# Logical Dynamics of Speech Acts

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**Summary:**  
Logical Dynamics of Speech Acts explores the interplay between linguistic actions and their social effects. This paper contributes to the understanding of how speech acts are embedded in social contexts, highlighting the importance of context in interpreting and evaluating the intended effects of communicative actions. It discusses the role of logical structures in underpinning the dynamics of these acts, offering insights into the formal aspects of speech act theory. The work also considers the implications of these dynamics for the broader field of social interaction, suggesting new avenues for research in the philosophy and ethics of social reality.

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**Detailed Analysis:**  
The paper begins by outlining the foundational concepts of speech act theory, drawing on earlier works by Bluhm, Gumperz, and other seminal figures in the field. It then proceeds to a detailed analysis of specific speech acts, employing logical frameworks to elucidate their dynamics. The author presents a series of case studies, each illustrating how logical analysis can reveal underlying patterns of social interaction. These case studies cover a range of contexts, from everyday conversations to more complex social settings, demonstrating the universality of the principles discussed.

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**Conclusion and Implications:**  
The conclusion emphasizes the importance of integrating logical analysis with sociological understanding to gain a comprehensive view of speech acts. The implications of this work extend beyond the scope of philosophy and ethics, touching on areas such as rhetoric, communication studies, and social psychology. Further research directions are suggested, encouraging scholars to explore the intersection of logical and social dynamics in more depth.

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**References:**  
Logical Dynamics of Speech Acts

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Treating Speech Acts as Acts

If we are to take the notion of speech act seriously, we must be able to treat speech acts as acts.

By characterizing speech acts in terms of dynamic changes they bring about, it becomes possible to treat them within a general theory of action.

Perlocutionary Acts as Acts

Perlocutionary acts are acts that really produce “real effects” upon the attitudes and the actions of addressees (Austin 1975, pp. 101-3).

Locutionary act: He said to me ‘Shoot her!’ meaning by ‘shoot’ shoot and referring by ‘her’ to her.

Illocutionary act: He urged (advised, ordered, etc.) me to shoot her.

Perlocutionary act (a): He persuaded me to shoot her.
Perlocutionary act (b): He got me to shoot her.

Illocutionary Acts: a Problem

Illocutionary acts do not directly affect brute facts, except for those physical conditions involved in the production of sounds and written symbols.

Nor do they directly affect attitudes and actions of addressees.

A Gap

For instance, intuitively, a command

“See to it that $\phi$!”

makes worlds where $\phi$ holds preferred over those where it does not – at least, if we accept the preference induced by the issuer of the command. (Van Benthem & Liu, to appear)

Illocutionary Acts as Acts

Illocutionary acts affect institutional facts.

How is it possible to capture changes in institutional facts?
Example 1: On A Hot Day (1)
Suppose you are reading an article on logic in the office you share with your boss and a few other colleagues. While you are reading, the temperature of the room rises, and it is now above 30 degrees Celsius.

Example 1: On A Hot Day (2)
Then, suddenly, you hear your boss's voice. She said, "Open the window!"
She commanded you to open the window.
What effects does her command have on the current situation?

Example 1: On A Hot Day (3)
Her act of commanding did not affect the state of the window directly.
Nor did it affect the number of alternatives you have. It is still possible for you to turn on the air conditioner, to ignore the heat, or to open the window.

Example 1: On A Hot Day (4)
But it has now become impossible for you to choose alternatives other than that of opening the window without going against your obligation.
It is now obligatory upon you to open the window, although it was not so before.

Acts of Commanding
Acts of commanding seem to affect deontic status of possible courses of actions.
We have considered how we can capture the kind of changes acts of commanding bring about in Yamada (2007a, b).

The Idea of Command Logic
We model changes acts of commanding bring about in terms of a new update logic. We combine a multi-agent variant of the language of monadic deontic logic with a dynamic language to talk about the situations before and after the issuance of commands, and the commands that link those situations.
The results
Although the resulting language inherits various inadequacies from the language of monadic deontic logic, some interesting principles are captured and seen to be valid nonetheless.

