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Public Investment Criteria and Optimal Taxation in a Polluted Environment

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Abstract

This paper studies an optimal policy of public investments and commodity taxation which maximizes a sum of discounted generational utilities in an environment polluted by production and/or consumption activities. It is shown that (1) the optimal discount rate for pollution control (productive) public investment is (not generally) equal to the social time preference rate; (2) if pollution is caused by production processes, then the optimal wage and interest tax formulae partly consist of a weighted average of the Ramsey tax and the Pigovian tax; and (3) the introduction of public debt does not always warrant aggregate production efficiency.

1. *Introduction*

The exploration of an optimal policy of public investment and commodity taxation which maximizes a sum of discounted generational utilities in a neoclassical growth model with overlapping generations has been worked out by Pestieau (1974). The optimal public policy derived is summarized as follows: (1) the second-best discount rate for public investment along the steady state path is not only equal to the social time preference rate but also can be expressed as a weighted average of the market rates of interest facing consumers and producers in the traditional opportunity cost approach to the social discount rate;[1] (2) the wage and interest tax rates depart from marginal productivities of labor and private capital according to Ramsey's optimal taxation structure; (3) there is inefficiency in production, namely, the rates of return on private and public capital will differ, though aggregate production efficiency can be attained if public investment can be financed by public bonds as well as taxes.

However, in the Pestieau model, other important factors of the distortion inherent in a competitive economy except indirect taxation financing public investment, such as externalities, were not considered. It is well known that when externalities are present, indirect taxation (the so-called Pigovian taxation) can be used as a tool for correcting inefficiency in a competitive allocation of resources. Therefore, it may be interesting to examine how the above-mentioned results obtained by Pestieau must be modified by the presence of externalities, because the objective of indirect taxation in this case is to control externalities as well as to finance public investment. The

purpose of this paper is to incorporate *environmental* externalities into the Pestieau model of public investment and to study the criteria for *productive* and *pollution control* public investments and the optimal structure of wage and interest taxes in an environment polluted by waste.[2] The paper is organized as follows:

Section 2 sets up the basic model and introduces the underlying assumptions for consumer behavior, production, environmental pollution, the role of government, and the intergenerational welfare function. Environmental externalities are now characterized as a *stock* of pollution which has negative external effects on the utilities of younger and older generations. This implies that the stock of pollution has a public "bads" property. It is assumed that new pollutants are discharged through private and public capital and labor factors, in production processes, and/or with consumption activities of the two generations.[3] Though these new pollutants are accumulated in the environment, a part of the stock of pollution deteriorates naturally as we observe empirically. It is also possible for the government to slow the accumulation of pollution by devoting some expenditure to anti-pollution activities: pollution control public investment. Thus, where environmental pollution is present, the government must strive to obtain an efficient intertemporal allocation of resources subject to the pollution stock accumulation equation as well as to its budget constraint and to the demand and supply relations of the private sector.

Section 3 derives the optimal criteria for two kinds of public investments and the optimal structure of wage and interest taxes in a polluted environment by using dynamic programming. Neither a lump-sum redistribution policy nor the public debt is now available to the government. The following results will be shown:

First, if public capital in production processes is a source of pollution, then the second-best discount rate for *productive* public investment is *not* equal to the social time preference rate, because marginal social damage of productive public capital must be added to opportunity costs. On the contrary, the marginal social benefit by *pollution control* public investment should be discounted with the social time preference rate, whether environmental pollution is caused by production processes or by consumption activities.

Second, if the wage tax is confined to the Pigovian tax for some reason, then the familiar weighted average formula for public investment continues to hold in a modified form even in the presence of pollution: (i) marginal social damages of private and public capital in production processes are deducted from their marginal productivities, respectively, on the one hand; and (ii) marginal social damage by consumption activities of the two generations is reflected in the weights of market rates of interest facing consumers and producers on the other.

Third, if pollution is caused by production processes, then the optimal wage (interest) tax formula consists of a weighted average of efficiency terms familiar from the Ramsey optimal taxation theory and a marginal social damage of labor (private

capital) according to the Pigovian taxation principle, plus an additional term representing a failure of the optimal allocation of capital between private and public sectors. However, if pollution is caused by consumption activities, then such a weighted average property is reflected in the optimal wage tax formula alone.

Section 4 extends the analysis to allow for public borrowing and the creation of public debt. It will be shown that in this case the *net* marginal productivities of private and public capital, from which their marginal social damages are deducted, are equated through the social time preference rate. Therefore, since an optimal allocation of capital between the two sectors can be attained, additional term in the optimal tax formulae above vanishes. However, this does not always imply efficiency in production, because if the marginal pollutants of private and public capital in production processes are different, so are the rates of return on the two kinds of capital. Thus, in a polluted environment, the introduction of public debt does not generally warrant an aggregate production efficiency.

