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Optimum Tariffs in Vertical Product Differentiation

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Since H. G. Grubel and P. J. Lloyd (1975) have emphasized the importance of intra-industry trade, significant contributions have been made both theoretically and empirically. Until recently, however, much interest has focused on horizontal product differentiation (see E. Helpman (1981), P. Krugman (1980), and C. Lawrence and P. T. Spiller (1983), for instance). Following Linder's idea, both H. Flam and E. Helpman (1987a) and R. E. Falvey and H. Kierzkowski (1987) presented models of vertical product differentiation, where income distribution plays an essential role. In the Flam-Helpman model (FH, model, for short), they employ the Hotelling's assumption, that is, each consumer, whose preference is of the same type, purchases one unit of differentiated products. This suggests that consumers belonging to a high (low) income class, which is defined later, will demand a high (low, correspondingly) quality product. The FH model explains North-South trade and the phenomenon of product cycles advocated by R. Vernon. When we apply the FH model to investigate a case of optimum tariffs, we face a difficult problem of choosing an index of social welfare for the country concerned. As indicated in FH, there exists a continuum of consumers on the interval of income class $[0, 1]$, over which the distribution of their skills is reflected by effective units of labor and the distribution of their population is defined. It has been shown in FH that which income class a particular consumer belongs to determines what quality of a differentiated product he chooses. In our model as well as the FH model, there are basically three kinds of products available: homogeneous products, low quality (differentiated) products, and high quality (differentiated) products. Therefore, if we succeed in treating the aggregate of consumers as a representative consumer, he either consumed only one unit of low or high quality products, together with homogeneous products. This means, in a simplified way to answer an optimum tariff analysis, that first we should solve the aggregation problem, so that social welfare functions of both the North and the South involve utilities of two representative consumers, respectively belonging to low and high income classes. In this paper, we assume that each consumer has the same preference revealed by the Cobb-Douglas type utility function, which satisfies the Gorman polar form (see H. Varian (1984)). Therefore, according to income levels of consumers either high or low, we select two types of representative consumers, whose marginal utilities of income are equivalent among consumers within the same income classes. Using this, we form implicit social welfare func-

tions, which enable us to derive optimum tariff formulas.

Some of the main results are noteworthy. While optimum tariff formulas in the case of vertical product differentiation are closely related to the traditional formula, a major difference is associated with the change-of-commodity effect, which does not appear in the traditional one (see W. Novshek and H. Sonnenschein (1979)). It will be shown that this effect is amplified by a technological difference between the North and the South.

This paper consists of the following five sections. Section 1 presents the basic framework of our model under free trade, most of which is based on the FH model. Section 2 modifies it to the one involving tariffs. Section 3 investigates comparative static analysis. By using results obtained in Section 3, Section 4 considers welfare effects of tariff and derives optimum tariff formulas for both the North and the South. Concluding remarks are briefly stated in Section 5. Most of our computational results are dealt with in the Appendixes.

1. A Basic Model (Free Trade Case)

Assume Ricardian technology, where labor is the only factor of production. There are two different kinds of commodities in our model: homogeneous goods and differentiated goods. While the same technology is available for producing homogeneous products, the North and the South respectively employ different technologies to produce differentiated products. The North is assumed to possess a comparative cost advantage for production of high quality products, while the South, for production of low quality products. Equations (1. 1) and (1. 2) respectively express unit cost functions for the North and the South.

(1. 1) $c(z) = (az + b)w.$

(1. 2) $c^*(z) = (a^*z + b^*)w^*,$

where z, w, and w* respectively stand for quality, for northern and for southern wage rates. Asterisks are attached to southern variables. Underlying production technologies implicit in equations (1. 1) and (1. 2) are of the Leontief type. Equation (1. 1), for instance, means that production of one unit of differentiated products with quality z uniquely corresponds to $az + b$ units of effective labor.[1] It also implies that in order to realize quality z, the amount of "fixed cost", bw, is required.[2] Because of cost advantage in each country, we assume:

(C1) $bw > b^*w^*$ and $aw < a^*w^*$.

Figure 1 demonstrates comparative cost advantages in the north and the South.[3] Under a competitive situation, $p(z) = c(z)$ and $p^*(z) = c^*(z)$, where $p(z)$ denotes the price of a commodity of quality z. As assumed in FH (1987a), we take for granted

the case where $w > w^*$. Since the same technology is available for producing homogeneous goods both in the North and the South, free trade means that the North does not produce any homogeneous goods and only imports them from the South. Let us measure homogeneous goods in terms of effective labor units and normalize their price as $p_y = w^* = 1$, where p_y stands for the price of homogeneous goods. Since most analyses are argued parallel with each other in both countries, we mainly concentrate our analysis on the northern case.

Each worker belongs to a particular income class $h \varepsilon [0, 1]$, which reflects a level of his skill. Over the interval of distance [0, 1], both distributions of effective labor and population are defined. They are respectively denoted by f(h) and n(h) as those defined by FH. Equation (1. 3) supplies a utility function of the consumer in income class h.

(1. 3) $u_h = y_h^{\alpha} z_h^{1-\alpha}$.

It is well-known that this type of direct utility function yields the same type of indirect utility function, which satisfies the Gorman polar form (see H. Varian (1984)). Since each consumer purchases one unit of differentiated products (the Hotelling's assumption), there are two kinds of consumers in essence: those consumers who demand differentiated goods produced either in the South or in the North. Consider a respresentative consumer who buys differentiated goods produced in the South. (From the reason stated below, we call them low quality products.) Equation (1. 4) shows his budget constraint.

(1. 4) $I_L^p = Y_L + p_z^* Z_L$,

where $I_L^p = \int_0^{hd} (I_h - b^*) Nn(h)\, dh$, $Y_L = \int_0^{hd} y_h Nn(h)\, dh$, $Z_L = \int_0^{hd} z_h Nn(h)\, dh$

and $p_z^* = a^*$. h_d corresponds to a critical level of income class and its determination is demonstrated below. When the representative consumer buys homogeneous goods Y_L and quality Z_L, his disposable income quals I_L^p. The price of quality z under our production technology is equalized to $p_z^* = a^*$.

Equation (1. 5) shows the indirect utility function for the representative consumer over the interval of income class $[0, h_d]$.

(1. 5) $V_L(p_z^*, I_L^p) = v^L I_L^p$,

where $v^L = \alpha^{\alpha}(1-\alpha)^{1-\alpha} p_z^{*-(1-\alpha)}$, $I_L^p = \int_0^{hd} I_h^p Nn(h)\, dh$ and $I_h^p = I_h - b^*$.

Similarly, consider a representative consumer who purchases high quality products made in the North. Equations (1. 6) and (1. 7) respectively show his budget constraint and indirect utility function.

(1. 6) $I_H^D = Y_H + p_z Z_H$.

(1. 7) $V_H(p_z, I_H^D) = v^H I_H^D$,

where $\int_{hd}^{1}$

$[I_h - bw]Nn(h)dh$, $Y_H = \int_{hd}^{1} y_h Nn(h)\,dh$, $Z_H = \int_{hd}^{1} z_h Nn(h)dh$,

$p_z = aw$, and $v^H = \alpha^{\alpha}(1-\alpha)^{1-\alpha} p_z^{-(1-\alpha)}$.

