Logical Dynamics of Speech Acts

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Treating Speech Acts as Acts

If we are to take the notion of speech act seriously, we must be able to treat speech acts as acts.

By characterizing speech acts in terms of dynamic changes they bring about, it becomes possible to treat them within a general theory of action.

Perlocutionary Acts as Acts

Perlocutionary acts are acts that really produce ‘real effects’ upon the attitudes and the actions of addressees (Austin 1975, pp. 101-3).

Locutionary act: He said to me `Shoot her!' meaning by `shoot' shoot and referring by `her' to her.

Illocutionary act: He urged (advised, ordered, etc.) me to shoot her.

Perlocutionary act (a): He persuaded me to shoot her.
Perlocutionary act (b): He got me to shoot her.

Illocutionary Acts: a Problem

Illocutionary acts do not directly affect brute facts, except for those physical conditions involved in the production of sounds and written symbols.

Nor do they directly affect attitudes and actions of addressees.

A Gap

For instance, intuitively, a command

``See to it that ϕ!''

makes worlds where ϕ holds preferred over those where it does not – at least, if we accept the preference induced by the issuer of the command. (Van Benthem & Liu, to appear)

Illocutionary Acts as Acts

Illocutionary acts affect institutional facts.

How is it possible to capture changes in institutional facts?

Example 1: On A Hot Day (1)

Suppose you are reading an article on logic in the office you share with your boss and a few other colleagues. While you are reading, the temperature of the room rises, and it is now above 30 degrees Celsius.

Example 1: On A Hot Day (2)

Then, suddenly, you hear your boss's voice. She said, ``Open the window!.''

She commanded you to open the window.

What effects does her command have on the current situation?

Example 1: On A Hot Day (3)

Her act of commanding did not affect the state of the window directly.

Nor did it affect the number of alternatives you have. It is still possible for you to turn on the air conditioner, to ignore the heat, or to open the window.

Example 1: On A Hot Day (4)

But it has now become impossible for you to choose alternatives other than that of opening the window without going against your obligation.

It is now obligatory upon you to open the window, although it was not so before.

Acts of Commanding

Acts of commanding seem to affect deontic status of possible courses of actions.

We have considered how we can capture the kind of changes acts of commanding bring about in Yamada (2007a, b).

The Idea of Command Logic

We model changes acts of commanding bring about in terms of a new update logic. We combine a multi-agent variant of the language of monadic deontic logic with a dynamic language to talk about the situations before and after the issuance of commands, and the commands that link those situations.

The results

Although the resulting language inherits various inadequacies from the language of monadic deontic logic, some interesting principles are captured and seen to be valid nonetheless.

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Related Paradigms

- Speech Act Theory (Austin, Searle)

- Propositional Dynamic Logic
- Update Semantics (Veltman)
- Logic of Public Announcements (Plaza, Groeneveld & Gerbrandy, Baltag, Moss & Solecki, Kooi & van Benthem, etc.)

- Deontic Logic

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Logics of Public Announcements

The Static Base: Epistemic Logics

Adding Announcement Modalities ↓ ↑ Translation along Reduction Axioms

The Dynamic Extension:

Logics of Public Announcements

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The Same Strategy Works for Acts of Commanding

The Static Base

The static language: $\mathcal{L}_{MDL^+}$

Multi-agent deontic logic: MDL^+

Adding Command Modalities ↓ ↑ Translation along Reduction Axioms

The Dynamic Extension

The language of Command Logic: $\mathcal{L}_{CL}$

Eliminative Command Logic: ECL

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The Language of Deontic Logic

Op : It is obligatory that p.

Pp : It is permitted that p.

Fp : It is forbidden that p.

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Relativisation

$O_i p$: It is obligatory upon i that p.

$P_i p$: It is permitted of i that p.

$F_i p$: It is forbidden of i that p.

where ***i*** is an agent in a given set ***I*** of agents.