Finally, section 5 offers a summary and some concluding remarks.

2. *Formulation of the Basic Model*

Let us now consider an overlapping generations economy where individuals are alike except for their ages, and who live for two periods, working during the first and retiring in the second. For the sake of simplicity, it is assumed that there is only one person in each generation.[4] The basic assumptions we make on consumer behavior, production, environmental pollution, role of the government, and the intergenerational welfare function are described as follows:

2.1. *Consumer Behavior*

The preference of an individual born at period t is assumed to be represented by the additively separable utility function:

$$(1) \qquad U = U^1(c_t^1,\ n_t,\ P_t) + U^2(c_{t+1}^2,\ P_{t+1}) \ ,$$

Where $U^1(.)$ and $U^2(.)$ stand for utilities of the first and second periods, respectively; c_t^1 and c_{t+1}^2 consumption in the first and second periods, respectively, of the individual's life; P_t and P_{t+1} the stocks of pollution at periods t and $t+1$, respectively; and n_t the amount of work he performs during the first period. The utility function displays decreasing positive marginal utilities for c_t^1, c_{t+1}^2, $-n_t$, $-P_t$ and $-P_{t+1}$. The stock of pollution at period t has a public "bads" property because each additional unit of pollution is harmful for both younger and older generations living in the period concerned.

Let us describe the utility maximization behavior of an individual born at period t. We denote the wage rate at that period and the next period rate of interest by w_t and r_{t+1}, respectively. The individual is assumed to have perfect foresight regarding r_{t+1}.

His budget constraint is then

$$(2) \quad c_t^1 + q_{t+1}c_{t+1}^2 = w_t n_t \quad \text{where} \quad q_{t+1} \equiv 1/(1 + r_{t+1}) \ ,$$

which is the price to generation t of second period consumption in terms of that in the first period. He must choose the optimal allocation between labor and leisure and between immediate consumption and savings in order to maximize his utility function (1), subject to his budget constraint (2). The first-order conditions for a maximum are given by:

$$(3) \quad U_{c_t}^1/U_{c_{t+1}}^2 = 1/q_{t+1} \quad \text{and} \quad U_{n_t}^1/U_{c_t}^1 = -w_t \ ,$$

where subscripts express derivatives with respect to variables.

From these conditions, the quantities c_t^1, c_{t+1}^2, and n_t can be represented as functions of w_t, q_{t+1}, P_t and P_{t+1}, and hence one can define demand functions for c_t^1 and c_{t+1}^2 and a supply function for n_t. Reintroducting these demand and supply functions in (1), one can define an indirect utility function in terms of w_t, q_{t+1}, P_t, and P_{t+1}:

$$(4) \quad V_t = V_t(w_t, q_{t+1}, P_t, P_{t+1})$$

Assuming now that the demand and supply functions for c_t^1, c_{t+1}^2, and n_t do not depend on P_t and P_{t+1},[5] then the derivatives of V_t are given by:

$$(5) \quad V_{w_t} = \gamma_t n_t, \quad V_{q_t} = -\gamma_t c_{t+1}^2, \quad \gamma_t = U_{c_t}^1 \ , \ V_{P_t} = U_{P_t}^1, \ V_{P_{t+1}} = U_{P_{t+1}}^2 \ ,$$

where γ_t is the private marginal utility of income.

The savings by generation t is defined by:

$$(6) \quad s_t \equiv w_t n_t - c_t^1 \ .$$

Thus, generation t makes up the supply side of the capital market for period $t+1$ and of the labor market for period t.

2.2. *Production*

The unchanging technology of the economy is assumed to be specified by an aggregate production function:

$$(7) \quad y_t = F(k_t, g_t, n_t) \ ,$$

where y is the output; k the capital stock in the private sector; g the *productive* capital stock in the public sector; and n the labor supplied. The production function is assumed to be homogeneous of degree one in all of its arguments, concave and differentiable with positive first partial derivatives.

A private entrepreneur in period t wishes to employ both savings of generation t and the labor of the next generation $t+1$ for production in period $t+1$. He is

assumed to be a price taking profit maximizer, so that labor and private capital are paid their marginal products. Since he faces a constant returns to scale technology (7), maximum profits are zero in equilibrium.

