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A Dynamic Version of the Falvey-Kierzkowski North-South Trade Model

Hiroshi ONO

1. Introduction

Recent years have witnessed many publications on intra-industry trade, which the traditional Heckscher-Ohlin-Samuelson framework finds difficult to consistently harmonize with inter-industry trade. Early steps in theoretical development emphasized imperfectness in the state of markets (for instance, monopolistic competition) (see Dixit and Stiglitz (1977), Helpman and Krugman (1985), Lawrence and Spiller (1983), Lancaster (1979), and so on). These contributions seem to succeed in explaining two-way trade among developed countries. However, they can not illustrate a phenomenon on product cycle in North-South trade. At least there are two explicit theoretical approaches on North-South trade. Based on Vernon's product cycle hypothesis, one of the promising approaches on North-South trade is given by P.Krugman (1979), which has been extended by Dollar (1986) and Thursby and Jensen (1986). These models emphasize learning process of the South, because the North is an innovator and the South an imitator with finite time lags. The other one is called the "product quality cycle" approach, where income distribution is introduced to demonstrate why high quality goods are exported from the North and low quality goods, from the South. Both Flam and Helpman (1987) and Falvey and Kierzkowski (1987) presented quite similar models in essence and succeeded in static analyses.

The purpose of this paper is to dynamise a "product quality cycle" approach and to derive policy implications from our analysis. We employ the Falvey-Kierzkowski model with the spirit of Flam-Helpman. While both models are based on the general equilibrium approach, there are distinct differences to be noted.

(1). The Falvey-Kierzkowski (FK) model contains two factors of production: labor and capital: while the Flam-Helpman (FH) model has only one factor of production: labor.

(2). FK investigate several possible technological differences between North and South, while FH examine the case where the North and South have an identical production function in homogeneous goods and differing ones in differentiated goods.

(3). FH consider only a normal case where wage rate in the North is greater than that in the South, which automatically excludes production of homogeneous goods in the North.

(4). On consumers' preferences, FK assume that consumers can choose both quantity and quality of differentiated goods, as well as of homogeneous goods. In contrast, FH assume that consumers purchase one unit of differentiated goods. That is, they choose only quality of the differentiated goods. With these differences in mind, we make our model as simple as possible. In order to dynamize it, we have two factors of production: labor and capital. However, we assume identical production technology in production of homogeneous goods, because our main concern is related to the differentiated goods sector. Then as assumed in FH, we examine the "normal" case. Furthermore, this paper investigates the case where consumers purchase one unit of differentiated goods. These modifications of the FK model may suggest that our model rather is an extension of the FH model with explicitly introducing capital.

Our paper consists of six sections. Section 2 demonstrates the basic framework of the model. Section 3 examines relationships of equilibrium in the factor markets, in the commodity markets, and in balance of trade. Short-run analyses are conducted in Section 4, where both equal wage rate case and differing wage rate cases are shortly discussed. Section 5 shows some of the comparative statics results. In section 6 we examine trade variations along the dynamic path. Finally, our main conclusions are stated in Section 7.

2. The Model

2. 1. Producers

Since our model presented here is basically similar to the FK model, the explanation will be minimized. Two kinds of commodities are available both in the North and South: homogeneous goods and differentiated goods. In producing the former, both North and South share the same Ricardian technology. The homogeneous product is assumed to be both consumption goods and investment goods. In contrast to this, differentiated products of quality z are subject to the Heckscher-Ohlin type, for simplicity, the Leontief-type fixed coefficient technology. In the North, x units of quality z, denoted by x (z), will be produced as follows:

$$(1.\ 1)\quad x\ (z)\ =\ \min.\ \left\{\frac{Lx}{a},\ \frac{Kx}{bz}\right\}$$

Similarly, production technology of the South will be given by equation (1.2).

$$(1.\ 2)\quad x^*(z)\ =\ \min.\ \left\{\frac{Lx^*}{a^*},\ \frac{Ly^*}{bz^*}\right\}$$

Since production technology is of linear homogeneity and the markets are competitive, the price of differentiated products with quality z equals their unit costs.

$$(1.\ 3)\quad p\ (z)\ =\ c\ (z)\ =\ aw+brz$$

(1. 4) $p^* (z^*) = c^* (z^*) = a^*w^* + b^*r^*z^*$

Assume as in FK that $aw > a^*w^*$ and $br < b^*r^*$. Then there exists quality $\bar{z}$, where it is indifferent to producing a commodity of quality $\bar{z}$ either in the North or South. Using $\bar{z}$, we can express the price of p (z) as equation (5).

(1. 5) $p(z) \begin{cases} a^*w^* + b^*r^*z & \text{for } z < \bar{z} \\ aw + brz & \text{for } z > \bar{z} \end{cases}$

As assumed in the FH model, we adopt the simplest form of production function for the homogeneous product: $Y = Ly$. Similarly, for the South: $Y^* = Ly^*$. Now that its price is normalized as unity, we have either the situation where (1) both North and South produce homogeneous goods at $w = w^* = 1$, or (2) the North ceases to produce the homogeneous commodity and only the South continues to produce it at $w^* = 1$. While our main concern is with the second case, the first case is briefly discussed in Section 4.

2. 2. Consumers

As suppliers in factors of production, consumers possess different levels of skills, which are reflected by a difference in the endowments of both effective labor supply and initial capital stock. Skills are continuously arranged in the unit interval of income classes. The density functions 1 (h), n (h), and f (h) respectively stand for the distribution of effective labor units of the population and of initial capital endowment, over income classes in the North. The same notations with asterisks attached are employed in the case of the South. Equation (2. 1) shows the average income of a person with income level h[1].

(2. 1) $I_h = w\tilde{1}(h) + rk\tilde{f}(h)$,

where r and k respectively stand for the rental from capital service and the overall capital labor ratio, and $\tilde{1}(h) = 1(h) / n(h)$ and $\tilde{f}(h) = f(h) / n(h)$. Equation (2. 2) supplies the budget equation of a person in income class h.

(2. 2) $I_h^d - y - p(z) = 0$,

where I_h^d stands for disposable income of a person in income class h. To avoid confusion, hereafter, we will omit subscript h. With a constant saving ratio, we assume a simple saving function: $I^d = (1-s) I$. In this paper we assume that every consumer always demands one unit of the differentiated goods. It is probable that persons belonging to a very low income class could not afford to purchase even one unit of differentiated goods. In that case their whole income will be spent only for homogeneous goods. Since this possibility only adds a minor complication, we leave this for interested readers.

Using relation (1. 5), we can alternatively express budget equation (2. 2) as equation (2. 3).

$$\text{(2. 3)}\quad \begin{aligned} I_h^d - y - (a^* + b^* r^* z) &= 0 \quad \text{for } z < \bar{z} \\ I_h^d - y - (aw + brz) &= 0 \quad \text{for } z > \bar{z} \end{aligned}$$

Budget equation (2. 3) suggests that when a consumer purchases homogeneous goods and chooses quality z, his disposable income is rather equal to either $I^d - a^*$ or $I^d - aw$.

For simplicity, we assume the Cobb-Douglas type of utility function.(2)

$$U(y, z) = y^{\frac{1}{1+a}} \quad z^{\frac{a}{1+a}}$$

This utility function yields the Gorman form of an indirect utility function, which is useful in aggregation (see H. Varian (1984)). Equations (2. 4) and (2. 5) respectively show the demand for homogeneous goods and the quality level which a consumer with income class h obtains.

$$\text{(2. 4)}\quad y = \frac{1}{1+a}(I^d - A)$$

$$\text{(2. 5)}\quad z = \frac{a}{1+a}\frac{(I^d - A)}{B}$$

where A and B respectively denote aw and br for $I^d \geqq I_o^d$, and a^* and b^*r^* for $I < I_o^d$. Figure 1 shows the determination of level I_o^d, where a consumer with income I_o^d is indifferent to choose either goods produced in the North or South. z^+ and z^- in the Figure are respectively computed as :

$$z^+ = \frac{a}{1+a}\frac{I_o^d - aw}{br}$$

$$z^- = \frac{a}{1+a}\frac{I_o^d - a^*}{b^*r^*}$$

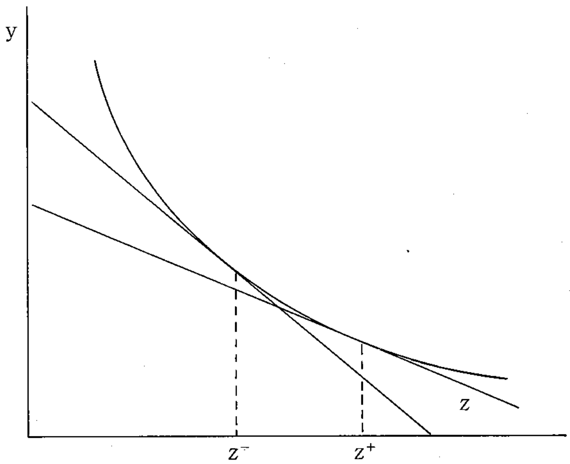

Figure 1

Substituting values of y and z respectively given in (2. 4) and (2. 5) into the utility function, we may express the indirect utility function either V (r, w, I^d) for $I^d \geqq I^d_o$ or V (r^*, w^*, I^d) for $I^d < I^d_o$.