Related Paradigms
- Speech Act Theory (Austin, Searle)
- Propositional Dynamic Logic
- Update Semantics (Veltman)
- Logic of Public Announcements (Plaza, Groeneveld & Gerbrandy, Baltag, Moss & Solecki, Kooi & van Benthem, etc.)
- Deontic Logic

Logics of Public Announcements
The Static Base: Epistemic Logics

Adding Announcement Modalities
The Dynamic Extension: Logics of Public Announcements

The Same Strategy Works for Acts of Commanding

The Static Base
The static language: $\mathcal{L}_{MDL}$
Multi-agent deontic logic: $\text{MDL}^*$

Adding Command Modalities
The Dynamic Extension
The language of Command Logic: $\mathcal{L}_{CL}$
Eliminative Command Logic: $\text{ECL}$

The Language of Deontic Logic
$\text{Op} :$ It is obligatory that $p$.
$\text{Pp} :$ It is permitted that $p$.
$\text{Fp} :$ It is forbidden that $p$.

Relativisation
$\text{Op}_i :$ It is obligatory upon $i$ that $p$.
$\text{Pp}_i :$ It is permitted of $i$ that $p$.
$\text{Fp}_i :$ It is forbidden of $i$ that $p$.

where $i$ is an agent in a given set $I$ of agents.
The Static Language $\mathcal{L}_{\text{MDL}^+}$

Definition 1  Take a countably infinite set $\text{Aprop}$ of proposition letters and a finite set $i$ of agents, $p$ ranging over $\text{Aprop}$ and $i$ over $I$. The multi-agent monadic deontic language with an alethic operator $\mathcal{L}_{\text{MDL}^+}$ is given by:

$$
\phi ::= \top \mid p \mid \neg \phi \mid \phi \land \psi \mid \Box \phi \mid O \phi
$$

$\mathcal{L}_{\text{MDL}^+}$ -models

Definition 2  By an $\mathcal{L}_{\text{MDL}^+}$-model, I mean a quadruple $M = \langle W^M, R^M, R^M, V^M \rangle$ where:

(i) $W^M$ is a non-empty set

(ii) $R^M \subseteq W^M \times W^M$

$\mathcal{L}_{\text{MDL}^+}$ -models (2/2)

(iii) $R^M(i)$ is a function that assigns a subset $R^M(i)$ of $R^M$ to each $i \in I$

(iv) $\mathcal{V}^M$ is a function that assigns a subset $\mathcal{V}^M(p)$ of $W^M$ to each $p \in \text{Aprop}$.

We usually abbreviate $R^M(i)$ as $R^M$. 

Truth definition for $\mathcal{L}_{\text{MDL}^+}$ (1/2)

Definition 3  Let $M$ be an $\mathcal{L}_{\text{MDL}^+}$-model and $w \in W^M$. If $p \in \text{Aprop}$, $i \in I$, and $\phi, \psi$ are sentences of $\mathcal{L}_{\text{MDL}^+}$, then

(a) $M,w \models \phi$ iff $w \in \mathcal{V}^M(p)$

(b) $M,w \models \top$

(c) $M,w \models \neg \phi$ iff $M,w \not\models \phi$

(d) $M,w \models (\phi \land \psi)$ iff $M,w \models \phi$ and $M,w \models \psi$

(e) $M,w \models \Box \phi$ iff for every $v$ such that $\langle w,v \rangle \in R^M$, $M,v \models \phi$

(f) $M,w \models O \phi$ iff for every $v$ such that $\langle w,v \rangle \in R^M$, $M,v \models \phi$

Completion of MDL

A sound and complete proof system for MDL+ is given in Yamada (2007a).

Thus:

Theorem 1: MDL is strongly complete with respect to the class of $\mathcal{L}_{\text{MDL}^+}$-models.
Example 1 in $\mathcal{L}_{\text{MDL}}$

The situations you were in before and after the issuance of your boss’s command in Example 1 can be represented by two related $\mathcal{L}_{\text{MDL}}$ -models $M$ and $N$ respectively. For example, we may say:

1. $M,s \models \neg O^p \land \neg O^{\neg p}$
2. $N,s \models O^p$

What we cannot say in $\mathcal{L}_{\text{MDL}}$

Sentences of $\mathcal{L}_{\text{MDL}}$ can be used to describe the situations before and after the issuance of your boss’s command.

But they cannot be used to talk about the act of commanding that links these situations.