2.3. *Market Equilibrium Conditions*

We assume that the output can be either consumed or invested, and that there is no depreciation for convenience. Then, the market equilibrium condition for goods is given by:

$$(8) \quad c_t^1 + c_t^2 + k_{t+1} + g_{t+1} + h_{t+1} = F(k_t, g_t, n_t) + k_t + g_t + h_t ,$$

where the left and right-hand sides are aggregate demand and supply of goods in period t, respectively; and h the *pollution control* capital stock in the public sector. The market equilibrium condition for capital in period t is given by:

$$(9) \quad s_t = k_{t+1} + b_t ,$$

where b_t stands for public bonds issued in period t.

2.4. *Environmental Pollution*

It is assumed that pollution is caused by production processes and/or by consumption activities. We denote new pollutants discharged into the environment by a differentiable function $E(k, g, n; c^1 + c^2)$ which has positive first partial derivatives with respect to all of its arguments. Fortunately, as we observe empirically, the stock of pollution deteriorates naturally, so that we assume now that pollution decays at a constant rate α. It is also possible for the government to slow the accumulation of pollution by devoting some expenditure to anti-pollution activities. The amount of pollution cleaned up is assumed to be a differentiable function $G(h)$ with a positive first derivative. Thus, the stock of pollution accumulates according to the equation:

$$(10) \quad P_{t+1} = E(k_t, g_t, n_t; c_t^1 + c_t^2) - G(h_t) + (1 - \alpha) P_t .$$

2.5. *The Role of the Government*

The government can impose taxes on wages and interest income; it can also choose the levels of productive and pollution control public investments it judges desirable; and further, it can issue one period public debt which bears the same interest as other capital. However, lump-sum redistribution policy is not now available for the government. Assuming that the government can recover the full imputed share for use of its productive capital, its budget constraint is then written:

$$(11) \quad g_{t+1} + h_{t+1} + (1 + r_t) b_{t-1} = \theta_t^w n_t + \theta_t^r k_t + (F_g + 1) g_t + h_t + b_t ,$$

where

$$(12) \quad \theta_t^w \equiv F_{n_t} - w_t \quad \text{and} \quad \theta_t^r \equiv F_{k_t} - r_t \ .$$

That is, the wage tax θ^w is defined as the difference between the marginal productivity of labor and the wage rate received by workers, and the interest income tax θ^r is defined in a similar way. However, since we can ignore the government budget constraint (11) from Walras' Law, we will not consider it explicitly in the subsequent analysis.

2.6. *The Intergenerational Welfare Function*

It is necessary to specify an intergenerational social welfare function in order to speak about a desired public investment and taxation policy for the government. We shall now adopt the following welfare function which is a sum of discounted generational utilities with a positive social time preference rate δ per period, though there is a belief that the government should not discount the welfare of future generations on ethical grounds:[6]

$$(13) \quad \sum_{t=0}^{\infty} (1+\delta)^{-t} V_t(w_t, q_{t+1}, P_t, P_{t+1}) \ ,$$

where the welfare of generation -1, in the second period of his life at period 0, is taken as given.

3. *Optimal Public Investment and Taxation Policy*

The government's objective in this model is to determine the amounts of productive and pollution control public investments, wage and interest tax rates, and public borrowing in each period in such a way as to maximize the social welfare function (13) subject to the feasibility condition (8), the capital market equilibrium condition (9), and the pollution stock accumulation equation (10). Therefore, noting that the determination of tax rates is equivalent to the determination of wage and interest rate, the intergenerational welfare maximization problem for the government can be formulated as follows

$$\underset{(g_{t+1},\, h_{t+1},\, w_t,\, q_{t+1},\, b_t)}{\text{Maximize}} \quad \sum_{t=0}^{\infty} (1+\delta)^{-t} V_t(w_t, q_{t+1}, P_t, P_{t+1}) \ ,$$

subject to

$$(14) \quad k_{t+1} = F(k_t, g_t, n_t) + k_t + g_t + h_t - c_t^1 - c_t^2 - g_{t+1} - h_{t+1} \ ,$$

$$(15) \quad P_{t+1} = E(k_t, g_t, n_t; c_t^1 + c_t^2) - G(h_t) + (1-\alpha) P_t \ ,$$

$$(16) \quad c_t^2 + g_{t+1} - g_t + h_{t+1} - h_t - b_t = F(k_t, g_t, n_t) - w_t n_t + k_t \ ,$$

where the last constraint (16) is derived from (6), (8) and (9), and c_t^1, c_t^2 and n_t are equilibrium consumptions and labor obtained from the consumer's utility maximization.