As proved in FH, consumers with income $I^d \geqq I^d_o$ will purchase commodities of quality z greater than z^+ and consumers with income $I^d < I^d_o$ will buy commodities of quality less than z^-. Now that I^d is equal to $(1-s)\ I_h$ and $I_h = wl\ (h) + rkf\ (h)$, I^d_o determines the values of both h_d satisfying $I^d_o = (1-s)\ [wl\ (h_d) + rk\ (h_d)]$ and $h_d{}^*$ with $I^d_o = (1-s)\ [w^*l^*\ (h_d{}^*) + r^*k^*f^*\ (h_d{}^*)]$.

3. Equilibrium Conditions

In the FH model, the North produces only differentiated goods of higher quality, which are demanded both in the North and South with income greater than I^d_o. We assume that a consumer with income greater than I^d_o always demands z^+. Now that every consumer with income class $h \in [h_d, 1]^U\ [h_d{}^*, 1]$ always demands one unit of the differentiated goods produced in the North, the total demand for differentiated goods produced in the North is equivalent to the number of people belonging to income class $h \in [h_d, 1]^U\ [h_d{}^*, 1]$. That is,

$$(3.\ 1)\quad TD_1 = \int_{hd}^{1} Nn\ (h)\ dh + \int_{h^*d}^{1} N^*n^*\ (h)\ dh,$$

where N and N* respectively stand for the population sizes of North and South.

3. 1. Equilibrium in the Factor Market

First, let us consider equilibrium in the factor markets of the North. In order to meet total demand given in equation (3. 1), labor must, following technology (1. 1), be allocated as follows:

$$(3.\ 1)'\quad L = a\ [\int_{hd}^{1} Nn\ (h)\ dh + \int_{h^*d}^{1} N^*n^*\ (h)\ dh]$$

For simplicity, let us assume $N = L$ and $N^* = L^*$. Then by letting $u = L/(L+L^*)$, equation (3. 2) shows, in per capita terms, equilibrium in the labor market of the North.

$$(3.\ 2)\quad u = a\ [u \int_{hd}^{1} n\ (h)\ dh + (1-u) \int_{h^*d}^{1} n^*\ (h)]$$

Now, bzx (z) indicates the supply of capital to meet domestic demand for differentiated products of quality z. Since consumers with income class $h \in [h_d, 1]^U\ [h_d{}^*, 1]$ demand differentiated goods produced in the North, equation (3. 3) describes equilibrium in the capital market.

$$(3.\ 3)\quad K = [\int_{hd}^{1} bzx\ (z)\ dh + \int_{h^*d}^{1} bz^*x^*\ (z^*)\ dh]$$

$$= b\ [N \int_{hd}^{1} zn\ (h)\ dh + N^* \int_{h^*d}^{1} z^*n^*\ (h)\ dh]$$

Equation (3. 3) is transformed in per capita expression.

$$(3.\ 4)\quad uk = b\ \{u\int_{hd}^{1} zn\ (h)\ dh+(1-u)\ \int_{h^*d}^{1} z^*n^*\ (h)\ dh\}$$

Similar argument will derive equilibrium conditions in the factor markets of the South.

$$(3.\ 5)\quad 1-u = a^*\ \{u\int_{0}^{hd} n\ (h)\ dh+(1-u)\ \int_{0}^{h^*d} n^*\ (h)\ dh\}$$

$$(3.\ 6)\quad (1-u)\ k^* = b^*\ \{u\int_{0}^{hd} zn\ (h)\ dh+(1-u)\ \int_{0}^{h^*d} z^*n^*\ (h)\ dh\}$$

3. 2. Equilibrium in the Commodity Markets

First, let us consider equilibrium in the commodity markets of the North. Since Nn (h) and N*n* (h) units of differentiated goods are respectively demanded by consumers in the North and South with income class $h \in [h_d, 1]^U\ [h_d^*, 1]$, equation (3. 7) will supply total demand for differentiated goods produced in North.

$$(3.\ 7)\quad u\int_{hd}^{1} p\ (z)\ n\ (z)\ dh+(1-u)\ \int_{h^*d}^{1} p^*\ (z^*)\ n^*\ (h)\ dh = u\ [w+rk]$$

The right hand side of equation (3. 7) shows the distributive income engaged in production of differentiated products in the North. As for equilibrium in the homogeneous goods sector, the demand side basically consists of three parts: domestic demand, foreign demand, and demand for capital accumulation. Equation (3. 8) describes equilibrium in the homogeneous goods market.

$$(3.\ 8)\quad \int_{0}^{1} yNn\ (h)\ dh+\int_{0}^{1}y^*N^*n^*\ (h)\ dh+s\int_{0}^{1} I_h\ Nn\ (h)\ dh+s^*\int_{0}^{1} I_h^*N^*n^*\ (h)\ dh = wLy+rKy+Ly^*+r^*Ky^*$$

This will be turned into an expression in per capita terms:

$$(3.\ 9)\quad u\ \{\int_{0}^{1} yn\ (h)\ dh+s\int_{0}^{1} In\ (h)\ dh\}+(1-u)\ \{\int_{0}^{1} y^*n^*\ (h)\ dh+s^*\int_{0}^{1} I^*n^*\ (h)\ dh\}= u\ \{wly+rlyky\}+(1-u)\ \{ly^*+r^*ly^*ky^*\}$$

3. 3. Equilibrium in the Balance of Trade

Since the North exports only differentiated products of high quality and imports both differentiated products of low quality and homogeneous goods, equilibrium in the trade balance will be given by equation (3. 10)

$$(3.\ 10)\quad (1-u)\int_{h^*d}^{1} p^*\ (z^*)\ n^*\ (h)\ dh-u\ \ [\int_{hd}^{1} p\ (z)\ n\ (h)\ dh-\int_{0}^{1} yn\ (h)\ dh-s\int_{0}^{1} In\ (h)\ dh]= 0$$

As explicitly stated in FH, equilibrium in the factor markets ensures the equilibria in both the commodity markets and in the balance of trade. For proof of this, see appendix 2.

4. Short-Run Analysis

As FH a priori assumed the case of equal wage rates, it is natural to presume people in the North receiving higher wages. However, since we are concerned with a dynamic aspect of trade between North and South, it is of interest first to investigate the possibility of wage equalization in the steady state.

4. 1. The Equal Wage Rate Case

Now that an equal wage rate is paid in both North and South, the North can produce homogeneous goods. Therefore, equilibrium conditions in the factor markets are slightly changed and are stated by the following four equations-the first two of which are related to activities in the North, and the latter two to activities in the South.

$$(4.\ 1)\quad u = a\ \{u\int_{hd}^{1} n\ (h)\ dh + (1-u)\int_{h^*d}^{1} n^*\ (h)\} + uly$$

$$(4.\ 2)\quad uk = b\ \{u\int_{hd}^{1} zn\ (h)\ dh + (1-u)\int_{h^*d}^{1} z^*n^*\ (h)\ dh\}$$

$$(4.\ 3)\quad 1-u = a^*\ \{u\int_{0}^{hd} (h)\ dh + (1-u)\int_{0}^{h^*d} n^*\ (h)\} + (1-u)ly^*$$

$$(4.\ 4)\quad (1-u)\ k^* = b^*\ \{u\int_{0}^{hd} n\ (h)\ dh + (1-u)\int_{0}^{h^*d} z^*n^*\ (h)\}$$

In the short-run, variables r, r*, ly and ly* are endogenously determined by equations (4. 1) through (4. 4). The system of the above equations clearly shows that once equilibrium values of r and r* are known, ly and ly* respectively are determined by equations (4. 1) and (4. 3). Therefore, we can concentrate attention on equations (4. 2) and (4. 4). Totally differentiating (4. 2) and (4. 4), and using well-known hat notation, we have the following matrix system:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \hat{r} \\ \hat{r}^* \end{bmatrix} = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} \hat{k} + \begin{bmatrix} b_{12} \\ b_{22} \end{bmatrix} \hat{k}^*$$

where $a_{11} = -\ [1-t_kq_r + t_kd_1\ (a_1 - s_k)/+t_k{}^*d_1{}^*a_1/\varepsilon_I{}^*]$, $a_{12} = t_k{}^*q_r{}^* + t_kd_1a_1/\varepsilon_I + t_k{}^*d_1{}^*\ (a_1 + s_k{}^*)/\varepsilon_I{}^*$, $a_{21} = T_kQ_r - T_kd_2(a_1-s_k)/\varepsilon_I - T_k{}^*d_2{}^*a_1/\varepsilon_I{}^*$, $a_{22} = -\ [1-T_k{}^*Q_r{}^* - T_kd_2a_1/\varepsilon_I - T_k{}^*d_2{}^*(a_1 + s_k{}^*)/\varepsilon_I{}^*]$, $b_{11} = 1-t_kq_r-t_kd_1s_k/\varepsilon_I$, $b_{21} = -\ [T_kQ_r + T_kd_2s_k/\varepsilon_I]$, $b_{12} = -\ [t_k{}^*q_r{}^* + t_k{}^*d_1{}^*s_k{}^*/\varepsilon_I{}^*]$, $b_{22} = 1 - T_k{}^*Q_r{}^*-T_k{}^*d_2{}^*s_k{}^*/\varepsilon_I{}^*$.

(For explanation on notations in a_{ij}s and b_{ij}s, see Appendix 1.)

It is shown that $\hat{r}/\hat{k}+\hat{r}/\hat{k}^* = -1$ and $\hat{r}^*/\hat{k}+\hat{r}^*/\hat{k}^* = -1$. It is also easy to prove that r^*/k^* is greater than -1. It should be noted that the effect of an increase in capital stock on the home interest rate is not symmetric in the sense that an increase in capital in the North might raise interest rate r, while an increase in k* results in a decrease in r^*. (For detailed discussions, see Appendix 3.) The above difference is created by income effects. The increase in k adds to income in every income class in the North and reduces the level of h_d. That is, more people can consume differentiated goods of high quality, which pushes up demand and tends to raise r. In contrast to this, the increase in k^* reduces demand for differentiated products produced in the South, because that increase in k^* adds to income of every income class in the South and reduces h_d^*.