The Language of the Logic of the Act of Commanding (Command Logic)

$O_si \xi$ : It is obligatory upon the agent $i$ to see to it that $\xi$.

$[i,\psi] \phi$ : After every act of commanding $i$ (the addressee) to see to it that $\psi$, $\phi$ holds.

$[i,\psi] O_i \xi$ : After every act of commanding $i$ to see to it that $\psi$, it is obligatory upon $i$ to see to it that $\xi$.

The Language of the Logic of Public Announcements

$K_i \psi$ : Agent $i$ knows that $\psi$.

$[A!] \phi$ : After every truthful public announcement that $A$, $\phi$ holds.

$[A!] K_i \psi$ : After every truthful public announcement that $A$, $i$ knows that $\psi$.

The Dynamified Language $\mathcal{L}_{\text{CL}}$

Definition 5. Take the same countably infinite set $\text{Aprp}$ of proposition letters and the same finite set $I$ of agents, $p$ ranging over $\text{Aprp}$ and $i$ over $I$ as before. The language of Command Logic $\mathcal{L}_{\text{CL}}$ is given by:

$\phi := \top | p | \neg \phi | \phi \land \psi | \top \phi | O_i \phi | [\pi] \phi$

$\pi := i \phi$
Truth definition for $L_{\text{CL}}$ (1/3)

Definition 4  Let $M$ be an $L_{\text{MDL}^*}$-model and $w \in W^M$. If $p \in \text{Aprop}$, $i \in I$, and $\phi, \psi$ are sentences of $L_{\text{CL}}$, then

(a) $M, w \vDash p$ iff $w \in V^M(p)$
(b) $M, w \vDash \top$
(c) $M, w \vDash \neg \phi$ iff $M, w \vDash \phi$

Truth definition for $L_{\text{CL}}$ (2/3)

(d) $M, w \vDash_{\text{ECL}} (\phi \land \psi)$ iff $M, w \vDash_{\text{ECL}} \phi$ and $M, w \vDash_{\text{ECL}} \psi$
(e) $M, w \vDash_{\text{ECL}} \Box \phi$ iff for every $v$ such that $(w, v) \in R^M_i$, $M, v \vDash_{\text{ECL}} \phi$
(f) $M, w \vDash_{\text{ECL}} O_i \phi$ iff for every $v$ such that $(w, v) \in R^M_i$, $M, v \vDash_{\text{ECL}} \phi$.

Truth definition for $L_{\text{CL}}$ (3/3)

(g) $M, w \vDash_{\text{ECL}} [\! [i \chi] \! ] \phi$ iff $M_{i, x}, w \vDash_{\text{ECL}} \phi$,

where $M_{i, x}$ is an $L_{\text{MDL}^*}$-model obtained from $M$ by replacing $R^M_i$ with the function $R^M_{i, x}$ such that for each $j \in I$:

(i) if $j \neq i$, $R^M_{i, x}(j) = R^M_i(j)$
(ii) if $j = i$, $R^M_{i, x}(j) = \{(x, y) \in R^M_i(i) \mid M, y \vDash_{\text{ECL}} \chi\}$.

On this truth definition

We abbreviate $\{(x, y) \in R^M_i(i) \mid M, y \vDash_{\text{ECL}} \chi\}$ as $R^M_i \chi_i^1$.

As we always have $R^M_i \chi_i^1 \subseteq R^M_i$, we also have $R^M_i \chi_i^1 \subseteq R^M_i$. Hence $M_{i, x}$ is guaranteed to be an $L_{\text{MDL}^*}$-model.

Boss’s Act of Commanding in MDL*

$M \quad \Rightarrow \quad N$

$t \quad s \quad u \quad p \quad q$

Boss’s Act of Commanding in ECL (Eliminative Command Logic)

$M \quad \Rightarrow \quad M_{a, p}$

$t \quad s \quad u \quad p \quad q$

$t \quad s \quad u \quad p \quad q$
Boss’s Act of Commanding in ECL (2)

CUGO Principle

If neither $O_i$’s nor $i$’s occur in $\phi$, we have $[i, \phi] O_i \phi$.

Usually, you should do what your boss commands you to do. In other word, commands usually generate obligations.

Why not CGO but CUGO

The following formula is not valid:

$[i, P, q] O_i P, q$.