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The Static Language $\mathcal{L}_{MDL^+}$

Definition 1 Take a countably infinite set *Aprop* of proposition letters and a finite set *I* of agents, *p* ranging over *Aprop* and *i* over *I*. The multi-agent monadic deontic language with an alethic operator $\mathcal{L}_{MDL^+}$ is given by:

$$\phi ::= \top \mid p \mid \neg\phi \mid \phi \wedge \psi \mid \Box\phi \mid O_i\,\phi$$

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$\mathcal{L}_{MDL^+}$ -models (1/2)

Definition 2 By an $\mathcal{L}_{MDL^+}$ -model, I mean a quadruple $M = \langle W^M, R_A^M, R_D^M, V^M \rangle$ where:

(i) W^M is a non-empty set
(ii) $R_A^M \subseteq W^M \times W^M$

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$\mathcal{L}_{MDL+}$ -models (2/2)

(iii) R_D^M is a function that assigns a subset $R_D^M(i)$ of R_A^M to each $i \in I$
(iv) V^M is a function that assigns a subset $V^M(p)$ of W^M to each $p \in$ *Aprop.*

We usually abbreviate $R_D^M(\,i\,)$ as R_i^M.

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Truth definition for $\mathcal{L}_{MDL^+}$ (1/2)

Definition 3 Let *M* be an $\mathcal{L}_{MDL^+}$ -model and $w \in W^M$. If $p \in$ *Aprop*, $i \in I$, and ϕ, ψ are sentences of $\mathcal{L}_{MDL^+}$, then

(a) $M,w \models_{MDL^+} p$ iff $w \in V^M(p)$
(b) $M,w \models_{MDL^+} \top$
(c) $M,w \models_{MDL^+} \neg\phi$ iff $M,w \not\models_{MDL^+} \phi$

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Truth definition for $\mathcal{L}_{MDL^+}$ (2/2)

(d) $M,w \models_{MDL^+} (\phi \wedge \psi)$ iff $M,w \models_{MDL+} \phi$ and $M,w \models_{MDL^+} \psi$
(e) $M,w \models_{MDL^+} \Box\phi$ iff for every *v* such that $\langle w,v \rangle \in R_A^M$, $M,v \models_{MDL^+} \phi$
(f) $M,w \models_{MDL^+} O_i\,\phi$ iff for every *v* such that $\langle w,v \rangle \in R_i^M$, $M,v \models_{MDL^+} \phi$.

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Completeness of MDL$^+$

A sound and complete proof system for MDL$^+$ is given in Yamada (2007a). Thus:

Theorem 1: MDL is strongly complete with respect to the class of $\mathcal{L}_{MDL^+}$ -models.

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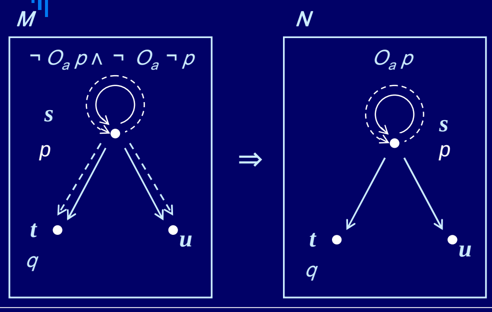
Example 1 in $\mathcal{L}_{MDL^+}$

The situations you were in before and after the issuance of your boss's command in Example 1 can be represented by two related $\mathcal{L}_{MDL^+}$ -models M and N respectively. For example, we may say:

(1) $M,s \vDash_{MDL^+} \neg O_a p \wedge \neg O_a \neg p$

(2) $N,s \vDash_{MDL^+} O_a\, p$

Boss's Act of Commanding



What we cannot say in $\mathcal{L}_{MDL^+}$

Sentences of $\mathcal{L}_{MDL^+}$ can be used to describe the situations before and after the Issuance of your boss's command.

But they cannot be used to talk about the act of commanding that links these situations.

The Language of the Logic of Public Announcements

$K_i\,\psi$: Agent i knows that ψ.

$[A!]\,\phi$: After every truthful public announcement that A, ϕ holds.

$[A!]K_i\psi$: After every truthful public announcement that A, i knows that ψ.

The Language of the Logic of the Act of Commanding (Command Logic)

$O_i\xi$: It is obligatory upon the agent i to see to it that ξ.

$[!_i\psi]\,\phi$: After every act of commanding i (the addressee) to see to it that ψ, ϕ holds.

$[!_i\psi]\,O_i\xi$: After every act of commanding i to see to it that ψ, it is obligatory upon i to see to it that ξ.