4. 2. The Differing Wage Case

This case has already been discussed in Section 3. Since homogeneous goods are solely produced by labor and differences in technologies are assumed, the North would not produce any amount of homogeneous goods. Letting $ly = 0$ in equation (4. 1) will state equilibrium in the labor market of the North. The other three equations still hold in this case.

$$(4.\ 1)' \quad u = \{u \textstyle\int_{hd}^{1} n\ (h)\ dh + (1-u) \int_{h^*d}^{1} n^*\ (h)\ dh\}$$

We then have four equations: (4. 1)', (4. 2), (4. 3) and (4. 4); and four variables: r, r^*, w and ly^*. However, as ly^* appears only in equation (4. 3), we can, by using equations (4. 1)', (4. 2) and (4. 4), determine equilibrium values of r, r^* and w. Totally differentiating these equations, we obtain the following matrix system denoted by (M):

$$(M) \quad \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} \hat{w} \\ \hat{r} \\ \hat{r}^* \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} \hat{\gamma}$$

where $m_{11} = -[v\ (a_2 - s_w)/\varepsilon_I + v^* a_2/\varepsilon_I^*]$, $m_{12} = -[v\ (a_1 - s_k)/\varepsilon_I + v^* a_1/\varepsilon_I^*]$, $m_{13} = va_1/\varepsilon + v^*\ (a_1 + s_k^*)/\varepsilon_I^*$, $m_{21} = t_k q_w - t_k^* q_0^* - t_k d_1\ (a_2 - s_w)/\varepsilon_I$, $m_{22} = -[1 - t_k q_r + t_k d_1\ (a_1 - s_k)/\varepsilon_I + t_k^* d_1^* a_1/\varepsilon_I^*]$, $m_{23} = t_k^* q_r^* + t_k d_1 a_1/\varepsilon_I^*$, $m_{31} = T_k Q_r - T_k d_2\ (a_2 - s_w)/\varepsilon_I - T_k^* d_2^* a_2/\varepsilon_I^*$, m_{32} $T_k Q_r - T_k d_2\ (a_1 - s_k)/\varepsilon_I - T_k^* d_2^* a_1\ \varepsilon_I$, $m_{33} = -[1 - T_k^* Q_r^* - T_k d_2 a_1/\varepsilon_I - T_k^* d_2^*\ (a_1 + s_k^*)/\varepsilon_I^*$.

γ stands for a parameter. Before proceeding comparative static analysis, we assume rather strong sufficient conditions for stability. (For adjustment mechanism taken here, see Appendix 3.)

$$(S\,1) \quad m_{ij} < 0$$

$$(S\,2) \quad S_{ij} = \begin{vmatrix} m_{ii} & m_{ij} \\ m_{ji} & m_{jj} \end{vmatrix} > 0 \qquad i \neq j = 1, 2, 3$$

(S 3) $S = \left| m_{ij} \right| < 0$

4. 3. Comparative Statics

Table 1 states the results of performing comparative statics, where a positive (or negative) sign indicates that a change in a parameter value moves in the same (or opposite) direction with a resulting change of variable.
(For detailed analysis, see Appendix 4.)
Let us consider an increase in capital. This increase pushes up per capita income for any income class and reduces the level of h_d, which implies that more people with higher income demand differentiated goods of higher quality. This will result in increasing wage rates. While demand for differentiated goods both of high and low qualities are increased, the supply side pushed up by capital increase will dominate the demand side in the North, which reduces rental rate r. Since there will be excess demand for differentiated goods of low quality, r* will be excess demand for differentiated goods of low quality, r* will be raised. The case of an increase in k* is not symmetrically discussed. The increase in k* raises per capita income for any income class in the South and reduces h_d*. However, since the increase in k* lessens r*, the South can produce differentiated goods at lower cost and would push up the level of h_d*, which reduces demand for differentiated goods of high quality, and diminishes w. An increase in the saving ratio, either s or s*, gives similar effects on w, r and r*. The increase in s reduces disposable income for any income class in the North and decreases demand. Therefore, these declines force both factor employment and factor prices to be reduced. The same argument essentially holds for the case of a change in s*. As for labor-saving technical progress, we might have very interesting results. First, suppose that it occurs in the North, which means in our model a decline in a. The decline in a reduces both the unit cost for producers producing differentiated goods and the demand for capital in producing differentiated goods of high quality, which decreases r in the North. However, a decline in a also creates positive income effects, because disposable income for consumers purchasing y and quality z is equal to I^d-aw. These two effects, namely cost effect and income effect, work countervailingly against each other and yield ambiguous signs on w. In the case of labor-saving technical progress in the South, the results are a little different. A decrease in a* reduces the unit cost in producing lower quality goods and also increases income for any income class in the South. Therefore, the demand for differentiated goods of lower quality is definitely increased, which pushes up r* and under normal conditions, this technical progress reduces demand for higher quality goods and decreases both w and r. This result essentially is the same as Flam-Helpman's.

Finally, let us consider capital-saving technical progress. When it occurs in the North, it reduces the unit cost in producing differentiated goods of high quality. Furthermore, that decline increases the demand for high quality goods and pushes up the utility level, resulting in a decline in I_0^d. Therefore, the demand for high quality goods is increased, which raises both w and r. These increases in factor prices in the North benefit every income class and increase the demand for low quality goods, too. Then follows an increase in r*. In the case of technical progress in the South (a decline in b*), the exact opposite reasoning will yield opposite signs in the case of a decline in b.

5. Long-Run Analysis

As already discussed in the previous section, irrespective of whether w is equal to or greater than unity, we have obtained, under normal conditions, the following qualitative functional relationships:

$$r = r\underset{-\ \ -}{(k\ ,\ k^*)}$$

$$r^* = r^*\underset{+\ \ -}{(k\ ,\ k^*)}$$

where the sign of either + or − indicates partial derivatives. Now let us consider a dynamic aspect of the model. Equations (5. 1) and (5. 2) respectively demonstrate dynamic variations in capital stocks in both North and South.

$$(5.1)\quad \dot{k} = s\ [w\ (k, k^*) + r\ (k, k^*)\ k] - mk = \phi\ (k, k^*)$$

$$(5.2)\quad \dot{k}^* = s^*\ [1 + r^*\ (k, k^*)\ k^*] - mk^* = \psi\ (k, k^*)$$

where m stands for the rate of population growth. For simplicity, we assume that both North and South have the same population growth. Let us assume rather strong sufficient conditions for stability.

$$(C\,1)\quad \phi_k < 0,\ \psi_k{}^* < 0,$$

where $\phi_k = \partial\phi/\partial i$ and $\psi_k = \partial\psi/\partial i$ for i = k, k*

Condition (C 1) guarantees that the Routh-Hurwitz conditions hold. Since $\phi_k{}^* < 0$ and $\psi_k > 0$, the slopes of both $\dot{k} = 0$ line and $\dot{k}^* = 0$ line respectively have the following properties:

$$\left.\frac{dk^*}{dk}\right|_{\dot{k}=0} = -\frac{\phi_k}{\phi_k{}^*} < 0$$

$$-\left.\frac{dk^*}{dk}\right|_{\dot{k}=0} = -\frac{\psi_k}{\psi_k{}^*} > 0$$

Therefore, we can draw Figure 2.

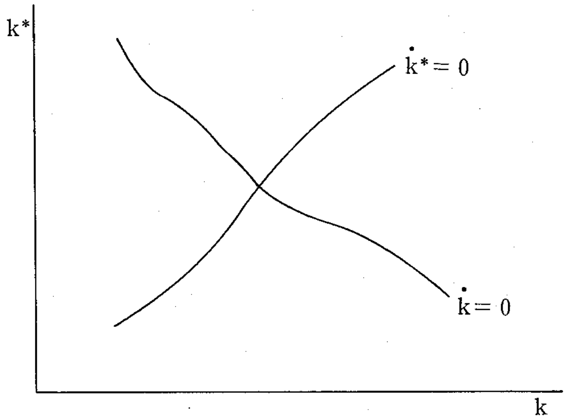

Figure 2

When we unite both cases of $w = 1$ and $w > 1$, Figure 2 should be corrected. Since $\hat{w}/\hat{k} = -\hat{w}/\hat{k}^*$, the iso-wage rate curve $w(k, k^*) = 1$ should be a straight line. For analytical convenience, we examine the case of the iso-wage rate curve being a 45° straight line. Then we can draw Figures 3 a and 3 b[3]. Note that line $\dot{k} = 0$ kinks downward at $k = k^*$, because of the existence of $w(k, k^*)$. We can see from Figures 3 a and 3 b that there is no possibility of multiple equilibria. In the case of Figure 3 a, wage differentials between North and South persist even in the long-run. The essential difference between Figures 3 a and 3 b is that when both the $\dot{k} = 0$ line and $\dot{k}^* = 0$ line pass through the iso-wage rate line, desirable capital stock configurations (k, k^*) are different. In case of Figure 3 a, the $\dot{k} = 0$ line supplies larger values of both k and k* than the $\dot{k}^* = 0$ line does. The opposite holds in the case of Figure 3 b. In the following, let us consider the case of Figure 3 a. (The

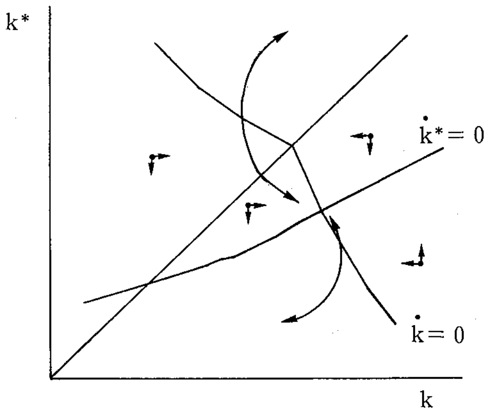

Figure 3a

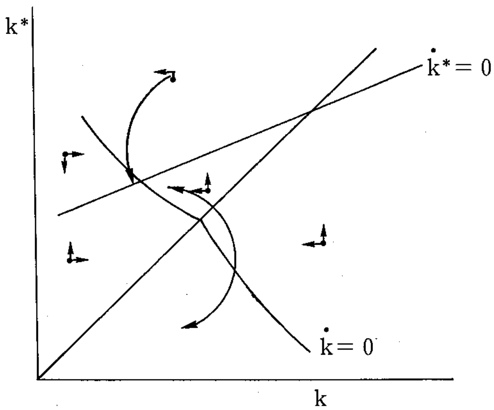

Figure 3b

case of Figure 3 b is similarly investigated without any difficulty.)

