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**Tax Incidence in Dynamic Economies with
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November, 2007

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Tax Incidence in Dynamic Economies with Externalities and Endogenous Labor Supply*

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Abstract

This paper examines the long-run incidence of factor income taxes and expenditure taxes in an infinitely lived representative agent growth model which allows both for production externalities and for endogenous labor supply. The novelty of this paper is its investigating of how the long-run incidence of taxes is affected by indeterminacy of equilibria that is caused mainly by nonseparable preferences between consumption and leisure. We show that the effects of the taxes on steady state welfare as well as the steady state levels of consumption, capital, and employment are all negative regardless of whether a steady state is determinate or indeterminate in an exogenous growth model. By contrast, in an endogenous growth model those distortionary taxes are growth and welfare enhancing in both a determinate steady state featuring the unconventional slope of the labor supply curve and an indeterminate steady state featuring its conventional slope.

Keywords: Tax incidence, externalities, indeterminacy and endogenous labor supply

JEL classifications: H41, F13, D62

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1 Introduction

In this paper we examine the long-run incidence of three types of tax instruments: capital income taxes, labor income taxes and consumption taxes, which are accompanied by compensating lump-sum transfers, while allowing both for endogenous labor supply and for production externalities in an infinitely lived representative agent growth model. We show that the results of long-run tax incidence (i.e., the effects on the functional distribution of income among factors or among individuals, or on welfare costs) are significantly affected by the emergence of indeterminate equilibrium that is caused by nonseparable preferences between consumption and leisure coupled with production externalities, when the model generates sustained endogenous growth.

Dynamic tax incidence has been studied in various versions of the growth models in the framework of an intertemporal optimizing representative agent. In the standard neoclassical growth model where physical capital is the only factor that can be accumulated (e.g. Turnovsky 1982, Becker 1985, Sinn 1987 and Judd 1987), both capital and labor income taxes reduce the steady state level of income, but have no effect on the net return to capital and thus no growth effect in the long run. On the other hand, the literature on endogenous growth models has reexamined the long-run incidence in a framework where both capital and labor (human capital) are reproducible factors under constant-returns-to-scale accumulation technology (e.g. Pecorino 1993, Devereux and Love 1994, and Milesi-Ferretti and Roubini 1998). This literature shows that in general both labor and capital income taxes reduce the long-run growth rate. Indeed, these two distortionary taxes effectively act as a tax on human and non-human capital incomes, respectively, thereby discouraging an incentive to either accumulation.

These findings have been derived using a standard version of exogenous and endogenous growth models in which a dynamic equilibrium path is uniquely determined. Recent advances in macroeconomics, on the other hand, have highlighted the importance of self-fulfilling prophecies such as ‘sunspots’ and ‘animal spirits’ in explaining economic fluctuations. Models

of indeterminacy of converging paths would provide a useful vehicle to account for the business cycle and other macroeconomic phenomena without having to rely on random shocks to economic fundamentals. In particular, non-uniqueness of equilibrium can arise straightforwardly in dynamic general equilibrium settings once the assumptions of perfectly competitive markets and constant-returns-to-scale are dropped. Although several authors investigate how the presence of fiscal or monetary policy alters the likelihood of indeterminacy (e.g. Guo and Lansing 1998), neither author in this area has explored the policy implications for indeterminacy, despite the fact that the emergence of indeterminacy may potentially change its implications.

Drawing on both strands, this paper investigates how the emergence of indeterminacy affects the long-run incidence of taxes in both exogenous and endogenous growth models. In our models with and without endogenous growth, indeterminate steady state equilibria are mainly caused by the assumption of nonseparable utility between consumption and leisure as in Bennett and Farmer's (2000) model. In view of Samuelson's correspondence principle, one may reasonably expect that the dynamic behavior of such a growth model in the neighborhood of a steady state is directly linked to its comparative statics properties. More specifically, since the Jacobian matrix of the dynamic system evaluated at a steady state would be of opposite signs depending on whether the steady state is determinate or indeterminate, the long-run comparative statics properties may be reversed if indeterminacy occurs. If this conjecture were correct, the outcome of dynamic tax incidence may be significantly affected by the emergence of indeterminacy.

In the existing literature, Pelloni and Waldmann (2000) is mostly closely related to our study. This paper considers the effects of taxes combined with various forms of government expenditures in the endogenous growth model featuring the indeterminacy of equilibrium paths. Our model differs from theirs in five important ways. First, unlike their tax experiments, we consider three types of taxation: a capital income tax, a labor income tax, and a consumption tax, all of which are accompanied by compensating lump-sum transfers. This method of analysis is very common in the literature on tax incidence, in part because of analytical convenience, but also because it

intends to isolate the pure incentive effect of the tax by nullifying the negative income effect. Owing to this virtue, this decomposition method serves in highlighting the efficiency aspects of taxation. Second, we compare between the results of tax incidence in exogenous growth models and those in endogenous growth models, retaining the same isoelastic utility and Cobb-Douglas production functions, but differentiating only the degree of capital externalities. Such a comparison enables one to clarify the essential role of endogenous growth in the context of dynamic tax incidence. Third, we allow the labor Frisch supply curve to slope down. Since Pelloni and Waldmann (2000) have used the model with nonseparable, strictly concave preferences, this important extension is left unexplored. In particular, they show that if the market equilibrium is indeterminate, capital income taxes increase a balanced growth rate. Their paradoxical finding conveys the impression that the adverse effects of distortionary taxes will be overturned if the indeterminacy of equilibrium occurs. Our exercises show that this may not always be the case. Fourth, we investigate how the degree of production externalities or increasing returns at the aggregate level *quantitatively* affects the results of tax incidence, which has not been little attention in their analysis nor others. Ignoring productivity spillover when they exist may lead to a substantial underestimation of the actual impacts of tax changes, and, therefore, larger degrees of externalities may dramatically alter the incidence outcomes as well as the welfare implications of tax policy indicated by the traditional analysis of tax incidence. Fifth, Pelloni and Waldmann assume that there are no pre-existing taxes for analytical convenience, which means that a tax increase is just an introduction of a new tax, setting all other taxes to zero, whereas we allow for such preexisting taxes. In other words, their analysis would be inappropriate in examining the effects of real-world taxes that are far from ‘small’ ones, a point made by Friedlaender and Vandendorpe (1976), and many others in the literature on static tax incidence analysis.

Section 2 first describes the behavior of households, firms and the government in an exogenous growth model whose long-run growth rate is exogenously given, and then investigates its stability properties. Section 3 derives the results of steady state tax incidence. Section 4 constructs an endoge-

nous growth version of the basic model by increasing the parameter value representing the magnitude of capital externalities so as to eliminate diminishing returns to capital. Section 5 analyzes the incidence of taxation along a balanced growth path. Section 6 concludes the paper. Some mathematical derivations will be given in the appendices.

2 The model

2.1 Firms

There is a continuum of identical competitive firms in the economy, with total number normalized to one. The representative firm i produces output using a constant returns-to-scale, Cobb-Douglas technology:

$$Y_i = K_i^a L_i^b X, \quad a + b = 1, \quad (1)$$

where K_i and L_i are capital stock and labor hours employed by firm i , respectively. The term X represents productive externalities that are taken as given by each firm. Following Benhabib and Farmer (1994), we further specify these externalities as

$$X = K^{\alpha-a} L^{\beta-b}, \quad a < \alpha \leq 1, \quad b < \beta < 1 \quad \text{and} \quad \alpha + \beta > 1, \quad (2)$$

where K and L represent the aggregate stock of capital and the aggregate labor hours, respectively.¹ In a symmetric equilibrium, all firms take the same action such that for all i , $Y_i = Y$, $K_i = K$ and $L_i = L$. Substituting (2) into (1), we can obtain the following social production technology:

$$Y = K^\alpha L^\beta. \quad (3)$$

¹Benhabib and Farmer (1994) show that the assumption $\beta > 1$ is needed to deliver an indeterminate steady state. However, this assumption is difficult to reconcile with empirical facts, as pointed out by a number of recent empirical papers. One of the reasons why we here adopt nonseparable utility is to avoid such criticism.

We shall analyze two cases where $\alpha < 1$ and $\alpha = 1$ separately. The first case corresponds to an exogenous growth model, while the second case corresponds to an endogenous growth model in which capital externalities are strong enough to generate sustained endogenous growth.²

Given such external effects, the competitive firms maximize their profits thus yielding

$$r = a(Y/K) = aK^{\alpha-1}L^\beta, \quad (4a)$$

$$w = b(Y/L) = bK^\alpha L^{\beta-1}, \quad (4b)$$

where r and w denote the pre-tax rates on the return to capital and on the real wage, respectively.