Now there exist marginal consumers at income class h_d, who are indifferent to purchase either low quality goods or high quality goods. As shown in FH, the amount of wage earnings at income class h_d is defined by equation (1. 8).

(1. 8) $I_d = wLf(h_d) / Nn(h_d)$.

With this income level, equation (1. 9) determines the critical level of h_d.

(1. 9) $V^L[I_d - b^*] = V^H[I_d - bw]$.

Finally, let us consider the equilibrium condition in the northern labor market.

(1. 10) $L = \int_{hd}^{1} a_L(Z_h)\,Nn(h)\,dh + \int_{hd*}^{1} a_L(Z_h)\,N^*n^*(h)\,dh$,

where $a_L(z_h)$ denotes input coefficient in producing one unit of differentiated goods with quality z_h. The first term and the second term in the RHS in euqation (1. 10) respectively show the aggregate demand for labor to meet domestic and foreign demands for high quality products. Substituting $a_L(z_h) = az_h + b$ into equation (1. 10), we derive equation (1. 11).[4]

(1. 11) $L = aZ_H + aZ_H^* + b\{\int_{hd}^{1} Nn(h)\,dh + \int_{hd*}^{1} N^*n^*(h)\,dh\}$.

A similar argument holds for the South. The only difference between the North and the South, except for equilibrium in the labor market, is to attach an asterisk notation to northern variables. Equation (1. 12) states equilibrium in the labor market of the South.

(1. 12) $L^* = L_y^* + a^*Z_L + a^*Z_L^* + b^*\{\int_0^{hd} Nn(h)\,dh + \int_0^{hd*} N^*n^*(h)\,dh\}$.

2. Tariff Imposition

Suppose that the North and the South respectively impose ad valorem tariff, t and t*, on imported differentiated goods. Let T denote the amount of per capita allocatioin of tariff revenues in the North. Without any notational confusion, we employ the same notation h_d as presenting a critical income class, whose determina-

tion is given below.

First, consider a representative consumer over the interval of income class $[0, h_d]$. Equation (2. 1) is his indirect marginal utility function.

$$(2.\ 1)\quad V_L = V^L I_L^D,$$

where $v^L = \alpha^\alpha (1-\alpha)^{1-\alpha} [(1+t)\, p_z^*]^{-(1-\alpha)}$, $I_h^D = I_h + T - (1+t)\, b^*$,

and $I_L^D = \int_0^{hd} I_h^D Nn\,(h)\, dh.$

Because of his Cobb-Douglas type preference, equations (2. 2) respectively show demands for homogeneous goods and low quality goods.

$$(2.\ 2)\quad \begin{aligned} Y_L &= \alpha\ I_L^D, \\ Z_L &= (1-\alpha)\, I_L^D/(1+t)\, p_z^*, \end{aligned}$$

where $p_z^* = a^*$.

Similarly, consider the behavior of a representative consumer over the interval of income class $[h_d, 1]$, who purchases high quality products. Equation (2. 3) indicates his indirect utility function.

$$(2.\ 3)\quad V_H = V^H I_H^D,$$

where $v^H = \alpha^\alpha (1-\alpha)^{1-\alpha} p_z^{-(1-\alpha)}$, $I_h^D = I_h + T - bw$, and

$I_H^D = \int_{hd}^1 IhDNn\,(h)\, dh.$

Correspondingly, demand functions are given by equations (2. 4).

$$(2.\ 4)\quad \begin{aligned} Y_H &= \alpha\ I_H^D, \\ Z_H &= (1-\alpha)\, I_H^D/p_z, \end{aligned}$$

where $p_z = aw$.

The value of h_d corresponds to a critical income level, with which marginal consumers are indifferent to purchase either low quality products or high quality products. That is,

$$(2.\ 5)\quad V^L[I_d + T - (1+t)\, b^*] = v^H\, [I_d + T - bw].$$

I_d is defined by equation (1. 8).

Aggregate tariff revenue, NT, is defined by equation (2. 6).

$$(2.\ 6)\quad NT = t\ I_M,$$

where

$$I_M = \int_0^{hd} p^*(z)\, Nn(h)\, dh = \text{northern imports.}$$

Substituting into equation (2. 6) $p^*(z) = P_z^* z + b^*$, we have equation (2. 7).

$$\text{(2. 7)} \quad I_M = p_z^* Z_L + b^* \int_0^{hd} Nn(h)\, dh.$$

Formal expression for equilibrium in the labor market is the same as given by equation (1. 11).

$$\text{(2. 8)} \quad L = aZ_H + aZ_H^* + b\left\{\int_{hd}^{1} Nn(h)\, dh + \int_{hd*}^{1} N^* n^*(h)\, dh\right\}.$$

Note, however, that values of both h_d and $h_d{}^*$ are changed by the imposition of tariff.

Now, examine the South. Equation (2. 9) describes the indirect utility function of a representative consumer, who demands low quality products.

$$\text{(2. 9)} \quad V_L^* = V^{*L} I^{*D}_L,$$

where $v^{*L} = \alpha^\alpha (1-\alpha)^{1-\alpha} p_z^{*-(1-\alpha)}$, $I_h^{*D} = I_H^* + T^* - b^*$, and

$$I_L^{*D} = \int_0^{hd*} I_h^{*D} N^* n^*(h)\, dh.$$

T^* stands for per capita allocation of tariff revenues. Equation (2. 10) show demands for homogeneous goods and differentiated products.

$$\text{(2. 10)} \quad \begin{aligned} Y_L^* &= \alpha I^{*D}_L, \\ Z_L^* &= (1-\alpha) I_L^{*D}/p_z^*. \end{aligned}$$

In correspondence to equations (2. 9) and (2. 10), a representative consumer for high quality products, whose indirect utility function is given by equation (2. 11), purchases homogeneous goods and high quality goods stated by equations (2. 12).

$$\text{(2. 11)} \quad V_H^* = V^{*H} I^{*D}_H.$$

$$\text{(2. 12)} \quad \begin{aligned} Y_H^* &= \alpha I_H^{*D}, \\ Z_H^* &= (1-\alpha) I^{*D}_H/(1+t^*)p_z, \end{aligned}$$

where $v^{*H} = \alpha^\alpha (1-\alpha)^{1-\alpha} [(1+t)\, p_z]^{-(1-\alpha)}$, I_h^{*D}
$= I_h^* + T^* - (1+t^*)bw$, and I_H^{*D}
$= \int_{hd*}^{1} I_h^{*D} N^* n^*(h)\, dh.$

Equations (2. 13) and (2. 14) determine both h_d^* and I_d^*.

(2. 13) $v^{*L}\{I_d^* + T^* - b^*\} = v^{*H}\{I_d^* \; T^* - (1 + t^*)\,bw\}.$

(2. 14) $I_d^* = L^*f^*(h_d^*)/N^*n^*(h_d^*).$

Equation (2. 15) shows equilibrium in the labor market.