When the North enjoys a higher wage rate in the steady state, how can this situation possibly be altered? This is the problem of comparative statics.

5. 2. Comparative Static Analysis

Let us write our system given equations (5. 1) and (5. 2) as:

(5. 3) $\quad k = \phi\,(k, k^*;\ \gamma)$

(5. 4) $\quad k^* = \psi\,(k, k^*;\ \gamma)$

where γ stands for a parameter. Recalling the discussions in Section 4, we shall here investigate three cases, in which we observed definite sign conditions.

(1) Saving Ratio

It has been seen that a change in either s or s* had the same qualitative effects on factor prices. Therefore, we do not need to separate these effects. Suppose the saving ratio in the North rises. Then the $\dot{k} = 0$ line and the $\dot{k}^* = 0$ line both shift downward. As a result, k* definitely declines, while steady state k can move in either direction. It is of interest to observe that a rise in the saving ratio in the South yields the same results. In order to increase steady state capital stock in the South, this rather suggests that a policy of stimulating consumption should be taken by the South, which will boost distributive income both in the South and in the North. It should be noted that there is no tendency for wage differentials between North and South to disappear. (See Figure 4)

(2) Labor-Saving Technical Progress in the South

When the South pursues labor-saving technologies, then both factor prices, w

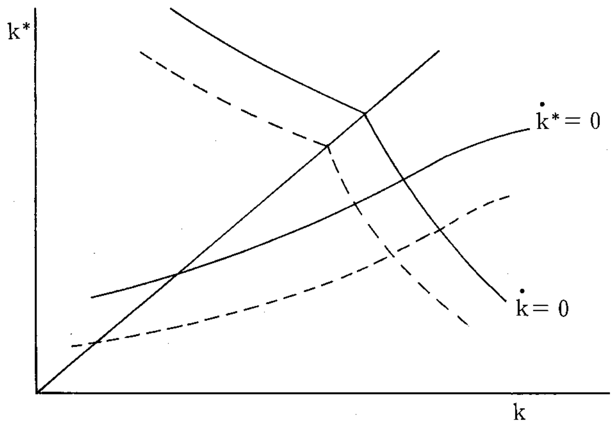

Figure 4

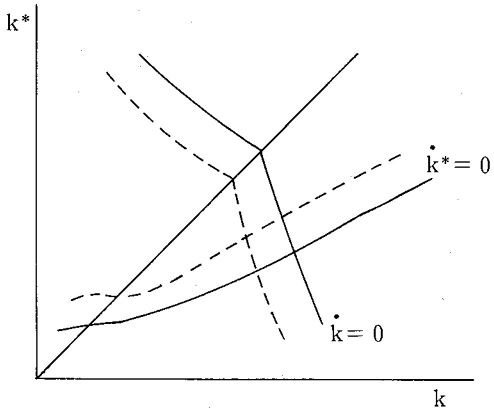

Figure 5

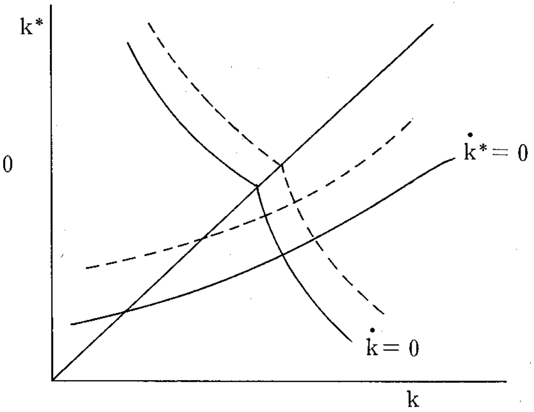

Figure 6

and r in the North, will be decreased and factor price r* in the South increased. Therefore, the $\dot{k} = 0$ line shifts upward. While at the new equilibrium steady state k* might stay at the same level, steady state k clearly is decreased and wage differentials tend to be tapered off (see Figure 5).

(3) Capital-Saving Technical Progress

Suppose that capital-saving technology occurs in the North, which raises factor prices w, r and r*. Therefore, both the $\dot{k} = 0$ and $\dot{k}^* = 0$ lines shift upward, resulting in an increase in steady state k*. This result follows from the distributive income effect that the increase in factor incomes arouses demand for both differentiated goods and homogeneous goods, of which the latter is solely produced in the South. Therefore, these expansions in production increase income in the South and add to capital stock. This result also suggests that capital-saving technical progress in the South turns out to reduce steady state k*, because of a decline in distributive factor income. (See Figure 6)

6. Trade Variations

Let us consider how the volume of trade will be influenced by the dynamic movement of the system. We define three kinds of trade concepts. The first is intra-industry trade, whose volume is given by equation (6. 1).

(6. 1) $IT = u \int_0^{hd} p\ (z)\ n\ (h)\ dh$

IT indicates trade activity within the same categories of commodities. The second type is inter-industry trade, which is expressed by equation (6. 2).

(6. 2) $HO = u \int_0^1 yn\ (h) + su \int_0^1 In\ (h)\ dh$

HO shows a traditional Heckscher-Ohlin type trade. Finally, the third concept is defined as the total volume of trade, which is equivalent to the sum of imports and exports. Therefore,

$$TT = (1-u)\int_{h^*d}^{1} p^*(z^*)\, n^*(h)\, dh + u\int_{0}^{hd} p(z)\, n(h)\, dh$$

$$= 2\,IT + HO$$

Let us consider the dynamic changes in those three different kinds of trade volume. We can compute the following, which shows that intra-industry trade essentially depends on a relative change in k and k*.

$$\frac{d\,(IT)}{dk} = \frac{u\alpha}{1+\alpha}(1-s)\left[A\frac{dw}{dk} + Br\left(1+\frac{\hat{r}}{\hat{k}}\right)\right]\left(1-\frac{\hat{k}^*}{\hat{k}}\right)$$

where $A = \int_0^{hd} l\,(h)\,dh$ and $B = \int_0^{hd} f\,(h)\,dh$.

Since the bracketed term associated with A and B is positive, a change in intra-industry trade caused by capital growth would totally depend on relative sizes of capital stock changes in the North and South. That is, when changes in k and k* take different signs along a dynamic time path, then intra-industry trade always increases. On the other hand, when both k and k* increase but k* increases at a faster rate than k, then intra-industry trade will be reduced. It turns out that this conclusion holds true in the other two cases as well.

We can compute:

$$\frac{d\,(HO)}{dk} = U\left\{\frac{1+\alpha s}{1+s}\frac{dw}{dk} + r\left(1+\frac{\hat{r}}{\hat{k}}\right) - \frac{ac}{1+\alpha}\frac{dw}{dk}\right\}\left(1-\frac{\hat{k}^*}{\hat{k}}\right)$$

where $C = \int_{hd}^{1} n\,(h)\,dh$.

The first bracketed term in the r. h. s. of the above equation would probably take a positive sign. Then, from the definition of TT, it is clearly shown that the sign of d (TT)/dk takes the same sign of $(1-\hat{k}^*/\hat{k})$.

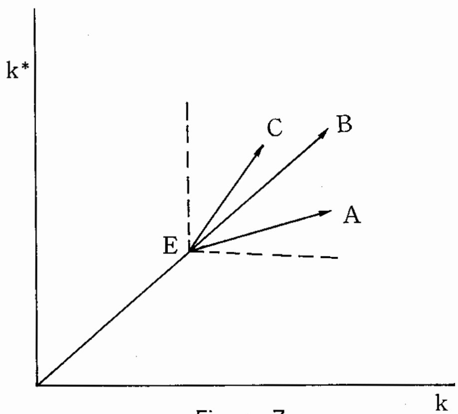

Figure 7

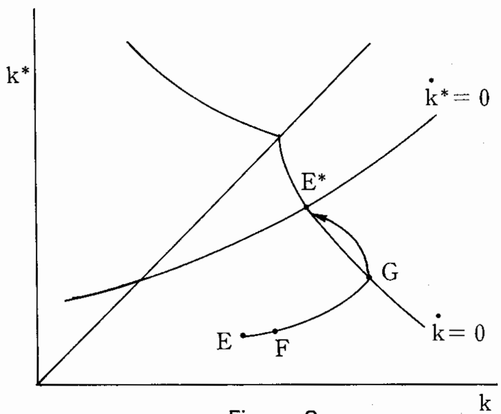

Figure 8

Suppose an initial point is given at point E in Figure 7. Then straight line OB is the extension of OE. When the economy moves toward direction A, then all three categories of the balance of trade increase. On the contrary, when the economy moves toward direction C, they decrease. Keeping this in mind, we can predict changes in the balance of trade in such case as the economy moving along a dynamic path in Figure 8. When the economy moves along EF as in Figure 8, then the balance of trade is increased. After the economy passes through point F toward G, the trade balance is decreased, because faster accumulation in capital stock in the South discourages distributive factor income. Then the balance of trade is increased after the economy moves from G toward the new equilibrium point E*.