If $\psi$ involves deontic operators or command operator indexed with $i$, the truth of $\psi$ at $w$ in $M$ does not guarantee the truth of $\psi$ at $w$ in $M_i, \phi$.

An Open Question

Let $S_{CGO}$ be the set of sentences $\phi$ such that we have $ECL [i, \phi] O_i \phi$. Let $S_{i, free}$ be the set of $E_{MDL}$-sentences in which no deontic operators for $i$ occur.

As $O_i \phi \rightarrow O_i \phi \in S_{CGO}$, we have $S_{i, free} \subseteq S_{CGO}$.

But exactly how large $S_{CGO}$ is is an open question.

Reduction Axioms

(1) $[i, \phi] p \leftrightarrow p$ where $p \in Aprop$

(2) $[i, \phi] \top \leftrightarrow \top$

(3) $[i, \phi] \rightarrow \psi \leftrightarrow \neg[i, \phi] \psi$

(4) $[i, \phi] (\psi \land x) \leftrightarrow ([i, \phi] \psi \land [i, \phi] x)$

(5) $[i, \phi] \Box \psi \leftrightarrow \Box[i, \phi] \psi$

(6) $[i, \phi] O_i \psi \leftrightarrow O_i ([i, \phi] \psi)$

(7) $[i, \phi] O_i \psi \leftrightarrow O_i [i, \phi] \psi$ where $i \neq j$.

Translation function $t$ (1/2)

$t(p) = p$

$t(\top) = \top$

$t(\neg \phi) = \neg t(\phi)$

$t(\phi \land \psi) = t(\phi) \land t(\psi)$

$t(\Box \phi) = \Box t(\phi)$

$t(O_i \phi) = O_i t(\phi)$
Translation function $t$ (2/2)

$\langle i, \phi \rangle \rho$ = $\rho$

$\langle i, \phi \rangle \top$ = $\top$

$\langle i, \phi \rangle \neg \psi$ = $\neg \langle i, \phi \rangle \psi$

$\langle i, \phi \rangle (\psi \land x)$ = $\langle i, \phi \rangle \psi \land \langle i, \phi \rangle x$

$\langle i, \phi \rangle \psi$ = $\psi$

$\langle i, \phi \rangle \Omega \psi$ = $\Omega_i \langle i, \phi \rangle \psi$

$\langle i, \phi \rangle \Omega \psi$ = $\Omega_i \langle i, \phi \rangle \psi$ where $i \neq j$.

$\langle i, \phi \rangle [\Omega \psi] x$ = $\langle i, \phi \rangle [\psi] \langle i, \phi \rangle x$ for any $j \leq l$

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Proof System for ECL

The proof system for ECL (Eliminative Command Logic) contains all the axioms and all the rules of the proof system for MDL+, and in addition all the reduction axioms listed above and the following rule:

$\Psi$

$\frac{\langle i, \phi \rangle \psi}{[\Omega \psi] \psi}$

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Completeness

The completeness of ECL is derived from the completeness of MDL+.

Theorem 2
ECL is strongly complete with respect to the class of $\mathcal{L}_{MDL^*}$-models.

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Built-in assumptions 1

Commands are assumed to be always eliminative.

Proposition 2 We have:

$\models_{ECL} [\Omega \psi] P_i \phi \rightarrow P_i \phi$.

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Example 2

Suppose you are in a combat troop and now waiting for your captain’s command to fire. Then you hear the command, and it has become obligatory upon you to fire. But before that, you were not permitted to fire. This forbiddance is now no longer in force. Thus it seems that after his command, you are permitted to fire, at least in the sense of lack of forbiddance.

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Built-in assumptions 2

Commands of the form $[\Omega \psi]$ is assumed to have no effects on deontic accessibility relations for any agent other than $i$.

Corollary 2 If $\psi \in S_{i,\text{free}}$, then:

$M, w \models_{ECL} \psi$ iff $M_{1,\psi}, w \models_{ECL} \psi$.
**Built-in assumptions 3**

Commands are assumed to have no preconditions.