2.2 Households with non-separable preferences

There is a unit measure of identical infinitely lived households, each of whom maximizes its lifetime utility:

$$\int_0^\infty \frac{[cV(l)]^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt, \quad (5)$$

where $V(l) \equiv (1-l)^\chi$, c and l are respectively the individual's consumption and hours worked, σ (> 0 but $\sigma \neq 1$) denotes the inverse of the intertemporal elasticity of substitution in consumption, and ρ (> 0) is the subjective rate of time preference.³ Also, $1 - \chi(1 - \sigma)$ expresses the inverse of the *effective* intertemporal elasticity of substitution in leisure.⁴ When σ tends to one, by continuity, the instantaneous utility function reduces to $\ln [cV(l)]$.

²When $\alpha > 1$, growth is explosive and thus we do not analyze this case.

³Alternatively, we may assume that $V(l) \equiv \exp\left(-\frac{l^{1+\gamma}}{1+\gamma}\right)$, which has been used by Bennett and Farmer (2000). Nevertheless, our essential results remain valid.

⁴Hintermaier (2003) points out that when the instantaneous utility function in (5) displays a strictly concave property as long as $\sigma > \chi/(1 + \chi)$, the Frisch labor supply curve is positively sloped. In contrast, we allow for any non-negative value of σ (i.e., the instantaneous utility function is quasi-concave), so that the Frisch labor supply curve may not be nonnegative; see footnote 6 in further detail.

The budget constraint faced by the representative household is given by

$$\dot{k} = (1 - \tau_w)wl + (1 - \tau_k)(r - \delta)k + z - (1 + \tau_c)c, \quad k(0) \text{ given}, \quad (6)$$

where δ and z stand for the depreciation rate and transfer payments which are rebated to households in a lump-sum fashion, respectively. The variables τ_k , τ_w and τ_c represent the tax rates applied to capital income, labor income and consumption, respectively.

The current value Hamiltonian function can be written as:

$$H(c, l, k, \lambda) \equiv \frac{[c(1-l)^x]^{1-\sigma} - 1}{1-\sigma} + \lambda[(1 - \tau_w)wl + (1 - \tau_k)(r - \delta)k + z - (1 + \tau_c)c],$$

where λ represents the shadow price of capital holdings. The first-order conditions for this problem are given by

$$(c(1-l)^x)^{-\sigma}(1-l)^x - \lambda(1 + \tau_c) = 0, \quad (7a)$$

$$-(c(1-l)^x)^{-\sigma}c\chi(1-l)^{x-1} + \lambda(1 - \tau_w)w = 0, \quad (7b)$$

$$\dot{\lambda} - \rho\lambda = -\lambda(1 - \tau_k)(r - \delta), \quad (7c)$$

together with the given initial level of capital stock k_0 and the transversality condition $\lim_{t \rightarrow \infty} e^{-\rho t}\lambda(t)k(t) = 0$.

Since we have assumed that the instantaneous utility function in (5) is quasi-concave, we need to check whether the solution to the household's optimization problem given by (7a), (7b) and (7c) is also sufficient for maximization. To do so, we first apply the implicit function theorem to (7a) and (7b) to get the functions $c(\lambda)$ and $l(\lambda)$, respectively, given λ . Next, we make use of Proposition 8 of Arrow and Kurz (page 49; 1970) in that a sufficient condition for maximization is that the maximized Hamiltonian $H^*(k, \lambda) \equiv \max_{c,l} H(c, l, k, \lambda)$ is concave for k given λ . Since the functions $c(\lambda)$ and $l(\lambda)$ depend on λ but not on k , the maximized Hamiltonian func-

tion $H^*(k, \lambda)$ associated with our household's optimization problem is linear in k given λ , and hence concave in k . As a result, the sufficient condition for maximization is satisfied.

To focus on the problem at hand, we rule out a market for government bonds and government expenditures. We focus on the *differential incidence* of taxes in that the government's flow budget should be balanced at each point in time through adjusting the size of transfer payments to households, when the government changes each of the tax parameters.⁵ Its flow budget constraint is thus expressed by:

$$Z = \tau_w wL + \tau_k (r - \delta) K + \tau_c C, \quad (8)$$

where Z stands for total transfers to households.

3 Steady state tax incidence

Since we focus on a symmetric perfect-foresight equilibrium, households know the future paths of factor prices, taxes as well as transfer payments when they decide how much to consume, work and invest over their lifetime. In this equilibrium the aggregate consistency condition requires that $k = K$, $c = C$, $l = L$, and $z = Z$.

Dividing (7b) by (7a) and taking into account (4b) results in

$$\chi \frac{c}{1-l} = \frac{b(1-\tau_w)}{1+\tau_c} k^\alpha l^{\beta-1}. \quad (9)$$

This condition requires that the marginal rate of substitution between consumption and leisure should be equated to the real wage rate adjusted for the consumption and wage taxes at each point in time. Combining (9)

⁵The analysis of differential tax incidence is concerned with the general equilibrium effects on prices, output, factor returns and welfare of substituting one tax for another (or lump-sum transfers), while keeping the budgetary scale constant. Such tax experiments can be straightforwardly implemented in an exogenous growth model, while a balanced growth incidence analysis should be conducted in an endogenous growth model, because in the latter model all state variables grow indefinitely at a positive constant rate, as does the scale of the government budget.

with (7a), we can rewrite the functions $c(\lambda)$ and $l(\lambda)$ as $c(k, \lambda, \tau_c, \tau_w)$ and $l(k, \lambda, \tau_c, \tau_w)$.

Substitution of $c(k, \lambda, \tau_c, \tau_w)$ and $l(k, \lambda, \tau_c, \tau_w)$ into (6) and (7c), together with (4a), (4b) and (8), yields

$$\dot{k} = k^\alpha l(k, \lambda, \tau_c, \tau_w)^\beta - c(k, \lambda, \tau_c, \tau_w) - \delta k, \quad (10a)$$

$$\dot{\lambda} = \lambda[\rho - (1 - \tau_k) \{a k^{\alpha-1} l(k, \lambda, \tau_c, \tau_w)^\beta - \delta\}]. \quad (10b)$$

Taking a linear approximation of (10a) and (10b) around the steady state, we have

$$\begin{bmatrix} \dot{k} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} \alpha \hat{k}^{\alpha-1} \hat{l}^\beta + \hat{k}^\alpha \beta \hat{l}^{\beta-1} l_k - c_k - \delta \\ -\hat{\lambda} (1 - \tau_k) a (\alpha - 1) \hat{k}^{\alpha-2} \hat{l}^\beta - \hat{\lambda} (1 - \tau_k) a \hat{k}^{\alpha-1} \beta \hat{l}^{\beta-1} l_k \\ \hat{k}^\alpha \beta \hat{l}^{\beta-1} l_\lambda - c_\lambda \\ -\hat{\lambda} (1 - \tau_k) a \hat{k}^{\alpha-1} \beta \hat{l}^{\beta-1} l_\lambda \end{bmatrix} \begin{bmatrix} \hat{k} - k \\ \hat{\lambda} - \lambda \end{bmatrix}, \quad (11)$$

where c_j and l_j ($j = k, \lambda$) represent the partial derivatives with respect to the argument j , and the notation $\hat{\cdot}$ denotes the steady state value of the corresponding variable. The steady state values of c and k satisfy the following steady state conditions:

$$\hat{k}^\alpha l(\hat{k}, \hat{\lambda}, \tau_c, \tau_w)^\beta = c(\hat{k}, \hat{\lambda}, \tau_c, \tau_w) + \delta \hat{k}, \quad (12a)$$

$$\frac{\rho}{1 - \tau_k} + \delta = a \hat{k}^{\alpha-1} l(\hat{k}, \hat{\lambda}, \tau_c, \tau_w)^\beta. \quad (12b)$$

To identify the qualitative nature of the model's dynamics, we should investigate the signs of the determinant and the trace of the matrix appearing on the right-hand side of (11). The determinant of the Jacobian matrix, denoted by $R(\hat{l})$, is given by:

$$R(\hat{l}) \equiv [\rho + (1 - \tau_k) \delta] (1 - \alpha) \frac{\Delta(\hat{l})^{-1} \hat{c}}{1 - \hat{l} \frac{\hat{c}}{\hat{k}}}, \quad (13)$$

where

$$\Delta(\hat{l}) \equiv \sigma \left[\beta - 1 - \frac{\hat{l}}{1 - \hat{l}} \right] + (1 - \sigma) \chi \frac{\hat{l}}{1 - \hat{l}} = \sigma \left[\beta - 1 - \left(1 + \frac{\sigma - 1}{\sigma} \chi \right) \frac{\hat{l}}{1 - \hat{l}} \right].$$