(2. 15) $L^* = L_y^* + a^*(Z_L + Z_L^*) + b^*\{\int_0^{hd} Nn(h)\,dh + \int_0^{hd} N^*n^*(h)\,dh\}.$

Finally, equation (2. 16) gives a formula for tariff revenues.

(2. 16) $N^*T^* = t^*I_M^*,$

where

(2. 17) $I_M^* = p_z Z_H^* + bw\int_{hd*}^{1} N^*n^*(h)\,dh.$

Suppose that t and t* respectively are manipulated by either authorities and are given to private economies. Then, note that both I_L^D and I_H^D are functions of w, T, and h_d, and that both I_L^{*D} and I_H^{*D} are functions of T* and h_d^*. We have five variables and five equations: (2. 5), (2. 6), (2. 8), (2. 13), and (2. 16).[5] Since both T and T* become functions of w, h_d, and h_d^*, we reduce the number of endogenous variables into three basic ones: w, h_d, and h_d^*.

3. Comparative Static Analysis

By referring to the discussion in Appendix B, the impact of tariff on w, h_d and $h_d{}^*$ are stated as the following matrix system:

(3. 1) $$\begin{pmatrix} c_{11} & c_{12} & 0 \\ c_{21} & 0 & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} \begin{pmatrix} \hat{w} \\ \hat{h}_d \\ \hat{h}_d^* \end{pmatrix} = \begin{pmatrix} d_1 \\ 0 \\ d_3 \end{pmatrix} \hat{t} + \begin{pmatrix} 0 \\ d_2 \\ d_4 \end{pmatrix} \hat{t}^*.$$

where caret notation indicates percentage changes in associated variables. Appendix C discusses definitions of c_{ij}'s and d_i's and their signs.

$C_{11} > 0$, $C_{12} < 0$, $C_{21} > 0$, $C_{23} < 0$, $C_{31} < 0$, $C_{32} < 0$,
$C_{33} < 0$, $d_1 > 0$, $d_2 < 0$, $d_3 < 0$, $d_4 > 0$.

Using these sign conditions, we know the sign of the determinant of the above matrix.

$\Delta = c_{12}c_{23}c_{31} - c_{11}c_{32}c_{23} - c_{12}c_{21}c_{33} < 0.$

Fist, consider the case where the North imposes tariff. That is, $\hat{t} > 0$ and $\hat{t}^* = 0$.

Proposition 1.

Tariff by the North yields the following results:

$$\hat{w} > 0,\ \hat{h}_d \gtreqless 0, \text{ and } \hat{h}_d^* > 0.$$

(See Appendix C for detailed derivation.)

The imposition of tariff on imported low quality products stimulates the demand for high quality products and lowers a critical level of h_d. This change is caused by the behavior of marginal consumers. Following W. Novshek and H. Sonnenschein (1979), we may call this the change-of-commodity effect. Since an increase in the demand for high quality products raises the wage rate in the northern labor market, all consumers in the North, also receiving tariff revenues, enjoy increases in wage income, which make marginal consumers to switch their demand for differentiated products from low to high quality. (We may call this the income effect.) In a competitive situation, however, an increase in the wage rate pulls up the price of high quality products through an increase in cost of production. We call this the cost effect. While both the change-of-commodity effect and the income effect tend to lower h_d, the cost effect reacts against these two effects. We assume here that the latter two effects dominate the cost effect and that tariff reduces h_d. As for the effect on h_d^*, an increase in the wage rate affects southern consumers only through the cost effect, which makes them switch their demand from high to low quality.

Next, consider the case where the South imposes tariff on imported high quality products ($\hat{t}^* > 0$ and $\hat{t} = 0$).

Proposition 2.

An increase in tariff by the South results in the following:

$$\hat{w} < 0,\ \hat{h}_d < 0 \text{ and } \hat{h}_d^* \gtreqless 0.$$

Tariff by the South, through the change-of-commodity effect, reduces the demand for high quality products, and raises h_d^*. Furthermore, since tariff decreases disposable income for consumers purchasing high quality products, there will be a switch of the demand for differentiated products from high to low quality. However, a decrease in the demand for high quality products lowers the wage rate, which gives rise to the cost effect and tends to lower h_d^*. While a change in h_d^* is generally ambiguous, we also assume that both the change-of-commodity effect and the income effect dominate the cost effect and that h_d^* becomes greater.[6]

4. Optimum Tariffs

In Section 3, rates of tariff, t and t*, are treated exogenously. This section derives optimum tariff formulas both for the North and the South. We assume that there is no retaliation against tariff imposition. First, we show that either country

probably has an incentive to impose a small tariff. Then, we derive optimum tariff formulas.

Consider the North, whose social welfare function consists of utilities of two representative consumers.

$$(4.1)\quad W = W(V_L, V_H).$$

Equation (4. 2) shows the effect of tariff on the northern social welfare.

$$(4.2)\quad \frac{dw}{dt} = \frac{\partial w}{\partial V_L}\frac{dV_L}{dt} + \frac{\partial w}{\partial V_H}\frac{dV_H}{dt}.$$

Since V^L and V^H respectively equal marginal utilities of income for representative consumers of the low income class and of the high income class,

$$\partial W/\partial V_L = 1/V^L \text{ and } \partial W/\partial V_H = 1/V^H.$$

Totally differentiating V_L in equation (2. 1) with respect to t, we derive equation (4. 3).

$$(4.3)\quad \frac{dV_L}{dt} = -V^L P_z^* Z_L + b^*\int_0^{hd} N_n(h)\,dh - \left(\int_0^{hd} I_h N_n(h)\right)\frac{\hat{w}}{dt}.$$
$$- T\left(\int_0^{hd} Nn(h)\,dh\right)\frac{\hat{T}}{dt} - I_d^L Nn(h_d)\frac{\hat{h}_d}{dt}.$$

Similarly, by using equation (2. 3),

$$(4.4)\quad \frac{dV_H}{dt} = -v_H\left\{\left[p_z Z_H + bw - \int_{hd}^{1} I_h Nn(h)\,dh\right]\frac{\hat{w}}{dt} - \left(\int_{hd}^{1} Nn(h)\,dh\right)\frac{dT}{dt}\right.$$
$$\left. + I_d^H Nn(h_d)\frac{dh_d}{dt}\right\}.$$

Therefore, substituting equations (4. 3) and (4. 4) into (4. 2) and using equation (2. 7), we obtain equation (4. 5).

$$(4.5)\quad \frac{dw}{dt} = \left[wL - \int_{hd}^{'} p(z)\,Nn(h)\,dh\right]\frac{\hat{w}}{dt} - \left[I_M - N\frac{dT}{dt}\right]$$
$$+ \left[bw - (1+t)\,b^*\right] Nn(h_d)\frac{dh_d}{dt}.$$

When we evaluate equation (4. 5) at $t = 0$ (under free trade), $N(dT/dt) = I_M$. Equation (4. 5) is rewritten as equation (4. 6).