Compare:

**PAL:** \( [\phi]K\psi \leftrightarrow (\phi \rightarrow [\phi] \psi) \)

**ECL:** \( [\psi] O\psi \leftrightarrow O(\phi \rightarrow [\psi] \psi) \)

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**Dead end principle**

We have:

\[ \models_{ECL} [i (p \land \neg p)] O, \phi. \]

Since \( R^{\#}_i (p \land \neg p) \downarrow \) is empty, no world is \( R^{\#}_i (p \land \neg p) \downarrow \)-accessible for \( i \). Absurd commands lead to an obligational dead end.

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**D axiom cannot be added**

We have:

\[ M_1(p \land \neg p), w \models_{ECL} O, (p \land \neg p) \land \neg P(p \land \neg p). \]

Thus \( O, \phi \rightarrow P, \phi \) cannot be added to ECL.

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**Restricted sequential conjunction principle**

If \( \phi, \psi \in S_{\text{free}} \), we have:

\[ \models_{ECL} \langle i, \phi \rangle \langle i, \psi \rangle X \leftrightarrow \langle i, \psi \rangle \langle i, \phi \rangle X. \]

Since \( R^{\#}_i (\phi \land \psi) \downarrow \) can be distinct from \( (R^{\#}_i \uparrow \phi) \downarrow \psi \downarrow \), unrestricted form of sequential conjunction principle is not valid.

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**Restricted order invariance principle**

If \( \phi, \psi \in S_{\text{free}} \), we have:

\[ \models_{ECL} \langle i, \phi \rangle \langle i, \psi \rangle X \leftrightarrow \langle i, \psi \rangle \langle i, \phi \rangle X. \]

Since \( (R^{\#}_i \uparrow \psi) \uparrow \psi \downarrow \) can be distinct from \( (R^{\#}_i \uparrow \phi) \uparrow \phi \downarrow \), unrestricted form of order invariance principle is not valid.

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**Another Interesting Validity**

We have:

\[ \models_{ECL} \langle i, \phi \rangle \langle i, \neg \psi \rangle O, \phi. \]

Everything comes to be obligatory if a pair of contradictory commands is issued. Thus you will be in an obligational dead end.
An Extension: ECL II

We may distinguish command issuers:

\[ [i_0] p \rightarrow [i_0] \neg p \rightarrow (O_{i_0} p \land O_{i_0} \neg p) \].

If an authority \( k \) commands you to see to it that \( \neg p \) after another authority \( j \) commands you to see to it that \( p \), then you will be in an obligational dilemma.

Example 3

A Contingent Dilemma (1/2)

\[ [i_0] p \rightarrow [i_0] q \rightarrow (O_{i_0} p \land O_{i_0} q) \].

If some very rapid means of transportation were available, it would be possible for you to obey both commands. But unfortunately, no such means of transportation happens to be available in the real world.

Logical Dynamics of Multi-agent Language Games

(1) Acts of promising can also be considered as deontic updaters, and acts of asserting can be considered as updaters of propositional commitments.

(2) Perlocutionary Acts can also be considered as updaters of systems of knowledge, belief and preference.

Why an obligational dilemma

Although no worlds are both \( R_{i_0} \)-accessible and \( R_{i_0} \)-accessible, there may be an \( R_{i_0} \)-accessible world and an \( R_{i_0} \)-accessible world in such a situation.

A Contingent Dilemma (2/2)

\[ [i_0] p \rightarrow [i_0] q \rightarrow (O_{i_0} p \land O_{i_0} q) \].

Treating Illocutionary Acts and Perlocutionary Acts Together

ECL II can be combined with DEUL (Dynamic Epistemic Upgrade Logic) of van Benthem & Liu (to appear).

DEUL can be interpreted as dealing with perlocutionary acts of getting addressees to prefer something.
The Language of Epistemic Preference Logic

\[ K_i \phi : \text{the agent } i \text{ knows that } \phi \]

\[ [\text{pref}]_i \phi : \text{in every world the agent } i \text{ considers at least as good as the current one } \phi \text{ holds} \]

\[ U \phi : \text{in every world } \phi \text{ holds} \]

The Language of Dynamic Epistemic Upgrade Logic

\[ [\psi]_i K_i \phi : \text{after every truthful public announcement that } \phi, \psi \text{ holds} \]

\[ [\phi!]_i K_i \phi : \text{after every truthful public announcement that } \phi, i \text{ knows that } \psi \text{ holds} \]

\[ [\# \phi]_i K_i \phi : \text{after every act of publicly suggesting } \phi, \psi \text{ holds} \]

\[ [\# \phi]_i \text{[pref]}_i \psi : \text{after every act of publicly suggesting } \phi, \text{ in every world } i \text{ considers at } \cdots \text{current one, } \psi \text{ holds} \]

The Need for Reinterpretation

As an act of suggesting \( \phi \) is an illocutionary act, it works as a trigger of preference upgrade only if we accept the preference induced by its issuer.