It follows from (13) that $sign[R(\hat{l})] = sign[\Delta(\hat{l})]$. This means that if $\Delta(\hat{l}) < 0$, then the system has a saddle-point property, whereas if $\Delta(\hat{l}) > 0$, then indeterminacy arises, provided that the trace of the Jacobian matrix, $\frac{\rho}{1 - \tau_k} -$

$\Delta(\hat{l})^{-1} \left(\frac{\rho}{1 - \tau_k} + \delta \right) Q(\hat{l})$, is negative, where

$$Q(\hat{l}) \equiv (\sigma - 1) \left[\beta - \chi \frac{\hat{l}}{1 - \hat{l}} \left\{ 1 - \alpha \delta \left(\frac{\rho}{1 - \tau_k} + \delta \right)^{-1} \right\} \right] + \frac{\alpha - a}{a} \frac{\sigma}{1 - \hat{l}} + \beta \tau_k.$$

It is also worth noting that the sign of $\Delta(\hat{l})$ implies the relative slope of the labor demand and Frisch labor supply curves.⁶ Since the slope of the labor demand curve is always negative by assumption $\beta - 1 < 0$, if the labor demand curve is steeper than the Frisch labor supply curve as illustrated in Fig. 1 (we may say that the labor demand curve and the Frisch labor supply curve cross with the ‘normal’ slopes), then $\Delta(\hat{l}) < 0$.

Conversely, if the Frisch labor supply curve is steeper than the labor demand curve as illustrated in Fig. 2 (i.e., the labor demand curve and the Frisch labor supply curve cross with the ‘wrong’ slopes), then $\Delta(\hat{l}) > 0$.

Totally differentiating (12a) and (12b) and rearranging, we obtain the long-run effects of changes in the respective taxes as follows:

⁶According to Bennett and Farmer (2000), the Frisch labor supply curve is defined as labor supply as a function of the real wage holding the marginal utility of consumption constant. By taking the logarithm of both sides of (7b) and substituting (9) for c in the resultant expression, this curve can be expressed by $\log w = [-1 + (1 - \sigma)\chi/\sigma] \log(1 - \hat{l}) - (1/\sigma) \ln \lambda + const.$ The slope of the Frisch labor supply curve evaluated at the steady state is given by $[1 + (\sigma - 1)\chi/\sigma][\hat{l}/(1 - \hat{l})] > 0$, while the slope of the labor demand curve is $\beta - 1 < 0$. It should also be noted that assumption $\beta < 1$ implies that the determinant condition for indeterminacy (13) (i.e. $\Delta(\hat{l}) > 0$) is met only when $\sigma < 1$, that is, the Frisch labor supply is negatively sloped.

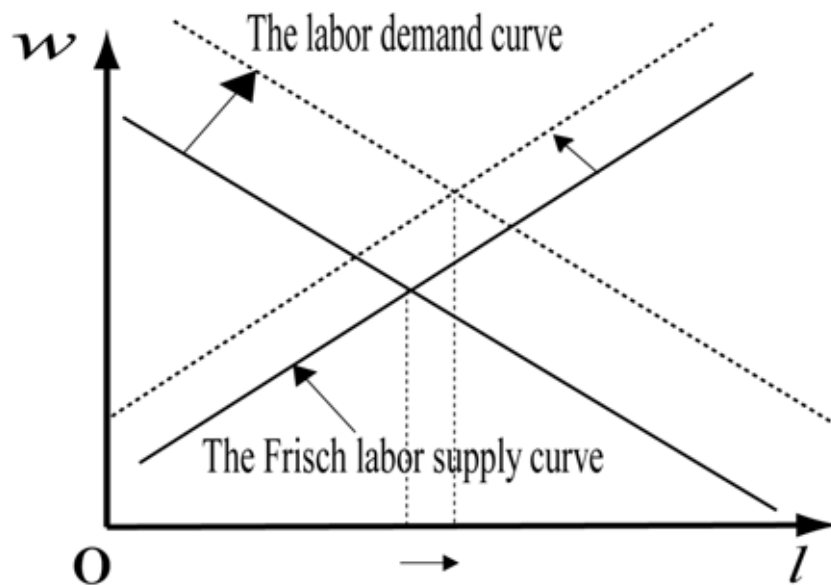


Figure 1: The labor demand curve and the Frisch labor supply curve cross with the ‘normal’ slopes (i.e. $\Delta(\hat{l}) < 0$).

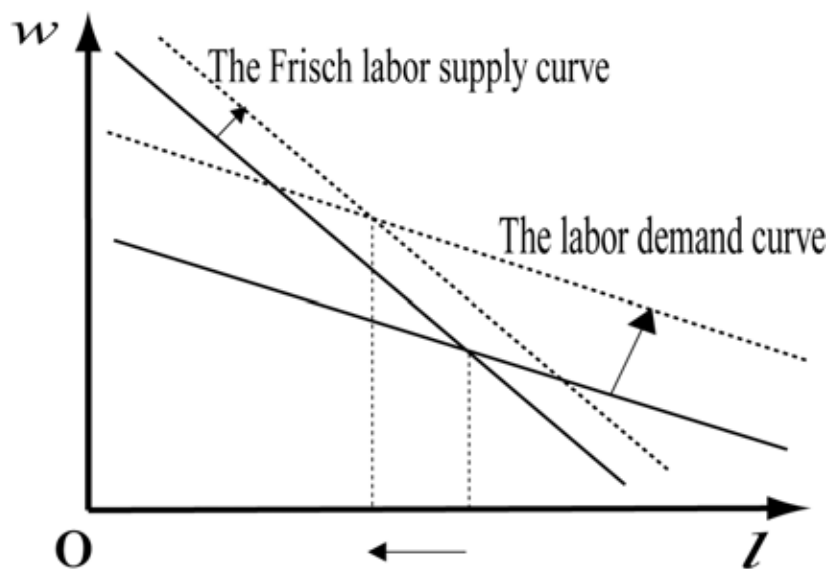


Figure 2: The labor demand curve and the Frisch labor supply curve cross with the ‘wrong’ slopes (i.e. $\Delta(\hat{l}) > 0$).

$$\frac{d\hat{k}}{d\tau_k} = -\frac{\hat{k}}{1-\tau_k} \frac{\rho}{\rho + (1-\tau_k)\delta} \frac{1}{1-\alpha} \left[\beta(1-\hat{l}) \frac{\delta\hat{k}}{\hat{c}} + 1 \right] < 0, \quad (14a)$$

$$\frac{d\hat{c}}{d\tau_k} = -\frac{\hat{c}}{1-\tau_k} \frac{\rho}{\rho + (1-\tau_k)\delta} \frac{1}{1-\alpha} \left[\beta(1-\hat{l}) \frac{\delta\hat{k}}{\hat{c}} + (\alpha\hat{k}^{\alpha-1}\hat{l}^\beta - \delta) \frac{\hat{k}}{\hat{c}} \right] < 0, \quad (14b)$$

$$\frac{d\hat{l}}{d\tau_k} = -\frac{\hat{l}}{1-\tau_k} \frac{\rho}{\rho + (1-\tau_k)\delta} (1-\hat{l}) \frac{\delta\hat{k}}{\hat{c}} < 0, \quad (14c)$$

$$\frac{dj}{d\tau_c} = -\frac{j}{1+\tau_c} \frac{\beta}{1-\alpha} (1-\hat{l}) < 0, \quad j = \hat{k}, \hat{c}, \quad (14d)$$

$$\frac{d\hat{l}}{d\tau_c} = -\frac{\hat{l}}{1+\tau_c} (1-\hat{l}) < 0, \quad (14e)$$

$$\frac{dj}{d\tau_w} = -\frac{j}{1-\tau_w} \frac{\beta}{1-\alpha} (1-\hat{l}) < 0, \quad j = \hat{k}, \hat{c}, \quad (14f)$$

$$\frac{d\hat{l}}{d\tau_w} = -\frac{\hat{l}}{1-\tau_w} (1-\hat{l}) < 0. \quad (14g)$$