$$(4.6)\quad \frac{dW}{dt} = E_x\frac{\hat{w}}{dt} + R\frac{\hat{h}_d}{dt},$$

where

$$E_x = wL - \int_{hd}^{1} p(z)\, Nn(h)\, dh = \text{northern exports and}$$

$R = [bw - (1 + t)\, b^*]\, Nn(h_d)\, h_d$. We assume that tariff does not violate the basic technological requirement, (C1). That is, $R > 0$. Since $p_z = aw$, the first term of the RHS in equation (4. 6) represents the terms of trade effect. As in traditional analysis, the terms of trade effect affect favorably the tariff-imposing country. The second term, which does not usually appear traditionally reflects the change-of-commodity effect. That is, while marginal consumers with a critical income I_d can enjoy the same level of utilities by purchasing either low or high quality products, the levels of their disposable income are different. On the one hand, when they buy low quality products, their disposable income equals $I_d + T - (1 + t)\, b^*$. On the other hand, when they produce high quality products, it equals $I_d + T - bw$. Therefore, when tariff forces marginal consumers to switch their demand for differentiated products from high to low quality, there will be surpluses in income equal to $bW - (1 + t)b^*$. R shows the aggregate of these surpluses.

Proposition 3.

A small tariff by the North influences her social welfare in two ways: (1) the terms of trade effect, which works favorably for the social welfare of a tariff-imposing country and (2) the change-of-commodity effect, which decreases social welfare.

Normally, where the former effect dominates the latter, a small tariff improves northern social welfare.

Let us derive the optimum tariff formula. Multiplying t to both sides of equation (4. 5) and arranging it, we obtain equation (4. 7).

$$\text{(4. 7)} \quad t\,(dW/dt) - E_x \varepsilon_w + tI_M\,(\varepsilon_T - 1) + R\varepsilon_d,$$

where $\varepsilon_w = \hat{w}/\hat{t}$, $\varepsilon_T = \hat{T}/\hat{t}$ and $\varepsilon_d = \hat{h}_d/\hat{t}$. ε_w, ε_d and ε_d respectively stand for elasticities of the terms of trade, tariff revenues, and income class of marginal consumers, in terms of tariff. Note that $\varepsilon_M = -\hat{I}_M / \hat{t}$ equals $\varepsilon_T - 1$. Using equation (B3) in Appendix B,

$$\text{(4. 8)} \quad \varepsilon_M = \varepsilon_T - 1 = m_w \varepsilon_w + m_d \varepsilon_d + m_t,$$

where $m_w > 0$, $m_d > 0$ and $m_t > 0$. (See Appendix B for definitions on m_w, m_d and m_t.) Since there exists a prohibitive tariff, optimum tariff will exist. Substituting equation (4. 8) into (4. 7), we derive the optimum tariff formula.

$$\text{(4. 9)} \quad t_0 = \frac{E_x}{I_M} \frac{\varepsilon_w + R'\varepsilon_d}{1 - m_w \varepsilon_w - m_d \varepsilon_d - m_t},$$

where $R' = R / E_x$. Note that the North exports high quality products and imports both low quality products and homogeneous products. Therefore, E_x means the volume of trade and I_M / E_x shows the relative significance of intra-industry trade.

Proposition 4 (Northern optimum tariff)
The rate of northern optimum tariff depends upon: (1) intra-industry trade activities (E_x / I_M), (2) the terms of trade effect (ε_w), (3) the change-of-commodity effect (ε_d), and (4) the technological difference between the North and the South (R'). The more elastic both the terms of trade and a change in income class, the greater the optimum tariff rate. This also holds for the larger technological difference between the North and South and smaller activity in intra-industry trade.

Equation (4. 9) suggests that the traditional optimum tariff formula should be modified in dealing with vertical product differentiation. Define ε the elasticity of demand for imports due to a change in the terms of trade: $\varepsilon = \hat{I}_M/\hat{w} = \varepsilon_M/\varepsilon_w \mid_{t = t_0}$. Then we rewrite equation (4. 9) as equation (4. 10).

$$(4.\ 10) \quad t_0 = (E_x/I_M)\,[1 + R'(\varepsilon_d/\varepsilon_w)]/\varepsilon.$$

when the technological difference is relatively small compared to the amount of exports ($R' = 0$), equation (4. 10) reduces to a similar form with the traditional optimum tariff formula. Still, however, intra-industry activity has an influence on the determination of optimum tariff.

Let us investigate the effect of northern tariff on southern welfare, which is demonstrated by equation (4. 11).

$$(4.\ 11) \quad t\,(dW^*/dt) = -\,I^*_M\varepsilon_w - t^*I^*_M(\varepsilon_w - \varepsilon^*_T) + R^*\varepsilon^*_d,$$

where $\varepsilon^*_d = \hat{h}^*_d/\hat{t}$, $\varepsilon_T = \hat{T}/\hat{t}$ and $R^* = [(1 + t^*)\,bw - b^*]\,N^*n^*\,(h^*_d)\,h^*_d > 0$. Referring to the discussion in Appendix B (particularly, see equation (B4)), we have;

$$\varepsilon_w - \varepsilon^*_T = (1 - m^*_W)\,\varepsilon_w + M^*_d\varepsilon^*_d > 0.$$

Since northern tariff raises the northern wage rate and increases the price of high quality products produced there, southern welfare will deteriorate through the terms of trade effect. However, since an increase in the price of high quality products makes marginal consumers to switch their demand for differentiated products from high to low quality, domestic production of low quality products in the South is expanded, which raises southern social welfare. However, if this change-of-commodity effect is relatively small, we have Proposition 5.

Proposition 5.
The imposition of tariff by the North will decrease southern social welfare, mainly

through the terms of trade effect.
Let $\tilde{W} = W + W^*$ be world welfare. By using equations (4. 7) and (4. 9), the effect of northern tariff on world welfare is computed as follows:

$$(4.\ 12)\quad t\,(d\bar{w}/dt) = -\,t^* I_M^* (1 - m_{W,}^*)\,\varepsilon_W + tI_M\varepsilon_M - T^* I_M^* m_d^* \varepsilon_d^* R\varepsilon_d + R^*\varepsilon_d^*.$$

Under a normal situation, $1 - m_W^* > o$, $\varepsilon_M < 0$, $\varepsilon_d < 0$ and $\varepsilon_d^* > 0$. Therefore, tariff likely decreases world welfare.

Similar procedures above will be applied for the case of tariff by the South. Equation (4. 13) demonstrates the effect of tariff by the South on her social welfare.

$$(4.\ 12)\quad dW^*/dt^* = N^*\,(dT^*/dt) - (1 + t^*)\,I_M^*\,(w/dt) - I_M^* + R^*\,(h_d^*/dt^*).$$

In case of a small tariff, $N^*\,(dT^*/dt^*) - I_M^* = 0$. As discussed in Appendix B, $dw/dt^* < 0$ and $dh_d^*/dt^* > 0$.

Proposition 6.(Small tariff by the South)
A small tariff improves southern welfare through two channels: (1) the terms of trade effect, and (2) the change-of-commodity effect. The latter effect will have an amplified effect on social welfare in case of the existence of a large technological difference between the North and the South.