7. Concluding Remarks

This paper presented a dynamic version of the FK model in the spirit of Flam-Helpman. We have shown that if equilibrium exists, it is then unique. Whether or not the wage rate in the South is equalized to that in the North depends on the relative size of steady state capital stocks in North and South. Only when steady state capital stock in the South is equal to or larger than that in the North, equalization in the wage rate is realized. Some of our results on comparative statics are interesting. We have investigated three cases: (1) changes in saving ratios, (2) labor-saving technology in the South, and (3) capital-saving technology in the South. In these three cases, steady state capital stock could change in either direction except for the second case. Suppose, for simplicity, that the steady state capital stock in the North remains unchanged. Then, what are welfare effects of these variations on the economy? Since our utility function is of the Gorman form, we may construct a social welfare function whose component consists of an individual utility function with the inverse of marginal utility income attached as weights[4]. Then, while steady state capital stock in the North is unchanged, it can benefit from capital-saving technical improvements in the South. However, when the South experiences labor-saving technical progress, it directly raises social welfare through raising utility belonging to income class $h \in [0, h_d]$, but a part will be offset by a decline in capital stock and we do not *a priori* know the net effect to be either positive or negative. On the other hand, while decline in the Northern saving ratio does not give a direct effect, it will raise Southern welfare through increasing an accumulation of capital stock in the South. As for social welfare in the South, both a decrease in saving ratios and capital-saving technical progress increase its capital stock. Since these changes also directly raise social welfare, it definitely increases social welfare in the South. However, in case of labor-saving technical progress, capital stock in the South could change in either direction. Suppose it remains its initial amount. Then labor-saving technical progress also raises social welfare in the South.

We have also shown how both intra-industry trade and inter-industry trade would be varied along the dynamic path of an economy moving toward steady state equilibrium.

Appendix 1. Explanation of Notation

$a_1 = (\alpha/1+\alpha)\, t\,(t-1)\,(aw-a^*)/I_o^d > 0$, $a_2 = awt/I_o^d$, $t = (br)^{-\alpha/(1+\alpha)} / [(br)^{-\alpha/(1+\alpha)} - (b^*r^*)^{-\alpha/(1+\alpha)}] > 0$, $\varepsilon_I = [wl^1(h_d) + rkf^1(h_d)]\, h_d / [wl(h_d) + rkf(h_d)]$ = elasticity of substitution in income class h_d, $\varepsilon_I^* = [l^{*1}(h_d^*) + r^*k^*f^{*1}(h_d^*)]\, h_d^* / [l^*(h_d) + r^*k^*f^*(h_d^*)$ = elasticity of substitution in income class h_d^*, $s_w = wl(h_d)/[wl(h_d) + rkf(h_d)]$ = labor share in income class h_d, $s_k = rkf(h_d)/[wl(h_d) + rkf(h_d)]$ = capital share in income class h_d, $s_w^* = l^*(h_d^*)/[l^*(h_d^*) + r^*k^*f^*(h_d^*)]$ = labor share in income class h_d^*, $s_k^* = r^*k^*f^*(h_d^*)/[l^*(h_d) + r^*k^*f^*(h_d)]$ = capital share in income level h_d^*, $T_k = u\int_{hd}^{1} zn(h)\,dh / [u\int_{hd}^{1} zn(h)\,dh + (1-u)\int_{h^*d}^{1} z^*n^*(h)\,dh]$, $t_k^* = (1-u)\int_{h^*d}^{1} z^*n^*(h)\,dh / [u\int_{hd}^{1} zn(h)\,dh + (1-u)\int_{h^*d}^{1} z^*n^*(h)\,dh] = 1 - t_k$, $q_w = \int_{hd}^{1} (\alpha/1+\alpha)(1/br)[(1-s)\,wl(h) - aw]\, n(h)\,dh / \int_{hd}^{1} zn(h)\,dh$, $q_r = \int_{hd}^{1} (\alpha/1+\alpha)[(1-s)\,rk/br]\, f(n)\, n(h)\,dh / \int_{hd}^{1} zn(h)\,dh$, $q_o = \int_{hd}^{1} (\alpha/1+\alpha)(aw/br)\, n(h)\,dh / \int_{hd}^{1} zn(h)\,dh$, $q_r^* = \int_{h^*d}^{1} (\alpha/1+\alpha)[(1-s)\,r^*k^*/br]\, f^*(h)\, n^*(h)\,dh / \int_{h^*d}^{1} z^*n^*(h)\,dh$, $q_o^* = \int_{h^*d}^{1} (\alpha/1+\alpha)(aw/br)\, n^*(h)\,dh / \int_{h^*d}^{1} z^*n^*(h)\,dh$, $q_s = (s/1-s)\int_{hd}^{1} (\alpha/1+\alpha)(I^d/br)\, n(h)\,dh / \int_{hd}^{1} zn(h)\,dh$, $q_s^* = (s^*/1-s^*)\int_{h^*d}^{1} (\alpha/1+\alpha)(I_d^*/br)\, n^*(h)\,dh / \int_{h^*d}^{1} z^*n^*(h)\,dh$, $T_k = u\int_0^{hd} zn(h)\,dh / [u\int_0^{hd} zn(h)\,dh + (1-u)\int_0^{h^*d} z^*n^*(h)\,dh]$, $T_k^* = (1-u)\int_0^{h^*d} z^*n^*(h)\,dh / [u\int_0^{hd} zn(h)\,dh + (1-u)\int_0^{h^*d} z^*n^*(h)\,dh]$, $Q_w = \int_0^{hd} (\alpha/1+\alpha)[(1-s)\,wl(h)/b^*r^*]\, n(h)\,dh / \int_0^{hd} zn(h)\,dh$, $Q_r^* = \int_0^{hd} (\alpha/1+\alpha)[(1-s)\,rkf(h)/b^*r^*]\, n(h)\,dh / \int_0^{hd} zn(h)\,dh$, $Q_r = \int_0^{h^*d} (\alpha/1+\alpha)[(1-s^*)\,r^*k^*f^*/b^*r^*]\, n^*(h)\,dh / ♀ z^*n^*(h)\,dh$, $Q_s = (s/1-s)\,\square\,(\alpha/1+\alpha)(I^d/b^*r^*)\, n(h)\,dh / \int_0^{hd} zn(h)\,dh$, $Q_o^* = \int_0^{h^*d} (\alpha/1+\alpha)(a^*/b^*r^*)\, n^*(h)\,dh / \int_0^{h^*d} z^*n^*(h)\,dh$, $Q_s^* = (s^*/1-s^*)\int_0^{h^*d} (\alpha/1+\alpha)(I_d^*/b^*r^*)\, n^*(h)\,dh / \int_0^{h^*d} z^*n^*(h)\,dh$, $v = un(h_d)\,h_d / [un(h_d) + (1-u)\,n^*(h_d^*)\,h_d^*]$, $v^* = (1-u)\,n^*(h_d^*)\,h_d^* / [un(h_d)\,h_d + (1-u)\,n^*(h_d^*)\,h_d^*]$, $d_1 = z^+n(h_d)\,h_d / \int_{hd}^{1} zn(h)\,dh$, $d_1^* = z^+n^*(h_d^*)\,h_d^* / \int_{h^*d}^{1} z^*n^*(h)\,dh$, $d_2^* = z^-n(h_d)\,h_d / \int_0^{hd} zn(h)\,dh$, $d_2^* = z^-n^*(h_d^*)\,h_d^* / \int_0^{h^*d} z^*n^*(h)\,dh$

Appendix 2. Equilibrium in Factor Markets Implies Equilibria in Both Commodity Markets and Balance of Trade.

First show that the equilibrium in factor markets implies equilibrium in commodity markets. Note that $p(z) = aw + brz$ for $h_d \geqq I_o^d$ and $p^*(z^*) = aw + brz^*$ for $I^{*d} \geqq I_o^d$. Multiplying w to equation (3. 2) and r to equation (3. 4) respectively and rearranging them, we have equilibrium in differentiated goods of high quality, equation (3. 7). Similarly, note that $p(z) = a^* + b^*r^*z$ for $I^d < I_o^d$ and $p^*(z^*) = a^* + b + r^*z^*$ for $I^{*d} < I_o^d$.

Then multiplying r^* to equation (3. 6) and adding it to equation (3. 5), we have equilibrium in differentiated goods of low quality.