We can solve this problem by using dynamic programming. Suppose now that the government has inherited at period t a private capital stock k_t, a stock of pollution P_t, and the policy parameters set in the preceding period $(g_t, h_t, w_{t-1}, q_t, b_{t-1})$. (It may be noted that these determine the welfare of generation $t-1$.) We can then introduce the state valuation function $J(k_t, g_t, h_t, P_t, w_{t-1}, q_t, b_{t-1})$ to represent the maximal level of social welfare discounted to period t obtainable given these initial conditions. The government maximizes by choosing k_{t+1}, g_{t+1}, h_{t+1}, P_{t+1}, w_t, q_{t+1}, and b_t subject to the constraints (14)–(16). If we now introduce the multiplier λ_t for the constraint (16), we can apply the principle of optimality of dynamic programming.[7] In view of the stationarity of the problem,

$$
\begin{aligned}
(17)\quad J(t) &\equiv J(k_t, g_t, h_t, P_t, w_{t-1}, q_t, b_{t-1}) \\
&\equiv \mathrm{Max}\,\{V_t(w_t, q_{t+1}, P_t, P_{t+1}) + \lambda_t[F(k_t, g_t, n_t) \\
&\quad - w_t n_t + k_t - c_t^2 - g_{t+1} + g_t - h_{t+1} + h_t + b_t] \\
&\quad + (1+\delta)^{-1} J(k_{t+1}, g_{t+1}, h_{t+1}, P_{t+1}, w_t, q_{t+1}, b_t)\}\ ,
\end{aligned}
$$

where k_{t+1} and P_{t+1} are given by (14) and (15), respectively.

In this section we will explore optimal criteria for two kinds of public investment and optimal wage and interest tax formulae when the government is not permitted to finance public investments by bonds. Thus, $b_t = 0$ holds identically. We assume throughout the subsequent analysis both that the optimal path exists and that it converges to a stationary state. The first-order conditions for maximization are obtained by differentiating the basic recursion relation (17) with respect to the parameters k_t, g_t, h_t, P_t, w_{t-1}, and q_t, and by differentiating the maximand with respect to government control variables g_{t+1}, h_{t+1}, w_t, and q_{t+1} and equating to zero:

$$(18.1)\quad J_k(t) = [(1+\delta)^{-1} J_k(t+1) + \lambda(t)][F_k(t) + 1] + M(t) E_k(t)\ ,$$

$$(18.2)\quad J_g(t) = [(1+\delta)^{-1} J_k(t+1) + \lambda(t)][F_g(t) + 1] + M(t) E_g(t)\ ,$$

$$(18.3)\quad J_h(t) = (1+\delta)^{-1} J_k(t+1) + \lambda(t) - M(t) G_h(t)\ ,$$

$$(18.4)\quad J_p(t) = V_{p_t}(t) + (1-\alpha) M(t)\ ,$$

$$(18.5)\quad J_w(t) = \{M(t) E_c(t) - [(1+\delta)^{-1} J_k(t+1) + \lambda(t)]\}\, c_w^2(t)\ ,$$

$$(18.6)\quad J_q(t) = \{M(t) E_c(t) - [(1+\delta)^{-1} J_k(t+1) + \lambda(t)]\}\, c_q^2(t)\ ,$$

$$(18.7)\quad (1+\delta)^{-1}[J_g(t+1)-J_k(t+1)]-\lambda(t)=0\ ,$$

$$(18.8)\quad (1+\delta)^{-1}[J_h(t+1)-J_k(t+1)]-\lambda(t)=0\ ,$$

$$\begin{aligned}(18.9)\quad & (1+\delta)^{-1}\{J_k(t+1)[F_n(t)n_w(t)-c_w^1(t)]+J_w(t+1)\}\\ &+\lambda(t)[F_n(t)n_w(t)-n(t)-w(t)n_w(t)]\\ &+M(t)[E_n(t)n_w(t)+E_c(t)c_w^1(t)]+V_w(t)=0\ ,\end{aligned}$$

$$\begin{aligned}(18.10)\quad & (1+\delta)^{-1}\{J_k(\mathrm{t}+1)[F_n(t)n_q(t)-c_q^1(t)]+J_q(t+1)\}\\ &+\lambda(t)[F_n(t)n_q(t)-w(t)n_q(t)]\\ &+M(t)[E_n(t)n_q(t)+E_c(t)c_q^1(t)]+V_q(t)=0\ ,\end{aligned}$$

where

$$(19)\quad M(t)\equiv V_{P_{t+1}}(t)+(1+\delta)^{-1}J_P(t+1)\ .$$

In this paper we are interested in the nature of optimal public policy in the stationary state. Here, eliminating J_w, J_q, J_h, and λ from the above first-order conditions in the stationary state and using (5), we obtain the following conditions:

$$(20.1)\quad J_k=(1+\delta)^{-1}J_g(F_k+1)+[U_P^2+(1+\delta)^{-1}J_P]E_k\ ,$$

$$(20.2)\quad J_g=(1+\delta)^{-1}J_g(F_g+1)+[U_P^2+(1+\delta)^{-1}J_P]E_g\ ,$$

$$(20.3)\quad J_g=(1+\delta)^{-1}J_g-[U_P^2+(1+\delta)^{-1}J_P]G_h\ ,$$

$$(20.4)\quad J_P=U_P^1+(1-\alpha)[U_P^2+(1+\delta)^{-1}J_P]\ ,$$

$$\begin{aligned}(20.5)\quad & (1+\delta)^{-1}\{J_k(n+wn_w-c_w^1)+J_g[F_nn_w-n-wn_w-(1+\delta)^{-1}c_w^2]\}\\ &+[U_P^2+(1+\delta)^{-1}J_P]\{E_nn_w+E_c[c_w^1+(1+\delta)^{-1}c_w^2]\}+\gamma n=0\ ,\end{aligned}$$

$$\begin{aligned}(20.6)\quad & (1+\delta)^{-1}\{J_k(wn_w-c_q^1)+J_g\{F_nn_q-wn_q-(1+\delta)^{-1}c_q^2]\}\\ &+[U_P^2+(1+\delta)^{-1}J_P]\{E_nn_q+E_c[c_q^1+(1+\delta)^{-1}c_q^2]\}-\gamma c=0\ .\end{aligned}$$

Let us first derive optimal wage and interest tax formulae. Using (2), (12), (20.1) and (20.4), conditions (20.5) and (20.6) can be finally rewritten:

$$\begin{aligned}(21)\quad & [q/(1+\delta)][\theta^r-\phi mE_k/(1-mE_c)][1+\phi mE_c/(1-mE_c)]c_w^2\\ &+[\theta^w-\phi m(E_n+E_cw)/(1-mE_c)]n_w=(1-\phi)n\ ,\end{aligned}$$

$$\begin{aligned}(22)\quad & [q/(1+\delta)][\theta^r-\phi mE_k/(1-mE_c)][1+\phi mE_c/(1-mE_c)]c_q^2\\ &+[\theta^w-\phi m(E_n+E_cw)/(1-mE_c)]n_q=-(1-\phi+\rho)c^2\ ,\end{aligned}$$

where

(23) $\phi \equiv (1 - mE_c)(1 + \delta)\gamma/J_g$,

(24) $m \equiv -[U_P^1 + (1 + \delta)U_P^2]/(\delta + \alpha)U_c^1$,

(25) $\rho \equiv [F_k - \delta - \phi mE_k/(1 - mE_c)]/(1 + \delta)$,

which can be interpreted as the so-called net social marginal valuation of income in the presence of environmental pollution,[8] the shadow price of pollution in terms of first period consumption reflecting the marginal valuation to pollution of younger and older generations: $-U_P^1/U_c^1$ and $-U_P^2/U_c^1$, and the distortional index which shows a failure of the optimal allocation of capital between private and public sectors, respectively.

The fundamental conditions for optimal wage and interest tax formulae are given by (21) and (22). In general it is not possible to solve these conditions for optimal taxes explicitly, since the quantities demanded or supplied depend on the taxes. However, it is illuminating to solve θ^w and θ^r from (21) and (22), although the resulting expressions still only give us implicit solutions. We can then conclude that the optimal tax formulae have the following form in a polluted environment:

(26) $\theta^w = (1 - \phi)X - \rho Z + \phi m(E_n + E_c w)/(1 - mE_c)$,

(27) $\theta^r = [(1 - \phi)Y + \rho W](1 + \delta)/\{q[1 + \phi mE_c/(1 - mE_c)]\} + \phi mE_k/(1 - mE_c)$,

where

(28) $X \equiv -(c_w^2 c^2 + nc_q^2)/(c_w^2 n_q - n_w c_q^2)$,

(29) $Y \equiv (nn_q + c^2 n_w)/(c_w^2 n_q - n_w c_q^2)$,

(30) $Z \equiv c^2 c_w^2/(c_w^2 n_q - n_w c_q^2)$ and $W \equiv c^2 n_w/(c_w^2 n_q - n_w c_q^2)$.

The terms X and Y are efficiency terms familiar from the theory of optimal taxation and can be expressed as the Ramsey inverse elasticity formulae in the case of independent demands: $c_w^2 = n_q = 0$. Especially,

(a) if $E_c = 0$, $E_k \neq 0$ and $E_n \neq 0$, then $\theta^w = (1 - \phi)X + \phi mE_n - \rho Z$
and $\theta^r = (1 - \phi)(1 + \delta)Y/q + \phi mE_k + (1 + \delta)\rho W/q$,

and

(b) if $E_c \neq 0$ and $E_k = E_n = 0$, then $\theta^w = (1 - \phi)X + \phi[mE_c/(1 - mE_c)]w - \rho Z$
and $\theta^r = [(1 - \phi)Y + \rho W](1 + \delta)/\{q[1 + \phi mE_c/(1 - mE_c)]\}$.