It should be noted that traditional analysis does not pay any attention on this latter effect. Using equation (4. 12), we obtain the optimum tariff formula for the South.

$$(4.\ 13)\quad t_0^* = (1/\eta^*) + R^{*\prime}\,(\eta_d^*/\eta_M^*),$$

where $\eta_d^* = \hat{h}_d^*/\hat{t}^*$, $\eta_W^* = -\,\hat{w}/\hat{t}^*$, $\eta_M^* = -\,(\hat{I}_M^*/w)/\hat{t}^*$, $R^{*\prime} = R^*/I_M^*$ and $\eta^* = \eta_M^*\,/\,\eta_W^*\,|_t^* - {}_{t0}^*$. As for the elasticity of demand for imports (η_M^*), it is defined in terms of labor units, because I_M^* only presents the nominal amount of imports.

Proposition 7. (Optimum tariff by the South)
The southern rate of optimum tariff depends upon not only the inverse of the elasticity of demand for imports(η^*), but also both elasticities of income class for marginal consumers and real imports, in terms of tariff. Furthermore, greater technological difference raises the value of the optimum tariff rate.

Since southern tariff works favorably for the southern terms of trade, a cheaper price of high quality products makes marginal consumers switch their demand for differentiated products from low to high quality, which causes income losses as stated above. Northern welfare is definitely deteriorated. In case of world welfare, tariff tends to reduce it. (See Appendix D for detailed analysis.)

5. Concluding Remarks

This paper has derived the optimum tariff formulas when both the North and the South are engaged in intra-industry trade of vertically differentiated products as well as inter-industry trade. They possess much in common with the traditional result. That is, the rate of optimum tariff is critically related to the inverse of the elasticity of demand for imports in a tariff-imposing country. Because of the simple form of the utility functions taken here, the optimum tariff formulas are basically associated with both the elasticity of the terms of trade (ε_w or η_w^*) and the elasticity of income class for marginal consumers (ε_d or η_d^*). While the elasticity of the terms of trade always improves social welfare in a tariff-imposing country and raises the optimum tariff rate, the elasticity of income class behaves in a distinct way. It reduces northern social welfare and raises southern social welfare. Furthermore, a technological difference between the North and the South affects the optimum tariff rate between them inversely. A greater technological difference makes the southern optimum tariff rate larger and the northern smaller.

Finally, it should be noted that our analysis totally depends upon a simplified assumption, where there is no retaliation by tariff between the North and the South. H. Johnson (1953) argued in his seminal article that even allowing retaliation, one country may raise her welfare by imposing tariff, while tariff war has great possibility of resulting in the prisoner's dilemma. Since we have shown in this paper that tariff imposed by either country likely reduces world welfare, there is no possibility of raising welfare for both countries at the same time. This suggests that even in vertical product differentiation, traditional wisdom must be applied in the case of retaliation. Detailed analysis on this aspect is left for future research. (See R. Riezman (1982).)

Appendix A. Demand for Qualities.

Equation (2. 2) is restated as equation (A1).

$$\text{(A1)} \quad Z_L = (1-\alpha)\, I_L^p/(1+t)\, p_z^*,$$

$$\text{where } I_L^p = \int_0^{hd} I_h^p Nn(h)\, dh,\ I_h^p = I_h + T - (1+t)\, b^*, \text{ and}$$

$$I_h = wLf(h)/Nn(h).$$

Totally differentiating equation (A1) and using caret notation for percentage changes in variables, we derive equation (A2).

$$\text{(A2)} \quad \hat{Z}_L = s_W^L \hat{w} + s_T^L \hat{T} - (s_W^L + s_T^L)\, [t/(1+t)]\, \hat{t} + \delta_L \hat{h}_d,$$

$$\text{where } s_W^L = [\int_0^{hd} I_h Nn(h)\, dh]/I_L^p,\ s_T^L = [\int_0^{hd} TNn(h)\, dh]/\, I_L^p,$$

$s_t^L = [\int_0^{hd} (1 + t)\, b^* Nn\,(h)\, dh]/I_L^D$, and δ_L
$= [I_d + T - (1 + t)\, b^*]\, Nn\,(h_d)\, h_d/I_L^D$. Note that $s_W^L + s_T^L - s_t^L = 1$.

Similarly, rewrite equation (2. 4) as equation (A3).

(A3) $Z_H = (1 - \alpha) I_H^D / p_Z$.

Total differentiation gives equation (A4).

(A4) $\hat{Z}_H = - s_T^H \hat{W} + s_t^H \hat{T} - \delta_H \hat{h}_d$,

where $s_W^H = [\int_{hd}^1 < I_h - bw)\, Nn\,(h)\, dh]/I_H^D$, $s_T^H = [\int_{hd}^1 TNn\,(h)\, dh]/I_h^D$,
and $\delta_H = [I_d + T - bw]\, Nn\,(h_d)\, h_d/I_H^D$. Note that $s_W^H + s_T^H = 1$.

By using equations (2. 10) and (2. 12), total variations in Z_L^* and Z_H^* respectively will be obtained as in equations in (A5) and (A6).

(A5) $\hat{Z}_L^* = s^{*L}_T \hat{T}^* + \delta_L{}^* \hat{h}_d$.

(A6) $\hat{Z}_H{}^* = - (1 + s_t^{*H}) \hat{w} - (1 + s_t^{*H})\, [t^*/(1 + t^*)]\, \hat{t}^* + s_T^{*H} \hat{T}^* - \delta_H{}^* \hat{h}_d^*$,
where $s_T^{*L} = [\int_0^{hd*} T^* N^* n^*\,(h)\, dh]/I_L^{*D}$,

$s_T^{*H} = [\int_{hd}^{*1} T^* N^* n^*\,(h)\, dh]/I_H^{*D}$,

$s_t^{*H} = [\int_{hd*}^1 (1 + t^*)\, bwN^* n^*\,(h)\, dh]\, I_H^{*D}$, $\delta_L^* = [I_d^* + T^* - b^*]$

$N^* n^*\,(h_d^*)\, h_d^*$, and $\delta_H^* = [I_d^* + T^* - (1 + t^*)\, bw]\, N^* n^*\,(h_d^*)\, h_d^*/I_H^{*D}$.

Appendix B. Variations in Tariff Revenues and Imports.

First, consider variations in northern imports. Using equations (2. 7) and (A2), we have:

(B1) $\hat{I}_M = s_m s_W^L \hat{w} + s_m s_T^L \hat{T} + (s_m \delta_L + s_F \delta_n)\, \hat{h}_d - s_m\, (s_W^L + s_T^L)\, [t/(1+ t)]\, \hat{t}$,

where $s_m = p_Z^* Z_L/I_M$, $s_F = [b^* \int_0^{hd} Nn\,(h)\, dh]/\, I_M$, and

$\delta_n = n\,(h_d)\, h_d/\int_0^{hd} n\,(h)\, dh$.

By totally differentiating equations (2. 17) and using (A6), total variations in southern imports will be expressed as in equation (B2).