(A 1) $u \int_0^{h^d} p(z)\, n(h)\, dh + (1-u) \int_0^{h^{*d}} p^*(z^*)\, n^*(h)\, dh + (1-u)\, ly =$
$(1-u)(1 - ly^* + r^*k^*)$

Adding equation (3. 7) to equation (A 1), we have:

$u \int_0^1 p(z)\, n(h)\, dh + (1-u) \int_0^1 p^*(z^*)\, n^*(h)\, dh +$
$(1-u)\, ly = u\,(rk + w) + (1-u)(r^*k^* + 1)$

Now individual budget constraints yield the following relations:

$u\ [(1-s) \int_0^1 In(h)\, dh - \int_0^1 p(z)\, n(h)\, dh - \int_0^1 yn(h)\, dh] = 0$, and
$(1-u)\ [(1-s^*) \int_0^1 I^*n^*(h)\, dh -$
$\int_0^1 p^*(z^*)\, n^*(h)\, dh - \int_0^1 y^*n^*(h)\, dh] = 0$

Using these relations, we have an equilibrium condition expressed by equation (3. 9). Similarly, it is easily shown that substitution of individual budget constraints into equation (3. 7) yields the balance of trade being equilibrated.

Appendix 3

(1) Effects of Change in k on r and r*
We can compute:

$$\frac{\hat{r}}{\hat{k}} = \frac{1}{\Delta}\begin{vmatrix} b_{11} & a_{12} \\ b_{21} & a_{22} \end{vmatrix} = -1 + \frac{1}{\Delta}\begin{vmatrix} b_{11} + a_{11} & a_{12} \\ b_{21} + a_{21} & a_{22} \end{vmatrix}$$

$$\frac{\hat{r}}{\hat{k}^*} = \frac{1}{\Delta}\begin{vmatrix} b_{12} & a_{12} \\ b_{21} & a_{22} \end{vmatrix} = -\frac{1}{\Delta}\begin{vmatrix} b_{11} + a_{11} + a_{12} & a_{12} \\ b_{21} + a_{21} + a_{22} & a_{22} \end{vmatrix}$$

where $\Delta = | a_{ij}$. Therefore
$(\hat{r}/\hat{k}) + (\hat{r}/\hat{k}^*) = -1.$
Furthermore,

$$\frac{\hat{r}}{\hat{k}} = \frac{1}{\Delta}\begin{vmatrix} b_{12} + a_{12} & a_{12} \\ b_{21} + a_{22} & a_{22} \end{vmatrix}$$

It is easily computed:

$b_{12} + a_{12} = a_1\ [(t_k d_1/\varepsilon) + (t_k^* d_1^*/\varepsilon_1^*] > 0,$ and
$b_{21} + a_{22} = a_1\ [(t_k d_2/\varepsilon_1) + (t_k^* d_1^*/\varepsilon_1^*)] > 0.$

Under a normal situation where $a_{22} < 0$, $\hat{r}/\hat{k}^* < 0$.
Therefore $\hat{r}/\hat{k} > -1$. Now $\hat{r}/\hat{k} = [a_{22}(b_{11} - a_{12}) - a_{12}(b_{21} - a_{22})]$ Note:

$$b_{11} - a_{12} = 1 - t_k q_r - t_k{}^* q_r{}^* - t_k d_1 (a_1 + s_k)/\varepsilon_l - t_k{}^* d_1{}^* (a_1 + s_k{}^*)/\varepsilon_l{}^*,$$

and

$$b_{21} - a_{22} = [1 - T_k Q_r - T_k{}^* Q_r{}^* - T_k d_2 (a_1 + s_k)/\varepsilon_l - T_k{}^* d_2{}^* (a_1 + s_k)/\varepsilon_l{}^*$$

When both $b_{11} - a_{12} > 0$ and $b_{21} - a_{22} > 0$, then $r/k < 0$.
However, when d_i's and $d_i{}^*$'s are not negligible, it is difficult to assume $b_{11} - a_{12} > 0$ and $b_{21} - a_{22} > 0$.

(2) Effects of Change in k* on r and r*

$$\frac{\hat{r}^*}{\hat{k}^*} = \frac{1}{\Delta}\begin{vmatrix} a_{11} & b_{12} \\ a_{21} & b_{22} \end{vmatrix} = -1 + \frac{1}{\Delta}\begin{vmatrix} a_{11} & b_{12} + a_{12} \\ a_{21} & b_{22} + a_{22} \end{vmatrix}$$
$$= -1 - (\hat{r}^*/\hat{k}).$$

Note that $b_{12} + a_{12} = a_1\ [(t_k d_1/\varepsilon_l) + (t_k{}^* d_1{}^*/\varepsilon_l{}^*] > 0$ and $b_{22} + a_{22} = a_1\ [(T_k d_2/\varepsilon_l) + (T_k{}^* d_2{}^*/\varepsilon_l{}^*)] > 0$. If $a_{11} < 0$ and $a_{21} > 0$, then $\hat{r}^*/\hat{k} > 0$, implying $\hat{r}^*/\hat{k}^* < 0$.

Appendix 4. Short-Run Comparative Statics

First consider the determination of I_o^d, which satisfies $V(r, w, I_o^d) = V(r^*, w^*, I_o^d)$. Solving this relation gives the value of I_o^d.

$$I_o^d = awt + a^* (1 - t) = I_o^d (w, r, r^* ; a, a^*, b, b^*)$$

Using hat notation, we can compute :

$$\hat{I}_o^d = a_2\hat{w} + a_1 (\hat{r} - \hat{r}^*) + a_1 (\hat{b} - \hat{b}^*) + a_2\hat{a} + (1 - a_2)\ \hat{a}^*.$$

Once I_o^d is determined, so is income class h_d, satisfying
$I_o^d = (1 - s)\ [wl\ (h_d) + rkf\ (h_d)]$. Similar computation with hat notation will finally yield equation (A 3).

$$\text{(A 3)}\quad \hat{h}_d = [(a_1 - s_k)/\varepsilon_l]\ \hat{r} - (a_1/\varepsilon_l)\ \hat{r}^* + [(a_2 - s_k)/\varepsilon_l]\ \hat{w} - (s_k/\varepsilon_l)\ \hat{k} + [(s/1 - s)(1/\varepsilon_l)]\ \hat{s} + (a_1/\varepsilon_l)(\hat{b} - \hat{b}^*) + (a_2/\varepsilon_l)\ \hat{a} + [(1 - a_2)/\varepsilon_l]\ \hat{a}^*$$

Note that $1 - a_2 < 0$. Similarly, $h_d{}^*$ is determined, satisfying $I_o^d = (1 - s^*)\ [l^* (h_d{}^*) + r^* k^* f^* (h_d{}^*)]$. Therefore, we have equation (A 4).

$$\text{(A 4)}\quad \hat{h}_d{}^* = (a_1/\varepsilon_l{}^*)\ \hat{r} - [(a_1 + s_k{}^*)/\varepsilon_l{}^*]\ \hat{r}^* + (a_2/\varepsilon_l{}^*)\ \hat{w} - (s_k{}^*/\varepsilon_l{}^*)\ \hat{k}^* + [(s^*/1 - s^*)(1/\varepsilon_l{}^*)]\ \hat{s}^* + (a_1/\varepsilon_l{}^*)(\hat{b} - \hat{b}^*) + (a_2/\varepsilon_l{}^*)\ \hat{a} + [(1 - a_2)/\varepsilon_l{}^*]\ \hat{a}^*$$

Totally differentiating (4. 1) and using hat notation, we have :

(A 5) $u\hat{a} - un\ (h_d)\ h_d\hat{h_d} - (1 - u)\ n^*\ (h_d^*)\ h_d^*\hat{h_d^*} = 0$

Substituting (A 3) and (A 4) into (A 5) gives equation (A 6).

(A 6) $m_{11}\hat{w} + m_{12}\hat{r} + m_{13}\hat{r}^* = -(vs_k/\varepsilon_l)\ \hat{k} - (v^*s_k^*/\varepsilon_l^*)\ \hat{k}^* + [(s/1 - s)(v/\varepsilon_l)]\ \hat{s} + [(s^*/1 - s^*)(v^*/\varepsilon_l^*)]\ \hat{s}^* + a_1\ [(v/\varepsilon_l) + (v^*/\varepsilon_l^*)]\ (\hat{b} - \hat{b}^*) - \{v' - a_2\ [(v/\varepsilon_l) + (v^*/\varepsilon_l^*)]\ \ I_o^d\ \hat{a} - (a_2 - 1)\ [(v/\varepsilon_l) + (v^*/\varepsilon_l^*)]\ \hat{a}^*.$

Totally differentiating equation (4. 2) and using hat notation, we have:

$$\hat{k} = -\hat{r} + t_kq_w\hat{w} - t_kq_o\hat{a} + t_kq_r\ (\hat{r} + \hat{k}) - t_k^*q_o^*\hat{w} - t_k^*q_o^*\hat{a}^* + t_k^*q_r^*\ (\hat{r} + \hat{k}^*) - t_kq_s\hat{s} - t_k^*q_s^*\hat{s}^* - t_kd_1\hat{h_d} - t_k^*d_1^*\hat{h_d}^*.$$

Substituting equations (A 3) and (A 4) into the above equation and rearranging it, we have equation (A 7).