Therefore, (a) if pollution is caused by production processes, then the optimal wage and interest tax formulae have in part the same two properties on the optimal tax formula of an externality-creating commodity pointed out by Sandmo (1975) in the *static* framework: (i) an additivity property of marginal social damage of that commodity and (ii) a weighted average property of two terms--the efficiency terms from the theory of optimal taxation, and the marginal social damage of that commodity. Thus, we confirm that the Pigovian taxation principle can be validated as part of a more comprehensive system of indirect taxation even in the polluted overlapping generations economy by waste discharged through the use of these factors in production processes. However, (b) if pollution is caused by consumption activities, the above two properties are reflected in the optimal wage tax formula alone, though the Pigovian interest tax is imposed on the interest income which is received and consumed by the older generation.

Next, optimal criteria for productive and pollution control public investments can be derived from (20.2), (20.3), and (20.4):[9]

$$(31) \quad F_g = \delta - \{[U_P^1 + (1+\delta)\, U_P^2]/(\delta + \alpha)\,[J_g/(1+\delta)]\}\, E_g > \delta \ ,$$

$$(32) \quad -\{[U_P^1 + (1+\delta)\, U_P^2]/(\delta + \alpha)\,[J_g/(1+\delta)]\}\, G_h = \delta \ .$$

Thus, if productive public capital is a source of pollution, then the optimal (second-best) discount rate for productive public investment is not equal to the social time preference rate because marginal social damage (cost) of productive public capital must be counted as opportunity cost as well, in addition to the social time preference rate.[10] Otherwise, the optimal discount rate will be equal to the social time preference rate, even if there may be marginal social damages of private capital, labor, and aggregate consumption. This is because these damages do not result in *additional* opportunity costs of public investment, though they as Pigovian taxes affect items of opportunity cost, as will be shown. On the contrary, marginal social benefit by pollution control public investment should be discounted with the social time preference rate, whether pollution is caused by production processes or by consumption activities.

Finally, let us now explore how the weighted average formula on public investment familiar in literature,[11] showing *items* of opportunity cost per unit in public investment, must be modified by the Pigovian taxes introduced for counteracting negative external effects by environmental pollution. From (21) and (22), we obtain the following equation:

$$\begin{aligned}(33) \quad & 1 - [\theta^w - \psi m\,(E_n + E_c w)]\,[(n_w/n) + (n_q/c^2)] \\ & = [q/(1+\delta)]\,(\theta^r - \psi m E_k)\,(1 + \psi m E_c)\,[(c_w^2/n) \\ & \quad + (c_q^2/c^2)] + (F_k + 1 - \psi m E_k)/(1+\delta) \ ,\end{aligned}$$

where

$$(34) \quad \psi \equiv (1+\delta)\,\gamma/J_g = \phi/(1-mE_c)\ .$$

Now, substituting $n_q = -(1+r)^2 n_r$ and $c_q^2 = -(1+r)^2 c_r^2$ into (33), solving it with respect to $1+\delta$, and then using (31), we can obtain finally the following equations:[12]

$$(35) \quad F_g + 1 - \psi m E_g = 1 + \delta$$

$$= \frac{(F_k + 1 - \psi m E_k)\,[1 - (1 + \psi m E_c)\,\varepsilon_r] + (1+r)\,[(1+\psi m E_c)\,\varepsilon_r]}{1 - [\theta^w - \psi m\,(E_n + E_c w)]\,\varepsilon_w/w}\ ,$$

where

$$(36) \quad \varepsilon_w \equiv (w/n)\,\{n_w - [(1+r)^2/c^2]\,n_r\}\ ,$$

$$(37) \quad \varepsilon_r \equiv [(1+r)/c^2]\,\{c_r^2 - [c^2/n(1+r)^2]\,c_w^2\}\ ,$$

which are the "total compensated variation elasticities" of labor and second period consumption with respect to wages and interest rate, respectively, introduced by Pestieau (1974).