(B2) $\hat{I}_M^* = (s_F^* - s_m{}^* s_t^{*H})\, \hat{w} + s_m{}^* s_T^{*H} \hat{T} - (s_m^* \delta_H^* + s_F^* \delta_n^*)\, \hat{h}_d^*$
$- s_m^*\, (1+ s^{*H}_t)[t^* / (1+ t^*)]\hat{t}^*$,

where $s_m^* = p_Z Z_H^*/I_M^*$, $s_F^* = [bw\int_{hd*}^{1} N^* n^*(h)\,dh]/I_M^*$, and

$\delta_n^* = n^*(h_d^*)\,h_d^*/\int_{hd*}^{1} n^*(h)\,dh.$

It should be noted that

$s_F^* - s_m^* s_t^{*H} = [\alpha bw\int_{hd*}^{1} N^* n^*(h)\,dh]/I_M^* > 0.$

Now, totally differentiating equation (2. 6) and using equation (B1), we derive total variation in tariff revenue.

(B3) $\hat{T} = m_W \hat{W} + m_d \hat{h}_d + m_t \hat{t},$

where $m_W = s_m s_W^L/(1 - s_m s_F^L) > 0$, $m_d = (s_m \delta_L + s_F \delta_n)/(1 - s_m s_T^L)$,

and $m_t = \{1 - s_m (s_W^L + s_T^L)[t/(1+t)]\}/(1 - s_m s_T^L) > 0.$

Similarly, by using equations (2. 16) and (B2),

(B4) $\hat{T}^* = m_W^* \hat{w} - m_d^* \hat{h}_d^* + m_t^* \hat{t}^*,$

where $m_W^* = (s_F^* - s_m^* s_t^{*H})/(1 - s_m^* s_T^{*L}) > 0,$

$m_d^* = (s_m^* \delta_H^* + s_F^* \delta_n^*)/(1 - s_m^* s_T^{*L}) > 0$, and

$m_t^* = \{1 - s_m^*(1 + s_t^{*H})[t^*/(1+t^*)]\}/(1 - s_m^* s_T^{*L}) > 0.$

Appendix C. Comparative Static Analaysis.

Totally differentiate equation (2. 5).

(C1) $-(1-\alpha)\hat{p}_z + (1-\alpha)[t/(1+t)]\hat{t} + q_1(\hat{w} + e\hat{h}_d) + q_2\hat{T} - q_3\hat{w}$
$= q_4(\hat{w} + e\hat{h}_d) + q_5\hat{T} - q_6[t/(1+t)]\hat{t},$

where $q_1 = I_d/I_d^H$, $q_2 = T/I_d^H$, $q_3 = bw/I_d^H$, $q_4 = I_d/I_d^L$, $q_5 = T/I_d^L$, $q_6 = (1+t)b^*/I_d^L$, and $e = d\log[f(h_d)/n(h_d)]/d\log h_d$. Note that $q_1 + q_2 - q_3 = 1$ and $q_4 + q_5 - q_6 = 1$. Substituting the relation, $p_z = w$, and equation (B3) into equation (C1), we derive equation (C2).

(C2) $c_{11}\hat{w} + c_{12}\hat{h}_d = d_1\hat{t},$

where $c_{11} = 1 - \alpha + q_4 + q_3 - q_1 + (q_5 - q_2)\, m_W$, c_{12}
$= (q_4 - q_1)\, e + (q_5 - q_2)\, m_d$,
and $d_1 = (1 - \alpha + q_6)\, [t/(1 + t)] + (q_2 - q_5)\, m_t$.
As for the sign of c_{11}, not a the following:

$$q_4 + q_3 - q_1 + (q_5 - q_2)\, m_W = \{(1 + t)\, b^* [I_d - bw] + bwT\, (1 - m_W)\}/I_d^H I_d^L$$

and

$$1 - m_W = \{1 - s_m\, (s_T^L + s_T^L)\}/(1 - s_m s_T^L) > 0.$$

Therefore, $c_{11} > 0$.

It should easily be derived that

$$c_{12} = [(1 + t)\, b^* - bw]\, [I_d e + Tm_d]/I_d^H I_d^L < 0.$$

The above argument implies that $q_4 - q_1 < 0$ and $q_5 - q_2 < 0$, which means $d_1 > 0$.

Similarly, total differentiation of equation (2. 13) yields the following equation:

$$(1 - \alpha + q_3^*)\, \hat{w} + (q_4^* - q_1^*)\, e^* \hat{h}_d^* + (q_5^* - q_2^*)\, \hat{T}^*$$
$$= -\, (1 - \alpha + q_3^*)\, [t^*/(1 + t^*)]\, t^*,$$

where $q_1^* = I_d^*/I_d^{*H}$, $q_2^* = T^*/I_d^{*H}$, $q_3^* = (1 + t^*)\, bw/I_d^{*H}$, $q_4^* = I_d^*/I_d^{*L}$, $q_5^* = T^*/I_d^{*L}$, and $e^* = d\log\, [f^*(h_d^*)/n^*\, (h_d^*)]/d\log h_d^*$

Substituting equation (B4) into the above equation, we have:

(C3) $C_{21}\hat{w} + c_{23}\hat{h}_d^* = d_2\hat{t}^*$,

where $c_{21} = 1 - \alpha + q_3^* + (q_5^* - q_2^*)\, m_W^*$, c_{23}
$= (q_4^* - q_1^*)\, e^* - (q_5^* - q_2^*)\, m_d^*$, and
$d_2 = -\, \{(1 - \alpha + q_3^*)\, [t^*/(1 + t^*)] + (q_5^* - q_2^*)\, m_t^*\}$.
It is clear that $c_{21} > 0$, for

$$q_3^* + (q_5^* - q_2^*)\, m_W^* = \{(1 + t^*)\, bw\, [I_d^* - b^* + T^*\, (1 - m_W^*)] + T^* b^*)/I_d^{*H} I^{*L}_d > 0.$$

In case of c_{23},

$$c_{23} = [b^* - (1 + t^*)\, bw]\, [I_d^* e^* - T^* m_d^*]/I_d^{*H} I_d^{*L}.$$

The first and the second terms in the RHS bracket in the above equation show the income distribution effect and the allocation effect of tariff revenues, respectively, created by a change in critical income class h_d^*. We assume $I_d^* e^* - T^* m_d^* > 0$, so that $c_{23} < 0$.

Similarly, the first term in the RHS of d_2 indicates a direct effect of a change in t^*, while the second term shows an indirect effect through a change in T^*. We

assume that the former effect dominates the latter. That is, $d_2 < 0$.

Finally, total differentiation of equation (2. 8) yields equation (C4).

(C4) $1_1 \hat{Z}_H + 1_2 \hat{Z}^*_H - 1_3\delta_n\hat{h}_d - 1_4\delta^*_n\hat{h}^*_d$,

where $1_1 = aZ_H/L$, $1_2 = aZ^*_H/L$, $1_3 = [b\int_{hd}^{1} Nn(h)\,dh]/L$,

$1_4 = [b\int_{hd*}^{1} N^*n^*(h)\,dh]/L, \delta_n = n(h_d)\,h_d/\int_{hd}^{1} n(h)\,dh$, and

$\delta^*_n = n^*(h^*_d)\,h^*_d/\int_{hd*}^{1} n^*(h)\,dh$.