Preferences relativized to individual agents can be considered as propositional attitudes. So acts of getting addressees to prefer something can be considered as perlocutionary acts.

The Language of Dynamic Deontic Epistemic Preference Logic (DDEPL)

Definition Take a countably infinite set \( A_{prop} \) of proposition letters and a finite set \( I \) of agents, with \( p \) ranging over \( A_{prop} \) and \( i,j \) over \( I \).

The language of DDEPL is given by:

\[ \phi ::= \top | p | \neg \phi | \phi \land \psi | U \phi | O_{ij} \phi | K_i [\text{pref}]_i U \phi | [\text{G-pref}]_{ij} \phi | \text{Annc } \phi \]

\[ \pi ::= \text{Com}_{ij} \phi \]

The Language of Dynamic Deontic Epistemic Preference Logic (DDEPL) (2)

\[ [\text{Com}_{ij} \phi]_i \psi : \text{after every successful act of commanding the agent } i \text{ by } j \text{ to see to it that } \phi, \psi \text{ holds} \]

\[ [\text{G-pref}_{ij} \phi]_i \psi : \text{after every successful } j \text{’s act of getting the agent } i \text{ to consider every world where } \phi \text{ holds at least as good as the current one, } \psi \text{ holds} \]

\[ [\text{G-pref}_{ij} \phi]_i \psi : \text{after every successful } j \text{'s act of getting the agent } i \text{ to consider every world where } \phi \text{ holds at least as good as the current one, } \psi \text{ holds} \]
What we Can Say in the Language of Dynamic Deontic Epistemic Preference Logic

\[ O_{\langle i \mid j \rangle} \phi \land \langle \text{pref} \rangle_i \rightarrow \neg \phi \]

It is obligatory upon an agent \( i \) with respect to \( j \) to see to it that \( \phi \), but \( i \) find some none-\( \phi \)-world at least as good as the current one.

What we Can Say in the Language of Dynamic Deontic Epistemic Preference Logic (2)

\[ \text{[Com}_{\langle i \mid j \rangle} \rho \land \langle \text{Com}_{\langle i \mid j \rangle} \rho \rangle \neg \langle \text{pref} \rangle_i \rho \]

Acts of commanding affect deontic status of possible courses of action, but not necessarily affect addressee’s preferences.

What we Can Say in the Language of Dynamic Deontic Epistemic Preference Logic (3)

\[ \text{[Com}_{\langle i \mid j \rangle} \rho \land \langle \text{G-pref}_{\langle i \mid j \rangle} \rho \rangle \langle \text{pref} \rangle_i \rho \]

Acts of commanding affect deontic status of possible courses of action, while acts of getting addressee to prefer something affect addressee’s preferences.

Deontic Epistemic Preference Model (1/2)

A deontic epistemic preference model is a tuple \( M = \langle W, \{ \sim_j \mid i \in I \}, \{ \varepsilon_j \mid i \in I \}, \{ \text{R}_{\langle i \mid j \rangle} \mid i, j \in I \}, V \rangle \), where

- \( W \) is a non-empty set of possible worlds
- \( \sim_j \) is a usual equivalence relation of epistemic accessibility for agent \( i \)
- \( \varepsilon_j \) is a reflexive transitive relation of preference ordering for agent \( i \)
- \( \text{R}_{\langle i \mid j \rangle} \) is a deontic accessibility for agent \( i \) with respect to \( j \)
- \( V \) is a valuation for proposition letters

Deontic Epistemic Preference Model (2/2)

- \( \sim_j \) is a reflexive transitive relation of preference ordering for agent \( i \)
- \( \text{R}_{\langle i \mid j \rangle} \) is a deontic accessibility for agent \( i \) with respect to \( j \)
- \( V \) is a valuation for proposition letters

Thank you for your attention!

The End