If for some reason, especially, the wage tax is confined to Pigovian taxes which are imposed on labor in production processes and on wages received and consumed by the younger generation: $\theta^w = \psi m\,(E_n + E_c w)$, then (35) reduces to

$$(38) \quad F_g + 1 - \psi m E_g = 1 + \delta$$

$$= (F_k + 1 - \psi m E_k)\,[1 - (1 + \psi m E_c)\,\varepsilon_r] + (1+r)\,[(1+\psi m E_c)\,\varepsilon_r]\ .$$

Therefore, in such a special case the familiar weighted average formula for public investment must be modified as follows: first, if pollution is caused by production processes, then marginal social damage of private capital is imposed on *producers* as Pigovian interest income tax. Thus, in this case their market rate of return becomes the net marginal productivity of private capital from which its marginal social damage is deducted: $F_k + 1 - \psi m E_k$. However, since such a tax is not imposed on consumers, their market rate of interest and the relative share of resources transferred from private consumption to the public sector remain $1+r$ and ε_r, respectively. Second, if pollution is caused by consumption activities, then the Pigovian tax is imposed on interest income which is received and consumed by the older generation. Though such a tax does not affect market rates of interest facing consumers and producers, it increases the relative share of resources from private consumption to the public sector. Therefore, in this case the weight becomes $(1 + \psi m E_c)\,\varepsilon_r$.

Thus, we find that the weighted average formula for public investment can be interpreted in terms of the opportunity cost principle into which the impact of Pigovian taxes on consumers and producers are incorporated consistently, even if this

results in environmental pollution.

4. *Public Borrowing*

The model is now extended to allow for public borrowing and the creation of public debt. The number of instruments in the hands of government is thus extended to five: wage and interest taxes, productive and pollution control public investments, and public borrowing. It is assumed that the public debt has a one-period maturity and that interest on bonds is paid at the same rate as private capital in order for savers to be willing to hold both debt and private capital in their portfolios.

The first-order conditions for welfare maximization with respect to bonds are obtained by differentiating the basic recursion relation (17) with respect to b_{t-1}, and by differentiating the maximand with respect to b_t and equating it to zero:

$$(39) \quad J_b(t) = 0 \quad \text{and} \quad (1+\delta)^{-1} J_b(t+1) + \lambda_t = 0 \ .$$

Thus, the multiplier λ_t is zero, so that constraint (16) becomes ineffective in this case. First-order conditions for the other variables are the same as eqs. (18) of the previous section. In the stationary state, from (18.7)

$$(40) \quad J_k = J_g \ ,$$

which shows that the shadow price of private capital in terms of utility is equal to that of productive public capital. Substituting this into (20.1) and then solving it with respect to F_k, we obtain the following equation:

$$(41) \quad F_k = \delta + \psi m E_k \ ,$$

which shows that the rate of return on private capital is equal to a sum of the social time preference rate and marginal social damage of private capital. On the other hand, optimal criteria for productive and pollution control public investments have exactly the same forms as eqs. (31) and (32) in section 3:

$$(42) \quad F_g = \delta + \psi m E_g \quad \text{and} \quad \psi m G_h = \delta \ .$$

Therefore, from (41) and (42)

$$(43) \quad F_k - \psi m E_k = F_g - \psi m E_g = \delta \ .$$

These equations assert that social marginal productivities of private and public capital, from which their marginal social damages are deducted, are equated through the social time preference rate. Thus, since the optimal allocation of capital between the private and public sectors can be attained by introduction of public debt, the term involving a distortional index ρ in the optimal tax formulae (26) and (27) in section 3 vanishes.

However, this does not always imply an aggregate efficiency in production,

because if marginal pollutants of private and public capital in production processes are different: $E_k \neq E_g$, so are the rates of return on the two kinds of capital: $F_k \neq F_g$. Thus, in a polluted environment, the introduction of public debt does not warrant an aggregate production efficiency in general.[13]

Finally, let us note that the first-best intertemporal allocation of resources cannot be attained even if the government is permitted to finance public investments by bonds as well as taxes, because (i) the market rate of interest r is not equal to the social rate of time preference δ, and (ii) the net social marginal valuation of income ϕ is not equal to unity. However, it can be easily shown that the first-best allocation can be attained if the government can use a lump-sum tax to the *older* generation as an additional instrument.[14] In this case, it is of course necessary that the optimal wage and interest tax formulae consist of the Pigovian taxes:

$$(44) \qquad \theta^w = m\,(E_n + E_c w)/(1 - mE_c) \quad \text{and} \quad \theta^r = mE_k/(1 - mE_c)\ .$$

It may be interesting to note that if pollution is caused by consumption activities ($E_c \neq 0$ and $E_k = E_n = 0$), then marginal social damage of aggregate consumption is reflected in the optimal wage tax formula alone and the optimal interest tax is zero. This is because the wage tax can affect not only directly first period consumption but also indirectly second period consumption through its effect on savings; whereas the interest tax can affect only second period consumption.