Substituting equations (A4) and (A6) into (C4), and using equations (B3) and (B4), we obtain equation (C5).

(C5) $c_{31}\hat{w} + c_{32}\hat{h}_d + c_{33}\hat{h}^*_d = d_3\hat{t} + d_4\hat{t}^*$,

where $c_{31} = -\{1_1 s^H_T(1 - m_W) + 1_2(1 + s^{*H}_t - s^{*H}_T m^*_W)\}$, $c_{32} = -\{1_1(\delta_H - s^H_T m_d) + 1_3\delta_n\}$, $c_{33} = -\{1_2(\delta^*_H + s^{*H}_T m^*_d) + 1_4\delta^*_n\} < 0$, $d_3 = -1_1 s^H_T m_t < 0$, and $d_4 = 1_2\{< 1 + s^{*H}_t)\,[t^*/(1 + t^*)] - s^{*H}_T m^*_t\}$. It is shown that $c_{31} < 0$, because

$1 + s^{*H}_t - s^{*H}_T m^*_W = \{\int_{hd*}^{1} [I_h + (1 - m^*_W)\,T^*]\,N^*n^*(h)\,dh\}/I^{*D}_H > 0$.

If direct effects dominate indirect ones,

$c_{32} < 0$ and $d_4 > 0$.

Using equations (C2), (C3) and (C5), we present our system as matrix representation.

$$\text{(C6)} \quad \begin{pmatrix} c_{11} & c_{12} & 0 \\ c_{21} & 0 & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} \begin{pmatrix} \hat{w} \\ \hat{h}_d \\ \hat{h}^*_d \end{pmatrix} = \begin{pmatrix} d_1 \\ 0 \\ d_3 \end{pmatrix} \hat{t} + \begin{pmatrix} 0 \\ d_2 \\ d_4 \end{pmatrix} \hat{t}^*$$

(1) The case where the North imposes tariff ($t > 0$, $t^* = 0$)
Using cramer's rule, we have the following results:

(C7) $\hat{w}/\hat{t} = \{c_{23}[d_3c_{12} - d_1c_{32}]\}/\Delta > 0$.
(C8) $\hat{h}_d/\hat{t} = \{d_1c_{23}c_{31} - d_3c_{11}c_{23} - d_1c_{21}c_{33}\}/\Delta > 0$.
(C9) $\hat{h}^*_d/\hat{t} = c_{21}\{d_1c_{32} - d_3c_{12}\}/\Delta > 0$,

where $\Delta = c_{12}c_{23}c_{31} - c_{11}c_{32}c_{23} - c_{12}c_{21}c_{33} < 0$.
(2) The case where the South imposes tariff ($\hat{t} = 0$, $\hat{t}^* > 0$)
We only show our own results.

(C10) $\hat{w}/\hat{t}^* = c_{12}\,[d_4c_{23} - d_2c_{33}]/\Delta < 0.$

(C11) $\hat{h}_d/\hat{t}^* = c_{11}\,[d_2c_{33} - d_4c_{23}]/\Delta < 0.$

(C12) $\hat{h}_d^*/\hat{t}^* = \{d_2c_{12}c_{31} - d_2c_{32}c_{11} - d_4c_{12}c_{21}\}/\Delta > 0.$

Appendix D. Optimum Tariff

First, let us show that small tariff improves social welfare in a tariff-imposing country. Consider the North, which maximizes its social welfare with respect to t.

Maximize $W(V_L, V_H)$ with respect to t.

Referring to equation (2. 1), we compute the effect of tariff on V_L.

(D1) $$dV_L/dt = -v^L\{p_Z^*Z_L + b^*\int_0^{hd} Nn(h)\,dh - [\int_0^{hd} I_h Nn(h)\,dh]\,w/dt - (\int_0^{hd} Nn(h)\,dh)(dT/dt) - I_d^L Nn(h_d)(dh_d/dt)\}.$$

Similarly, by differentiating equation (2. 3) with respect to t, the effect of tariff on V_H is derived as follows:

$$dV_H/dt = -v^H\{[p_zZ_H + bw\int_{hd}^1 Nn(h)\,dh]\,w/dt + I_d^H Nn(h_d)\,dh_d/dt - (\int_{hd}^1 I_h Nn(h)\,dh)\,w/dt - (\int_{hd}^1 Nn(h)\,dh)\,dT/dt\}.$$

Since marginal utility of income for a representative consumer either in low or high income class respectively equals v^L or v^H, the North assigns the following marginal evaluations on W.

$$\partial W/\partial V_L = 1/v^L \text{ and } \partial W/\partial V_H = 1/v^H.$$

Therefore,

(D2) $$dW/dt = (\partial W/\partial V_L)(dV_L/dt) + (\partial W/\partial V_H)(dV_H/dt) = [wL - \int_{hd}^1 p(z_h)\,Nn(h)\,dh]\,\hat{w}/dt - \{I_M - N(dT/dt)\} + R(\hat{h}_d/dt).$$

For a small change in t, $N(dT/dt)I_M$. Furthermore, note that the first bracketed term in the RHS of equation (D2) shows nothing but the amount of high quality export products by the North, i. e., E_x. That is,

$$dW/dt = E_x(\hat{w}/dt) + R(\hat{h}_d/dt).$$

When $t \neq 0$, by multiplying t to both sides of equation (D2), equation (D2) is rewritten as equation (D3).

(D3) $$t(dW/dt) = E_x(\hat{w}/\hat{t}) + tI_M[(\hat{T}/\hat{t}) - 1] + R(\hat{h}_d/\hat{t}).$$

We have two alternative ways to analyse the effect of tariff on social welfare.

(1) Using equation (B3), we divide welfare changes into the terms of trade effect($\hat{w}$ /

$\hat{t}$) and the change-of-commodity effect($\hat{h}_d/\hat{t}$).

$$t\,(dW/dt) = (E_x + tI_M m_W)\,(\hat{w}/\hat{t}) + tI_M\,(m_t - 1) + (tI_M m_d + R)\,(\hat{h}_d/\hat{t}).$$

(2) We relate our result to traditional analysis of optimum tariff. Note that $(\hat{T}/\hat{t}) - 1 = (\hat{I}_M / \hat{t})$. Then,

$$t\,(dW/dt) = E_x\,(\hat{w}/\hat{t}) + tI_M\,(\hat{I}_M/\hat{t}) + R\,(\hat{h}_d/\hat{t}).$$

Next, consider the effect of tariff by the North on southern social welfare. Let the southern social welfare function be $W^* = W^*(V^*_L, V^*_H)$. Using equations (2. 9) and (2. 11), we compute the effects on utilities of representative consumers for both low income class and high income class.