It is immediately clear from (14a)-(14g) not only that the factor $\Delta(\hat{l})$ does not appear in all of the above expressions, but also that the steady state effects of changes in the respective taxes on the long-run capital stock, consumption and labor supply (i.e., employment) are all *negative*. Stated differently, the long-run effects of the taxes do not hinge on whether the steady state is determinate or indeterminate in the present exogenous growth model (or equivalently, whether the Frisch labor supply curve may have the ‘normal’ or the ‘wrong’ slope). In addition, since the parameter σ does not appear in (14a)-(14g), the effects of the taxes are qualitatively and quantitatively unaffected by the (*pure*) intertemporal substitution parameter σ . In other words, in the class of CES utility functions the effects of the taxes remain the

same, irrespective of whether the utility function is separable (i.e., $\sigma = 1$) or nonseparable (i.e., $\sigma \neq 1$). This is mainly because under homothetic utility the consumption-leisure choice dictated by (9) is independent of σ .⁷

In summary, we have the following proposition:

Proposition 1

- (i) An increase (decrease) in each of the capital income, labor income and consumption taxes reduces (raises) the steady-state levels of capital, consumption and employment, regardless of whether the steady state is determinate or indeterminate;*
- (ii) as capital and/or labor externalities become larger, so do the effects of the respective taxes on capital and consumption in absolute value, but the effects of the respective taxes on employment remain the same; and*
- (iii) the magnitudes of not only the inverse of the intertemporal elasticity of substitution in leisure but also the elasticity of intertemporal substitution in consumption have no effect on capital, consumption and employment.*

The reason why the long-run effects of the taxes on capital stock and consumption are in magnitude positively associated with the degree of production externalities can best be explained with the aid of Fig.3. As α becomes larger, the long-run capital demand curve, $(1 - \tau_k) [a\hat{k}^{\alpha-1}\hat{l}^\beta - \delta]$, will be flatter (i.e., more elastic with respect to \hat{k}), while the long-run capital supply curve represented by the horizontal line at ρ remains the same as before. Since an increase in the capital income tax, τ_k , shifts the capital demand curve to the left, the resulting decrease in \hat{k} will be larger with α , as shown in Fig.3. As β becomes larger, on the other hand, the long-run capital demand curve becomes more responsive to variations in \hat{l} caused by the tax changes, and thus the effects on \hat{c} and \hat{k} will be more negative.

The long-run effects of the taxes are not affected by the stability properties of the steady state, both because of the infinitely elastic long-run supply

⁷It should be remarked that these properties may not robust under more general preferences, as shown in Appendix A. According to Appendix A, it appears that the effect of the capital income tax (i.e., (A4)) depends also on the sign of the determinant of the Jacobian matrix evaluated at the steady state, that is, the stability property of the steady state.

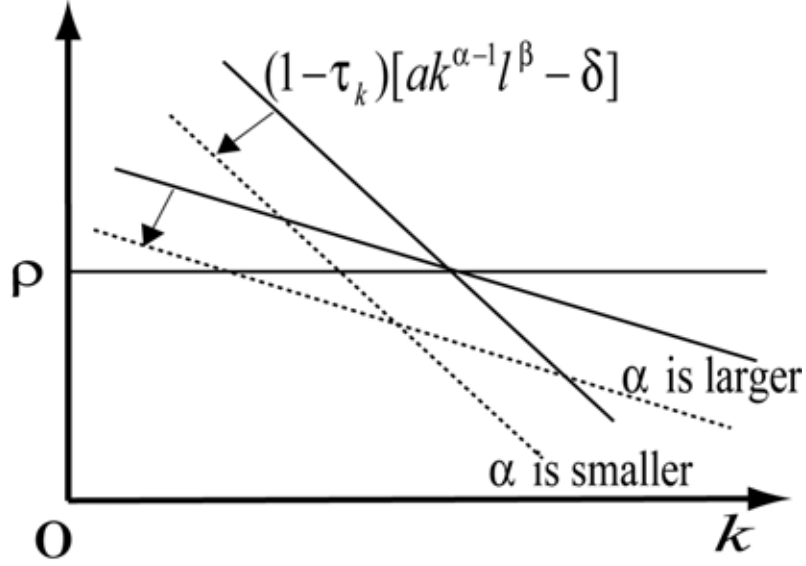


Figure 3: The effect of an increase in the capital income tax on the long-run capital stock.

curve of capital (i.e., the horizontal line at ρ) and because of the homogenous property of the Cobb-Douglas production function. Indeed, by manipulating (12a) and (12b), the long-run rental capital market equilibrium condition can be expressed by:

$$\rho = (1 - \tau_k) \left[a \left(\frac{\hat{c}}{\hat{k}} + \delta \right) - \delta \right], \quad (15)$$

which reveals that (15) solely determines the ratio \hat{c}/\hat{k} . Combining (15) with (9) results in

$$\chi \frac{\hat{l}}{1 - \hat{l}} = \frac{b(1 - \tau_w)}{1 + \tau_c} \left[1 + \delta \frac{\hat{k}}{\hat{c}} \right]. \quad (16)$$

It follows from (15) and (16) that the steady state-responses of employment to any tax changes remain invariant to differing values of α or β . Eq.(16) also indicates that given the ratio \hat{c}/\hat{k} determined by (15), an increase in τ_c or τ_w lowers the long-run level of employment since through (16), regardless of whether the steady state is determinate or indeterminate. The resulting reduction in \hat{l} decreases the marginal product of capital, thereby shifting the

long-run capital demand curve to the left. This movement causes \hat{k} and thus \hat{c} to fall.

There are several measures to evaluate long-run tax incidence (i.e., the long-run effects on income distribution) in the existing literature. Among them, the ratio of factor incomes, wl/rk , and the income share of capital, $rk/(rk + wl)$, remain invariant to any tax changes by virtue of Cobb-Douglas production technology. Instead, we shall adopt the lifetime utility of the representative agent (5) evaluated at the steady state, that is, $[(\hat{c}(1-\hat{l})^x)^{1-\sigma} - 1]/(1-\sigma)\rho$ (denoted by W_{ss}), as a measure for tax incidence.

Differentiating W_{ss} with respect to each tax rate and using (12a), (14a)-(14g) yields the followings:

$$\frac{dW_{ss}}{d\tau_k} = \frac{1}{\rho} [\hat{c}(1-\hat{l})^x]^{1-\sigma} \frac{1-\hat{l}}{1-\tau_k} \frac{\rho}{\rho + (1-\tau_k)\delta} \frac{\delta \hat{k}}{\hat{c}} \left[-\frac{\beta}{1-\alpha} + \chi \frac{\hat{l}}{1-\hat{l}} - \frac{1}{(1-\alpha)(1-\hat{l})} \frac{\alpha \hat{k}^{\alpha-1} \hat{l}^\beta - \delta}{\delta} \right] < 0, \quad (17a)$$

$$\frac{dW_{ss}}{d\tau_c} = \frac{1}{\rho} [\hat{c}(1-\hat{l})^x]^{1-\sigma} \frac{1-\hat{l}}{1+\tau_c} \left[-\frac{\beta}{1-\alpha} + \chi \frac{\hat{l}}{1-\hat{l}} \right] < 0, \quad (17b)$$

$$\frac{dW_{ss}}{d\tau_w} = \frac{1}{\rho} [\hat{c}(1-\hat{l})^x]^{1-\sigma} \frac{1-\hat{l}}{1-\tau_w} \left[-\frac{\beta}{1-\alpha} + \chi \frac{\hat{l}}{1-\hat{l}} \right] < 0. \quad (17c)$$

The negative sign of (17a)-(17c) can be verified by the following inequalities:

$$-\frac{\beta}{1-\alpha} + \chi \frac{\hat{l}}{1-\hat{l}} < -\frac{b}{1-a} + \frac{b(1-\tau_w) \hat{k}^\alpha \hat{l}^\beta}{(1+\tau_c) \hat{c}} = \frac{-(1+\tau_c) \hat{c} + (1-\tau_w) w \hat{l}}{1+\tau_c} < 0,$$

where the first inequality follows from the inequalities $a < \alpha$ and $b < \beta$ coupled with (9), while the last inequality follows from (6) (noting that $b = 1 - a$). An increase in any tax reduces labor supply, thereby improving welfare, whereas it decreases output due to the decreased labor supply and thus consumption, thereby depressing welfare. Nevertheless, the above results imply that the latter negative consumption effect unambiguously dom-

inates the former positive effect on leisure. It should also be noted that there is no tied relationship between the stability properties of the steady state and the welfare effects.