5. *Concluding Remarks*

The main results of this paper can briefly be summarized as follows: (1) if public capital in production processes is a source of environmental pollution, then the optimal (second-best) discount rate for productive public investment is not equal to the social time preference rate; (2) the marginal social benefit by pollution control public investment should be discounted with the social time preference rate, whether pollution is caused by production processes or by consumption activities; (3) if the wage tax is confined to the Pigovian tax, then the familiar weighted average criterion for public investment continues to hold in a modified form; (4) if pollution is caused by production processes, then the optimal tax formulae consist of a weighted average of the Ramsey tax and the Pigovian tax, plus an additional term by a failure of the optimal allocation of capital between the private and public sectors; (5) the optimal allocation of capital can be attained by the introduction of public debt, while this does not always warrant aggregate production efficiency in a polluted environment.

However, it must be noted that these results have been obtained under several assumptions. First, it has been assumed that the stock of pollution and the consumer's demands for leisure and the first and second period consumption are independent. This assumption may not be realistic. However, it is *not* crucial, because we can easily show that the relaxation of such an assumption never affects all

of the above-mentioned results, although it does change the shadow price of pollution. Second, the population growth rate has been assumed to be zero. This assumption may be a serious shortcoming of the model as in most optimal pollution control models, because important relationships between population growth and environmental pollution are ignored. However, the introduction of a growing population would greatly complicate an already complicated model, since it is not possible to rewrite the model in per capita terms as in most economic growth models due to the dependence of utility on the *total* stock of pollution. Finally, the assumption of homogeneous individuals that all people are alike with respect to preference and income stream has been set. Therefore, the problem of income distribution within each generation has been completely disregarded. However, a consideration of this problem is very important, because there may be a trade-off between intragenerational and intergenerational equity in general.

Footnotes

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1. Similar results were also found in a slightly different overlapping generations model by Yoshida (1986). However, there are some important differences between the two models in the formulation on public production, labor, taxation, social time preference, and fiscal policy. See Yoshida (1986).
2. A similar study was worked out by Sandmo (1975). He showed that in the presence of environmental externalities such as air and water pollution, the Pigovian taxation principle could be validated as part of a more comprehensive system of indirect taxation: the optimal tax formula of an externality-generating commodity has a weighted average property of two terms–the efficiency terms familiar from the theory of optimal taxation and the marginal social damage of that commodity. However, he did not treat the criterion problem for public investment, and his analysis was so static that the optimal accumulation problem of capital and pollution stocks was not considered. Therefore, our model may also be regarded as a dynamic version of his model.
3. This assumption is familiar in optimal pollution control literature. For example, see Plourde (1972) for the case of pollution caused by production processes and Forster (1973) for the case caused by consumption activities. However, optimal pollution control models seem to be all unsatisfactory because the role of the public sector in a modern mixed economy was either disregarded completely or not formulated sufficiently. Therefore, our model may also be regarded as an optimal pollution control model in a mixed economy.
4. An assumption of a fixed number of people does not essentially alter the results in

this paper.

5. This assumption implies that there is no complementarity and substitutability between private goods and the stock of pollution. A similar assumption for public goods was employed in a different context by Atkinson and Stern (1974).
6. See, for example, Pestieau (1974).
7. See also Atkinson and Sandmo (1980).
8. See, for example, Atkinson and Stiglitz (1980).
9. Using ϕ and m defined in (23) and (24) respectively, these criteria can be also expressed as follows:

$$F_g = \delta + [\phi m/(1 - mE_c)]\, E_g \quad \text{and} \quad [\phi m/(1 - mE_c)]\, G_h = \delta \ .$$

10. It was shown by Arrow and Kurz (1970, p.134) that similar result also holds in a neoclassical growth model without overlapping generations when the per capita *public capital stock* is made an argument for the utility function as well as for per capita consumption in addition. However, an important interaction between investment and financing behavior in the public sector and maximizing behavior in the private sector was not considered, because it was assumed that savings are a fixed fraction of income and that the labor force grows at a constant proportional rate.
11. See, for example, Sandmo and Dreze (1971), Pestieau (1974), and Yoshida (1986).
12. If there are no environmental externalities: $E_k = E_g = E_n = E_c = 0$, then (35) coincides with eq. (28) in Pestieau (1974, p.229).
13. If it is assumed that new pollutants are a fixed proportion by-product of production, namely, that they are joint products, then $E_k = F_k$ and $E_g = F_g$. In this case, since eqs. (43) reduce to

$$F_k = F_g = \delta/(1 - \phi m) \ ,$$

the introduction of public debt warrants aggregate production efficiency.

14. Since government debt and lump-sum tax to the *younger* generation are not independent because they affect jointly market equilibrium in the private sector, introduction of the latter becomes redundant. See Atkinson and Stiglitz (1980).

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