The Dynamified Language $\mathcal{L}_{CL}$

Definition 5 Take the same countably infinite set *Aprop* of proposition letters and the same finite set *I* of agents, *p* ranging over *Aprop* and *i* over *I* as before. The language of Command Logic $\mathcal{L}_{CL}$ is given by:

$$\phi ::= \top \mid p \mid \neg\phi \mid \phi \wedge \psi \mid \Box\phi \mid O_i\phi \mid [\pi]\phi$$
$$\pi ::= !_i\,\phi$$

Truth definition for $\mathcal{L}_{CL}$ (1/3)

Definition 4 Let M be an $\mathcal{L}_{MDL^+}$ -model and $w \in W^M$. If $p \in Aprop$, $i \in I$, and ϕ, ψ are sentences of $\mathcal{L}_{CL}$, then

(a) $M,w \vDash_{ECL} p$ iff $w \in V^M(p)$

(b) $M,w \vDash_{ECL} \top$

(c) $M,w \vDash_{ECL} \neg\phi$ iff $M,w \nvDash_{ECL} \phi$

Truth definition for $\mathcal{L}_{CL}$ (2/3)

(d) $M,w \vDash_{ECL} (\phi \wedge \psi)$ iff $M,w \vDash_{ECL} \phi$ and $M,w \vDash_{ECL} \psi$

(e) $M,w \vDash_{ECL} \Box\phi$ iff for every v such that $\langle w,v\rangle \in R^M_A$, $M,v \vDash_{ECL} \phi$

(f) $M,w \vDash_{ECL} O_i\phi$ iff for every v such that $\langle w,v\rangle \in R^M_i$, $M,v \vDash_{ECL} \phi$.

Truth definition for $\mathcal{L}_{CL}$ (3/3)

(g) $M,w \vDash_{ECL} [!_i\chi]\phi$ iff $M_{!_i\chi},w \vDash_{ECL} \phi$,

where $M_{!_i\chi}$ is an $\mathcal{L}_{MDL^+}$ -model obtained from M by replacing R^M_D with the function $R^{M_{!_i\chi}}_D$ such that for each $j \in I$:

(i) if $j \neq i$, $R^{M_{!_i\chi}}_D(j) = R^M_I(j)$

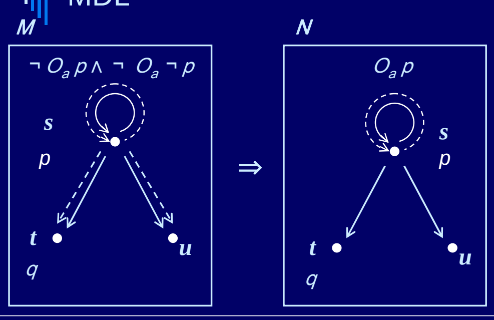
(ii) If $j = i$, $R^{M_{!_i\chi}}_D(j) =$ $\{\langle x,y\rangle \in R^M_i(i) \mid M,y \vDash_{ECL} \chi\}$.

On this truth definition

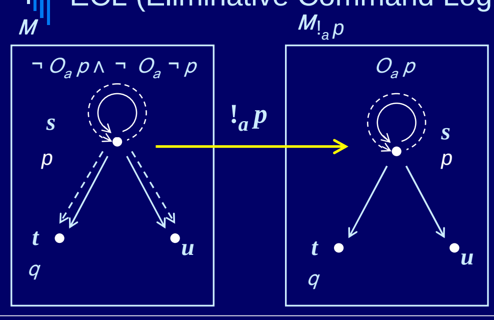
We abbreviate $\{\langle x,y\rangle \in R^M_D(i) \mid M,y \vDash_{ECL} \chi\}$ as $R^M_i{\upharpoonright}\chi^{\downarrow}$.

As we always have $R^M_i{\upharpoonright}\chi^{\downarrow} \subseteq R^M_i$, we also have $R^M_i{\upharpoonright}\chi^{\downarrow} \subseteq R^M_A$. Hence $M_{!_i\chi}$ is guaranteed to be an $\mathcal{L}_{MDL^+}$ -model.

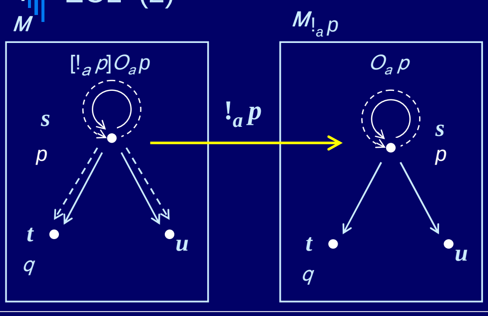
Boss's Act of Commanding in MDL$^+$



Boss's Act of Commanding in ECL (Eliminative Command Logic)



Boss's Act of Commanding in ECL (2)



CUGO Principle

If neither O_i's nor $!_i$'s occur in ϕ, we have

$[!_i \phi]\, O_i \phi$.