Proposition 2 *The effects of the respective taxes on steady state welfare are all negative. Moreover, the larger the magnitude of production externalities are, those negative effects are strengthened.*

Proposition 2 is apparently consistent with the findings of Chamley (1986) and Mino (2001) who discuss optimal capital income taxation in exogenous growth models. Chamley finds that in the absence of spillover effects the long-run distortionary taxes should be eventually eliminated, while Mino shows that in the presence of positive production externalities the capital income tax should be negative in the steady state (i.e., capital should be subsidized). In other words, irrespective of whether such externalities are present or not, higher rates of distortionary taxes unambiguously depress social welfare. More interestingly, since it follows from (15) and (16) that the steady state level of employment is independent of the degrees of externalities, larger degrees of increasing returns (i.e., $\alpha + \beta$) tend to magnify the detrimental effects of distortionary taxation on welfare. This is because as the degree of increasing returns is greater, the negative impact of the distortionary taxes on output [recall (12a)] will be reinforced, thereby magnifying the contractive effect on consumption but leaving the level of employment intact.

4 The case of endogenous growth

In this section we consider the case where the economy follows a balanced growth equilibrium path (call a BGE path). The model is identical to the basic model presented in Section 2 except for setting $\alpha = 1$, which is needed to generate sustained growth. Note also that when $\alpha = 1$, the Jacobian $R(\hat{l})$ vanishes so that the dynamics of the resulting model becomes one-dimensional.

Taking the time derivative of the logarithm of (7a), and substituting (7c) into the resultant expression, we obtain

$$\frac{\dot{c}}{c} = -\frac{1-\sigma}{\sigma} \frac{\chi}{1-l} \dot{l} + \frac{(1-\tau_k)(al^\beta - \delta) - \rho}{\sigma}, \quad (18)$$

where noting $r = al^\beta$. Eqs.(9) and (10a), together with $\alpha = 1$, are respectively rewritten as follows:

$$\frac{c}{k} = \frac{b(1-\tau_w)l^{\beta-1}(1-l)}{1+\tau_c} \chi, \quad (19a)$$

$$\frac{\dot{k}}{k} = l^\beta - \frac{c}{k} - \delta. \quad (19b)$$

Eqs.(18), (19a) and (19b) together completely characterize the dynamics of the present endogenous growth model.⁸

Before proceeding to comparative statics excises, we have to check the stability of the BGE in this model. To do this, we subtract (19b) from both sides of (18), together with (19a):

$$\frac{\dot{c}}{c} - \frac{\dot{k}}{k} = -\frac{1-\sigma}{\sigma} \frac{\chi}{1-l} \dot{l} + N(l), \quad (20)$$

where

$$N(l) \equiv \frac{(1-\tau_k)(al^\beta - \delta) - \rho}{\sigma} - l^\beta + \frac{b(1-\tau_w)l^{\beta-1}(1-l)}{1+\tau_c} \chi + \delta.$$

By taking the time derivative of the logarithm of (19a), on the other hand, we find

$$\frac{\dot{c}}{c} - \frac{\dot{k}}{k} = \left[\beta - 1 - \frac{l}{1-l} \right] \frac{\dot{l}}{l}. \quad (21)$$

Equating (21) with (20) and rearranging, we finally obtain

⁸Along the BGE the transversality condition is given by $\lim_{t \rightarrow \infty} k_0 c_0^{-\sigma} (1 - \hat{l})^{(1-\sigma)\chi} e^{-[\rho - (1-\sigma)g]t} (1 + \tau_c)^{-1} = 0$, which is equivalent to the condition that $\rho > g(1-\sigma)$, where k_0 and c_0 are the initial levels of capital and consumption, respectively.

$$\frac{\dot{l}}{l} = \sigma \frac{N(l)}{\Delta(l)}. \quad (22)$$

Since the BGE path is characterized by a situation where both c and k (and w) grow at the same rate (denoted by g), while leaving l constant (hence $\dot{l} = 0$). As a result, the BGE level of employment, \hat{l} , is obtained by setting $N(\hat{l}) = 0$:

$$\frac{(1 - \tau_k)(a\hat{l}^\beta - \delta) - \rho}{\sigma} = \hat{l}^\beta - \frac{b(1 - \tau_w)\hat{l}^{\beta-1}(1 - \hat{l})}{1 + \tau_c} - \delta. \quad (23)$$

Since \dot{l} in (22) depends only on l , not on k , in order to know the stability properties of the BGE, we only have to identify the sign of $d\dot{l}/dl$ evaluated at \hat{l} . Differentiating the right-hand side of (22) with respect to \hat{l} yields

$$\frac{d(\dot{l}/l)}{dl} = \sigma \frac{N'(\hat{l})\Delta(\hat{l}) - N(\hat{l})\Delta'(\hat{l})}{[\Delta(\hat{l})]^2} = \sigma \frac{N'(\hat{l})}{\Delta(\hat{l})}, \quad (24)$$

where noting $N(\hat{l}) = 0$ and

$$N'(\hat{l}) \equiv \left[\frac{(1 - \tau_k)a}{\sigma} - 1 \right] \beta \hat{l}^{\beta-1} + \frac{b(1 - \tau_w)\hat{l}^{\beta-2}(1 - \hat{l})}{1 + \tau_c} \left[\beta - 1 - \frac{\hat{l}}{1 - \hat{l}} \right]. \quad (25)$$

The sign of (24) is in general undetermined due to the ambiguous signs of both $\Delta(\hat{l})$ and $N'(\hat{l})$. Further inspection of (24) reveals that if both $\Delta(\hat{l})$ and $N'(\hat{l})$ are of the same sign, then $d\dot{l}/dl > 0$, and thus the fixed point \hat{l} is a repeller; consequently, the BGE is locally determinate. In contrast, if $\Delta(\hat{l})$ and $N'(\hat{l})$ are of opposite signs, then $d\dot{l}/dl < 0$, and thus the fixed point \hat{l} is an attractor; hence, the BGE is locally indeterminate.

In Figs. 4-7 the graph of (23) is drawn, where we can show that the curves corresponding to the left- and right-hand sides of (23) both slope up (see Appendix B). The intersection of these two curves gives us the common growth rate of consumption and capital on the vertical axis and the BGE level of labor supply on the horizontal axis. As shown in Appendix B, $N'(\hat{l}) > 0$

graphically implies that at the fixed point \hat{l} the curve \dot{k}/k cuts the \dot{c}/c curve from above as illustrated in Figs. 4 and 6, whereas $N'(\hat{l}) < 0$ implies the opposite as illustrated in Figs. 5 and 7. Taken together, we have:

Proposition 3 *When the labor demand curve and the Frisch labor supply curve cross with the ‘normal’ slopes (i.e., $\Delta(\hat{l}) < 0$), a BGE is locally indeterminate if $N'(\hat{l}) > 0$, while it is locally determinate if $N'(\hat{l}) < 0$. Conversely, when the labor demand curve and the Frisch labor supply curve cross with the ‘wrong’ slopes (i.e., $\Delta(\hat{l}) > 0$), the results are reversed.*

This proposition allows us to identify the two possible conditions for the emergence of an indeterminate BGE; namely, the signs of $\Delta(\hat{l})$ and $N'(\hat{l})$. This feature stands in sharp contrast to that of the exogenous growth model presented in the previous section, since in the exogenous growth model the likelihood of indeterminacy hinges only on the sign of $\Delta(\hat{l})$ (recall $sign[R(\hat{l})] = sign[\Delta(\hat{l})]$). Moreover, Pelloni and Waldmann (2000) assume strictly concavity of the utility function, and thereby rule out a case where the Frisch labor supply curve takes the ‘wrong’ slope (i.e., $\Delta(\hat{l}) > 0$). As a result, in their model the stability properties of the BGE are governed solely by the sign of $N'(\hat{l})$. By contrast, in our endogenous growth model which allows for quasi-concave preferences, the relative slope of the demand and supply curves in the labor market gives us another condition for generating indeterminacy, in addition to the sign of $N'(\hat{l})$.

5 A balanced growth path incidence

In this section we examine tax incidence along the BGE path of the endogenous growth model presented in the previous section. Since in endogenous growth models consumption and capital stock both grow indefinitely, we cannot compute the effects of the tax changes on these real variables. Instead, we first study the effects on employment and thus the growth rate of changes in the respective taxes along the BGE path. As a result, these results enable us to compute the effects on welfare along the BGE path.

