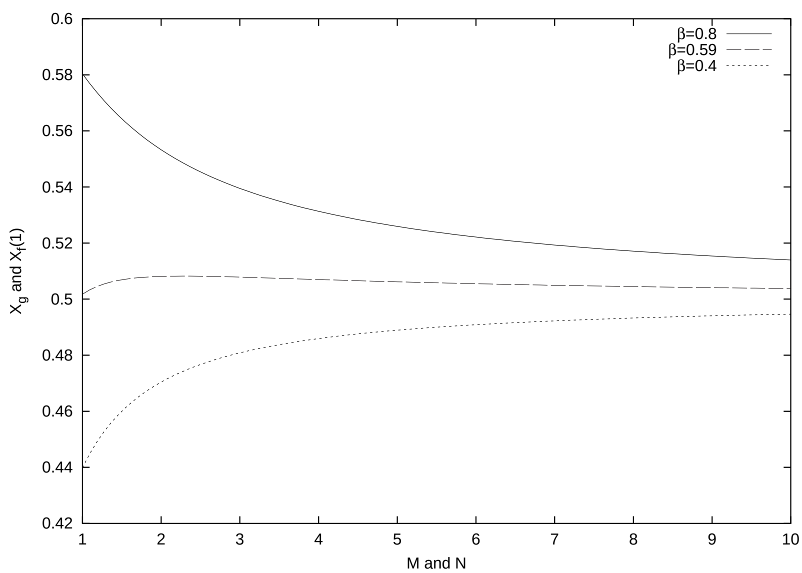

Figure 6: Growth rates; $X_f(1)$ and X_g.