Usually, you should do what your boss commands you to do. In other word, commands usually generate obligations.

Why not CGO but CUGO

The following formula is not valid:

$[!_i P_i q]\, O_i P_i q$.

If ψ involves deontic operators or command operator indexed with i, the truth of ψ at w in M does not guarantee the truth of ψ at w in $M!_i \phi$.

An Open Question

Let S_{CGO} be the set of sentences ϕ such that we have $\vDash_{\mathrm{ECL}} [!_i \phi]\, O_i \phi$. Let $S_{i\text{-free}}$ be the set of $\mathcal{L}_{\mathrm{MDL}^+}$ -sentences in which no deontic operators for i occur.

As $O_i \phi \to O_i \phi \in S_{\mathrm{CGO}}$, we have $S_{i\text{-free}} \subseteq S_{\mathrm{CGO}}$.

But exactly how large S_{CGO} is is an open question.

Reduction Axioms

(1) $[!_i \phi]\, p \leftrightarrow p$ where $p \in Aprop$
(2) $[!_i \phi] \top \leftrightarrow \top$
(3) $[!_i \phi] \neg \psi \leftrightarrow \neg [!_i \phi]\, \psi$
(4) $[!_i \phi] (\psi \wedge \chi) \leftrightarrow ([!_i \phi]\, \psi \wedge [!_i \phi]\, \chi)$
(5) $[!_i \phi] \Box \psi \leftrightarrow \Box [!_i \phi]\, \psi$
(6) $[!_i \phi]\, O_i \psi \leftrightarrow O_i (\phi \to [!_i \phi]\, \psi)$
(7) $[!_i \phi]\, O_j \psi \leftrightarrow O_j [!_i \phi]\, \psi$ where $i \neq j$.

Translation function t (1/2)

$t(p)$	=	p
$t(\top)$	=	$\top$
$t(\neg \phi)$	=	$\neg t(\phi)$
$t(\phi \wedge \psi)$	=	$t(\phi) \wedge t(\psi)$
$t(\Box \phi)$	=	$\Box t(\phi)$
$t(O_i \phi)$	=	$O_i\, t(\phi)$

Translation function t (2/2)

$t([!_i\phi]\, p) = p$

$t([!_i\phi]\, \top) = \top$

$t([!_i\phi]\, \neg\psi) = \neg t([!_i\phi]\, \psi)$

$t([!_i\phi]\, (\psi \wedge \chi)) = t([!_i\phi]\, \psi) \wedge t([!_i\phi]\, \chi)$

$t([!_i\phi]\, \Box\psi) = \Box t([!_i\phi]\, \psi)$

$t([!_i\phi]\, O_i\psi) = O_i(t(\phi) \rightarrow t([!_i\phi]\, \psi))$

$t([!_i\phi]\, O_j\psi) = O_j t([!_i\phi]\, \psi)$ where $i \neq j$.

$t([!_i\phi]\, [!_j\psi]\, \chi) = t([!_i\phi]\, t([!_j\psi]\, \chi))$ for any $j \in I$

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Proof System for ECL

The proof system for ECL (Eliminative Command Logic) contains all the axioms and all the rules of the proof system for MDL⁺, and in addition all the reduction axioms listed above and the following rule:

$$\frac{\psi}{[!_i\phi]\, \psi}$$

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Completeness

The completeness of ECL is derived from The completeness of MDL⁺.

Theorem 2
ECL is strongly complete with respect to the class of $\mathcal{L}_{\mathrm{MDL}^+}$-models.

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Built-in assumptions 1

Commands are assumed to be always eliminative.

Proposition 2 We have:

$$\models_{\mathrm{ECL}} [!_i\phi] P_i\phi \rightarrow P_i\phi .$$

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Example 2

Suppose you are in a combat troop and now waiting for your captain's command to fire. Then you hear the command, and it has become obligatory upon you to fire. But before that, you were not permitted to fire. This forbiddance is now no longer in force. Thus it seems that after his command, you are permitted to fire, at least in the sense of lack of forbiddance.

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Built-in assumptions 2

Commands of the form $!_i p$ is assumed to have no effects on deontic accessiblity relations for any agent other than i.