Totally differentiating (23) with respect to each tax rate and manipulating yields

$$\frac{d\hat{l}}{d\tau_k} = \frac{a\hat{l}^\beta - \delta}{\sigma} \left[N'(\hat{l}) \right]^{-1}, \quad (26a)$$

$$\frac{d\hat{l}}{d\tau_c} = \frac{\hat{l}^{\beta-1}(1-\hat{l})}{\chi} \frac{b(1-\tau_w)}{(1+\tau_c)^2} \left[N'(\hat{l}) \right]^{-1}, \quad (26b)$$

$$\frac{d\hat{l}}{d\tau_w} = \frac{\hat{l}^{\beta-1}(1-\hat{l})}{\chi} \frac{b}{1+\tau_c} \left[N'(\hat{l}) \right]^{-1}. \quad (26c)$$

It is immediately seen that the effect of a change in each tax rate on the BGE level of employment is governed only by the sign of $N'(\hat{l})$ rather than whether the BGE is determinate or indeterminate. It is also important to note that the qualitative as well as quantitative impacts of the taxes are sensitively influenced by the value of σ , unlike the exogenous growth model presented in Section 2. This difference stems from the fact that the parameter σ does not vanish in the balanced growth rate of consumption (18) coupled with $\dot{l} = 0$, whereas the steady state conditions (9), (12a) and (12b) in the exogenous growth model do *not* contain the parameter σ .

After substitution of (19a) for c/k in (19b), setting $g \equiv \dot{k}/k$ and differentiating the resultant expression with respect to each tax rate yields

$$\frac{dg}{d\tau_k} = \left[\beta \frac{\hat{l}}{1-\hat{l}} \chi - \frac{b(1-\tau_w)}{1+\tau_c} \left(\beta - 1 - \frac{\hat{l}}{1-\hat{l}} \right) \right] \frac{\hat{l}^{\beta-2}(1-\hat{l})}{\chi} \frac{d\hat{l}}{d\tau_k}, \quad (27a)$$

$$\frac{dg}{d\tau_c} = \frac{1-\tau_k}{1+\tau_c} \frac{a\beta\hat{l}^{2(\beta-1)}}{\sigma} \frac{1-\hat{l}}{\chi} \frac{b(1-\tau_w)}{1+\tau_c} \left[N'(\hat{l}) \right]^{-1}, \quad (27b)$$

$$\frac{dg}{d\tau_w} = \frac{1-\tau_k}{1-\tau_w} \frac{a\beta\hat{l}^{2(\beta-1)}}{\sigma} \frac{1-\hat{l}}{\chi} \frac{b(1-\tau_w)}{1+\tau_c} \left[N'(\hat{l}) \right]^{-1}. \quad (27c)$$

It is immediate that the effects of the respective taxes on the growth rate have the same signs as $d\hat{l}/d\tau_j$ ($j = k, c, w$) (i.e., $N'(\hat{l})$), respectively.

A higher capital income tax shifts the \dot{c}/c curve downward, while leaving the \dot{k}/k curve unchanged. When $N'(\hat{l}) > 0$, the new BGE will be located at

the northeast of the previous intersection, which features higher employment and a higher growth rate of the economy, as shown in Fig. 4. At the initial level of employment $\dot{k}/k > \dot{c}/c$, and the ratio c/k will decline. The sustained increase in k causes the labor demand curve to shift to the right, while the sustained increase in c causes the Frisch labor supply curve to shift to the left, see Figs.1 and 2.⁹ Since the growth rate of k is greater than that of c , the rightward movement of the labor demand curve is bigger than the leftward movement of the labor supply curve, as shown in Fig.1. Consequently, employment will rise when the labor demand curve and the Frisch labor supply curve cross with the ‘normal slopes’ (i.e., $\Delta(\hat{l}) < 0$). This movement is thus consistent with the new BGE featuring higher employment, so that employment is gradually rising to the new BGE level. Hence, it is stable and thus indeterminate. In contrast, when the labor demand curve and the Frisch labor supply curve cross with the ‘wrong slopes’ (i.e., $\Delta(\hat{l}) > 0$), employment falls in spite of the large increased labor demand, as shown in Fig.2. This movement of employment induces the economy to depart from the new BGE. Hence, the new BGE is unstable and thus determinate. Accordingly, when the tax is unanticipatedly increased in this economy, employment must instantaneously jump to its new BGE level.

Conversely, when $N'(\hat{l}) < 0$, the new BGE will be located at the southwest of the previous intersection, which entails lower employment and a lower growth rate in Fig.5. Furthermore, since at the initial level of employment $\dot{k}/k > \dot{c}/c$, the ratio c/k will decline. When $\Delta(\hat{l}) < 0$, employment rises for the same reason outlined as before. This movement induces the economy to depart away from the new BGE. As a result, employment should immediately jump to the new BGE level and thus the new BGE is locally determinate. When $\Delta(\hat{l}) > 0$, employment falls and thus gradually approaches the new BGE level; hence it is locally indeterminate.

When the consumption tax (or the labor income tax) is increased, which is accompanied by compensating lump-sum transfers, consumption becomes

⁹More precisely, since both c and k are increasing at the same rate along the BGE path, the labor demand and supply curves illustrated in Figs.1 and 2 keep moving up so that the real wage rate continues to rise, while leaving the BGE level of employment unchanged.

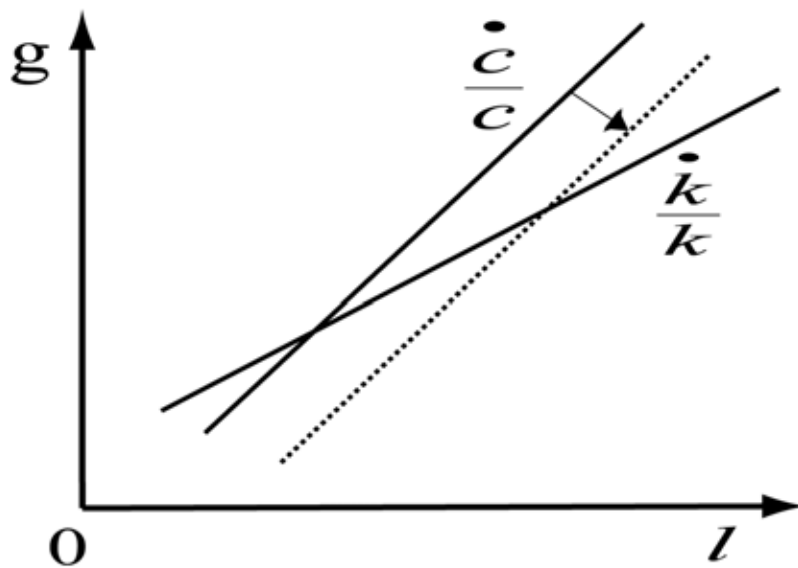


Figure 4: The effect of an increase in the capital income tax if $N'(l) > 0$.

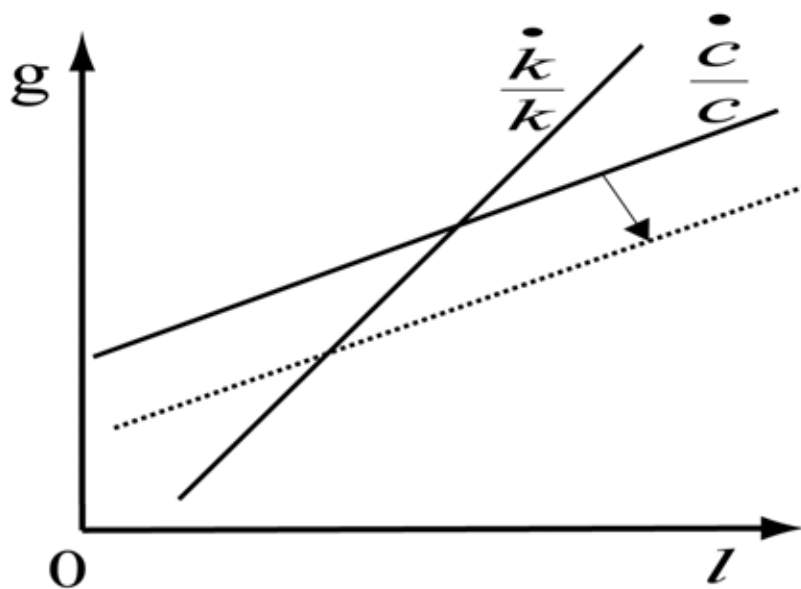


Figure 5: The effect of an increase in the capital income tax if $N'(l) < 0$.