(A 7) $m_{21}\hat{w} + m_{22}\hat{r} + m_{23}\hat{r}^* = [1 - t_kq_r - (t_kd_1s_k/\varepsilon_l)]\ \hat{k} - [t_k^*q_r + (t_k^*d_1^*s_k^*/\varepsilon_l^*)]\ \hat{k}^* + [t_k^*q_s^* + (t_k^*d_1^*/\varepsilon_l^*)(s^*/1 - s^*)]\ \hat{s}^* + [t_kq_s + (t_kd_1/\varepsilon_l)(s/1 - s)]\ \hat{s} + a_1\ [(t_kd_1/\varepsilon_l) + (t_k^*d_1^*/\varepsilon_l^*)]\ (\hat{b} - \hat{b}^*) + \{t_kq_o + t_kq_o^* + a_2\ [(t_kd_1/\varepsilon_l) + (t_k^*d_1^*/\varepsilon_l^*)]\ \hat{a} + (1 - a_2)\ [(t_kd_1/\varepsilon_l) + (t_k^*d_1^*/\varepsilon_l^*)]\ \hat{a}^*.$

Finally totally differentiating equation (4. 4), we have:

(A 8) $m_{31}\hat{w} + m_{32}\hat{r} + m_{33}\hat{r}^* = -[T_kQ_r + (T_kd_2s_k/\varepsilon_l)]\ \hat{k} + [1 - T_k^*Q_r^* - (T_k^*d_2^*s_k^*/\varepsilon_l^*)]\ \hat{k}^* + [T_kQ_s + T_kd_2\ (s/1 - s)(1/\varepsilon_l)]\ \hat{s} + [T_k^*Q_s^* + T_k^*d_2^*\ (s^*/1 - s^*)(1/\varepsilon_l^*)]\ \hat{s}^* + a_1\ [(T_kd_2/\varepsilon_l) + (T_k^*d_2^*/\varepsilon_l^*)]\ (\hat{b} - \hat{b}^*) + [a_2\ [(T_kd_1/\varepsilon_l) + (T_k^*d_1^*/\varepsilon_l^*)]\ \hat{a} + \{T_kQ_o + T_k^*Q_o^* - (a_2 - 1)\ [(T_kd_2/\varepsilon_l) + (T_k^*d_2^*/\varepsilon_l^*)]\}\ \hat{a}^*.$

These equations form the matrix system (M) given in this paper. We consider the following Walrasian adjustment in the factor markets.

(A 1) $w = \lambda_w\ \{a\ [u \int n\ (h)\ dh + (1 - u) \int n^*\ (h)\ dh] - u\}$

(A 2) $r = \lambda_r\ \{[u \int zn\ (h)\ dh + (1 - u) \int z^*n^*\ (h)\ dh] - uk\}$

(A 3) $r^* = \lambda_r^*\ \{b^*\ [u \int zn\ (h)\ dh + (1 - u) \int z^*n^*\ (h)\ dh] - (1 - u)\ k^*\}$

We have assumed rather strong sufficient conditions for stability given in this paper, (S 1), (S 2), and (S 3).

(1) Labor-Saving Technical Progress ($\gamma = a^*$)

In this case,

$c_1 = -(a_2 - 1)\ [v/\varepsilon_1 + x^*/\varepsilon_1^*] < 0$, $c_2 = -(a_2 - 1)\ [t_k d_1/\varepsilon_1 + t_k^* d_1^*/\varepsilon_1^*] < 0$, and $c_3 = T_k Q_0 + T_k^* Q_0^* - (a_2 - 1)\ [t_k d_2/\varepsilon_1 + t_k^* d_2^*/\varepsilon_1^*]$

The first two terms in the elements of C_3 present the income effect accrued by a change in a*, because its decline increases disposable income belonging to income class $h \in [0, h_d) \cup [0, h_d^*)$. The last term in c_3 shows the income effect caused by changes in both h_d and h_d^*. Here we assume $c_3 < 0$.
First, consider the effect of a change in a* on w.

$$\hat{w}^*/\hat{a}^* = \Delta_1/S_1$$

where

$$\Delta_1 = \begin{vmatrix} c_1 & m_{12} & m_{13} \\ c_2 & m_{22} & m_{23} \\ c_3 & m_{32} & m_{33} \end{vmatrix}$$

Note that

$$\Delta_1 = S_{23} + \begin{vmatrix} 0 & m_{12} + m_{13} & m_{13} \\ c_2 & m_{22} + m_{23} & m_{23} \\ c_3 & m_{32} + m_{33} & m_{33} \end{vmatrix}$$

From stability condition (S 2), the first term in the RHS of Δ_1 is negative. The second term expressed by the determinant is expanded as follows:

$$c_3 m_{23}\ (m_{12} + m_{13}) + c_2 m_{13}\ (m_{32} + m_{33}) - c_3 m_{13}\ (m_{22} + m_{23}) - c_2 m_{33}\ (m_{12}\ m_{13})$$

In the above, only one term among the four takes a positive value. We assume here $\Delta_1 < 0$. That is, $\hat{w}/\hat{a}^* < 0$. Next, consider the effect of a change in a* on r.

$$\hat{r}/\hat{a}^* = \Delta_2/S_1$$

where

$$\Delta_2 = c_2 S_{13} + \begin{vmatrix} m_{11} & c_1 & m_{13} \\ m_{21} & 0 & m_{23} \\ m_{31} & c_3 & m_{33} \end{vmatrix} < 0.$$

Therefore, $\hat{r}/\hat{a}^* > 0$.
Finally, consider the effect on r*.

$$\hat{r}^*/\hat{a}^* = \Delta_3/S_1$$

where

$$\Delta_3 = \begin{vmatrix} m_{11} & m_{12} & c_1 \\ m_{21} & m_{22} & c_2 \\ m_{31} & m_{23} & c_3 \end{vmatrix} = \begin{vmatrix} m_{11} & m_{12} & c_1 - m_{11} \\ m_{21} & m_{22} & c_2 - m_{21} \\ m_{31} & m_{32} & c_3 - m_{31} \end{vmatrix}$$

Note that $c_1 - m_{11} > 0$ and $c_3 - m_{31} > 0$. Expanding Δ_3 will yield nine terms, eight of which take a positive value, irrespective of the sign of $c_2 - m_{21}$. Therefore, we assume $\Delta_3 > 0$. That is, $\hat{r}^*/\hat{a}^* < 0$.

(2) Capital-Saving Technical Progress ($\gamma = b^*$)

In this case,

$$c_1 = -a_1 (v/\varepsilon_1 + v^*/\varepsilon_1^*),\ c_2 = -a_1 (t_k d_1/\varepsilon_1 + t_k^* d_1^*/\varepsilon_1^*), \text{ and}$$
$$c_3 = -a_1 (T_k d_2/\varepsilon_1 + T_k d_2^*/\varepsilon_1^*)$$

First, consider the effect of a change in b^* on w.

$$\hat{w}/\hat{b}^* = \Delta_4/S_1$$

where

$$\Delta_4 = \begin{vmatrix} c_1 & m_{12} & m_{13} \\ c_2 & m_{22} & m_{23} \\ c_3 & m_{32} & m_{33} \end{vmatrix} = \begin{vmatrix} c_1 & m_{12} - c_1 & m_{13} + c_1 \\ c_2 & m_{22} - c_2 & m_{23} + c_2 \\ c_3 & m_{32} - c_3 & m_{33} + c_3 \end{vmatrix}$$

Looking at a sign pattern in each element of Δ_4, we shall find under reasonable conditions that

$$\Delta_4 = \begin{vmatrix} - & + & + \\ - & - & + \\ - & + & - \end{vmatrix}$$

Expansion of Δ_4 would give eight positive terms and one negative term. We assume $\Delta_4 < 0$. Therefore, $w/b^* > 0$. Next, consider the effect of a change in b^* on r.

$$\hat{r}/\hat{b}^* > 0,$$

because

$$\Delta_5 = \begin{vmatrix} m_{11} & c_1 & m_{13} \\ m_{21} & c_2 & m_{23} \\ m_{31} & c_3 & m_{33} \end{vmatrix} = c_3 S_{12} + \begin{vmatrix} m_{11} & m_{12} - c_1 & c_1 \\ m_{21} & m_{22} - c_2 & c_2 \\ m_{31} & m_{32} - c_3 & 0 \end{vmatrix}$$

(3) Capital Stock in the North ($\gamma = k$)

In this case,

$$c_1 = -vs_k/\varepsilon_1,\ c_2 = 1 - t_k q_r - t_k d_1 s_k/\varepsilon_1, \text{ and } c_3 = -\left[T_k Q_r + T_k d_2 s_k/\varepsilon_1\right].$$

First, consider the effect of a change in k on w.

$$\hat{w}/\hat{k} = \Delta_7/S_1$$

where

$$\Delta_7 = \begin{vmatrix} c_1 & m_{12} & m_{13} \\ c_2 & m_{22} & m_{23} \\ c_3 & m_{32} & m_{33} \end{vmatrix} = c_1 s_{23} + \begin{vmatrix} 0 & m_{12} + m_{13} & m_{13} \\ c_2 & m_{22} + m_{23} & m_{23} \\ c_3 & m_{32} + m_{33} & m_{33} \end{vmatrix}$$

The first term of elements in Δ_7 is negative from using stability condition S 2. The second term would probably take a negative sign, since $m_{12} + m_{13} > 0$, $m_{22} + m_{23} < 0$ and $m_{32} + m_{33} < 0$. Therefore, $\hat{w}/\hat{k} > 0$.