Corollary 2 If $\psi \in S_{i\text{-free}}$, then:

$$M, w \models_{\mathrm{ECL}} \psi \text{ iff } M_{!i\phi}, w \models_{\mathrm{ECL}} \psi .$$

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Built-in assumptions 3

Commands are assumed to have no preconditions.

Compare:

PAL: $[\phi !]\, K_i \psi \leftrightarrow (\phi \rightarrow K_i [\phi !]\ \psi)$
ECL: $[!_i \phi]\, O_i \psi \leftrightarrow O_i (\phi \rightarrow [!_i \phi]\ \psi)$

Dead end principle

We have:

$$\vDash_{ECL} [!_i (p \wedge \neg p)]\, O_i \phi .$$

Since $R_i^M \restriction (p\wedge\neg p)^{\downarrow}$ is empty, no world is $R_i^M \restriction (p\wedge\neg p)^{\downarrow}$-accessible for i. Absurd commands lead to an obligational dead end.

D axiom cannot be added

We have:

$$M_{!_i (p\wedge\neg p)}, w \vDash_{ECL} O_i (p \wedge \neg p) \wedge \neg P_i (p \wedge \neg p) .$$

Thus $O_i\ \phi \rightarrow P_i\ \phi$ cannot be added to ECL.

Restricted sequential conjunction principle

If $\phi,\ \psi \in S_{i\text{-free}}$, we have:

$$\vDash_{ECL} [!_i\ \phi][!_i\ \psi]\,\chi \leftrightarrow [!_i\,(\phi \wedge \psi)]\,\chi .$$

Since $R_i^M \restriction (\phi \wedge \psi)^{\downarrow}$ can be distinct from $(R_i^M \restriction \phi^{\downarrow}) \restriction \psi^{\downarrow}$, unrestricted form of sequential conjunction principle is not valid.

Restricted order invariance principle

If $\phi,\ \psi \in S_{i\text{-free}}$, we have:

$$\vDash_{ECL} [!_i\ \phi][!_i\ \psi]\,\chi \leftrightarrow [!_i\ \psi][!_i\ \phi]\,\chi .$$

Since $(R_i^M \restriction \phi^{\downarrow}) \restriction \psi^{\downarrow}$ can be distinct from $(R_i^M \restriction \psi^{\downarrow}) \restriction \phi^{\downarrow}$, unrestricted form of order invariance principle is not valid.

Another Interesting Validity

We have:

$$\vDash_{ECL} [!_i\, p]\ (\,[!_i\ \neg p]\ O_i \phi\,).$$

Everything comes to be obligatory if a pair of contradictory commands is issued. Thus you will be in an obligational dead end.

An Extension: ECL II

We may distinguish command issuers:

$[!_{(i,j)}\, p]\ (\ [!_{(i,k)} \neg p]\ (O_{(i,j)}\, p \wedge O_{(i,k)} \neg p)\ ).$

If an authority *k* commands you to see to it that $\neg p$ after another authority *j* commands you to see to it that *p,* then you will be in an obligational dilemma.

Why an obligational dilemma

Although no worlds are both $R_{(i,j)}$ - accessible and $R_{(i,k)}$ - accessible, there may be an $R_{(i,j)}$- accessible world and an $R_{(i,j)}$ - accessible world in such a sitation.

Example 3 A Contingent Dilemma (1/2)

$[!_{(i,j)}\ p]\ (\ [!_{(i,k)}\ q]\ (O_{(i,j)}\ p \wedge O_{(i,k)}\ q)\ ).$

p : you attend the conference in Hakodate on May 8 2007.
q : you join the demonstration in São Paulo on May 8 2007.

You will be in an obligational dilemma, if *p* and *q* happen to be incompatible.

A Contingent Dilemma (2/2)

$[!_{(i,j)}\, p]\ (\ [!_{(i,k)}\, q]\ (O_{(i,j)}\, p \wedge O_{(i,k)}\, q)\ ).$

If some very rapid means of transportation were available, it would be possible for you to obey both commands. But unfortunately, no such means of transportation happens to be available in the real world.

Logical Dynamics of Multi-agent Language Games

(1) Acts of promising can also be considered as deontic updaters, and acts of asserting can be considered as updaters of propositional commitments.

(2) Perlocutionary Acts can also be considered as updaters of systems of knowledge, belief and preference.

Treating Illocutionary Acts and Perlocutionary Acts Together

ECL II can be combined with DEUL (Dynamic Epistemic Upgrade Logic) of van Benthem & Liu (to appear).