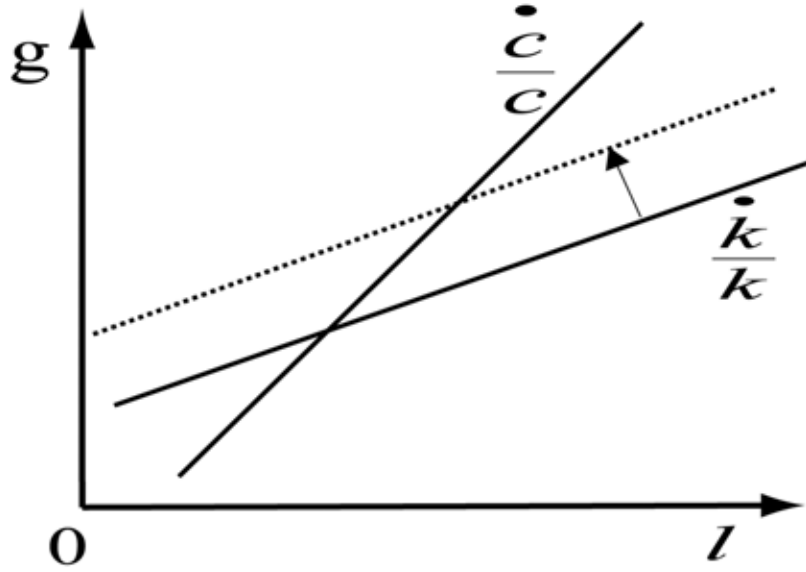


Figure 6: The effect of an increase in the consumption tax (or the labor income tax) if $N'(l) > 0$.

more expensive relative to leisure, thus inducing a substitution away from the demand for consumption to leisure. When $N'(\hat{l}) > 0$, the induced fall in the ratio c/k moves up the curve \dot{k}/k , which is implied by (19b), while leaving the curve \dot{c}/c unchanged, as shown in Fig.6. Consequently, the new BGE entails higher employment and a higher growth rate. When $\Delta(\hat{l}) < 0$ (resp., $\Delta(\hat{l}) > 0$), labor supply falls but employment gradually rises (resp., immediately jumps) to the new BGE level, and thus the new BGE is indeterminate (resp., determinate). Conversely, when $N'(\hat{l}) < 0$, although the dynamic movement towards the new BGE is still uncertain depending on the sign of $\Delta(\hat{l})$, an increase in τ_c or τ_w unambiguously has a negative impact on employment and thus on the growth rate along the BGE path; see Fig.7. Thus we have:

Proposition 4 *When the labor demand curve and the Frisch labor supply curve cross with the ‘normal’ slopes, an increase in any of the respective taxes raises (reduces) the BGE level of employment as well as the balanced growth rate if and only if the BGE is locally indeterminate (determinate). When the labor demand curve and the Frisch labor supply curve cross with the ‘wrong’*

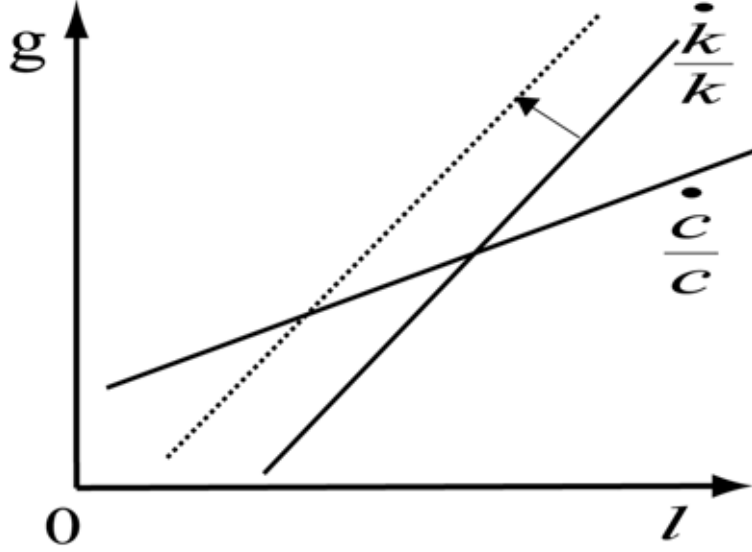


Figure 7: The effect of an increase in the consumption tax (or the labor income tax) if $N'(l) < 0$.

slopes, vice versa.

To evaluate tax incidence, we need to examine the welfare effects along the BGE in response to changes in the respective taxes. The level of welfare along the BGE path, denoted by W_{BG} , is obtained by substituting (19a) into c in (5) and manipulating:

$$W_{BG} \equiv \frac{1}{1-\sigma} \left[\frac{\{c_0(1-\hat{l})^x\}^{1-\sigma}}{\rho-g(1-\sigma)} - \frac{1}{\rho} \right], \quad (28)$$

where $c_0 (= k_0 b (1 - \tau_w) \hat{l}^{\beta-1} (1 - \hat{l}) / [\chi (1 + \tau_c)])$ is the endogenously determined initial level of consumption situated in the BGE path. Differentiating the right-hand side of (28) with respect to each tax rate results in

$$\frac{dW_{BG}}{d\tau_j} = \Gamma(\hat{l})^{-\sigma} \frac{(d\Gamma(\hat{l})/d\tau_j) [\rho - g(1 - \sigma)] + \Gamma(\hat{l}) (dg/d\tau_j)}{[\rho - g(1 - \sigma)]^2}, \quad j = k, c, w, \quad (29)$$

where $\Gamma(\hat{l}) \equiv c_0(1 - \hat{l})^x$. Appendix C demonstrates that $dW_{BG}/d\tau_j$ (for

$j = k, c, w$) have the signs of $d\hat{l}/d\tau_j$ (i.e., $N'(\hat{l})$), respectively. Therefore, we have

Proposition 5 *When the labor demand curve and the Frisch labor supply curve cross with the ‘normal’ slopes, an increase in any of the respective taxes increases (decreases) welfare along the BGE path if and only if the BGE is locally indeterminate (determinate). When the labor demand curve and the Frisch labor supply curve cross with the ‘wrong’ slopes, vice versa.*

Propositions 4 and 5 are a straightforward generalization of Propositions 2 and 3 of Pelloni and Waldmann (2000) in the following senses. First, they restrict their analysis to a case where the Frisch labor supply curve takes the ‘normal’ slope, while we analyze a case where the Frisch labor supply curve takes the ‘wrong’ slope as well. Our generalized results suggest that the outcomes of tax incidence are no longer uniquely tied to the stability properties of the economy, unlike Pelloni and Waldmann’s result, and thus their ‘paradoxical’ results may well occur even in a determinate steady state. Second, although their results have been derived in the economy where there are no preexisting taxes, Propositions 4 and 5 in our paper reveal that their results continue to hold even in the economy with pre-existing distortionary taxes. In fact, they analyze the first-order welfare effect, and thus they ignore the excess burden associated with pre-existing taxes. Accordingly, their welfare analysis tends to underestimate the welfare losses (or gains). Nevertheless, Proposition 5 claims that the welfare effects of capital income taxation derived by Pelloni and Waldmann still remain valid. Furthermore, our propositions 4 and 5 are different from the results of Uchijima (2005). He shows that the growth and welfare effects of tax changes in a similar model do not hinge on whether the BGE is locally indeterminate or determinate. Since his analytical focus lies on the global stability properties of the BGE, he has to impose more restrictive conditions in order to ensure its global uniqueness compared to our local analysis. This would end up distinguishing his comparative statics results from ours.

Propositions 4 and 5 seem counter-intuitive, probably because the conventional view is that tax substitution from nondistortionary taxes (such

as lump-sum taxes) to distortionary taxes usually has an adverse effect on welfare. In contrast, such substitution may improve welfare in the present endogenous growth model. This is in part because the change in the growth rate of the economy (i.e., consumption and capital) plays a dominant role in determining the ultimate effect on welfare, but also because the response of employment to the tax changes, which is positively associated with the growth rate, rests on the relative slope of the labor demand and supply curves. Hence, when the tax increase enhances the growth rate of consumption, the growth-promoting effect on welfare will outweigh the negative effect of the increased labor supply on welfare.