Next, consider the effect of a change in k on r.

$$\hat{r}/\hat{k} = \Delta_8/S_1$$

where

$$\Delta_8 = \begin{vmatrix} m_{11} & c_1 & m_{13} \\ m_{21} & c_2 & m_{23} \\ m_{31} & c_3 & m_{33} \end{vmatrix} = c_2 S_{13} + \begin{vmatrix} m_{11} & c_1 & m_{13} \\ m_{21} & 0 & m_{23} \\ m_{31} & c_3 & m_{33} \end{vmatrix}$$

The first term in the elements of Δ_8 takes a positive value and the second, a negative value. The latter comes from indirect income effects. While we can not *a priori* determine the sign of r/k, we assume r/k $<$ 0 as a proper case. Note that

r/k, we assume $\hat{r}/\hat{k} < 0$ as a proper case. Note that

$$\Delta_8 = -S + \begin{vmatrix} m_{11} & c_1 + m_{12} & m_{13} \\ m_{21} & c_2 + m_{22} & m_{23} \\ m_{31} & c_3 + m_{32} & m_{33} \end{vmatrix} = -S + \Delta_8$$

We know that $c_1 + m_{12} < 0$, $c_2 + m_{22} < 0$, and $c_3 + m_{32} < 0$. Using stability condition (S 2), we can show that $\Delta_8' < 0$. Therefore r/k > -1.

Finally, consider the effect of a change in k on r*.

$$\hat{r}^*/\hat{k} = \Delta_9/S > 0,$$

where

$$\Delta_9 = \begin{vmatrix} m_{11} & m_{21} & c_1 \\ m_{21} & m_{22} & c_2 \\ m_{31} & m_{32} & c_3 \end{vmatrix} = c_3 S_{12} + \begin{vmatrix} m_{11} & m_{21} + c_1 & c_1 \\ m_{21} & m_{22} + c_2 & c_2 \\ m_{31} & m_{32} & c_3 \end{vmatrix}$$

which would take a negative value.

(4) Capital Stock in the South ($\gamma = k^*$)
As for the effect of a change in k^* on w, we first note that

$$\hat{w}/\hat{k} = -\hat{w}/\hat{k} < 0$$

Next, consider the effect of a change in k^* on r. We can easily find the following relation to hold:

$$(\hat{r}/\hat{k}) + (\hat{r}/\hat{k}^*) = -1$$

Therefore, under a normal situation where $r/k < 0$, we have: $-1 < r/k^* < 0$. Finally, consider the following relation:

$$(\hat{r}^*/\hat{k}^*) + (\hat{r}^*/\hat{k}) = -1$$

However, since it was shown that $\hat{r}^*/\hat{k} > 0$, $\hat{r}^*/\hat{k}^* < -1$.

(5) Saving Ratio ($\gamma = s^*$)
In this case,

$$c_1 = (s^*/1 - s^*)(v^*/\varepsilon_1^*) > 0,\ c_2 = t_k{}^*q_s{}^* + (s^*/1 - s^*)(t_k{}^*d_1{}^*/\varepsilon_1{}^*) > 0, \text{ and}$$
$$c_3 = T_k{}^*Q_s{}^* + (s^*/1 - s^*)(T_k{}^*d_2{}^*/\varepsilon_1{}^*) > 0.$$

We can compute the following:

$$\hat{w}/\hat{s}^* = \Delta_{10}/S,\ \hat{r}/\hat{s}^* = \Delta_{11}/S,\ \hat{r}^*/\hat{s}^* = \Delta_{12}/S,$$

where

$$\Delta_{10} = c_1S_{23} + \begin{vmatrix} 0 & m_{12} + m_{13} & m_{13} \\ c_2 & m_{22} + m_{23} & m_{23} \\ c_3 & m_{32} + m_{33} & m_{33} \end{vmatrix},$$

$$\Delta_{11} = c_2S_{13} + \begin{vmatrix} m_{11} & c_1 & m_{13} \\ m_{21} & 0 & m_{23} \\ m_{31} & c_3 & m_{33} \end{vmatrix} > 0, \text{ and } \Delta_{12} = \begin{vmatrix} m_{11} & m_{12} & c_1 \\ m_{21} & m_{22} & c_2 \\ m_{31} & m_{32} & c_3 \end{vmatrix}$$

In case of Δ_{10}, expansion of the second term shows that three among the four terms are positive. Similarly, in case of Δ_{12}, its expansion reveals eight among the nine terms are positive. Therefore, we assume both Δ_{10} and Δ_{12} positive. This results in:

$$\hat{w}/\hat{s}^* < 0,\ \hat{r}/\hat{s}^*,0, \text{ and } \hat{r}^*/\hat{s}^* < 0.$$

(6) Labor-Saving Technical Progres in the North ($\gamma = a$)
In this case,

$$c_1 = a_2 \ [(v/\varepsilon_1) + (v^*/\varepsilon_1^*)] \ - u/\ [un(h_d)h_d + (1 - u)n^*(h_d^*)h_d^*],$$
$$c_2 = t_k q_0 + t_k^* q_0^* + a_2 \ [(t_k d_1/\varepsilon_1) + (t_k^* d_1^*/\varepsilon_1^*)] \text{ , and}$$
$$c_3 = a_2 \ [(T_k d_1/\varepsilon_1) + (T_k^* d_1^*/\varepsilon_1^*)] \ .$$

Let us consider the effect of a change in a on factor prices.

$$\hat{w}/\hat{a} = \Delta_{13}/S,\ \hat{r}/\hat{a} = \Delta_{14}/S, \text{ and } \hat{r}^*/\hat{a} = \Delta_{15}/S,$$

where

$$\Delta_{13} = c_1 S_{23} + \begin{vmatrix} 0 & m_{12} & m_{13} \\ c_2 & m_{22} & m_{23} \\ c_3 & m_{32} & m_{33} \end{vmatrix},\ \Delta_{14} = c_2 S_{13} + \begin{vmatrix} m_{11} & c_1 & m_{13} \\ m_{21} & 0 & m_{23} \\ m_{31} & c_3 & m_{33} \end{vmatrix},$$

$$\text{and } \Delta_{15} = \begin{vmatrix} m_{11} & m_{12} & c_1 \\ m_{21} & m_{22} & c_2 \\ m_{31} & m_{32} & c_3 \end{vmatrix} = \begin{vmatrix} - & - & + \\ + & - & + \\ + & + & + \end{vmatrix}$$

Suppose that an increase in labor productivity initially reduces the demand for labor, which tends to depress the wage rate. That is, the sign of c_1 is positive, because c_1 shows an excess supply of labor in the North, due to productivity improvement. While the direct effect of the rise in labor productivity tends to reduce the wage rate, the indirect effect, through an income increase in terms of $(I^d - aw)$, pushes up the demand for labor and tends to increase the wage rate. Thus, net effect of the change in a on w would be ambiguous. However, using a stability condition and signs of m_{ij} and c_j, we could probably predict that an increase in labor productivity would result in raising both Northern and Southern rates of interest.

(7) Capital-Saving Technical Progress(γ = b)

In this case,

$$c_1 = a_1 \ [(v/\varepsilon_1) + (v^*/\varepsilon_1^*)] \ ,\ c_2 = a_1 \ [(t\downarrow k d_1/\varepsilon_1) + (t\downarrow k^* d_1^*/\varepsilon_1^*)] \ ,$$
$$\text{and } c_3 = a_1 \ [(T\downarrow k d_2/\varepsilon_1) + (T\downarrow k^* d_2^*/\varepsilon_1^*)] \ .$$

We can compute the following :

$$\hat{w}/\hat{b} = \Delta_{16}/S,\ r/b = \Delta_{17}/S,\ r^*/b = \Delta_{18}/S,$$

where

$$\Delta_{16} = c_1 S_{23} + \begin{vmatrix} 0 & m_{12} & m_{13} \\ c_2 & m_{22} & m_{23} \\ c_3 & m_{23} & m_{33} \end{vmatrix} > 0,$$

$$\Delta_{17} = c_2 S_{13} + \begin{vmatrix} m_{11} & c_1 & m_{13} \\ m_{21} & 0 & m_{23} \\ m_{31} & c_3 & m_{33} \end{vmatrix}, \text{ and}$$

$$\Delta_{18} = c_3 S_{12} + \begin{vmatrix} m_{11} & m_{11} + c_1 & c_1 \\ m_{21} & m_{22} + c_2 & c_2 \\ m_{31} & m_{32} & 0 \end{vmatrix} > 0.$$

Capital-saving technical progress in the North decreases factor prices of production.

Footnotes

(1) Let L and N respectively stand for effective labor supply and population size. In this paper, assume $L = N$.

(2) This is a special case of Stone-Geary type utility function given in Falvey-Kierzkowski.

(3) The $\dot{k} = 0$ line kinks downward at $k = k^*$, because of the existence of w (k, k*).

(4) The envelope theorem ensures the following results:

$$\frac{dv}{da^*} = \begin{cases} -\lambda_h & h \in [0, h_d] \\ 0 & h \in [h_d, 1] \end{cases}$$

$$\frac{dv^*}{da^*} = \begin{cases} -\lambda^* & h \in [0, h_d^*] \\ 0 & h \in [h_d, 1] \end{cases}$$

$$\frac{dv}{db^*} = \begin{cases} -\lambda_h r^* & h \in [0, h_d] \\ 0 & h \in [h_d^*, 1] \end{cases}$$

$$\frac{dv^*}{db^*} = \begin{cases} -\lambda_h^* r^* & h \in [0, h_d^*] \\ 0 & h \in [h^{d*}, 1] \end{cases}$$

$$\frac{dv}{ds} = -\lambda_h I_h \quad h \in [0, 1]$$

$$dv^*/ds = 0$$

$$\frac{dv^*}{ds^*} = -\lambda_h^* I_h^* \quad h \in [0, 1]$$

$$dv/ds^* = 0$$

$$\frac{dv^*}{dk^*} = \lambda_h^* (1 - s^*) r^* f^* (h) \quad h \in [0, 1]$$

$$dv/dk^* = 0$$

where λ_h and $\lambda_h{}^*$ respectively show the inverse of marginal utility of income in both North and South.

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