$$dV^*_L/dt = v^{*L}\,\{(\int_0^{hd*} N^*n^*\,(h)\,dh)\,(dT^*/dt) + I^{*L}_d N^*n^*\,(h_d)\,(dh^*_d/dt)\}.$$

$$\text{(D4)}\quad dV^*_H/dt = -\,v^{*H}\,\{[(1-\alpha)\,I^{*D}_H + (1+t^*)\,bw\int_{hd*}^1 N^*n^*\,(h)\,dh]\,(w/dt) - (\int_{hd*}^1 N^*n^*\,(h)\,dh)\,(T/dt) + I^{*H}_d N^*n^*\,(h^*_d)\,(dh^*_d/dt)\}.$$

Using equation (D4), we derive impacts of northern tariff on southern social welfare.

$$\text{(D5)}\quad t\,(dW^*/dt) = -\,(1+t^*)\,I^*_M\,(\hat{w}/\hat{t}) + R^*\,(\hat{h}^*_d/\hat{t}) + N^*T^*\,(\hat{T}^*/\hat{t}),$$

where $I^*_M = \int_{hd*}^1 p\,(z)\,N^*n^*\,(h)\,dh$ = imports of high quality products by the South and $R^* = [(1+t^*)\,bw - b^*]\,N^*n^*\,(h^*_d)\,h^*_d > 0$.
By substituting equation (B4) into (D5),

$$\text{(D6)}\quad t\,(dW^*/dt) = -\,\{[1 + (1 - m^*_W)\,t^*]\,I^*_M\,(\hat{w}/\hat{t}) + [R^* - t^*I^*_M m^*_d]\,(\hat{h}^*_d/\hat{t})\}.$$

Note that $1 - m^*_W > 0$.

Suppose world welfare be $\bar{W} = W + W^*$. Tariff affects world welfare as follows:

$$\text{(D7)}\quad t\,(dW/dt) = -\,t^*I^*_M\,(\hat{w}/\hat{t}) + tI_M\,(\hat{I}_M/\hat{t}) + t^*I^*_M\,(\hat{I}^*_M/\hat{t}) + R\,(\hat{h}_d/\hat{t}) + R\,(\hat{h}^*_d/\hat{t}^*)$$

Note that $E_x = I^*_M$.

Next, investigate the case where the South imposes tariff. Equation (D8) shows the effect of tariff on southern social welfare.

$$\text{(D8)}\quad dW^*/dt^* = N^*\,(dT^*/dt^*) - (1+t^*)\,I^*_M\,(\hat{w}/dt) - IM^{**}$$

In case of small tariff, $N^*\,(dT^*/dt^*) = I^*_M$.
Therefore,

$$dW^*/dt^*|^*_{t-0} = -\,I^*_M\,(\hat{w}/dt^*) + R^*\,(\hat{h}^*_d/dt^*).$$

From equations (C10) and (C12),

$\hat{w}/dt^* < 0$ and $\hat{h}_d^*/dt^*$ 0.

If a change in t^* makes marginal consumers in the South to switch their demand from importables (high quality products) to domestic goods (low quality products), then tariff raises h_d^*, which we assume here. Then both the terms of trade effect and the change-of-commodity effect improve southern social welfare.

Let us derive an optimum tariff formula. Multiplying t^* to both sides of equation (D8) and defining the following elasticities:

$$\eta_M^* = -(\hat{I}_M^*/w)/\hat{t}^*,\ \eta_W^* = -\hat{w}/\hat{t}^*, \text{ and } \eta_d{}^* = \hat{h}_d^*/\hat{t}^*.$$

Using these elasticities, we rewrite equations (D8) and (D9) as follows:

$$\text{(D9)}\quad t^*(dW^*/dt) = I_M^*\eta_W^* - t^*I_M^*\eta_M^* + R^*\eta_d^*.$$

The optimum tariff rate is determined as:

$$t_0^* = (1/\eta^*) + (\eta_d^*/\eta_M^*)\,R^{*\prime},$$

where $\eta^* = \eta_M^*/\eta_W^*$ and $R^{*\prime} = R^*/\eta_M^*$.
Similar procedure will be applied for computing a change in northern social welfare due to southern tariff.

$$\text{(D10)}\quad t^*(dW/dt^*) = -E_x\eta_W^* + NT(\hat{T}/\hat{t}^*)\,R(\hat{h}_d/\hat{t}^*).$$

By substituting equation (B3) into (D10) and defining $\eta_d = -(\hat{h}_d/\hat{t}^*)$,

$$\text{(D11)}\quad t^*(dW/dt^*) = -(E_x + tI_Mm_W)\,\eta_W^* - R\eta_d < 0.$$

Northern social welfare is definitely deteriorated by southern tariff.

The effect of tariff on world welfare is expressed by equation (D12).

$$\text{(D12)}\quad t^*(d\bar{W}/dt^*) = -t^*I_M^*\eta_M^* - tI_Mm_W\eta_W - R\eta_d + R^*\eta_d^*.$$

Notes

(1) When we derive the optimum tariff formula in section 4, we associate it with the elasticity of demand for imports. There, we use the quantity of importables in terms of labor units.

(2) We may regard the part of bw in the unit cost function as research cost, which becomes sunk cost for existing firms (see Flam and Helpman(1987b)). In our competitive case, all firms are treated as newly entered ones. When we examine a monopoistically competitive case, we may distinguish cost functions between entrants and incumbent firms.

(3) Figure 1 only illustrates a possibility of production. As shown in FH(1987a), the commodity of quality z is not generally chosen by consumers.

(4) Equation (1. 11) will be rewritten as the following equation.

$$\{L - b\,[\int_{hd}^{1} Nn\,(h)\,dh + \int_{hd*}^{1} N^*n^*\,(h)\,dh]\}/a = Z_H + Z_H^*.$$

The LHS of this equation indicates the aggregate amount of high quality supply and the RHS, the aggregate amount of high quality demand. Therefore, the equation shows the equilibrium condition for high quality.

(5) As indicated in footnote 4, equilibrium in the labor market of the North implies equilibrium in the differentiated products market. However, we exclude equation (2. 15), because $L_y{}^*$ appears only in equation (2. 15) and can be solved as a residual.

(6) While tariffs affect both h and h_d^* in compound ways, in later sections we simply call the effect of tariffs on h_d and $h_d{}^*$ as the change-of-commodity effect.

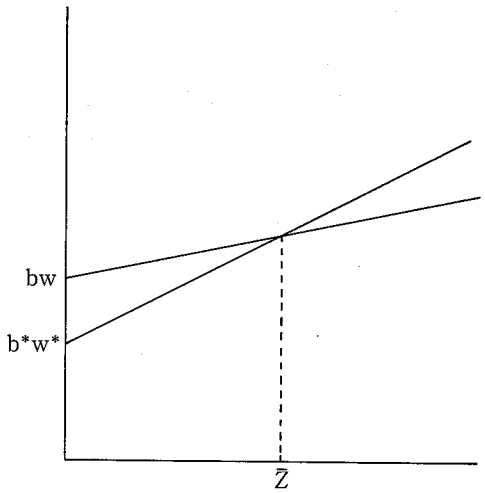

Fig. 1 Second-best optimum in the Diamond-Mirrlees model

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