6 Concluding remarks

This paper has investigated the relationship between the incidence of factor incomes and expenditures taxes and the properties of stability (i.e., a steady state is determinate or indeterminate) in an infinite-horizon, representative agent growth model with endogenous labor supply. The stability properties of the model characterized by the CES utility and Cobb-Douglas production functions do not matter for dynamic tax incidence as long as the model displays exogenous growth, whereas the stability properties do if it displays endogenous growth. Moreover, in the former model all of the taxes unambiguously depress economic activity, whereas in the latter model these taxes may potentially stimulate economic activity. In either of determinate and indeterminate steady states of the endogenous growth model analyzed here, the tax increase may promote growth and improve welfare, which contrasts with Pelloni and Waldmann's (2000) result in that the only indeterminate steady state delivers such a counter-intuitive outcome. We may, therefore, conclude that Samuelson's correspondence principle is no longer valid in the sense that the one-to-one relationship between the stability properties of the model and its comparative statics results are broken, at least in the economy structured by CES preferences and Cobb-Douglas technology.

Admittedly, all of the results in this paper depend on the specification of the model. As noted by King et al. (1988), the functional form of the

instantaneous utility functions that we have assumed is the most general one compatible with balanced growth and stationary labor supply. The specification of the utility and production functions we employ here could be justified by this restriction. Nevertheless, since it seems to us that the choice of these functions is crucial in determining the results of tax incidence, which is implied by Appendix A, it is valuable to examine dynamic tax incidence under other functional forms in order to examine the robustness of our incidence results.

Appendix A

Consider the economy whose dynamics is characterized as follows:

$$\dot{k} = f(k, l) - c - \delta k, \quad (\text{A1})$$

$$\dot{\lambda}/\lambda = \rho - (1 - \tau_k) [r(k, l) - \delta], \quad (\text{A2})$$

where $f(\cdot)$ represents the social production function, and the rest of the other variables are the same those in the text. Taking a linear approximation around the steady state yields

$$\begin{bmatrix} \dot{k} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} f_k + f_l l_k - c_k - \delta & f_l l_\lambda - c_\lambda \\ -\hat{\lambda}(1 - \tau_k)(f_{kk} + f_{kl} l_k) & -\hat{\lambda}(1 - \tau_k) f_{kl} l_\lambda \end{bmatrix} \begin{bmatrix} \hat{k} - k \\ \hat{\lambda} - \lambda \end{bmatrix}. \quad (\text{A3})$$

We carry out the comparative statics exercises with respect to, say, the capital income tax τ_k :

$$\partial \hat{k} / \partial \tau_k = \rho (f_l l_\lambda - c_\lambda) / (1 - \tau_k) |J|, \quad (\text{A4})$$

where $|J|$ is the determinant of the Jacobian matrix appearing on the right-hand side of (A3). Inspection of (A4) reveals that we need more information in order to identify the exact relationship between the sign of $|J|$ and the effect of capital income taxation on the long-run capital stock. If the sign of $f_l l_\lambda - c_\lambda$ remains unchanged when that of $|J|$ changes, there is a one-to-one relationship between the sign of $|J|$ and the effect of the capital income

tax. Nevertheless, it may or may not be true depending on the further specification of the model.

Appendix B

Since the slope of the \dot{c}/c curve is given by $\sigma^{-1}[(1 - \tau_k)(a\hat{l}^\beta - \delta) - \rho]$ along a BGE path (recall (18)), we differentiate it with respect to \hat{l} , thus yielding

$$\sigma^{-1}(1 - \tau_k)a\beta\hat{l}^{\beta-1} > 0, \quad (\text{B1})$$

which implies that the \dot{c}/c curve is positively sloped. As to the \dot{k}/k curve, substituting (19a) into (19b), we differentiate the resultant expression with respect to \hat{l} to obtain

$$\beta\hat{l}^{\beta-1} - \frac{b(1 - \tau_w)\hat{l}^{\beta-2}(1 - \hat{l})}{1 + \tau_c} \left[\beta - 1 - \frac{\hat{l}}{1 - \hat{l}} \right] > 0. \quad (\text{B2})$$

This implies that the \dot{k}/k curve is positively sloped as well.

Moreover, since the right-hand side of (25) can be rearranged as follows:

$$N'(\hat{l}) = \frac{(1 - \tau_k)a\beta\hat{l}^{\beta-1}}{\sigma} - \left[\beta\hat{l}^{\beta-1} - \frac{b(1 - \tau_w)\hat{l}^{\beta-2}(1 - \hat{l})}{1 + \tau_c} \left(\beta - 1 - \frac{\hat{l}}{1 - \hat{l}} \right) \right], \quad (\text{B3})$$

(B1) and (B2) imply that the first and second terms on the right-hand side of (B3) represent the slopes of the \dot{c}/c curve and the \dot{k}/k curve, respectively. Hence, it is seen that when $N'(\hat{l}) > 0$, the \dot{c}/c curve has a steeper upward slope compared to the \dot{k}/k curve, and vice versa.

Appendix C

Since the denominator of (29) is positive, the sign of (29) is determined according to that of its numerator:

$$(d\Gamma(\hat{l})/d\tau_j)[\rho - g(1 - \sigma)] + \Gamma(\hat{l})(dg/d\tau_j) \text{ for } j = k, c, w. \quad (\text{C1})$$

Recalling $\Gamma(\hat{l}) \equiv c_0(1 - \hat{l})^\chi = k_0 b(1 - \tau_w)(1 - \hat{l})^{1+\chi} \hat{l}^{\beta-1} / [\chi(1 + \tau_c)]$, we differentiate $\Gamma(\hat{l})$ with respect to τ_k to obtain

$$\frac{d\Gamma(\hat{l})}{d\tau_k} = \left[\beta - 1 - \frac{\hat{l}}{1 - \hat{l}}(1 + \chi) \right] \frac{\Gamma(\hat{l})}{\hat{l}} \frac{d\hat{l}}{d\tau_k}. \quad (\text{C2})$$

Substituting (23), (19a) and (19b) into ρ and g in the expression $\rho - g(1 - \sigma)$, respectively, results in

$$\rho - g(1 - \sigma) = (1 - \tau_k)(a\hat{l}^\beta - \delta) - \left[\hat{l}^\beta - \frac{b(1 - \tau_w)\hat{l}^{\beta-1}(1 - \hat{l})}{1 + \tau_c} - \delta \right]. \quad (\text{C3})$$

We substitute (27a), (C2) and (C3) into (C1) to obtain

$$\begin{aligned} & \left[\left\{ \beta - 1 - \frac{\hat{l}}{1 - \hat{l}}(1 + \chi) \right\} \left\{ (1 - \tau_k)(a\hat{l}^\beta - \delta) - \left(\hat{l}^\beta - \frac{b(1 - \tau_w)\hat{l}^{\beta-1}(1 - \hat{l})}{1 + \tau_c} - \delta \right) \right\} \right. \\ & \left. + \left\{ \beta \frac{\hat{l}}{1 - \hat{l}}\chi - \frac{b(1 - \tau_w)}{1 + \tau_c} \left(\beta - 1 - \frac{\hat{l}}{1 - \hat{l}} \right) \right\} \frac{\hat{l}^{\beta-1}(1 - \hat{l})}{\chi} \right] \frac{\Gamma(\hat{l})}{\hat{l}} \frac{d\hat{l}}{d\tau_k}. \end{aligned}$$

Further rearrangement gives rise to

$$\begin{aligned} & = \left[\left\{ \beta - 1 - \frac{\hat{l}}{1 - \hat{l}}(1 + \chi) \right\} \left\{ -\hat{l}^\beta(1 - a) - \tau_k(a\hat{l}^\beta - \delta) \right\} \right. \\ & \left. + \hat{l}^{\beta-2} \left\{ \beta - \frac{b(1 - \tau_w)}{1 + \tau_c} \right\} \right] \frac{\Gamma(\hat{l})}{\hat{l}} \frac{d\hat{l}}{d\tau_k}. \quad (\text{C4}) \end{aligned}$$

Since $\beta - 1 - [(1 + \chi)\hat{l}/(1 - \hat{l})] < 0$ and $\beta - [b(1 - \tau_w)/(1 + \tau_c)] > 0$, all terms within the square brackets of (C4) are positive. As a result, the signs of (C1) and thus $dW_{BG}/d\tau_k$ depend only on that of $d\hat{l}/d\tau_k$ (i.e., $N'(\hat{l})$). In an analogous manner, we can show that

$$\text{sign} \left[\frac{dW_{BG}}{d\tau_j} \right] = \text{sign} \left[\frac{d\hat{l}}{d\tau_j} \right] = \text{sign} \left[N'(\hat{l}) \right] \quad \text{for } j = c, w.$$

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