DEUL can be interpreted as dealing with perlocutionary acts of getting addressees to prefer something.

The Language of Epistemic Preference Logic

$K_i\phi$: the agent *i* knows that ϕ

$[pref]_i\phi$: in every world the agent *i* considers at least as good as the current one ϕ holds

$U\phi$: in every world ϕ holds

The Language of Dynamic Epistemic Upgrade Logic

$[\phi!]\psi$: after every truthfull public announcement that ϕ, ψ holds

$[\phi!]K_i\psi$: after every truthful public announcement that ϕ, *i* knows that ψ

$[\#\phi]\psi$: after every act of publicly suggesting ϕ, ψ holds

$[\#\phi][pref]_i\psi$: after every act of publicly suggesting ϕ, in every world *i* considers at ･･･current one, ψ holds

The Need for Reinterpretation

As an act of suggesting ϕ is an illocutionary act, it works as a trigger of preference upgrade only if we accept the preference induced by its issuer.

Preferences relativized to individual agents can be considered as propositional attitudes. So acts of getting addressees to prefer something can be considered as perlocutionary acts.

The Language of Dynamic Deontic Epistemic Preference Logic (DDEPL)

Definition Take a countably infinite set *Aprop* of proposition letters and a finite set *I* of agents, with *p* ranging over *Aprop* and *i, j* over *I* .
The language of DDEPL is given by:

$$\phi ::= \top \mid p \mid \neg\phi \mid \phi\wedge\psi \mid U\phi \mid O_{(i,j)}\phi \mid K_i \mid [\mathrm{pref}]_i \mid [\pi]\phi$$

$$\pi ::= \mathrm{Com}_{(i,j)}\phi \mid \text{G-pref}_{(i,j)}\phi \mid \mathrm{Annc}\ \phi$$

The Language of Dynamic Deontic Epistemic Preference Logic (DDEPL)

$K_i\phi$: the agent *i* knows that ϕ

$[pref]_i\phi$: in every world the agent *i* considers at least as good as the current one, ϕ holds

$U\phi$: in every world ϕ holds

$O_{(i,j)}\phi$: it is obligatory upon the agent *i* with respect to *j* to see to it that ϕ

The Language of Dynamic Deontic Epistemic Preference Logic (DDEPL) (2)

$[Com_{(i,j)}\phi]\psi$: after every successful act of commanding the agent *i* by *j* to see to it that ϕ, ψ holds

$[\text{G-}pref_{(i,j)}\phi]\psi$: after every successful *j*'s act of getting the agent *i* to consider every world where ϕ holds at least as good as the current one, ψ holds

What we Can Say in the Language of Dynamic Deontic Epistemic Preference Logic

$O_{(i,j)}\, \phi \wedge \langle pref \rangle_i\, \neg \phi$

It is obligatory upon an agent *i* with respect to *j* to see to it that ϕ, but *i* find some none-ϕ-world at least as good as the current one.

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What we Can Say in the Language of Dynamic Deontic Epistemic Preference Logic (2)

$[Com_{(i,j)}\, p]\, O_{(i,j)}\, p \wedge \langle Com_{(i,j)}\, p \rangle \neg [pref]_i\, p$

Acts of commanding affect deontic status of possible courses of action, but not necessarily affect addressee's preferences.

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What we Can Say in the Language of Dynamic Deontic Epistemic Preference Logic (3)

$[Com_{(i,j)}\, p]\, O_{(i,j)}\, p \wedge [G\text{-}pref_{(i,j)}\, p][pref]_i\, p$

Acts of commanding affect deontic status of possible courses of action, while acts of getting addressee to prefer something affect addressee's preferences.

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Deontic Epistemic Preference Model (1/2)

A deontic epistemic preference model is a tuple $M = \langle W, \{\sim_i \mid i \in I\}, \{\lesssim_i \mid i \in I\}, \{R_{(i,j)} \mid i, j \in I\}, V \rangle$, where

- W is a non-empty set of possible worlds
- $\sim_i$ is a usual equivalence relation of epistemic accessibility for agent *i*

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Deontic Epistemic Preference Model (2/2)

- $\lesssim_i$ is a reflexive transitive relation of preference ordering for agent *i*
- $R_{(i,j)}$ is a deontic accessibility for agent *i* with respect to *j*
- V is a valuation for proposition letters

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Thank you for your attention!

The End

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